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LETTER FROM MR. GOLDSCHMIDT TO THE EDITOR.

Paris, 1857, June 3.

I MUST not delay to inform you of the discovery of the 44th asteroid (my sixth), which I found on the night of May 27. The planet resembles a star of the 10.11 magnitude. Approximate positions are :

	Paris M. T.	α	δ
1857 May 27	11 ^h 0 ^m	15° 15' 58"	— 12° 8'
" June 1	10 55	15 12 9.2	—

The latter place depends upon a comparison with LALANDE 28036, which was too distant for determining the declination ; and I therefore compared it with an anonymous star of the 10 magnitude, which will be determined at the Observatory here. This comparison gave :

Paris M. T.	Planet — Star
10 ^h 32 ^m	$\Delta\alpha = 30^s$ $\Delta\delta = 2' 56''$

The comparisons on May 31 were not successful, partly on account of the want of fixity in my telescope, which has only an ordinary mounting, and partly by reason of the too great distance of the comparison-star. I will, however, append them.

	Paris M. T.	α	δ
May 31	11 ^h 15 ^m	15 ^h 12 ^m 43 ^s	—
	15 50	—	— 12° 3'

HERMANN GOLDSCHMIDT.

The forty-third Asteroid, discovered at the Radcliffe Observatory by Mr. POGSON, has received the name *Ariadne*.

G.

ON THE METHOD OF FINDING THE LONGITUDE OF A PLACE BY TRANSITS OF THE MOON AND A STAR OVER THE SAME VERTICAL CIRCLE.

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1. In the use of small (portable) instruments for determining time, longitude and latitude, those methods of observation are in general to be preferred which are independent of graduated circles. In illustration of this remark, we may cite the methods of finding the latitude by transits over the prime vertical ; of finding the time by equal altitudes ; the latitude by the difference of the zenith distances of two stars, one north and the other south of the zenith, this difference being measured by the micrometer and level ; the latitude by equal zenith distances of two stars, one of the stars or

both being off the meridian, but at azimuths differing nearly 180° ; and finally the method of finding the longitude from equal altitudes of the moon and a star, developed by Dr. OUDEMANS in the *Astr. Journal*, No. 93.

In the article of Dr. OUDEMANS just referred to, a remark of Professor KAISER is quoted to the effect "that a traveling astronomer can very easily multiply his determinations in a single night, by observing transits of the moon and neighboring stars across the same vertical;" and it is also added that "Mr. DE LANGE, during his stay in Leyden, made use of this method, and found that in a latitude as high as that of Leyden it offers no difficulty, while it might readily be foreseen that it would be attended with great advantages." As no one appears to have published formulas for the reduction of such observations, I am induced to offer here my own, which, however, are so naturally suggested by the nature of the observations, that it is possible that they will be found to be essentially the same as those employed by the astronomers above alluded to.

2. To prepare for the observation, it is necessary first to determine with all possible precision the correction and rate of the chronometer.

For the comparison with the moon, a well determined star is to be selected, nearly in the same parallel with the moon, and not far distant in right-ascension. A preliminary computation of the hour-angles and altitudes corresponding to assumed azimuths (say 15° , 10° and 5° from the meridian) will serve to enable the observer to point the instrument properly. The formulas for this purpose are too well known to be repeated here.

3. The Altitude and Azimuth Instrument, and the Ertel Universal Instrument, are peculiarly adapted for these observations; but the common portable transit-instrument is even preferable when it is furnished with a small horizontal circle, by which it may be approximately placed at a given azimuth*. The observations may be conducted as follows: 1° The instrument being mounted and the horizontal axis leveled, the reading of the horizontal circle corresponding to the azimuth 0° is to be found by any convenient method; for example, by pointing the telescope upon a slow-moving star at the computed time of its meridian transit. Great ac-

curacy in this operation is of course unnecessary, since we use the horizontal circle merely as a *finder*. 2° The telescope is then pointed for the first assumed azimuth, and the altitude of either the moon or the star, whichever precedes. 3° The inclination of the horizontal axis is measured with the striding level. 4° The times of transit of the first object, moon or star, over the several threads, are noted by the chronometer. 5° The approximate apparent zenith-distance of the object is read from the vertical circle, the observer having caused the object to cross the middle thread near its intersection with the fixed horizontal thread. 6° The telescope is pointed for the altitude of the second object, star or moon, observing carefully not to disturb the instrument in azimuth; and if there is time, the level of the axis is to be re-examined. 7° The times of transit of the second object over the vertical threads are noted by the chronometer. 8° The approximate zenith distance of the object is read from the vertical circle. 9° The inclination of the horizontal axis may be again observed.

The repeated examination of the level, here recommended, is not always necessary, but the traveling astronomer is not unfrequently compelled to employ rather unstable supports for his instrument; and it will be a safe rule to observe the inclination of the axis for both the moon and the star, since the correction for this inclination is very easily computed.

In order to eliminate any error of collimation, or inequality of pivots, the rotation axis of the telescope is now to be reversed; and another observation, similar to the first, is to be taken at another assumed azimuth. In this way the observations may be repeated from the extreme eastern to the extreme western azimuth, which, however, should not exceed about 15° . The observations on opposite sides of the meridian should be arranged in symmetrical pairs; that is, for each observation at the east azimuth $-A$, there should be a corresponding one at the west azimuth $+A$, and at these two observations the rotation axis should have reversed positions.

4. A rigorous reduction of the observations requires, first, that the transits over the different threads should be severally reduced to the middle thread; for which purpose, the theory of the altitude- and azimuth-instrument furnishes a suitable formula. If, however, the threads are symmetrically placed, and all are employed, it will suffice (unless the observations are extended greatly beyond the limits above recommended) to take the means of the observed times for the times hereafter designated by T and T' . The slight theoretical error which this procedure involves will be eliminated, if the observations are arranged symmetrically with respect to the meridian.

In order farther to reduce the observations, let us denote by

* The Zenith Telescope, the application of which in the method of finding the longitude by equal altitudes of the moon and star is pointed out by Dr. GOULD in a note to Dr. OUDEMANS' article (*A. J.* Vol. iv, p. 165), may also be applied to the method here considered. It is necessary to observe, however, that as the instrument is now constructed, it cannot be used at the zenith-distances required in both these methods. A very simple modification in the arrangement of the level will obviate the difficulty.

- T, T' , the chronometer times of the transit of the moon's limb and the star, respectively;
 $\Delta T, \Delta T'$, the corresponding corrections of the chronometer;
 i, i' , the inclination of the rotation axis, given by the level, for the moon and the star respectively: *positive* when the *left-hand* pivot is the highest;
 c , the collimation constant, *positive* when the sight-line of the middle thread tends to the *right* of the collimation axis;
 α, α' , the moon's and the star's right ascensions;
 δ, δ' , " " declinations;
 t, t' , " " hour angles;
 z, z' , " " true zenith distances;
 q, q' , " " parallactic angles;
 A, A' , " " true azimuths from the south towards the west;
 φ , the latitude of the place of observation;
 ω , the assumed west longitude;
 $\delta\omega$, the correction of this longitude.

The moon's α and δ are to be taken from the Ephemeris for the Greenwich time $T + \Delta T + \omega$ (expressed in mean solar time), and also the quantities

- $\Delta\alpha$, the increase of the moon's right ascension in one second of sidereal time;
 $\Delta\delta$, the increase, towards the north, of the moon's declination in one second of sidereal time;
 π , the moon's equatorial horizontal parallax;
 S , the moon's geocentric semidiameter.

The quantities A, z, q are now to be computed for the time $T + \Delta T$, and A', z', q' for the time $T' + \Delta T'$. We do not require q' in the computation of the correction of the longitude; but as it is employed in the differential formulas which we shall give in the next paragraph, it will be well to find it at the same time with the other auxiliary quantities. Since A and A' are required with all possible precision, logarithms of not less than six decimal places are to be employed in their computation; but for z, q, z' and q' , four decimal places will be sufficient. The following formulas for this purpose will be found to be as convenient as any others.

For the moon:

$$t = T + \Delta T - \alpha,$$

For the star:

$$t' = T' + \Delta T' - \alpha',$$

$$\left. \begin{aligned} \tan M &= \tan \delta \sec t, \\ \tan A &= \frac{\tan t \cos M}{\sin(\varphi - M)}, \end{aligned} \right\} \text{with six decimals;} \quad \left\{ \begin{aligned} \tan M' &= \tan \delta' \sec t', \\ \tan A' &= \frac{\tan t' \cos M'}{\sin(\varphi - M')}, \end{aligned} \right.$$

$$\left. \begin{aligned} \tan N &= \cot \varphi \cos t, \\ \tan q &= \frac{\tan t \sin N}{\cos(\delta + N)}, \end{aligned} \right\} \text{with four decimals;} \quad \left\{ \begin{aligned} \tan N' &= \cot \varphi \cos t', \\ \tan q' &= \frac{\tan t' \sin N'}{\cos(\delta' + N')}, \end{aligned} \right.$$

$$\tan z = \cot(\delta + N) \sec q, \quad \tan z' = \cot(\delta' + N') \sec q',$$

in which the angles A and q are to be so taken that $\sin A$ and $\sin q$ shall have the same sign as $\sin t$.

If the instrumental readings of the apparent zenith distances are reliable, we may omit the computation of z and z' ; but it will be better always to compute them for the sake of verification.

The true azimuth of the moon's limb will be found by applying to the computed azimuth A the correction

$$\pm \frac{S}{\sin z} \quad (\text{lower sign for second limb}).$$

If we assume (as we may in these observations) that the horizontal parallax of the limb is the same as that of the centre of the moon, the apparent azimuth of the limb is found by applying the correction

$$\frac{\rho \pi (\varphi - \varphi') \sin 1'' \sin A'}{\sin z}$$

in which $\varphi - \varphi'$ is the reduction of the latitude, and ρ is the radius of the terrestrial spheroid for the latitude φ . We here write A' , which is the star's true azimuth, instead of the moon's apparent azimuth; since, by the nature of the observation, these are very nearly equal quantities.

To correct rigorously for the collimation and level of the instrument, we must have the apparent zenith distances, which for this purpose will be found with more than sufficient precision by the formulas

$$\text{Moon's app. zen. dist.} = z_1 = z + \pi \sin z - \text{refraction};$$

$$\text{Star's " " " " } = z'_1 = z' - \text{refraction};$$

and the reductions of the observed instrumental azimuths are, according to known formulas,

$$\text{for the moon, } \frac{c}{\sin z_1} + \frac{b}{\tan z_1};$$

$$\text{for the star, } \frac{c}{\sin z'_1} + \frac{b'}{\tan z'_1}.$$

Hence the computed instrumental azimuths will be, for the moon,

$$A_1 = A \pm \frac{S}{\sin z} + \frac{\rho \pi (\varphi - \varphi') \sin 1'' \sin A'}{\sin z} - \frac{c}{\sin z_1} - \frac{b}{\tan z_1},$$

and for the star,

$$A'_1 = A' - \frac{c}{\sin z'_1} - \frac{b'}{\tan z'_1}.$$

If now the longitude and the other elements of the computation are correct, we shall find A_1 and A'_1 to be equal; but if the elements of the computation are incorrect, let ζ denote the difference of these computed azimuths, or

$$\zeta = A_1 - A'_1, \quad (1)$$

then we are to find, first, how the correction of the longitude depends upon ζ .

The effect of the error in the longitude appears simply in this, that we have taken α and δ for the Greenwich sidereal time $T + \Delta T + \omega$, when we should have taken them for the time $T + \Delta T + \omega + \delta\omega$: hence

α requires the correction $\Delta\alpha \cdot \delta\omega$,

δ " " $\Delta\delta \cdot \delta\omega$;

and hence, also,

t " " $-\Delta\alpha \cdot \delta\omega$;

and these corrections must produce the correction $-\zeta$ in the moon's azimuth. The differential formulas of spherical trigonometry give us

$$dA = \frac{\cos \delta \cos q}{\sin z} 15 dt + \frac{\sin q}{\sin z} d\delta$$

whence we infer

$$-\zeta = -\frac{\cos \delta \cos q}{\sin z} 15 \Delta\alpha \cdot \delta\omega + \frac{\sin q}{\sin z} \Delta\delta \cdot \delta\omega$$

so that if we put

$$\alpha = \frac{15 \cos \delta \cos q}{\sin z} \Delta\alpha + \frac{\sin q}{\sin z} \Delta\delta \quad (2)$$

we have

$$\delta\omega = \frac{\zeta}{\alpha} \quad (3)$$

5. But the quantity ζ results not only from the error in the assumed longitude, but also from the errors of observation and of the Ephemeris. In order to obtain a general expression in which the effect of each source of error may be represented, let us denote by

$\delta\alpha, \delta\delta, \delta\pi, \delta S$, the corrections of the elements taken from the Ephemeris for the date of the observation;

$\delta\alpha', \delta\delta'$, the corrections of the star's place;

$\delta T, \delta T'$, the corrections for errors in the observed times;

δAT , the correction of AT ;

$\delta\varphi$, the correction of the latitude.

If, when the corrected values of all the elements (that of the longitude included) are substituted in the above computation, A , and A' , become $A + dA$, and $A' + dA'$, we ought to find

$$A + dA = A' + dA',$$

which, compared with (1), gives

$$\zeta = -dA + dA', \quad (4)$$

We have then to find expressions for dA , and dA' , in terms of the above named corrections. By differentiating the formula for A , we find

$$dA = dA \pm \frac{dS}{\sin z} + \frac{\varrho(\varphi - \varphi') \sin 1'' \sin A'}{\sin z} d\pi$$

in which we neglect not only the effects of changes of z as practically insensible, since the observations are not very remote from the meridian, but also the effect of an error in A' upon the parallax in azimuth, since this quantity itself is a quantity of the second order.

We have already found, in the preceding paragraph, the expression for dA taken with reference to δ and t : the differential with reference to φ is

$$-\cot z \sin A d\varphi$$

so that the total differential taken with reference to δ , t and φ , is

$$dA = \frac{15 \cos \delta \cos q}{\sin z} dt + \frac{\sin q}{\sin z} d\delta - \cot z \sin A d\varphi$$

Consequently, also,

$$dA' = dA' = \frac{15 \cos \delta' \cos q'}{\sin z'} dt' + \frac{\sin q'}{\sin z'} d\delta' - \cot z' \sin A' d\varphi$$

Since $t = T + AT - \alpha$, we have

$$dt = dT + dAT - d\alpha$$

where dT and dAT may be exchanged for δT and δAT ; but $d\alpha$ is composed of two parts: 1° of the actual correction of the Ephemeris; 2° of $\Delta\alpha(\delta\omega + \delta T + \delta AT)$, resulting from our having taken α for the uncorrected time. Hence we obtain

$$dt = \delta T + d\delta T - \delta\alpha - \Delta\alpha(\delta\omega + \delta T + \delta AT)$$

which is to be substituted in the expression for dA .

The correction $d\delta$ is also composed of two parts, namely,

$$d\delta = \delta\delta + \Delta\delta(\delta\omega + \delta T + \delta AT)$$

Further we have simply $d\delta = \delta\delta'$, and

$$dt' = \delta T' + \delta AT' - \delta\alpha'$$

but as a very brief interval elapses between the observation of the moon and the star, the error in the *rate* of the chronometer may be neglected, and we may take $\delta AT' = \delta AT$, or

$$dt' = \delta T' + \delta AT - \delta\alpha'$$

These substitutions being made in the value of dA , and dA' , the expression for ζ takes the following form:

$$\begin{aligned} \zeta = & \left(\frac{15 \cos \delta \cos q}{\sin z} \Delta\alpha - \frac{\sin q}{\sin z} \Delta\delta \right) \delta\omega - \left(\frac{15 \cos \delta \cos q}{\sin z} (1 - \Delta\alpha) + \frac{\sin q}{\sin z} \Delta\delta \right) \delta T + \frac{15 \cos \delta' \cos q'}{\sin z'} \delta T' \\ & - \left(\frac{15 \cos \delta \cos q}{\sin z} (1 - \Delta\alpha) + \frac{\sin q}{\sin z} \Delta\delta - \frac{15 \cos \delta' \cos q'}{\sin z'} \right) \delta AT + \frac{15 \cos \delta \cos q}{\sin z} \delta\alpha - \frac{\sin q}{\sin z} \delta\delta \mp \frac{dS}{\sin z} \\ & - \frac{\varrho(\varphi - \varphi') \sin 1'' \sin A'}{\sin z} \delta\pi - \frac{15 \cos \delta' \cos q'}{\sin z'} \delta\alpha' + \frac{\sin q'}{\sin z'} \delta\delta' + (\cot z \sin A - \cot z' \sin A') d\varphi. \end{aligned}$$

In the last term we may put A' for A , and the computation will be facilitated by introducing the auxiliaries

$$f = \frac{15 \cos \delta \cos q}{\sin z}, \quad f' = \frac{15 \cos \delta' \cos q'}{\sin z'}$$

$$a = f \Delta\alpha - \frac{\sin q}{\sin z} \Delta\delta$$

by means of which we shall have

$$\begin{aligned} \zeta &= a\delta\omega + (a-f)\delta T + f'\delta T' + (a-f+f')\delta AT \\ &+ f\delta\alpha - \frac{\sin q}{\sin z}\delta\delta - f'\delta\alpha' + \frac{\sin q'}{\sin z'}\delta\delta' \\ &\mp \frac{\delta S}{\sin z} - \frac{q(q-q')\sin 1'' \sin A'}{\sin z} \delta\pi + \frac{\sin(z'-z)\sin A'}{\sin z \sin z'} \delta\varphi \end{aligned} \quad (5)$$

With the aid of this equation, we can now trace the influence of each source of error :

1° An error δT in the observed time of the moon's transit, produces in $\delta\omega$ the error

$$\left(1 - \frac{f}{a}\right) \delta T$$

When $A = 0$, we have $q = 0$, and then this error is

$$\left(1 - \frac{1}{\Delta\alpha}\right) \delta T = \delta T - \frac{\delta T}{\Delta\alpha}$$

which is therefore the effect of an error in a meridian transit, and is nearly the mean effect of errors of observations equidistant from the meridian.

2° An error in the star's transit produces nearly the same effect (without regard to sign) as an error in that of the moon, when the observations are not too far from the meridian.

3° Since f and f' are nearly equal, we see that when different stars are employed, the mean of whose declinations is nearly the same as the moon's, the mean value of $f-f'$ will be nearly zero, and will have no sensible effect upon the term in δAT ; so that the effect of an error δAT in the chronometer correction upon ζ will be simply $a\delta AT$, and upon the longitude simply δAT .

4° The tabular errors of right ascension produce nearly the same effect as errors in the observed times of transit; and since the essential signs of f and f' are always positive, these errors can be eliminated only by finding the values of $\delta\alpha$ and α' from direct observations. The value of $\delta\alpha$ can be found from any meridian observations of the moon at a standard observatory, taken during the semilunation in which the observations for longitude are made, by PEIRCE'S method. (U.S. Coast Survey Report for 1854, Appendix, p. 115*.)

5° Since $\sin q$ has nearly equal numerical values with opposite signs for observations equidistant from the meridian, the effect of errors of declination (whether of the moon or of the star) will be nearly eliminated from the mean of such observations.

6° The error of the moon's semidiameter will be eliminated if observations of both limbs are combined, in equal number.

7° The error of the horizontal parallax is eliminated by combining observations equidistant from the meridian.

8° The error in the latitude will have no effect when the observations are in the meridian : in other cases, the effect is zero when $z = z'$, which occurs when the star's and moon's declinations are equal. Hence an error in the latitude is eliminated by employing different stars in such positions that the mean of their declinations is nearly the same as that of the moon.

These considerations prove that the mean result of observations, properly grouped according to this method, is subject to no errors which do not equally affect meridian observations; while by the repetition of the observations, we reduce the error of observation below that of a single meridian transit. The method commends itself especially to the traveling astronomer, who may wish to obtain an accurate result in the shortest possible time.

It ought to be observed, however, that it would be a waste of time to make a very large number of observations on the same night. It appears that one portion of the error to which an observer is liable in noting the time of the moon's transit over a thread is nearly constant for the same day, and depends on the moon's age. If then we take in one night a sufficient number of transits over vertical circles to reduce the mean error of observation to an amount considerably below this constant error, we have exhausted the method for this night, and any further multiplication of observations would be useless. The point at which this limit is to be established must depend on the skill of the observer, the power of his instrument, the stability of its mounting, and even on atmospheric conditions; so that there is here abundant room for the exercise of his judgment. I think we may safely say, however, that cases will be rare in which more than six complete observations, three on each side of the meridian, will sensibly increase the accuracy of the mean result. With good instruments and average skill, two on each side of the meridian ought to suffice.

6. It remains to be considered, how the observations on the same night are to be combined, so as to obtain the most probable mean, having regard to their relative weight; for not only will the coefficient of $\delta\omega$ in our equation (5) have different values for the different observations, but the weights of the observations will vary with the distance from the meridian and the number of threads upon which the transits may happen to be observed. The preceding discussion shows that the terms involving $\delta\alpha$ and $\delta\alpha'$ are the only ones which are not either eliminated, or reduced to a minimum by a proper combination of observations. Omitting $\delta\alpha'$ as being very small, and having probably different signs for different stars, we may consider our equations of condition under the simple form

$$a\delta\omega = \zeta - f\delta\alpha. \quad (6)$$

For all the observations on the same day, $\delta\alpha$ is to be regarded as constant; so that having computed ζ , f and a for each observation, we have the equations

$$\begin{aligned} a_1 \delta\omega &= \zeta_1 - f_1 \delta\alpha, \\ a_2 \delta\omega &= \zeta_2 - f_2 \delta\alpha, \\ a_3 \delta\omega &= \zeta_3 - f_3 \delta\alpha, \\ &\text{etc.} \end{aligned}$$

To find expressions for the weight of one of these equations, let

ε = the mean error of a transit of the *moon* over a single thread;

ε' = the mean error of a transit of a *star* over a single thread;

v, v' = the number of threads on which the moon and the star are observed, respectively:

then the mean errors of T and T' are

$$\frac{\varepsilon}{\sqrt{v}}, \quad \frac{\varepsilon'}{\sqrt{v'}}$$

and the effect of these errors upon ζ are, severally, according to our equation (5),

$$(a-f) \frac{\varepsilon}{\sqrt{v}}, \quad f' \frac{\varepsilon'}{\sqrt{v'}}$$

for which, since a is small in comparison with f , while f' is nearly equal to f , we may here take

$$-\frac{f\varepsilon}{\sqrt{v}}, \quad \frac{f\varepsilon'}{\sqrt{v'}}$$

and the combined effect of these errors produces in ζ the mean error

$$f \sqrt{\left(\frac{\varepsilon^2}{v} + \frac{\varepsilon'^2}{v'}\right)}$$

and the weight of the equation of condition formed from ζ is therefore proportional to

$$\frac{vv'}{f^2(v\varepsilon'^2 + v'\varepsilon^2)}$$

It has heretofore been the practice of astronomers, in estimating the relative weights of moon-transits, to regard the mean error of the transit of the moon as the same as that of a star; that is, to take $\varepsilon = \varepsilon'$. As we are here considering only that part of the error which is not constant, this assumption is probably not so far wrong as to require us to go beyond it in actual practice. We shall, however, put $\varepsilon\varepsilon' = k\varepsilon'^2$; and then regarding ε'/ε as constant, we may express the weight of an observation by the formula

$$p = \frac{vv'}{f^2(v + kv')} \quad (7)$$

in which we may take $k = 1$, unless a special investigation in an individual case may furnish a different value*.

* From a discussion of a number of observations of moon culminations by the experienced observers at Greenwich, I have found for one of the observers $k = 1.3$. For one of the Washington observers, I found for the moon's first limb, $k = 0.88$; but for the second limb, $k = 1.06$.

We are now to multiply our equations respectively by $\sqrt{p_1}, \sqrt{p_2}, \sqrt{p_3}$, etc. as found by this formula for each observation; and then multiplying them respectively by the coefficient of $\delta\omega$, their sum, according to the usual notation of the method of least squares, is

$$[aap] \delta\omega = [ap\zeta] - [afp] \delta\alpha$$

The immediate result of a single night's work, uncorrected for the error of the ephemeris, is

$$\delta\omega = \frac{[ap\zeta]}{[aap]}, \quad \text{with the weight } [aap] \quad (8)$$

to which, when $\delta\alpha$ has been found, we have to apply the correction

$$- \frac{[afp]}{[aap]} \delta\alpha.$$

The unit of weight here corresponds to the error ε' : when this is known, we have for a single result (on one day),

$$\text{Mean error of } \delta\omega = \frac{\varepsilon'}{\sqrt{[aap]}}$$

which, however, is to be increased on account of the error in the correction of the ephemeris. If then

$$e = \text{the mean error of } \delta\alpha$$

we have

$$\text{Mean error of } \delta\omega = \sqrt{\frac{\varepsilon'\varepsilon'}{[aap]} + \left(\frac{[afp]}{[aap]}\right)^2 e^2} \quad (9)$$

Results on different days may be combined by employing weights assigned according to the mean errors thus determined.

In practice, however, the following simpler process will give results not sensibly differing from those furnished by the above. If we put $\frac{a}{f} = A\alpha$, which is nearly its mean value for observations on the same day when they are equidistant from the meridian, and also put

$$p' = \frac{vv'}{v + kv'} \quad (10)$$

we have

$$p = \left(\frac{A\alpha}{a}\right)^2 p'$$

and then $A\alpha$ being regarded as constant, we have

$$[ap\zeta] = (A\alpha)^2 [p' \frac{\zeta}{a}]$$

$$[aap] = (A\alpha)^2 [p']$$

so that by (8) we have

$$\delta\omega = \frac{[p' \frac{\zeta}{a}]}{[p']} \quad (11)$$

If then we find from each observation the value of $\delta\omega$ by the formula

$$\delta\omega = \frac{\zeta}{a}$$

we can find the general mean of these values by the usual process, employing for the relative weights the formula (10). But the weight of this mean, when it is combined with the means of different days, must still be denoted by $[aap]$ or $(A\alpha)^2 [p']$.

7. It may be well to add here a single example, in order to illustrate the use of our general formulas, and to show the values which the coefficients of equation (5) obtain.

At the U.S. Naval Academy, May 9, 1857, the transits of the moon's second limb, and of σ *Scorpii*, were observed over the same vertical circle at an approximate azimuth of 10° east of the meridian, with an Ertel Universal Instrument of 15 inches focal length, as follows :

Chronometer : Level : Collimation :
 $\text{D II. T} = 16^{\text{h}} 11^{\text{m}} 30^{\text{s}}.17$ $\epsilon = +2''.2$ $c = 0$
 σ *Scorp. T'* = $16^{\text{h}} 27^{\text{m}} 49^{\text{s}}.83$

These times are the means of three threads. The correction of the chronometer on sidereal time was $-55^{\text{m}} 9^{\text{s}}.16$ at 13^{h} , with an hourly rate of $-0^{\text{s}}.32$.

The latitude is $\varphi = 38^\circ 58' 53''.5$, and the assumed longitude, $\omega = 5^{\text{h}} 5^{\text{m}} 55^{\text{s}}$.

With these data, we find first

$$\begin{aligned} \Delta T &= -15^{\text{m}} 9^{\text{s}}.89 & \Delta T' &= -55^{\text{m}} 9^{\text{s}}.97 \\ T + \Delta T &= 15^{\text{h}} 16^{\text{m}} 20^{\text{s}}.28 & T' + \Delta T' &= 15^{\text{h}} 32^{\text{m}} 39^{\text{s}}.86 \end{aligned}$$

The Greenwich time corresponding to the sidereal time $T + \Delta T$ is May 9, $17^{\text{h}} 10^{\text{m}} 27^{\text{s}}.09$; for which we take from the Ephemeris the following quantities :

$$\begin{aligned} \alpha &= 15^{\text{h}} 54^{\text{m}} 45^{\text{s}}.32 & \delta &= -24^\circ 42' 54''.4 \\ \Delta \alpha &= 0^{\text{s}}.03513 & \Delta \delta &= -0''.127 \\ \pi &= 54' 9''.2; & S &= 14' 47''.2; \end{aligned}$$

and from the B. A. Catalogue,

$$\alpha' = 16^{\text{h}} 12^{\text{m}} 32^{\text{s}}.62 \quad \delta' = -25^\circ 14' 58''.5.$$

The computation of the formulas of Article 4 gives then

$$\begin{aligned} A &= -9^\circ 40' 51''.2 & A' &= -9^\circ 57' 25''.4 \\ \log \tan q &= n 9.1626 & \log \tan q' &= n 9.1769 \\ z &= 64^\circ 19'.5 & z' &= 64^\circ 54'.1 \\ p \sin z &= +48.8 & & \\ \text{Refraction} &= -2.1 & \text{Refraction} &= -2.1 \\ z_1 &= 65^\circ 6'.2 & z'_1 &= 64^\circ 52'.0 \\ -b \cot z_1 &= -1''.0 & -b \cot z'_1 &= -1''.0 \\ -\frac{S}{\sin z} &= -16' 24''.2 & & \\ \log q &= 9.9994 & \frac{q(\varphi - \varphi') \sin \pi \sin A'}{\sin z} &= -2''.0 \\ \varphi - \varphi' &= 11' 15'' & & \\ A_1 &= -9^\circ 57' 18''.4 & A'_1 &= -9^\circ 57' 26''.4 \\ \zeta &= +8''.0. \end{aligned}$$

For the coefficients of the equation (5), we find

$$\begin{aligned} f &= 14.963 & f' &= 14.818 \\ a &= 0.505 & & \\ a - f &= -14.458 & a - f + f' &= +0.360 \\ -\frac{\sin q}{\sin z} &= +0.160 & \frac{\sin q'}{\sin z'} &= -0.164 \\ \text{cosec } z &= 1.109 & & \\ -\frac{q(\varphi - \varphi') \sin 1'' \sin A'}{\sin z} &= +0.0006; & & \\ \frac{\sin(z' - z) \sin A'}{\sin z \sin z'} &= -0.0021. \end{aligned}$$

The equation is therefore

$$\begin{aligned} +8''.0 &= 0.505 \delta \omega - 14.458 \delta T + 14.818 \delta T' + 0.360 \delta \Delta T \\ &\quad + 14.963 \delta \alpha + 0.160 \delta \delta \\ &\quad - 14.818 \delta \alpha' - 0.164 \delta \delta' \\ &\quad + 1.109 \delta S + 0.0006 \delta \pi - 0.0021 \delta \varphi; \end{aligned}$$

whence

$$\begin{aligned} \delta \omega &= +15^{\text{s}}.8 + 28.62 \delta T - 29.34 \delta T' - 0.713 \delta \Delta T \\ &\quad - 29.63 \delta \alpha - 0.32 \delta \delta \\ &\quad + 29.34 \delta \alpha' + 0.32 \delta \delta' \\ &\quad - 2.20 \delta S - 0.001 \delta \pi + 0.004 \delta \varphi. \end{aligned}$$

OBSERVATIONS OF EUPHROSYNE.

MADE WITH THE FILAR MICROMETER OF THE WASHINGTON EQUATORIAL.

By JAMES FERGUSON.

[Communicated by Lieutenant MAURY.— Corrected for refraction.]

EUPHROSYNE.

Date.	M. T. Washington.	No. of Comp.	Comparison-Star.	③1 — *		③1's apparent	
				$\Delta \alpha$	$\Delta \delta$	α	δ
1857, April	24 10 30 21.3	11	A. Z. 301, 59	+0 58.44	+2 22.55	14 42 4.39	-23 37 4.50
	25 10 4 30.6	14	" 301, 59	+0 0.55	+1 1.48	14 41 6.50	23 38 25.68
	27 10 0 21.3	10	" 301, 59	-1 59.47	-1 33.14	14 39 6.50	23 41 0.41
	30 9 48 13.4	11	" 301, 47	+1 22.70	+8 42.18	14 36 5.14	23 44 13.12
May	12 10 8 9.7	4	" 301, 35	+0 34.49	+5 31.73	14 24 12.59	23 50 23.08
	22 10 16 14.9	4	" 301, 20	+2 44.86	+2 19.39	14 15 15.65	23 48 48.35
	24 9 39 26.0	12	" 301, 20	+1 10.36	+3 0.10	14 13 41.16	23 48 7.77
	* 25 10 7 43.2	4	" 301, 20	+0 22.22	+3 21.06	14 12 53.03	-23 47 46.81

* The right-ascensions of the two last comparisons are observed with the electric apparatus of Mr. Bond, now first introduced in the observatory.