Letter to the Editor

Large-scale chaos in the solar system

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Abstract. Numerous integrations of the solar system have been conducted, with very close initial conditions, totaling an integration time exceeding 100 Gyr. The motion of the large planets is always very regular. The chaotic zone explored by Venus and the Earth is moderate in size. The chaotic zone accessible to Mars is large and can lead to eccentricities greater than 0.2. The chaotic diffusion of Mercury is so large that its eccentricity can potentially reach values very close to 1, and ejection of this planet out of the solar system resulting from close encounter with Venus is possible in less than 3.5 Gyr.

Key words: Celestial mechanics – solar system – planets – chaotic phenomena – instabilities – diffusion

1990, 1992) this is reduced to 16, and to 15 after taking into consideration the conservation of angular momentum.

of freedom system, when a surface of section is sufficient

to obtain this information. For three degree of freedom sys-

tems, this is considerably more difficult, but can be achieved

in some sense (Laskar, 1993, Dumas and Laskar, 1993). Such

a global view was also achieved in the study of the long time

evolution of the obliquities of the planets (Laskar et al., 1993,

Laskar and Robutel, 1993), but for the complete solar system,

this task seems too complicated at present. Indeed, the solar

system, including all its major planets (from Mercury to Nep-

tune), after performing the reduction of center of mass is a $3 \times 8 = 24$ degree of freedom system. After averaging over the mean longitudes of the planets (Laskar, 1985, 1986, 1989,

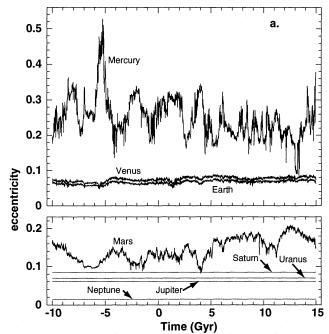
1. Introduction

The discovery of the chaotic behavior of the orbital motion of the inner planets in the solar system (Laskar, 1989) was obtained using a numerical integration of the averaged equations of motion over 200 Myr. Nevertheless, over such a long. but still limited time span, it was not possible to evaluate the possible variations of the planetary orbital elements over the age or expected lifetime of the solar system, that is about 5 Gyr. On the other hand, due to considerable improvement in the last three years, direct numerical integrations of the motion of the complete solar system have now been performed up to 100 Myr (Quinn et al., 1991, Sussman and Wisdom, 1992) and have confirmed its chaotic behavior, but are still far from the 5 Gyr goal. Furthermore, since the motion of the solar system is chaotic with a lyapunov time of about 5 Myr (Laskar, 1989, Sussman and Wisdom, 1992), there is no hope of tracking precisely the real motion of the solar system over more than 10 to 20 Myr. An integration of the solar system over 5 Gyr can thus only be considered as an indication of its possible behavior, and cannot pretend to be the description of its actual motion. The problem thus becomes more complicated, as a single integration is no longer sufficient. Since very close initial conditions can lead to completely different behavior after 100 Myr, one would like to have a more global view of the dynamics of the solar system and to study its behavior for all initial conditions. This is easy to do for a two degree

2. Evolution of the solar system on Gyr time scales

Even without describing the global dynamics of this 15 degree of freedom dynamical system, we can still get an idea of the geography of the chaotic zone where our solar system evolves. For this purpose, we can follow a numerical integration over a very long time span, even larger than the age of the solar system. This solution will act as a scout exploring the chaotic domain to which the solar system belongs. We can also send a number of similar explorers, starting from nearby initial conditions. At present, this appears to be the only way to obtain some understanding of the possible long time evolution of the solar system, and to provide some bounds on its behavior.

The equations of motion used here are the averaged equations which were previously used for the demonstration of the chaotic behavior of the solar system. They include the Newtonian interactions of the 8 major planets of the solar system (Pluto is neglected), and relativistic and Lunar corrections (Laskar, 1985, 1989, 1990). The numerical solution of these averaged equations showed excellent agreement when compared over 4400 years with the numerical ephemeris DE102 (Newhall *et al.*, 1983, Laskar, 1986), and over 3 millions years with the numerical integration performed by Quinn, Tremaine and Duncan (Quinn *et al.*, 1991, Laskar *et al.*, 1992). Similar



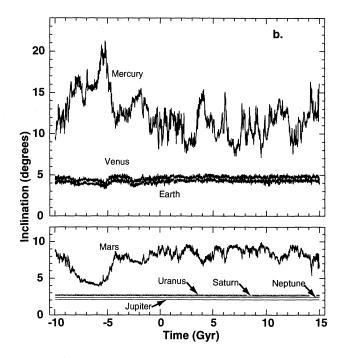


Fig. 1a and b. Numerical integration of the averaged equations of motion of the solar system 10 Gyr backward and 15 Gyr forward. For each planet, the maximum value obtained over intervals of 10 Myr for the eccentricity (a) and inclination (in degrees) from the fixed ecliptic J2000 (b) are plotted versus time. For clarity of the figures, Mercury, Venus and the Earth are plotted separately from Mars, Jupiter, Saturn, Uranus and Neptune. The large planets behavior is so regular that all the curves of maximum eccentricity and inclination appear as straight lines. On the contrary the corresponding curves of the inner planets show very large and irregular variations, which attest to their diffusion in the chaotic zone.

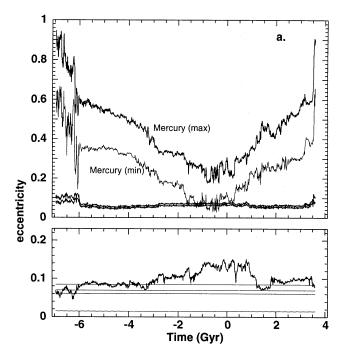
agreement was observed with subsequent numerical integration by Sussman and Wisdom (1992).

These averaged equations can thus be considered as a very good approximation of the real motion of the solar system. In particular, they are well suited for understanding the main features of its global dynamics. This system of equations was obtained with dedicated computer algebra and contains 153824 monomial terms of the form $\alpha z_1 z_2 z_3 z_4 z_5$ (Laskar, 1985). It was constructed in a very extensive way, containing all terms up to second order with respect to the masses, and up to 5th degree in eccentricity and inclination, but it appears that many of these terms are of very small amplitude. In order to improve the efficiency of the numerical integration, less than 50 000 terms were retained. The accuracy of the resulting system was tested against the full equations by direct comparison over 10 Myr. For all variables, the discrepancies are smaller than 10⁻⁵ after 10 Myr. This is much less than discrepancies with direct numerical integrations (Laskar et al., 1992), or among direct numerical integrations (Sussman and Wisdom, 1992). The evaluation of this simplified system is very efficient, since fewer than 6000 monomials need to be evaluated because of symmetries. Numerical integration is carried out using an Adams method (PECE) of order 12 and with a 250-year stepsize. The integration error is measured by integrating the equations back and forth over 10 Myr. It amounts to 3 10^{-13} after 10^7 years (40 000 steps), and behaves like t^{1.4}. Ignoring the chaotic behavior of the orbits, this would give a numerical error of only $4\ 10^{-9}$ after 10 Gyr. All integrations were performed on an IBM RS6000/370 workstation and took 1 day of CPU time per Gyr.

The equations were integrated over 10 Gyr backward, and 15 Gyr forward. It may seem strange to try to track the orbit of the solar system over such an extended time, longer than the age of the universe, but one should understand that it is done in order to explore the chaotic zone where the solar system evolves, and after 100 Myr, can give only an indication of what can happen. On the other hand, if there is a sudden increase of eccentricity for one planet after 10 Gyr, this still tells us that such an event could probably also occur over a much shorter time, for example in less than 5 Gyr. In the same way, what happens in negative time can happen as well in positive time.

In order to follow the diffusion of the orbits in the chaotic zone, one needs quantities which behave like action variables, that is quantities which will be constant for a regular (quasiperiodic) solution of the system. Such quantities can be given by the quasi frequencies obtained by frequency analysis (Laskar, 1990, 1993, Dumas and Laskar, 1993), but here I preferred variables more directly related to the physical behavior of the orbits, and plotted the evolution of the maximum eccentricity and inclination attained by each planet during intervals of 10 Myr (Fig. 1).

The behavior of the large planets is so regular that all the corresponding curves appear as straight lines (Fig. 1). On the



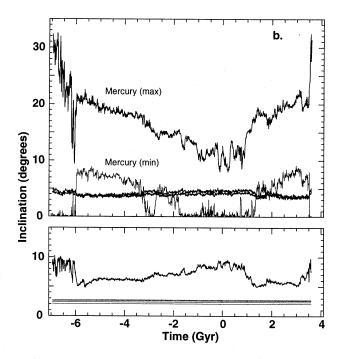


Fig. 2a and b. Orbit of the solar system leading to very large values for the eccentricity of Mercury, and possibility of escape at -6.6 Gyr and +3.5 Gyr. The plotted quantities are the same as in Fig.1, except for Mercury, where minimum eccentricity and inclination over 10 Myr are also plotted. During all the integrations, the motion of the large planets is very regular.

contrary the maxima of eccentricity and inclination of the inner planets show very large and irregular variations, which attest to their diffusion in the chaotic zone. The diffusion of the eccentricity of the Earth and Venus is moderate, but still amounts to about 0.02 for both planets. The diffusion of the eccentricity of Mars is large and reaches more than 0.12, leading to values higher than 0.2 for the eccentricity of Mars. For Mercury, the chaotic zone is so large (more than 0.4) that it reaches values larger than 0.5 at some time. The behavior of the inclination is very similar.

Strong correlations between the different curves appear in figure 1. Indeed, as the solar system wanders in the chaotic zone, it is dominated by the linear coupling among the proper modes of the averaged equations (Laskar, 1990), which induces a very similar behavior for the maximum eccentricity and inclination of Venus and the Earth. This coupling is also noticeable in the solution of Mars. On the other hand, an angular momentum integral exists in the averaged equations, and explains why when Mercury's eccentricity and inclination increase, the similar quantities for Venus, the Earth and Mars decrease. Thus it appears that, despite the small values of the inner planets' masses, the conservation of angular momentum plays a decisive role in limiting their excursions in the chaotic zone. In particular this should give some limitation on the diffusion of the more massive planets, Venus and the Earth.

3. The possible escape of Mercury

After this first experiment, let us play another game. At some time, Mercury suffered a large increase in eccentricity (fig. 1) rising up to 0.5. But this is not sufficient to cross the orbit of Venus. The question then arises whether it is possible for Mercury to escape from the solar system in a time comparable to its age. A first attempt to answer this was made by slightly changing the initial conditions for the planets. Indeed, because of the chaotic behavior, very small changes in the initial conditions lead to completely different solutions after 100 Myr. Using this, I decided to change only one coordinate in the position of the Earth, amounting to a physical change of about 150 meters (10^{-9} in eccentricity). The full system was integrated with several of these modified solutions, but it led to similar (although different) solutions. In fact, it should not be too easy to get rid of Mercury, otherwise it would be difficult to explain its presence in the solar system.

I thus decided to guide Mercury somewhat towards the exit. A first experiment was done for negative time: for 2 Gyr, the solution is left unchanged, then, 4 different solutions are computed for 500 Myr, in each of which the position of the Earth is shifted by 150 meters, in a different direction (due to the exponential divergence, this corresponds to a change smaller than Planck's length in the original initial conditions).

The solution which leads to the maximum value of Mercury's eccentricity is retained up to the nearest entire Myr, and is started again. In 18 of such steps, Mercury attains eccentricity values close to 1 at about -6 Gyr (Fig 2) when the solution

enters a zone of greater chaos, with lyapunov time ≈ 1 Myr, giving rise to much stronger variations of the orbital elements of the inner planets. In such a long computation, which goes far beyond the horizon of predictibility of the system, negative time is equivalent to positive time, but in order to be more convincing, a second solution was also computed in positive time, without the initial 2 Gyr period, and with changes in initial condition of only 15 meters instead of 150 meters. As anticipated, this led to a similar increase in Mercury's eccentricity, this time in only 13 steps and about 3.5 Gyr (Fig 2).

While the eccentricity increases, the inclination of Mercury can change very much (Fig 2b). In order to check whether these changes in Mercury's eccentricity can really lead to an orbit crossing with Venus, the relative positions of the intersection of the orbits of Mercury and Venus with their line of nodes was computed around 3.5 Gyr, at a date which seems critical (Fig.3). Most of the time, on the line of nodes, the orbit of Mercury stays inside the orbit of Venus, but at some times, the orbit of Mercury crosses the orbit of Venus. This lasts a few thousand years, and during that time, the two planets can experience a close encounter which can lead to the escape of Mercury or to collision.

It should be said that for very high eccentricity of Mercury, the model used here no longer gives a very good approximation to the motion of Mercury, but it is very important to know that in this approximation, the chaotic zone allows the escape of a planet from the solar system in a time smaller than the expected life of the solar system, due to diffusion in the chaotic zone. Even more, in this averaged system, the degrees of freedom corresponding to semi major axes and mean longitudes are removed, but in the real system the addition of these extra degrees of freedom will probably lead to even stronger chaotic behavior, as in general, addition of degrees of freedom increases the stochasticity of the motion.

4. Conclusion

In all the integrations which are reported here, the large planet motions are always very regular, while the diffusion of the inner planets' orbits in the chaotic zone is much larger than what was already seen over 200 Myr (Laskar, 1990, 1992). Combining the results of the different integrations (Figs. 1 and 2), it appears that the possible diffusion of Venus and the Earth over 5 Gyr is larger than 0.06 in eccentricity, and 1 degree in inclination. For Mars, it amounts to about 0.16 in eccentricity and 6 degrees in inclination, while for Mercury it is so large for the eccentricity that ejection of this planet out of the solar system resulting from close encounter with Venus (or collision) is possible in less than 3.5 Gyr.

The diffusion of the solar system in its chaotic zone is also sufficient to drive Mars' eccentricity above 0.2 in a few Gyr. If this happened in the past, it should be of great importance for understanding past climates of this planet.

The difference in behavior between the large planets and the inner planets is very striking (Fig.1ab). One reason for

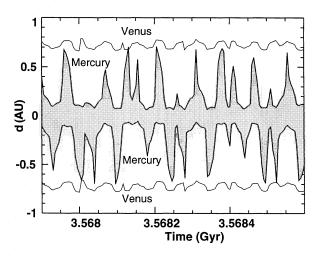


Fig. 3. Relative position of the orbits of Mercury and Venus computed at the line of intersection of their two orbital planes (line of nodes) for a selected date of the integration plotted in Fig.2. The distance from the Sun of the orbit on the line of nodes (in AU) is plotted against the date (in Gyr). Most of the time, on the line of nodes, the orbit of Mercury stays inside the orbit of Venus, but at some times, the orbit of Mercury crosses the orbit of Venus. This phenomenon lasts a few thousand years, and during that time, the two planets can experiencee close encounter which can lead to the escape of Mercury, or to collision.

this is probably that the large planets system is not perturbed much by the inner planets, and thus can be thought of as a system with only 8 degrees of freedom, instead of 16 for the whole system. Another possibility is that if the outer planets were less regular, then the inner planets' motion would be so chaotic, that the Earth would suffer changes too large in its orbit to ensure climatic stability on its surface.

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