Annu. Rev. Astron. Astrophys. 1993. 31: 433–71

THE FAINT END OF THE STELLAR LUMINOSITY FUNCTION

Michael S. Bessell and Guy S. Stringfellow[1]

Mount Stromlo and Siding Spring Observatories, The Australian National University, ACT 2611, Australia

KEY WORDS: luminosity function, low-mass stars, brown dwarfs, white dwarfs, mass-luminosity relation

1. INTRODUCTION

The observed stellar luminosity function (SLF) of field stars near the Sun is probably the most basic of data concerning stars in the Galaxy; the SLF is defined as the number of stars per cubic parsec within an absolute magnitude interval $M_i \pm \delta M_i$ where i denotes either bolometric or a specific bandpass and typically $\delta M_i = 0.5$ mag. It forms the basis for calculating the observed mass densities for a variety of stars thus enabling the derivation of the total observed stellar mass densities for comparison with dynamically derived mass densities. It is also used to reconstruct the initial mass function (IMF) for faint stars and to explore the star formation history of the disk and halo.

It is of great interest to measure stellar luminosity functions for different subgroups of stars: for main sequence stars with different metallicities, for degenerate white dwarfs, and for globular clusters, open clusters, and active star forming regions (T-associations). Comparison of these luminosity functions provides information about many different aspects of astronomy such as star formation scenarios, stellar evolution, cooling theories of white dwarfs, the evolution of stellar populations in galaxies, and the formation of galaxies by the disruption of clusters.

In this review we summarize the most recent measurements of the SLFs below $M_V = 5$; this regime includes hydrogen burning main sequence stars with masses between $\sim 0.08\ M_\odot$ and $1\ M_\odot$, substellar objects with masses below the mimimum main-sequence mass (i.e. the brown dwarfs, hereafter

[1] Presently at Center for EUV Astrophysics, University of California, Berkeley, California 94720.

0066–4146/93/0915–0433$02.00

BDs), and low luminosity stellar remnants. We discuss the wide ranging observational and theoretical studies which underpin such fundamental work. Also discussed are the various relations that are necessary to transform observed LFs into theoretical LFs and mass functions, namely the color-color, color-bolometric correction, color-temperature, color-luminosity, and mass-luminosity relations. For stars more massive than about 0.1 $M_\odot$ these relations are well established empirically, but for the faintest and coolest stars the relations must of necessity be theoretically predicted. Finally, we discuss future prospects and current trends of ongoing work in this field.

In discussing the subgroups of stars in the Galaxy with different ages, compositions, velocity dispersions, scale heights, and abundance ranges, words like *Pop I*, *Pop II*, *old disk*, *young disk*, *thin disk*, *thick disk*, *bulge*, *halo*, *spheroid*, and *corona* are used. Unfortunately, as a result of the increase in knowledge of the complexities of the Galaxy there is now some confusion in the use of the various terms (see e.g. Sandage 1986, Wyse & Gilmore 1988, and Norris 1992 for discussion of this point).

In this review, we shall call the *old disk*, that material which is supported by rotation, has an exponential scale hight of $\sim$250 pc in the vertical direction, a circular velocity at the Sun of $\sim$220 km s^{-1}, a total velocity dispersion of $\sim$40 km s^{-1}, and a probable age of about 10 Gyr. It is also often called the *thin disk*. The *young disk* is material which is similarly rotationally supported, but has a scale height of $\sim$50 pc, a velocity dispersion of $\sim$25 km s^{-1}, and an age less than 1 Gyr. It is also called the *young thin disk*. The disk material which comprises 80% of the luminous material in the Galaxy has a metal abundance compared with that of the Sun's of between +0.2 and −0.5 dex.

There has been much discussion on how to best describe the distribution of the material in our Galaxy and in external galaxies outside the thin exponential disk and we refer you to the above references. We will call the flattened spheroidal luminous material with an $r^{1/4}$ law magnitude profile, the *halo* or *spheroid*. At the solar circle it has a density of only 0.1 to 0.2% that of the disk. It is essentially supported by kinetic motions and has a velocity dispersion of $\sim$190 km s^{-1}; its rotation is <30 km s^{-1}. It has a probable age of 12–20 Gyr and its mean metallicity is about −1.6 dex. There is some evidence that in addition to the spheroid there exists material with a scale height of $\sim$1 kpc which is partly rotationally supported, and is often referred to as the *thick disk*.

In order to explain the flat rotation curve of the distant disk material, the presence of material with a density profile of r^{-2} has been proposed. This dynamically deduced material is called the *dark halo* or *dark corona* and could comprise remnants of the protogalactic material, although its extent and constituency is unknown.

2. DATA AND THEORY UNDERPINNING THE SLF

In this section we discuss the extensive databases and aspects of stellar evolution theory that underpin the analysis of the SLF. It is obviously necessary to be able to distinguish between different kinds of stars when making star counts; the methods used are discussed in Section 2.1. Distances need to be assigned to each of the stars. For the nearest stars, distances are obtained directly from parallaxes but for more distant stars and for large numbers of stars, photometric parallaxes are used. Section 2.2 and Section 2.3 detail the databases and relations used. The color-temperature and color-bolometric correction relations, which are required to relate observed color magnitude diagrams to theoretical H-R diagrams and when dealing with the bolometric luminosity functions, are detailed in Section 2.4 and Section 2.5. The mass-luminosity (M-L) relations that are necessary to convert the observed SLF to an inferred mass function, or in constructing theoretical LFs from the models are discussed in Section 2.6, and in subsequent sections.

2.1 *Distinguishing Between Different Star Types*

Old disk and spheroid (halo) dwarfs are usually separated by their space motions or their metallicity. The metallicity is normally assessed photometrically by ordering stars, with the same V − I or R − I color, by some color such as U − B or B − V, which is sensitive to line-blanketing.

White dwarfs are identified in several ways. Where proper motions are available, white dwarfs can easily be distinguished from other dwarfs by their excessively high transverse velocities which result from the assumption that they have the luminosity of normal dwarfs with the same color; otherwise, they are identified by their different spectral appearance from main sequence stars due to their much higher ($\sim$1000 times higher) atmospheric pressures. Hot white dwarfs (15,000 K $> T_{\rm eff} >$ 7000 K) can be identified spectroscopically or photometrically from their unique spectra in the blue; cooler white dwarfs generally have colors which are indistinguishable from those of normal dwarfs or subdwarfs and blue-yellow spectra are essential. Most known cool white dwarfs have continuous spectra (low metal abundance Z, He atmospheres) or extremely broad depressions (higher Z, He atmospheres) due to strong metal lines (MgI, NaI, CaI) or C molecular bands. If the atmospheres are H-rich, H lines are present, but pressure broadening limits the visibility of the H lines to lower members of the Balmer series than Hδ. Wickramasinghe et al (1977) and Bessell & Wickramasinghe (1979) discuss these problems. Because of the extremely low abundance of metals in the atmosphere of H-rich white dwarfs, stars showing very strong MgH and CaH bands are cool subdwarfs (Bessell 1982, Hartwick et al 1984) and not white dwarfs. The general properties of white dwarfs have most recently been reviewed by Liebert (1980), Sion (1986), and Weidemann (1988). Leggett (1989) summarizes the IR observations of white dwarfs and their fundamental properties.

When attempting to extend the identification of white dwarfs and M dwarfs to temperatures lower than those hitherto discovered, one necessarily extrapolates the appearance of the known dwarf stars using predictions based on theoretical model atmospheres; in addition, we know what the spectra of extremely cool M giants look like. We anticipate that in cooler M dwarfs the TiO, VO, and FeH bands will continue to strengthen as will the NaI, CaI, and KI lines; the I−K, H−K, and K−L colors will continue to increase; and the dramatic rise in the spectral energy distribution, as seen in the 0.8–1.0 μm range in the coolest known M dwarfs, will shift to somewhat longer wavelengths.

In the case of degenerate white dwarfs, the physics of the atmospheres are less straightforward due to the uncertainty of the photospheric composition and the extreme high pressures. We anticipate that the coolest white dwarfs will have continuous spectra in the visible but will have redder U−B and B−V colors than blackbodies with the same R−I or V−K color due to convection and Rayleigh scattering. However, although the coolest stars will be very old (D'Antona & Mazzitelli 1990) and all the metals should have sunk in the atmospheres, if traces of metals were to be still present in the atmospheres these would give rise to extremely broad absorption lines which would greatly depress the continuum (e.g. LP701-29). Such stars will have very unusual spectra compared to normal K and M dwarfs, although their UBV colors could be similar. Reduced motions are clearly the best way for isolating a prospective subset of white dwarfs in a large sample of stars, although final confirmation can only be attained through spectral analysis.

In order to discriminate between the lowest mass hydrogen burning M dwarfs and the substellar brown dwarfs in the field we must look for indirect indications of age. This of course precludes any direct determination of the star's mass with reasonably secure and meaningful errors. Age is an important discriminant because theoretical calculations predict that brown dwarfs and the lowest mass hydrogen burning dwarfs can occupy very similar positions in the HR diagram but at very different times. Brown dwarfs cool rapidly down tracks that parallel closely the main-sequence defined by the M dwarfs with mass close to the hydrogen burning limit. Space motions are one indication of age, although not a very sensitive one for ages less than 10^8 years. Chromospheric activity has often been used as another indicator—the utility of which has been shown for solar type stars (cf Skumanich 1986, also the review by Haisch et al 1991). However, the long-term correlation with age for late-type stars, in particular the M dwarfs, remains unclear. The cool M dwarfs, including the extreme ones, do exhibit flaring (VB10: Herbig 1956; LHS 2397a: Bessell 1991; VB8: Tagliaferri et al 1990). In the case of LHS 2397a the energy emitted in the UBV bands overwhelms that present during quiescent phases, resulting in immense color changes (Stringfellow & Bessell 1993, in preparation). Such variability could pose serious problems in subsequent analysis, particularly if only a single observation exists. Hence, it is important to obtain repeated observations, photometric and spectroscopic, over both short and long timescales in order to distinguish average properties during active and quiescent phases. As no brown dwarf has yet been unequivocally identified, it is uncertain whether such activity exists in brown dwarfs.

Strong LiI at 670.8 nm is another indicator of extreme youth, although it is perhaps impossible to see the line in the strong CaH and TiO bands of late M dwarfs. Stringfellow (1989, and in preparation) discusses the potential of lithium and deuterium as diagnostic probes based on his evolutionary calculations of low mass stars and brown dwarfs; see also Rebolo et al (1992). His time-dependent depletion curves of deuterium and lithium are shown in Figures 1*a* and 1*b*, respectively. Each curve corresponds to a given mass. Generally speaking, once burning begins, complete destruction soon follows. The onset of destruction is dependent upon mass, with more massive models depleting their deuterium/lithium before the lower mass models initiate theirs. For each element, a minimum mass exists for which no destruction occurs over a Hubble time; this limit is sensitive to the input physics, most notably the atmospheric opacities. For deuterium the limit is $\sim 0.01\ M_{\odot}$ (Stringfellow 1989; Burrows et al 1989, hereafter BHL), and for lithium it is $\sim 0.06\ M_{\odot}$ (Stringfellow 1989, Pozio 1991). Both of these mass-abundance limits lie in the brown dwarf mass regime, with the lithium one lying close to but below the hydrogen burning mass limit itself. The first search for lithium in the above context has recently been reported by Magazzu et al (1993). They find no evidence of lithium in either component of GL 473AB. As lithium is preserved in brown dwarfs with mass $\lesssim 0.06\ M_{\odot}$, the apparent absence of it in GL 473AB indicates that both components should have masses greater than $0.06\ M_{\odot}$. Heintz (1989) had reported earlier that the components of this binary were 0.059 and $0.051\ M_{\odot}$ with an error of $\pm 0.01\ M_{\odot}$; the errors could afford an upper mass just exceeding $0.06\ M_{\odot}$ which would have depleted at least some lithium if older than a few billion years, but still lie well within the brown dwarf regime. Recent IR speckle measurements of GL 473AB were made by Henry et al (1992) who determine higher masses than Heintz which are well outside the brown dwarf realm and fully consistent with the apparent absence of lithium. Only if lithium had been found would the substellar nature of GL 473AB been greatly stengthened, although a fairly young age also could then have been ascribed to account for the (unobserved) presence of lithium. Stringfellow (1993, in preparation) points out that at the cool atmospheric temperatures expected in the extreme M dwarfs and brown dwarfs it is possible that any unburnt lithium may be locked up in molecules, such as LiH, which may have their own unique spectroscopic signature; similarly for deuterium. Additional observations and atmospheric modeling are required in order to fully exploit this potentially powerful diagnostic tool.

In young clusters and T-associations where constraints can be placed on the age of cluster members, theoretical evolutionary tracks can be used to distinguish between the low mass stars that have yet to reach the vicinity of the main sequence and those fainter brown dwarfs that are rapidly cooling to lower luminosities along an apparent extension of the old disk main sequence (e.g. Hubbard et al 1990, Stringfellow 1991a,b). This is discussed further below.

Liebert & Probst (1987), Greenstein (1989b), and Stevenson (1991) have recently reviewed the properties of the faintest and the lowest mass stars and the state of our knowledge concerning them.

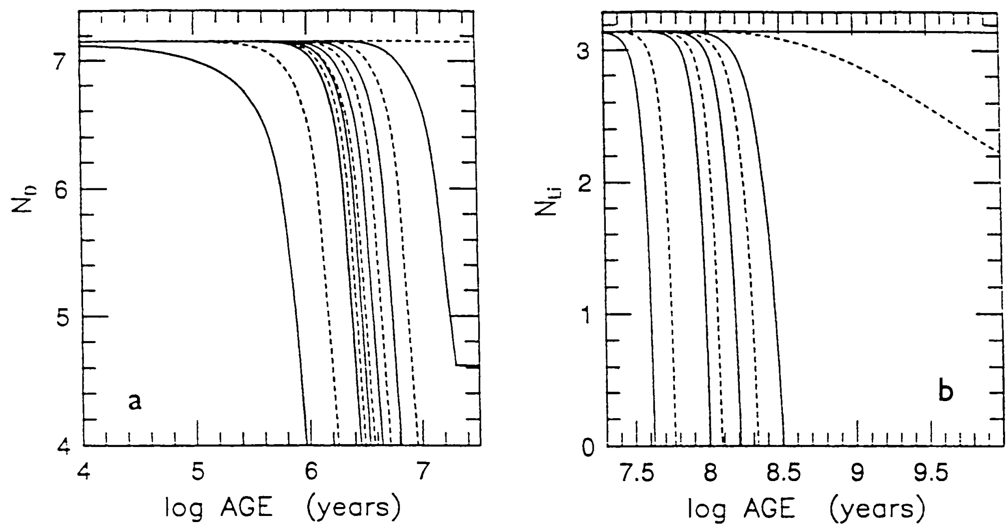

Figure 1 The destruction of deuterium (*a*) and lithium (*b*) with age. Each curve depicts an evolutionary sequence for a specific mass. Starting at the left these are: (*a*) 0.20, 0.15, and 0.10 to 0.01 in intervals of 0.01 $M_\odot$; (*b*) 0.20, 0.15, 0.10, 0.09, 0.08, 0.075, 0.70, and 0.06 $M_\odot$. Taken from Stringfellow (1989) for the IG opacity sequence. Alternating solid and dashed curves are interspersed for clarity. The mass fractions of deuterium and lithium used in the calculations were $X_D = 2.5 \times 10^{-5}$ and $X_{Li} = 10^{-8}$, respectively. The deuterium abundance is defined as $N_D = 12.0 + \log$ (D/H) where H is the number density on a solar scale with H = 12.0, and similarly for lithium.

2.2 *Colors and Spectral Types*

Photometric and spectroscopic data are available for many dwarf stars. Leggett (1992) has published a very useful compilation of all properties of M dwarfs. Some other compilations use: (*a*) photometry—Eggen 1979, 1980 (RI); Weis 1984, 1988 (VRI); Leggett & Hawkins 1988 (VRIJHK); Greenstein 1989a (V k R* I*); Monet et al 1992 (VI); Bessell 1990a (BVRI), 1991 (VRIJHK); (*b*) spectral types—Wing 1979, personal communication; Boeshaar 1976; and (*c*) atlases of digital spectra—Gunn & Stryker 1983; Turnshek et al 1985; Bessell 1982, 1991; Kirkpatrick et al 1991. From such material, reliable color-color and color versus spectral-type relations can be derived (e.g. Bessell 1991, Leggett 1992). The different passbands of different photometric systems can cause complications, but these are mostly understood (e.g. Bessell 1990b, Bessell & Brett 1988). Particular care must be taken, however, to ensure that the photographic magnitudes are correctly related to the photoelectric and CCD magnitudes for the coolest M stars.

The spectral type for M dwarfs is based mainly on the strength of the TiO and VO bands which dominate the absorption spectrum in the 5000 Å to 9000 Å region and therefore have the dominant influence on the V, R, and I magnitudes. The good correlations between spectral type and VRI colors and the tight VRI color-color relations are therefore not surprising even though studies of G and K dwarfs in the solar neighborhood indicate a large range in metallicity of $0.3 > [M/H]_{\odot} > -0.5$. Metal deficient M dwarfs can be detected photometrically by their redder $B-V$ and bluer $J-H$ colors for a given $R-I$ or $V-I$ color and spectroscopically by their strong MgH, CaH, and FeH bands and strong CaI, NaI, and KI lines compared to their TiO bands.

2.3 *Color–Luminosity Relations*

The derivation of absolute magnitudes is a critical part of determining the luminosity function. For the closest stars, distances are obtained directly by parallax measurement; for the more distant field stars, secondary techniques such as photometric or spectrocopic parallaxes are used. Distances to star clusters are derived by fitting their unevolved main sequences to the main sequence defined by the Hyades cluster and by field stars with individual parallax measurements. Because the position of the main sequence shifts with metallicity, the main-sequence fitting must take account of the relative metallicities of the stars. VandenBerg & Bell (1985) and VandenBerg et al (1983) have calibrated these shifts theoretically.

Parallaxes are listed in the Yale Parallax Catalogue (van Altena 1992). Several new parallaxes have been reported by Ianna (1993), including a few of the faintest known stars. Parallaxes and colors for subsets of the best data are given by Monet et al (1992) and Gliese & Jahreiss (1992). Figure 10 in Monet et al (1992) shows the M_V versus V−I diagram for the faintest dwarfs; brighter dwarfs are shown in Figure 2 of Harrington et al (1978). The colors V−R, V−I, V−K, etc for the individual parallax stars can be used to produce similar absolute magnitude-color relations such as M_K versus I−K; however, it is probably more accurate to use a mean M_V versus V−I relation from Monet et al and transform it into any other absolute magnitude versus color diagram by using averaged color-color relations.

R−I or V−I are excellent indicators of spectral type and luminosity in old disk dwarfs earlier than M6, but for later M dwarfs, photoelectric V−I and R−I colors "saturate" and even become bluer for some stars. The effect is less pronounced in the natural photographic (V: 103aD+GG495; R: IIIaF+OG590; I: IVN+RG9) and CCD VRI systems but still results in a loss in sensitivity in the M_V versus V−I relation. It is therefore imperative to obtain at least K photometry for all stars redder than $V-I = 4.2$ or $R-I = 2.0$ and to use I−K or I−J colors to derive precise photometric parallaxes for the faintest M dwarfs.

Although high order polynomials are necessary for best fits to the data, adequate linear relations over restricted magnitude ranges can be derived from Monet et al (1992). Examples are: for $5 < M_V < 18$, $M_V = 3.07 + 3.314\ (V-I)$; for $5 < M_V < 10$, $M_{bol} = 1.89 + 0.654 M_V$; and for $8 < M_V < 18$, $M_{bol} = 2.53 + 0.586 M_V$. For $M_I > 6$, $M_{bol} = 1.93 + 0.817 M_I$. For the reddest stars with $2.8 < I-K < 4.6$ ($M_I > 12$), $M_I = 5.76 + 2.06(I-K)$ and $M_{bol} = 8.67 + 1.092(I-K)$. We caution that any such relations must be used intelligently, and extrapolations even just outside the bounds for which they were fit will likely yield incorrect results; linear relations are a bit more flexible in this respect. Higher order polynomials are given by Monet et al (1992) and Tinney (1993).

For Pop II stars, Richer & Fahlman (1992) used a sample of parallax stars discussed by Eggen (1973) to derive $M_V = 4.27 + 3.377\ (V-I)$. The extreme halo stars of Monet et al (1992) are 1 mag fainter than this relation. The mean space velocity for the Eggen sample is 239 km s^{-1} whereas for the five stars with measured radial velocities considered by Monet et al, it is 427 km s^{-1}. From our experience also, the extreme subdwarfs, those stars with the largest reduced motions, have abundances less than −1.8 dex compared to abundances greater than −1.2 dex for the brighter parallax subdwarfs. The theoretical main-sequence HR diagrams of VandenBerg et al (1983) show that the luminosity decrease with metallicity ceases in the hotter subdwarfs for metallicities below approximately −1.2

dex but continues to decrease in those subdwarfs cooler than 4000 K. In addition, the lack of TiO absorption in the cooler subdwarf causes the V−I and R−I colors to be much bluer than near-solar composition M dwarfs of the same temperature, whilst the hotter dwarfs with different metallicities have similar V−I colors at the same temperature. Richer & Fahlman (1992) consider it more likely that their stars have representative compositions more similar to those of Eggen's sample.

2.4 *Color–Temperature Relations*

A reliable effective temperature calibration is necessary in order to compare theoretical evolutionary tracks with observed color-magnitude diagrams. Model atmospheres (e.g. R. L. Kurucz 1992, personal communication) and the infrared flux method (Blackwell et al 1980, 1990) provide a good temperature scale for dwarfs hotter than 4500 K; between 4500 K and 3000 K there is general agreement between empirical temperature scales and model atmospheres predictions, but below 3000 K the models are less secure and the temperature scale is rather uncertain.

Fundamentally-derived effective temperatures are available for only two eclipsing binary M dwarfs, YY Gem (M0.5, 3770 K) and CM Dra (M4.5, 3150 K); see Popper (1980) and Habets & Heintze (1981). Effective temperatures for M dwarfs have been estimated from a variety of blackbody or graybody fitting techniques [Greenstein et al 1970, Petterson 1980, Reid & Gilmore 1984, Berriman & Reid 1987, Doyle & Butler 1990 (DB), Stringfellow 1991a, Berriman et al 1992 (BRL), Tinney et al 1993]. These techniques can only provide approximations to the stellar effective temperature but should provide very reliable temperatures when used in conjunction with the fundamental temperatures.

Model atmospheres for M dwarfs have been constructed by Mould (1976), Allard (1991), Kui (1991), and Brett & Plez (1993). For M dwarfs with spectral types earlier than M5 or temperatures hotter than 3000 K there is reasonable agreement between most investigators and the empirical temperature scale. Spectral fits by Kui (1991) support the blackbody-based temperature scales of Bessell (1991), Monet et al (1992), Stringfellow (1991a,b, and in preparation), and Berriman et al (1992), but the fits by Kirkpatrick et al (1992) are up to 300 K hotter and Brett & Plez (1993) also predicts temperatures hotter by 250 K for K7 to M3 stars. For later spectral types, there is more disagreement. For example, for GL 406 (M5.5): 2550 K (BRL), 2580 K (DB), 2600 K (Stringfellow 1991a), 2800 K (Petterson 1980), 2600 K (adopted by Monet et al); and from model atmosphere fitting: 2750 K (Kui), 2875–3000 K (Kirkpatrick et al).

However, for stars cooler than 3000 K, there are serious differences between the model fluxes and the stellar fluxes between 1 and 4.5 μm, which make the coolest model atmospheres somewhat suspect (Kui 1991). Until the reason for these discrepancies is understood, the temperature scale of the M dwarfs cooler than 3000 K can hardly be supported by model predictions.

But the models can provide insights into the relevance of blackbody temperature fitting. Using model atmosphere fluxes, Kui (1991) examined the relation between blackbody temperatures derived by fitting the slope of the far-red continuum between 0.75 and 1.5 μm, the blackbody temperature from fitting the integrated fluxes, and the effective temperatures. Firstly, she found that both methods were sensitive to abundance (low abundance models yield higher blackbody temperatures), although the far-red slope was the least sensitive to abundance for temperatures above 3000 K and gave reliable temperatures about 100 K lower than the effective temperatures. The integrated flux fitting technique gave temperatures up to 400 K cooler for models hotter than 3250 K, but gave reasonable results between 3000 and 2500 K. In summary, combining the empirical temperature scale for M dwarfs hotter than 3000 K with blackbody integrated flux fits for cooler M dwarfs provides a useful working temperature scale. Improvements to model atmospheres for the cooler models may increase temperatures slightly.

As noted above in Section 2.3, it is important to use the most sensitive or reliable color for temperature determination. $R-I$, $V-I$, or $V-K$ are good for near–solar metallicity stars earlier than M5.5 but for later stars, $I-J$, $I-K$, or $K-L^*$ are better. For the most accurate temperature derivation, the abundance should be known, as all the usual temperature calibrations are also sensitive to the metallicity.

2.5 *Color–Bolometric Correction Relations*

As well as a reliable effective temperature calibration, bolometric corrections are required in order to compare theoretical evolutionary tracks with observed color-magnitude diagrams. Bolometric corrections for cool stars are most often calculated by integrating under the total energy curve using the effective wavelengths and zero-magnitude fluxes for each of the standard photometric bands UBVRIJHKL* (Bessell & Brett 1988, Bessell 1990b). Correction beyond L* (3.85 μm) can be made by assuming a Rayleigh-Jeans tail for the flux distribution. The zero point of the bolometric magnitude can be defined by adopting some bolometric magnitude and luminosity for the Sun (e.g. Bessell & Wood 1984). Bolometric corrections have been derived for M dwarfs by Veeder (1974), Reid & Gilmore (1984), Greenstein (1989a), Bessell (1991), and Stringfellow (1991a), and can be derived from Petterson (1980), and Doyle & Butler (1990). For

$0 < V-I < 4.5$, $BC_I = -0.145+1.836(V-I)-1.447(V-I)^2+0.503(V-I)^3 -0.086889(V-I)^4+0.00563077(V-I)^5$. For $4.5 < M_K < 10.5$, Tinney et al (1992; hereafter TMR) give $BC_K = 1.405+0.173\ M_K$.

Integrating the broadband magnitudes in the IR overestimates the IR flux as there is H_2O absorption between the JHKL bands which is very strong in the late M dwarfs. This absorption will cause the derived values of M_{bol} to be too bright, but by only a few percent for M5 dwarfs. Based on experience with late spectral type mira variables with strong H_2O bands, the difference could be up to 0.1 mag for a dwarf with temperature below 2000 K. It is obviously important to obtain complete IR spectra for the coolest M dwarfs to derive more precise bolometric magnitudes.

Kurucz (1979) provides bolometric corrections to V for the hotter dwarfs. The tabulated BC_V values should be made fainter by 0.124 mag to place them on the scale where $BC_{V\odot} = -0.07$.

2.6 *Mass–Luminosity Relations*

Popper (1980) reviewed the status of fundamental stellar mass determinations. Subsequent to that review, Liebert & Probst (1987) discussed the masses of very low mass stars including several additional stars with masses below 0.1 $M_\odot$. More recently, Henry et al (1992) have shown that the published orbit of the lowest mass binary W424 (GL 473ab; FL Vir) is incorrect and the low masses 0.059 $M_\odot$ and 0.051 $M_\odot$ previously derived for the components (Heintz 1989) are wrong and the masses are instead $0.13 \pm 0.01\ M_\odot$ and $0.12 \pm 0.01\ M_\odot$ (see also Section 2.1).

Smith (1983) derived empirical mass-luminosity relations from data in Popper (1980) and found that this M-L relation could be adequately represented by two relations: $L/L_\odot = (M/M_\odot)^4$ for $M/M_\odot > 0.4$ and $L/L_\odot = 0.23(M/M_\odot)^{2.3}$ for $M/M_\odot < 0.4$. (The additional star GL234B ≡ Ross 614B from Liebert & Probst with a mass of $\sim 0.08\ M_\odot$ lies near the low mass regression line.) Kroupa et al (1990; hereafter KTG) have discussed in detail the shape of the mass–absolute visual magnitude relation. They identify points of inflection in this relation near $M_V = 7$ and $M_V = 12$ as resulting from opacity changes due to H^- and H_2 respectively and suggest that further structure is to be expected at lower luminosities when other opacities, such as grain opacity, become dominant. The break point between the two empirical M-L relations derived by Smith (1983) can also be identified with the same switch from mainly atomic hydrogen to mainly molecular hydrogen and the dominance of convection, but as we do not expect any comparable dramatic changes in the equation of state or adiabatic gradient at lower temperatures we do not expect another dramatic change in the mass-luminosity relation from this source. However, as the hydrogen burning limit is approached from above, the slope of the mass-luminosity relation must steepen as the luminosity undergoes a precipitous drop. Theoretical M-L relations predict this rapid turnover to begin just below 0.1 $M_\odot$.

The results from theoretical modeling have been used to compare theoretical and empirical M-L relations and to predict M-L relations for even lower mass stars and substellar objects (e.g. D'Antona & Mazzitelli 1985; BHL; Stringfellow 1989, 1991b; Dorman et al 1989; Laughlin & Bodenheimer 1993). The largest differences between the authors concern the temperatures and radii of the models more so than the luminosities, and the theoretical mass-luminosity relations are quite similar. Many of the differences existing between these calculations can be traced to the various input physics used (chemical composition, atmospheric opacities, equation of state, etc). Nevertheless, differences in the slope of the M-L relation can impose various structures in the derived LFs, although sometimes these are rather subtle, or as in the case of the BD regime, presently untestable. In general, as the minimum main-sequence mass is approached from higher masses, the luminosity drops precipitously over a small increment in mass; for the mass interval 0.15–0.10 $M_\odot$, $M_{\rm bol}$ drops by just over 1 mag, compared to the interval 0.10–0.05 $M_\odot$ where $M_{\rm bol}$ drops by $\sim$7 mag! Lower atmospheric opacities steepen this trend, whereas larger opacities tend to smooth out this luminosity drop over a larger mass interval and lessen somewhat the overall magnitude of the drop. The minimum main-sequence mass also increases with lower opacities so that this drop occurs at lower values of $M_{\rm bol}$. It is the manner in which the drop begins and when it starts to smooth out at fainter luminosities (i.e. inflection points in the M-L relation) that produces all the significant and interesting structure in the LF.

In Table 1 those few members of the faint end of the main sequence with good data sets (including a broad range of photometry, trigonometric parallaxes, and in many cases optical and/or IR spectra) are presented along with a subsample of somewhat brighter more massive stars. The photometric data have been taken from Bessell (1991) and the trigonometric parallaxes (π) from Monet et al (1992) unless otherwise noted. The reduced proper motion $H_{\rm R}(=\rm R_{LHS}+5+5\log\mu)$ is listed in column 2. We have used the basic information discussed throughout this section to derive additional properties listed in Table 1 for these stars. This table should prove a useful reference throughout the remainder of our discussion. [Following the completion of this review important papers Tinney et al (1993) and Tinney (1993) appeared which contain several additional faint stars similar to LHS 2065 and 2294. These stars are not included in Table 1.1.]

Table 1 Faint end of the main sequence

Star*	H_R	π (mas)	d (pc)	I−K	I	M_I	BC_I[1]	$\log L/L_{\odot}$[2]	T_{eff}[1]
LHS 36	21.9	419[a]	2.4	3.37	9.50	12.61	−0.40	−2.997	2753
LHS 68	20.4	305[a]	3.3	2.96	8.64	11.06	−0.08	−2.504	2876
LHS 191	22.6	58.4	17.1	2.99	13.96	12.79	−0.10	−3.187	2865
LHS 248	20.0	275[a]	3.6	3.28	10.54	12.74	−0.17	−2.529	2775
LHS 330	22.6	39.6[a]	25.3	2.96	14.38[c]	12.37	−0.08	−3.028	2876
LHS 429	20.9	154.5	6.5	3.46	12.18	13.12	−0.47	−3.171	2732
LHS 474	23.7	170.1	5.9	4.05	12.84	13.99	−0.99	−3.313	2611
LHS 2026	21.0	50.8	19.7	3.16	14.32	12.85	−0.23	−3.160	2809
LHS 2065	21.7	117.3	8.5	4.58	14.54	14.89	−1.49	−3.472	2453
LHS 2397a	21.9	70.0	14.3	4.12	14.87	14.10	−1.05	−3.331	2595
LHS 2471	21.7	70.3	14.2	3.35	13.66	12.89	−0.38	−3.115	2758
LHS 2924	22.7	90.8	11.0	4.62	15.30	15.09	−1.53	−3.536	2437
LHS 2930	23.8	103.8	9.6	3.57	13.30	13.38	−0.56	−3.238	2709
LHS 3003	21.4	152.4	6.6	3.63	12.53	13.44	−0.62	−3.242	2697
M18[3]	...	41.0[b]	24.4	3.98	16.34	14.40	−0.92	−3.502	2626
M19[3]	...	32.4[b]	30.9	4.02	16.20	13.75	−0.96	−3.228	2618
ESO 207-61[4]	...	65.4[b]	15.3	4.02	16.23	15.31	−0.96	−3.852	2618
RG0050[5]	...	39.4[b]	25.4	4.10	16.65	14.63	−1.03	−3.550	2600

* LHS 36 ≡ GL 406, LHS 68 ≡ GL 866AB, LHS 429 ≡ VB 8, LHS 474 ≡ VB 10, LHS 248 ≡ GJ 1111.
[1] color-T_{eff}-BC_I relations, based on I−K, from Bessell (1991).
[2] assumes $M_{bol\odot} = 4.72$.
[3] Hawkins & Bessell (1988).
[4] Ruiz et al (1991).
[5] Reid & Gilmore (1981).
[a] Gliese & Jahreiss (1979).
[b] Ianna (1993).
[c] Monet et al (1992).

3. THE DISK LUMINOSITY FUNCTION

3.1 *The Solar Neighborhood*

A summary of the techniques and problems associated with the derivation of the faint end of the SLF in the solar neighborhood is given by Jahreiss (1987); discussions on Malmquist-type biases in such studies are given by Stobie et al (1989) and Kroupa et al (1993). Several different aproaches have been taken using two distinct methods. The first method is straightforward and simply counts the numbers of stars of different absolute magnitude in particular volume elements; the second method, the reduced-motion method, uses the apparent magnitudes and proper motions of stars with usually no direct distance estimates for them.

The reduced-motion method has been used by Luyten (1968) with data for 4000 proper motion stars from plates taken with the Palomar Schmidt telescope. Schmidt (1983) discusses problems associated with such derivations based on proper motion surveys and Reid (1984) concludes that the contamination by distant high velocity subdwarfs in Luyten's samples leads to a substantial excess in the space densities of low luminosity dwarfs. For these reasons, the more direct star-counting method is to be preferred.

This method has been very successfully applied to northern hemisphere ($\delta > -20°$) samples of stars listed in the original catalog of Gliese (1969) and its updates (Gliese & Jahreiss 1979, 1992; Wielen 1974; Wielen et al 1983; Jahreiss 1987). The luminosity function so derived is considered to be complete for stars brighter than $M_V = 10$ within 10 pc but samples progressively smaller volume elements at fainter magnitudes; consequently, it is considered to be very reliable for $M_V < 12$ but almost certainly underestimates the density for the faintest magnitudes. An additional qualification is that the faintest stars in the catalog are almost all proper motion selected so are biased to stars with large tangential velocities. Figure 2 shows these LFs for stars fainter than $M_V = 4$ (the solid symbols); note the Wielen Dip near $M_V \approx 7$ ($M_I \approx 5$) and the slow decline in the luminosity function for luminosities below $M_V \approx 12$ ($M_I \approx 9$). Luminosity functions for complete samples within 5 pc have also been derived by Dahn et al (1986) and Henry & McCarthy (1990).

To better sample the faintest stars without any kinematical bias, special photometric surveys have been made by several authors using Schmidt plates. These surveys cover quite restricted areas of the sky but are complete to much fainter limiting apparent magnitude than the whole sky catalogs. Gilmore & Reid (1983) determined photometric distances for 12,500 stars down to V = 19 mag or I = 18 mag in a field of 18.35 deg^2 near the SGP; Hawkins & Bessell (1988) surveyed to a fainter limit using R and I plates over 85 deg^2 in the SGP and $b = 45°$ fields; Stobie et al

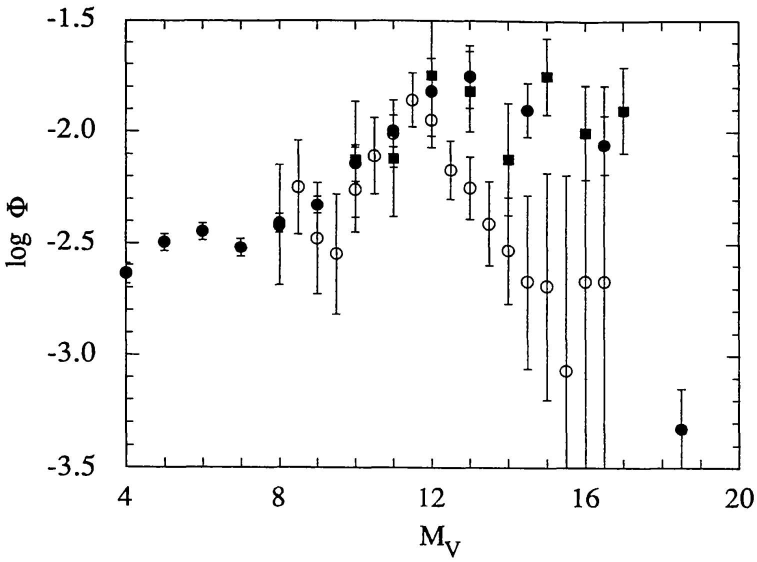

Figure 2 The log visual luminosity function (units: pc^{-3} mag^{-1}). Filled circles are points due to Wielen et al (1983) and Jahreiss (1987); filled squares are points due to Dahn et al (1986); open circles are the photometrically derived NGP luminosity function of Stobie et al (1989).

(1989) analyzed 21.5 deg^2 at the NGP using V and I plates, and finally TMR are surveying 320 deg^2 with R and I plates.

In these analyses, the V – I, R – I, and I – K colors are used to determine the absolute magnitude via absolute magnitude versus color relations (see Section 2.3) calibrated with parallax stars (e.g. Monet et al 1992, Harrington et al 1978). Many of the photometric surveys for the intrinsically faintest stars use the V/V_{max} method developed by Schmidt for quasar statistics and used by him to determine the LF of halo stars (Schmidt 1975). In this technique, one calculates for every star in the sample the maximum distance r_{max} to which it would be detected given the absolute magnitude of the star and the limiting magnitude of the sample. The inverse volume $1/V_{max}$, calculated from r_{max}, is the contribution of the star to the stellar density.

Stobie et al (1989) consider the photometrically derived LFs in the range $8 < M_V < 17$ mag ($7 < M_{bol} < 12.3$) and discuss the effects of Malmquist-type biases. Good general agreement was found between all the photometrically derived LFs and with the Wielen LF for $M_V < 12$, but in the range $12 < M_V < 17$ the photometrically derived LF (ϕ_{ph}) is 2 to 5 times smaller than the local LF (ϕ_{loc}) of Wielen and Dahn et al (1986) (see Figure 2). Dahn et al suggested that this disrepancy may result from close binary systems being unresolved with Schmidt plate resolutions at the sampling distances of more than 100 pc used in the photometric surveys. That is, ϕ_{ph} refers to systems and ϕ_{loc} refers to single stars.

Malkov (1987) was one of the first to qualitatively examine the effects of unresolved binaries on the luminosity function. This problem of unresolved binaries and the SLF has also been modeled quantitatively. Kroupa et al (1991) use the maximum likelihood method to simultaneously fit ϕ_{loc} and ϕ_{ph} and solve for the binary fraction and the indices of their adopted power-law mass function. They get the best fit with all stars as binaries and the lowest acceptable model fit for 43% binaries, which coincidentally is the percentage of M dwarf binaries found by Fischer & Marcy (1992). In a complementary study, Reid (1991), starting with a ϕ_{loc} and adopting the frequency (between 30% and 50%) and nature of binaries in the local star samples for stars with $4.5 < M_V < 10.5$, simulated the effects of unresolved binaries and derived equivalent ϕ_{ph}. He concludes that although binaries will clearly not be detected by the deep photometric surveys, the number of missed stars is partly compensated for by changes resulting from uncertainty in the photometric parallax calibration combined with the increased luminosity of the binaries and corresponding increased effective distance limit; as a consequence, unresolved binaries account for only a 25–30% shortfall (Reid 1992). The main explanation for the observed differences between ϕ_{ph} and ϕ_{loc} must clearly be a local excess of low luminosity stars and Reid suggests that as the number of local stars is small, it may simply be a sampling error.

Eggen (1993), in an interesting study to isolate members of the Hyades cluster and the Hyades supercluster within 20 parsecs of the Sun using the convergent point method, found that the fraction of binary stars in the cluster increased from 50% at M2 to 66% at M4.5 and an overall fraction of 56% binaries amongst the K and M dwarfs. In the supercluster, the fraction of binaries amongst the same kind of dwarfs was 34%. Eggen suggests the different percentage as being possibly due to the evaporation of fainter and less massive single stars from the cluster.

Tokovinin (1992) has independently estimated the frequency of low mass companions to red dwarfs by observing radial velocities for 200 dwarfs over a 5 year period. He finds far fewer binaries (10%) amongst dwarfs later than M5 than amongst earlier type (M2–K5) dwarfs (30–60%) and a complete absence of companions with masses between 0.02 and 0.08 $M_\odot$. A similar mass cutoff in companions had been suggested by Henry & McCarthy (1990) from their investigation of the IR luminosity function. These data appear to support one or more of the following propositions: 1. if very low-mass binaries form through direct protostellar collapse and fragmentation, then due to their low binding energy they are subsequently disrupted as the molecular cloud out of which they formed is dispersed and dynamical interactions occur; 2. that very low-mass binaries are unable to form directly through protostellar collapse due to unfavorable cloud properties (low rotation, large magnetic fields, etc); 3. that very low-mass binaries do not form through direct dynamical capture. If valid, these strengthen the case that the entire difference between ϕ_{ph} and ϕ_{loc} does not result solely from missed stars in binary systems.

However, there is some evidence that most of the shortfall in ϕ_{ph} could result from unexpected differences in the scale height of M dwarfs with different absolute magnitudes. Following the discovery by Hawkins (1987) and Hawkins & Bessell (1988) that the velocity dispersion of the faintest M dwarfs in their photometric samples were low and more typical of A stars than K dwarfs, Hawkins (1988) analyzed the scale height of all the dwarfs in these samples within 500 pc which have luminosities in the range $8 < M_R < 14$. The stars were analyzed in 1 mag luminosity bins. He obtained good fits to exponential distributions for each bin and showed convincingly that the stars in the two lowest luminosity bins (12–13, 13–14) had scale heights of about 85 pc compared to over 200 pc for the more luminous M dwarfs. Such differences in scale height would clearly result in the local LF exceeding the photometrically derived LF.

Kroupa et al (1993) have examined in detail the claim for a variation in apparent disk scale height as a function of absolute magnitude. They note that in Hawkin's data the sample distance limit decreases with increasing absolute magnitude, and they also derive similar variations in scale height from their sample of K dwarfs by doing linear regressions over restricted distances. They have also analyzed the kinematics of the 5.2 pc sample of M dwarfs and find no difference in the distribution of the U, V, and W components between the stars in different absolute magnitude bins suggesting no difference in age down to $M_V \sim 14$.

However, the faintest known M dwarfs identified from the LHS Catalogue (Luyten 1979)—LHS 2065, 2397a and 2924—are not included in the analysis. For these stars the reduced motions indicate tangential velocities at least 4 times lower than for the early M dwarfs (Bessell 1991), implying that they are much younger.

In the most recent study of a photometrically based sample of stars which was specifically designed to tie down the space density of the faintest stars, TMR compared the bolometric luminosity functions of three photometric surveys (Reid 1987, Hawkins & Bessell 1988, TMR) and the 5.2 pc sample of Henry & McCarthy (1990). They find, within the sampling statistics, that all four surveys agree and the luminosity function clearly peaks near $M_{bol} = 10$ and falls off dramatically to fainter luminosities. By extending the LF over a bolometric magnitude fainter than the 5.2 pc sample and by improving the precision by an order of magnitude, they show conclusively that the LF does not increase for luminosities below $M_{\rm bol} = 12.5$—a possibility hinted at by some previous analyses (Reid 1987, Leggett & Hawkins 1988), but instead appears to flatten out at a level of about 8% of the peak; however, the error bars remain large for data in this region rendering extrapolation of the LF indeterminate. The bolometric LFs for the faintest stars are shown in Figure 3.

In order to extract the mass function underlying the stellar luminosity function one needs to examine first of all the mass-luminosity relation. It is clear that changes in the shapes of the mass-luminosity relation and the underlying opacity-temperature relation must be reflected in the shape of the luminosity function *unless the mass function has compensatory changes.* Kroupa et al (1991) in fact show that within the constraints of the empirical M-L relation and the observed LF, all the observed structure in the stellar luminosity function is due to the M-L relation and that the stellar mass function is smooth. On the other hand, using theoretical M-L relations, TMR suggest that the mass function flattens for masses below 0.3 $M_\odot$.

The empirical mass-luminosity relation appears to be reasonably well established for masses above 0.1 $M_\odot$ (see references in Section 2.6) although the metallicities of the binaries are uncertain and must be

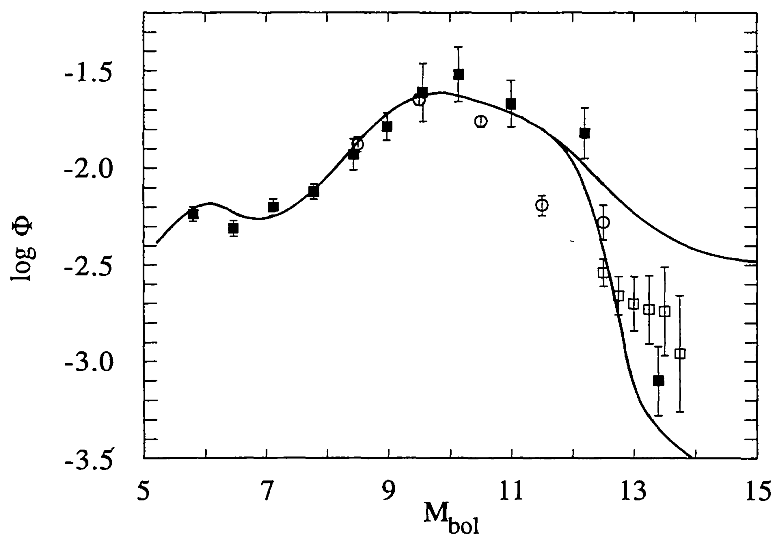

Figure 3 Comparisons of the log bolometric luminosity function for the faintest stars (units: pc^{-3} mag^{-1}). Filled squares are due to Wielen et al (1983) and Jahreiss (1987); open circles and open squares are the photometrically derived luminosity functions from Hawkins & Bessell (1988) and TMR, respectively. The upper curve is a theoretical curve derived for an age of 10^9 yrs and the lower curve for an age of 10^{10} yrs (based on a theoretical mass function of the form $dN/dm \propto m^{-\alpha}$, where $\alpha = 1.01$ for m between 0.5 and 0.03 $M_\odot$).

addressed before decisive conclusions can be drawn. The recent theoretical M-L relations are in quite good agreement (Section 2.6) although differing in some of the details. There are, however, larger differences between the observed and theoretical M-L relations for masses between 0.2 and 0.4 $M_{\odot}$.

The detailed analysis of Dorman et al (1989) and KTG show that for masses between 0.45 and 0.20 $M_{\odot}$ the theoretical models are under-luminous by about 0.5 mag or 60%. In this mass regime the surface opacities and structure of the star undergo significant change as the mass is lowered. Hydrogen recombines to form H_2, the size of the radiative core drops rapidly, and the star becomes fully convective; these effects are also sensitive to the metallicity since in low metal stars hydrogen recombination occurs at higher effective temperatures. This causes the slope of the M-L relation to change abruptly as the stars become hotter and more luminous than the trends shown by 1.0–0.5 $M_{\odot}$ stars; however, the observations suggest that the predicted luminosity increase is too low. A possible explanation for this could be that the theoretical models have too low surface pressures. Dorman et al (1989) indicate that higher luminosities and temperatures would in fact be derived if their models were corrected for the higher surface pressures indicated by nongray model atmospheres.

For lower mass stars there is another problem. The stellar models for masses between 0.10 and 0.07 $M_{\odot}$ predict an extremely rapid change in the mass-luminosity and mass-temperature relations to lower luminosities and temperatures as the nuclear energy generation in the core falls below that required to establish thermal equilibrium. As noted by BHL, $L \propto M^{20}$ between 0.07 and 0.09 $M_{\odot}$ compared to $L \propto M^{2.3}$ between 0.1 and 0.4 $M_{\odot}$. The error bars on the masses for stars near 0.08 $M_{\odot}$ are so large that there are essentially no empirical data for stars with mass below 0.1 $M_{\odot}$, but there is indirect evidence from the observed velocity dispersions and scale heights for the faintest M dwarfs that the theoretical predictions for the transition objects in this mass range may be amiss. All recent models (e.g. D'Antona & Mazzitelli 1985, Stringfellow 1989, BHL) predict that main sequence stars with ages of 10^{10} yrs should exist with masses down to $\sim 0.075\ M_{\odot}$ and magnitudes of $M_{bol} \approx 16$ (i.e. transition objects). This does not appear to be the case because, as discussed above, all the M dwarfs found with $M_{bol} > 13$ may be no older than 5×10^8 yrs. As stated by BHL "the age is as important a feature as the luminosity and effective temperature when comparing theory with observation and diagnosing the family of low mass objects."

The only model of BHL that fits the contraints of age and luminosity of the faintest red stars is the low opacity model E. This has a minimum MS mass of 0.09 $M_{\odot}$ and a minimum MS luminosity of $M_{bol} \approx 13$. The low opacities are however, quite unrealistic for these solar-like composition M dwarfs and the effective temperatures of the models are much too high. What is needed is a way to maintain the steeper drop in luminosity in the M-L relation for the transition objects (as in the BHL E models) while incorporating the representative low temperature opacities. Perhaps by using more realistic nongray model atmospheres this effect can be achieved. However, the construction of model atmospheres in this temperature regime is not straightforward. Recent cool model atmosphere grids for M dwarfs (Allard 1991, Kui 1991) may not be reliable for temperatures below 3000 K because they do not match the observed 1–3 μm spectra of cool dwarfs. The models appear to have too low a continuous opacity in the IR. Efforts to understand the problem are continuing.

To give an idea of the significant effect that the proposed change to the mass-luminosity relation with age can have on the luminosity function we have calculated two luminosity functions using the empirical mass-luminosity relation for stars above 0.1 $M_\odot$ and the predicted mass-luminosity relations for the E models of BHL for the $m < 0.1$ $M_\odot$ stars and brown dwarfs. We use a power-law form for the mass function, $dN/dm \propto m^{-\alpha}$ where m is the stellar mass and α is the fitting parameter determined by the data. Note that this form of the mass function allows different values of the slope to be fit to the data in separate regions if and where a change of slope is warrented. Kroupa & Tout (1991) suggest such a change in slope for the field SLF, and we have used their values in the construction of our luminosity functions; $\alpha = 2$ for $1.5 > m > 0.5$ $M_\odot$, and $\alpha = 1.01$ for $0.5 > m > 0.03$ $M_\odot$. Extrapolation of the relation to lower masses than 0.03 $M_\odot$ will affect only the derived luminosity function below $M_{\rm bol} = 19$ and the contribution to the integrated mass density by the lower mass objects is insignificant. Figure 3 shows the derived LFs for 10^9 yrs (upper curve) and 10^{10} yrs (lower curve). The difference between the curves is due to the fact that between 10^9 and 10^{10} yrs, the E models for masses below 0.09 $M_\odot$ quickly cool to fainter than $M_{\rm bol} = 18$. The lower curve obviously fits the character of the observed luminosity function rather well. All of the models predict an upturn in the LF fainter than a $M_{\rm bol}$ of 17, and for a flat mass function these models predict comparable numbers of objects at $M_{\rm bol} \sim 19$–20 as the peak of the LF at $M_{\rm bol} \sim 10$–11.

This uncertainty and the secular nature of the theoretical M-L relation for low mass stars thwart attempts to extract the mass function from the luminosity function. We can, however, still place some limits on the mass and luminosity functions. KTG showed that the observed M-L relation and observed LF were consistent with an underlying smooth and monotonically rising mass function, which if linearly extrapolated to masses below 0.1 $M_{\odot}$ resulted in a contribution of < 0.008 $M_{\odot}$ pc^{-3} from all objects below 0.35 $M_{\odot}$ compared to 0.05 $M_{\odot}$ pc^{-3} from more massive stars. That is, there is no evidence that low mass stars contribute much at all to the "local missing mass." An even stronger statement was made by TMR who used theoretical M-L relations and found that rather than a monotonically rising mass function, a flat or possibly falling mass function was indicated by their faint star counts, which would result in an even smaller contribution to the local mass density from the lowest mass stars. However, the 10^{10} yr curve shown in Figure 3 was constructed from a monotonically increasing mass function (m^{-1}) and it is clear that the data which include those of TMR can accomodate such a fit. Tinney (1993) also discusses the mass function at faint luminosities in some detail.

Although the observed luminosity function appears now to be well defined, at least down to $M_{\text{bol}} \sim 13$–14, the nature of the lowest luminosity stars must be clarified. Indirect evidence from the kinematics and space distribution of the faintest M dwarfs suggests that most of the stars are young. The smaller scale heights for the late M dwarfs also helps explain differences between the luminosity function derived for very nearby stars and for more distant stars. If the younger ages inferred for the faintest M dwarfs are correct, and there are no old stars in the range $M_{\text{bol}} \sim 13$–17, then evolutionary models for the transition objects need be reassessed as they predict the existence of stars of various ages at faint luminosities. It is clearly very important to follow up these questions and derive the velocity dispersions and scale heights for the M dwarfs in the TMR and other new surveys and to re-examine the interior and atmospheric modeling of cool dwarfs.

3.2 *The Young Disk Population*

Recent studies have begun to probe the faint end of the SLF in a variety of open clusters and in active star forming regions associated with dark cloud complexes. Several principal reasons have stimulated the search into this previously unexplored domain, each having its own caveat and problems.

1. The field low-mass stellar population is believed to have originated within open clusters. Generally, open clusters tend to become unbound as the major star formation episode subsides and any remaining cloud material is dispersed. The cluster stars themselves will then disperse with time as encounters with molecular clouds, the potential of the Galactic disk, and with individual stars occur. Thus, searching in young stellar clusters is a logical place to expect to find the lowest-mass stars. Mass segregation within the cluster is also likely, with the high-mass

stars confined to the cluster center and the low mass stars being dispersed to larger distances than those where they formed, wherever that may be. Hence, deep surveys of large areas centered upon the cluster core are required as are proper motions if field stars are to be purged from the sample. Photographic plates have been, and in many instances remain, the best means of surveying large areas although the development of larger format CCDs, which are much more sensitive, may soon supplant the former.

2. Even the lowest mass stars and brown dwarfs are bright and hot enough to be detectable in their youth with present technology.
3. The age and distance of clusters can be constrained, thereby placing reasonable bounds on the properties of the faintest members. Nevertheless, all stars may not be coeval (e.g. Herbig 1962a; Eggen 1976, 1977; Stahler 1985; Stringfellow 1991a) and plausible age spreads in older clusters ($\gtrsim$Hyades age) may render these bounds academic and produce scatter in the H-R diagram. Binaries along the lower main sequence can contaminate the isochrones of young clusters with ages $\gtrsim 5 \times 10^7$ yrs (e.g. the Pleiades) further confusing identification of cluster members. Extreme youth introduces additional concerns. If low mass stars form through accretion from a circumstellar disk (see Shu et al 1987) then transients in the post-star formation period could alter the observational characteristics of the stars; e.g. a remnant disk not yet fully dissipated could provide additional reddening over that intrinsic to the star, leading to an underestimate of its mass; if actively accreting material from this disk at a low rate then the star's apparent luminosity and spectral appearance (continuum veiling by the boundary layer) could also be altered. Of course, these effects become more severe as the SLF is pursued in even younger populations where star formation remains active. In principle, these effects could be corrected for, but require a much more extensive set of observations than usually acquired in order to deconvolve the actual stellar properties (e.g. Basri & Bertout 1989, Hartigan et al 1989). In the nearest clusters (i.e. the Hyades) the internal dispersion in distance along the line-of-sight can significantly alter the structure of the LF, particularly at the faint end (Stringfellow 1991b). However, a recent proper motion survey by Reid (1992) and analysis using the convergent point method by Eggen (1993) has begun to clarify the faint end of the Hyades LF.

Theoretical isochrones indicate that for ages older than $\sim 3 \times 10^8$ yrs the cooling tracks of brown dwarfs are virtually indistinguishable from the main-sequence locus while for younger ages the lower main sequence lies below the cooling tracks. This can be seen from the theoretical H-R diagram (Figure 4) showing young isochrones and the cooling track of a 0.07 $M_\odot$ brown dwarf (Stringfellow 1991b). Brown dwarfs begin to separate from stars in luminosity at the age of the Hyades (BHL, Hubbard et al 1990, Stringfellow 1991b), which produces an initial drop and subsequent rise or flattening in the LF (for $M_{bol} > 15$ or $M_K > 12$) with decreasing mass depending on the slope of the IMF used. However, contamination by background main-sequence field stars can make identification problematical.

Below we summarize recent observations of the SLF in the young disk population. Zinnecker et al (1993) present a complementary review of the issues discussed in this section, delving deeper into the star formation aspect of the problem, including higher mass stars, than considered here.

3.2.1 ACTIVE REGIONS OF STAR FORMATION The SLF in the Rho Ophiucus dark cloud (160 pc, $m-M \approx 6.0$) has been measured repeatedly over the years, each time extending the observations to either larger areas or fainter limiting magnitudes. Lada & Wilking (1984) presented the first bolometric LF obtained solely from ground-based data. Wilking et al (1989) extended

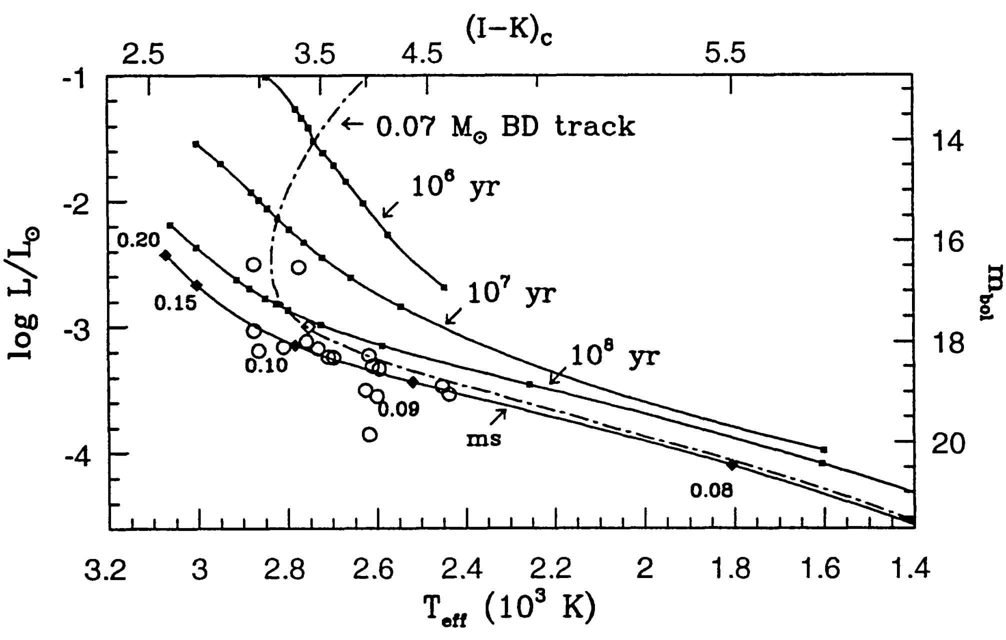

Figure 4 The theoretical H-R diagram indicating the main sequence, young isochrones, and the evolutionary track of a 0.07 $M_\odot$ brown dwarf (*dot dashed curve*). The right ordinate gives the apparent bolometric magnitude at the Pleiades distance. Also indicated by open circles are data contained in Table 1, including most of the intrinsically faintest known stars for which complete data exist. These have been transformed using the I−K color-T_{eff}-BC_I relations of Bessell (1991). GL 406 is represented by an open diamond. New trigonometric parallaxes have been taken from Ianna (1993).

this survey by a factor of ~ 3 and included *IRAS* data; the K-band data were obtained by aperture scanning. Source confusion and completeness are potential problems with such a study. Wilking et al were able to classify the individual sources based on their broadband spectral energy distribution into those which are heavily obscured and presumably actively accreting protostars (Class I sources) and the T Tauri stars (Class II sources). The LF of the Class II sources is roughly Gaussian, peaking at $\log L/L_{\odot} = 0.0$, suggesting a lower mass limit of $\gtrsim 0.2\ M_{\odot}$ depending on the age ascribed (see Figure 4). Barsony et al (1989) surveyed a 144 arcmin^2 region in the cloud core (a small subset of the above survey) to a limiting magnitude $m_K \sim 14$ using an IR array. A much smaller area (~ 20 arcmin^2) mostly contained within the above study was also similarly surveyed by Rieke et al (1989) with a completeness limit of $m_K \sim 14.5$. While Rieke et al found a LF that peaked around $m_K \sim 8.5$, Barsony et al found the LF continued to rise to the limit of the survey. Although no direct comparison of sources in the overlapping regions have been reported, the difference has been attributed to spatial variation in the LF within the cloud. Burton (1992) conducted an even deeper array survey ($m_K \sim 16.8$) of the Barsony et al region in the H and K bands and found the LF continued to rise to his detection limit. The study of Greene & Young (1992), which surveyed a 650 arcmin^2 area in the J, H, and K bands to $m_K \sim 13$ using an array, confirm the spatial variation of the LF. This latter global LF is flatter between $9.5 \leqslant m_K \leqslant 13$ than the above studies, undoubtedly due to more complete areal coverage, although the photometric errors associated with this study are quite large.

Rieke & Rieke (1990) and Burton (1992) present dereddened sources from which they estimate M_K. Keeping in mind that extinction corrections are large, uncertain, and undoubtedly vary within the cloud, an absolute $M_K \gtrsim 6$ ($\log L/L_{\odot} < -1.3$) lies well within the substellar domain for an age of $\sim 10^6$ yrs (see Figures 3 and 4 of Stringfellow 1991b).

Theoretical LFs for various slopes α of a power-law mass function of the form $dN/dm \propto m^{-\alpha}$ where m is the mass have been constructed by Stringfellow (1991b; for M_K) and Burrows et al (1993; for $\log L_{\rm bol}$) for Rho Oph. These indicate that at $\sim 10^6$ yrs for $M_K > 5$ (corresponding to $m \lesssim 0.20\ M_{\odot}$) the LF rises with no turnover down to $0.01\ M_{\odot}$ for values of $\alpha > 1$, and basically flat for values near 1. From models computed by Mazzitelli, Zinnecker et al (1993) find a peak in the LF at $\sim 0.3\ M_{\odot}$ which they identify as being due to deuterium burning. As the maximum mass computed in the other investigations is $0.20\ M_{\odot}$, this feature is not evident in their LFs. Curious though is the 2×10^6 yr LF of Zinnecker et al which peaks near $M_K \sim 5$ and falls off at fainter values; Zinnecker et al use a Miller-Scalo mass function which is approximately flat below $0.3\ M_{\odot}$.

This corresponds to $\alpha \sim 1$ in the other formulations which behave similarly (note the minimum mass utilized by Zinnecker et al is 0.080 $M_{\odot}$).

Several other regions have also been studied, although not as well as Rho Oph. Small regions (25″) have been imaged in the K band around selected members of the Taurus cloud by Forrest et al (1990). They found a total of 20 faint objects distributed around 13 of the 26 stars searched. Of these 9 are quite red and were tentatively identified as brown dwarfs of very low mass. Optical and IR spectra were obtained by Stauffer et al (1991a) for a subset of these, several of which are proper motion members. While only one of these clearly showed molecular absorption features, none appear to have the sort of temperatures expected of brown dwarfs. Note that in the models of Stringfellow (1991a,b) brown dwarfs of age 10^6 yrs have $T_{\rm eff} \lesssim 2800$ K, and using recent color-$T_{\rm eff}$ relations Bessell (1991) found $T_{\rm eff} \sim 2750$ K for GL 406; the more massive brown dwarfs at this age may not have spectra that appear cooler than the intrinsically faintest known field stars (see Figure 4). The principal difference for ages between 10^6 to $\sim 10^8$ yrs is a lower surface gravity for the younger stars/BDs. However, as cautioned above this is an age where other processes involving star formation can occur; for example, many of the Taurus T Tauri stars show evidence of lower level accretion, with spectra that can be strongly veiled (Basri & Bertout 1989, Hartmann & Kenyon 1990). Thus, such results can only be viewed as inconclusive, and such stars remain viable, although perhaps less attractive, brown dwarf candidates.

Other star forming regions that have been recently studied include LkHα 101 (Barsony et al 1991; 800 pc) and R Coronae Australis (Burton 1992; 130 pc). In the former study the LF appears to decline toward fainter $m_{\rm K}$, whereas the later is consistent with the results discussed above for Rho Oph with the LF rising to the limit in $m_{\rm K}$. Both investigations propose a notable number of potential brown dwarf candidates.

3.2.2 YOUNG OPEN CLUSTERS The Pleiades cluster (7×10^7 yrs, $m - M \approx 5.5$) is the most thoroughly studied young open cluster. Skrutskie et al (1989) and Zuckerman & Becklin (1987) surveyed the immediate surroundings of Pleiades main sequence stars and white dwarfs, respectively; neither found any low mass companions—thereby constraining the binary mass ratio in the cluster. Jameson & Skillen (1989) and Stauffer et al (1989) surveyed regions of 175 arcmin^2 in R and I and 870 arcmin^2 in V and I respectively using CCDs. Several very red stars were found in each survey with one star in common. As shown by Stringfellow (1991a) interpretation of the results when transformed into the theoretical domain for comparison with evolutionary tracks are not unique, depending on the colors and set of transformations used. It is worth noting that prior to these studies faint members of the Pleiades population were discovered in flare star searches; large color variation due to strong chromospheric activity may still be a common occurence in clusters this young. Prosser et al (1991) have investigated the emission activity-age relation for Pleiades, Hyades, and field stars and indeed confirm a mean decrease in activity with increasing age, but also find that at any given age a wide range of activity is exhibited. This is discussed by Stauffer et al (1991b) who have determined the faint end of the LF through a proper motion membership study of photographic plates down to $V \approx 18$, or $M_V \approx 12.5$, although the data fainter than $M_V \sim 10.5$ remain seriously incomplete due to insufficient areal coverage. They find that the LF continues to rise down to M_V of 10, and is not significantly different from the field SLF. Hambly & Jameson (1991) used Schmidt survey plates to measure the LF in the inner 1.5° radius and deduce the mass function (MF) for the Pleiades. They infer that a few, to perhaps many, brown dwarfs reside within the cluster and find a mass function which either continues to rise at the faint end or is flat, depending on whether or not those objects detected in the I band only, are cluster members. Thereafter Hambly et al (1991) determined proper motions using POSS and Schmidt plates as first and second epoch measurements, respectively. They find a large population of low mass Pleiades members, 10 or more of which could be brown dwarfs. The LF turns over at $M_I \sim 9.5$ and declines thereafter somewhat slowly. The resulting MF appears to peak near 0.125 $M_\odot$ and begins to turnover—a flat MF adequately represents the data as it stands (Stringfellow 1991b, Burrows et al 1993), implying that a nontrivial number of brown dwarfs should exist.

Another study by Simons & Becklin (1992) obtained deep I and K band images of selected regions in the Pleiades and control fields located about 5° from the cluster center. The total areas surveyed were 200 arcmin^2 and 75 arcmin^2 with detection limits of 20 mag and 17 mag, respectively. The control sample was scaled up to the Pleiades sample and used to subtract out field stars. Analysis was made in the H-R diagram and a total of 22 ± 10 Pleiades members of very low mass were found, about half of which lie in the BD domain. The precise identification of the candidates are a bit sensitive to the age of the isochrones used, the model isochrones utilized, and moreso to the color-$T_{\rm eff}$-BC scale adopted. Simons & Becklin find a LF that continues to rise nearly to the detection limit; a turnover may occur in the last magnitude bin ($M_I \sim 14$). What is most striking about their LF though is the apparent deficiency of low mass stars with mass around 0.2 $M_\odot$—only one cluster member was identified in their survey. The results imply a very steep mass function ($\alpha \approx 2.8$) and a very large number of BDs.

Additional surveys need to be conducted over larger areas to fainter magnitudes along with proper motion studies. Followup work on the low mass members found also needs to be performed in order to clarify the issues discussed above.

3.2.3 INTERMEDIATE-AGE OPEN CLUSTERS Like the Pleiades, the Hyades ($\sim 6 \times 10^8$ yrs, $m-M = 3.3$) has been extensively studied, mostly with photographic plates because of the even more extended area subtended by the cluster. Reid (1992) has conducted a proper motion analysis using V Schmidt plates, covering an area of ~ 112 deg^2 down to V ~ 19 ($M_V \sim 15.5$). He finds that the bright end of the Hyades LF closely resembles that of the solar neighborhood with a broad peak near $M_V \sim 12$, and that significant mass segregation has indeed occured within the Hyades. Leggett & Hawkins (1988) used R and I Schmidt plates to identify red stars in the central region of the Hyades for which they then obtained JHK photometry. The optical and IR LFs they derived indicate a peak around 0.2 $M_\odot$, in agreement with the field LF, and potential flattening at fainter magnitudes. Leggett & Hawkins (1989) extended this study by obtaining JHK photometry for those stars detected only in the I band in their previous study. A total of 49 probable members were found in these surveys, including 12 very faint stars. Hubbard et al (1990) and Stringfellow (1991b) have compared the above LF with their models and find that a nearly flat mass function ($\alpha \sim 1.0$) represents the data best, although the agreement found is rather poor overall. Stringfellow (1991b) points out that dispersion in the photometric parallaxes can significantly alter the shape of the Hyades LF compared to that using a mean distance for all stars. Proper motions, especially for the faint stars, are essential as are additional spectroscopic and photometric analysis.

Bryja et al (1992) have performed a proper motion study using different epoch POSS plates of a $6° \times 6°$ field in the Hyades and found 12 very faint candidates whose motions (directions and magnitude) reflect that of the Hyades core. These apparent cluster members have bolometric luminosities comparable to those found by Leggett & Hawkins (1989), but their I$-$K colors are much bluer. Their position within the H-R diagram (Figure 4) places them significantly below the main sequence. If it were not for their proper motions these would normally be interpreted as being background field stars. Inclusion of these stars in the Hyades LF of Leggett & Hawkins (see Figure 7 of Stringfellow 1991b) changes markedly the structure of the LF at the faint end; the dips are filled in and the slope of the mass function that best fits the combined data gives $\alpha \lesssim 2.0$ as opposed to the flatter mass function implied previously. However, the proper motions should not be regarded as definitively establishing Hyades membership as several brighter stars previously found to share the Hyades proper motions were later identified spectroscopically as being subdwarfs not physically associated with the Hyades. We urge more complete analysis prior to reaching final conclusions about the nature of these objects.

Praesepe ($\sim 4 \times 10^8$ yrs, $m-M = 6.0$) has properties similar to that of the Hyades (about the same age, metallicity, and kinematics). Although the faintest stars cannot be as easily probed as the Hyades stars due to the greater distance, it is not much farther than the Pleiades and so the potential for probing the faint end of the SLF is great. As a start in this direction we mention the study by Jones & Stauffer (1991) which measured proper motions for this cluster down to V $\sim$ 18 mag ($M_V \sim 12.0$). Once again, this study found that the cluster LF closely matches the nearby LF. By means of Monte-Carlo simulations Kroupa & Tout (1991) showed that the LFs of the Hyades and Praesepe clusters are consistent with the LF for the disk field stars and consistent with the same underlying two power-law segment mass function with the break at 0.5 $M_\odot$. Future searches would benefit from using the R, I, and IR bands rather than the traditional V band searches if one intends to sample into the brown dwarf regime.

4. THE POPULATION II LUMINOSITY FUNCTION

4.1 *The Globular Clusters*

Deep I-band imaging has been used to derive luminosity functions for 7 globular clusters (Fahlman et al 1989; Richer et al 1990, 1991). The low luminosity limits range from $M_I \approx 11.3$ ($M_{bol} \approx 11.2$) in NGC 6397 [$(m-M)_I = 12$] to $M_I \approx 9$ ($M_{bol} \approx 9.5$) in M5 [$(m-M)_I \approx 14.3$]. The technical difficulties associated with star counts at faint magnitudes, especially in a crowded globular cluster field, are formidable and one should treat with caution star counts for apparent magnitudes below $m_V \approx 24$, $m_I \approx 22$. Drukier et al (1988) discuss the large completeness corrections that were made to star counts at these magnitudes in M13. In Figure 5 the normalized luminosities are shown together with the solar neighborhood LF from Stobie et al (1989). For ease of comparison the LFs have been normalized to approximately the same values between $M_I = 5.5$ and 6.5.

The luminosity functions are all approximately flat between $M_I \sim 5$ and 7 and turn up at lower luminosities. The V−I color is between 0.5 and 0.7 at $M_I = 3.5$, the approximate turnoff luminosity in globular clusters. Most of the globular cluster LFs begin to increase at $M_I \approx 7$—about 1 mag brighter than the turnup for the solar neighborhood LF. The LF of the metal-rich cluster M71 is intermediate. D'Antona (1987) has shown that the flattening in the M-L relations due to formation of H_2 occurs at

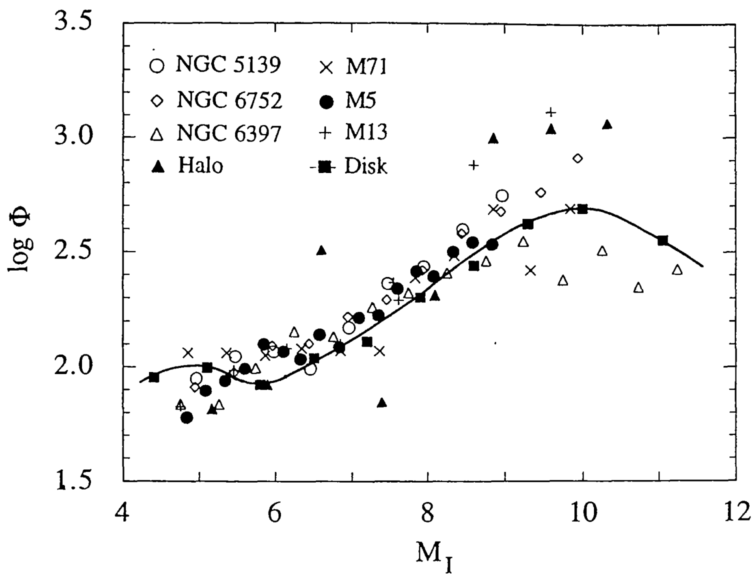

Figure 5 The normalized log I-band luminosity functions (units: pc^{-3} mag^{-1}) for six globular clusters (Fahlman et al 1989; Richer et al 1990, 1991) and a halo field (Richer & Fahlman 1992) in comparison with the disk luminosity function of Wielen et al (1983) (shown as the solid curve). Error bars not shown for clarity.

systematically higher luminosity in metal-poor stars than in solar composition stars and as a consequence, the M-L induced increase in the LF should also occur at higher luminosity, as observed.

Using an appropriate theoretical M-L relation, Richer et al (1991) derived mass functions for the globular clusters and found very steep mass functions for stars with masses less than 0.3 $M_\odot$. Most significantly, they found a strong correlation between the slope of the mass function and the predicted destruction time of the cluster due to interactions with the bulge and disk. They suggest that those clusters—NGC 6752 and M13—with the steepest LF (and hence steepest mass function) represent the initial luminosity functions for low mass stars in globular clusters which are later reduced by evaporation of many of the lowest mass stars due to cluster-disk interactions. However, as pointed out above, the theoretical M-L relation for the near-solar composition stars with masses below 0.4 $M_\odot$ are in need of revision so it is probable that the low metal models have similar problems. In addition, the lowest luminosity star counts have large incompleteness corrections. Therefore, although the differences between the LF and mass function of NGC 6397 and 6752 are almost certainly real, one cannot yet claim with the same degree of certainty that the underlying mass functions of NGC 6752 and the disk stars in the solar neighborhood are significantly different.

4.2 *Population II Field Stars*

Richer & Fahlman (1992) have also attempted to measure the luminosity function of the stars in the halo (spheroid). From V, R, and I CCD frames of a 44 arcmin^2 field at high Galactic latitude they selected those stars with heights exceeding 1200 pc above the Galactic plane. Distances were assigned using a M_I versus V$-$I relation intermediate between the old disk and the extreme halo stars of Monet et al (1992). The data were restricted to objects with V $\lesssim$ 25, R $\lesssim$ 24.5, and I $\lesssim$ 24.5 and are considered complete for point sources. Stars were separated from galaxies by means of color-color relations as well as by shape. Compact galactic nuclei and quasars with stellar colors will always be counted as stars in this procedure and we must assume that some of the faintest objects have been misidentified as stars. The halo (spheroid) LF is also shown in Figure 5. Eggen (1983; Figure 1) also derived the luminosity function for field halo stars on the basis of his complete sample of stars with annual proper motions greater than 0.7 arcsec and brighter than visual magnitude 15. Eggen's LF is very similar to that of Richer & Fahlman and shows a 2 times excess of halo stars over field stars for $9 < M_I < 11$.

The number density of stars found as a function of galactocentric distance was in excellent agreement with the $r^{1/4}$ law assumed for the halo (spheroid) by Bahcall et al (1983). There was no evidence in the observed density distribution at distances greater than 40 kpc, for the large contribution of faint stars that should have been seen were the massive corona (halo) (r^{-2} density law) in the form of low mass ($m \lesssim 0.2\ M_\odot$) hydrogen burning stars (Bahcall 1986). The luminosity function of the halo appears very similar to that of some of the globular clusters such as M13, and the derived mass function was similarly steep for masses below 0.5 $M_\odot$; however, similar caution applies to the literal interpretation of these data because of uncertainties in the mass-luminosity relation for low mass stars as discussed above.

In the disk the mass function can be represented as discrete power laws over different mass intervals: $dN/dm \propto m^{-\alpha}$, where $\alpha \approx 2$ for $0.5\ M_\odot \lesssim m \lesssim 1\ M_\odot$ and for the lowest mass stars $\alpha \approx 1.1$ for $0.08\ M_\odot \lesssim m \lesssim 0.5\ M_\odot$ (KTG). In contrast, Richer et al (1991) derive $\alpha \approx 3.6$ for globular clusters and 4.5 ± 1.2 for the halo (spheroid) (Richer & Fahlman 1992). Accepting the steep mass function at face value, Richer & Fahlman (1992) show that a factor of 10 increase in the halo mass density, which is within the constraints set by the Galactic rotation curve, would occur by extending the integration over the unseen mass in the range $0.056\ M_\odot \lesssim m \lesssim 0.17\ M_\odot$. That is, for such a steep index, the mass function of the halo is cutoff at $\sim$0.05 $M_\odot$. However, in order to explain the 100 times increase in density required to account for the massive dark halo as low mass stars and brown dwarfs, an even steeper mass function is required, or the mass function itself must be extended to much lower masses. With all the uncertainties involved it is premature to dismiss low mass stars and brown dwarfs as the major contributor to the "missing mass" in the halo, but the evidence for the proposal is not strong.

It is difficult to see how better data than that of Richer & Fahlman (1992) can be obtained because the major uncertainties in the analysis result from the large corrections applied to the data at the faintest magnitudes; however, the second epoch Schmidt surveys will provide a wealth of proper motion data which together with colors should enable a kinematically defined sample of very faint halo stars in the solar neighborhood to be identified. This sample will go much fainter intrinsically than the Richer & Fahlman sample and the Eggen proper motion sample and should enable the relative contributions of the disk, halo (spheroid), and dark halo to be better understood.

5. THE WHITE DWARF LUMINOSITY FUNCTION

The luminosity function of white dwarfs has been dealt with in detail elsewhere. The review on the cooling of white dwarfs by D'Antona & Mazzitelli (1990) summarized the status of the observed LF in 1990. Liebert et al (1989) derived the LF of the local disc and halo (spheroid) shown in Figure 6. Because the luminosity function for the faintest stars was drawn from a proper motion limited sample, it was known to have missed stars with low tranverse velocities. Weidemann (1991) also discussed this incompleteness and suggests a correction factor of 1.5 at $M_{\rm bol} = 15$. Boyle (1989) from a sample of WDs identified in a deep QSO survey obtained a scale height of 275 ± 50 pc for the DA white dwarfs.

The turnover in the LF at $M_{\rm bol} = 15.2$ has been interpreted as a result of the finite age of the Galactic disk (e.g. Mestel 1952, Schmidt 1959, Winget et al 1987, Iben & Laughlin 1989). Initial analysis derived an age of 9.3 ± 2 Gyr for the age of the disk (Winget et al 1987), which was younger than other estimates of 12–14 Gyr. The fit of the LF was rediscussed by Yuan (1989) who concluded that ages of 6–14 Gyr are not ruled out, while Wood (1991) suggested a range of 8 to 11 Gyr. Noh & Scalo (1990) examined the sensitivity of the white dwarf luminosity function to the local star formation history.

D'Antona & Mazzitelli (1990) discuss weaknesses in the theoretical modeling of the cooling of white dwarfs and caution that the cooling timescale between $12 \lesssim M_{\rm bol} \lesssim 16$ is still poorly known and as a consequence one cannot assign any firm age to the disk stars. They also note

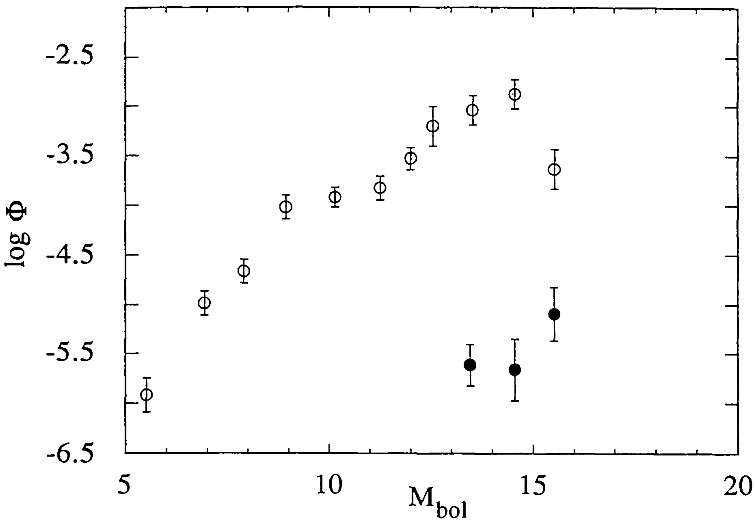

Figure 6 The log bolometric luminosity function (units: pc^{-3} mag^{-1}) for white dwarfs from Liebert et al (1988, 1989). The open circles are for the disk WDs; the closed circles are for the several large tangential velocity, presumably halo, WDs.

that the observed luminosity function should be better defined at the faintest limits by making deeper surveys and by attempting to better predict the energy distribution of cool white dwarfs.

The apparent conflict between the age of the white dwarfs and the age of the halo stars could be reconciled by the scenario of the collapse of the Galaxy and the formation of the Galactic disk presented by Burkert et al (1992). Their chemodynamical models suggest that following the formation of a hot halo (corona) at the end of a protogalactic phase of massive star formation, collapse occurs to a spheroid (scale height 1 kpc) where, because of energy dissipation, it pauses for between 4–6 Gyr before collapsing further to a disk of scale height 250 pc. This would then permit a difference between the age of most of the globular clusters and the age of the disk white dwarfs.

In a provocative paper, Tamanaha et al (1990) have suggested that much of the dynamically inferred dark halo mass density could comprise faint white dwarfs—remnants of a brief star formation burst at a rate 800 times the mean disk star formation rate, which took place more than 12 Gyr ago. These stars should be seen about 1 mag in I fainter than the magnitude of the turnover in the current WD LF, even if they comprise only 1% of the dark matter. Theoretical luminosity and discovery functions for halo white dwarfs have also been calculated by Mochkovitch et al (1990). CCD surveys to fainter limiting magnitudes are being undertaken in attempts to measure the spheroid white dwarf luminosity function and compare them with predictions. However, as emphasized by D'Antona & Mazzitelli

(1990) and Mochkovitch et al (1990), there are still significant uncertainties in the physics of white dwarf evolution on which the predictions are based.

Finally, Weidemann (1991) has discussed the total space density of white dwarfs and plausible models for the evolution of the disk. He derives a white dwarf birthrate that is in good agreement with the planetary nebula birthrate of $2.4 \pm 0.3 \times 10^{-12}$ pc^{-3} yr^{-1}.

6. INITIAL MASS FUNCTIONS

Much of the discussion above concerning the luminosity functions could not avoid mentioning implicitly or explicitly the underlying mass functions. After all, one of the main reasons for measuring the LF is to derive the initial mass function. The mass functions for faint disk dwarfs derived by KTG and TMR indicated that the IMF for low mass stars ($<0.5\ M_{\odot}$) was much less steep than the Miller & Scalo (1979) or Salpeter (1955) mass functions for more massive stars and therefore even extrapolation with the same slope to the lowest masses would not yield a significant increase in the local mass density of the disk. This is consistent with there being no local dark matter (Kuijken & Gilmore 1991). On the other hand, mass functions inferred by Richer and colleagues from the luminosity functions of halo stars in globular clusters and the field indicated a steepening of the IMF and the possibility of a many times increase in the mass density of halo stars from the contribution of very low mass stars. We pointed out the uncertainties, both observational and theoretical, that underlie these proposals. This is the place that future observations and analyses are crucial.

Gilmore & Roberts (1988) highlight the diversity of stellar luminosity functions and point out that models adopting a varying power-law slope or bimodal IMFs cannot fit the range of LFs such as the LFs of T-associations and old disk and halo field and cluster stars. Scalo (1986) reviewed the uncertainties in constructing the IMF; useful contributions concerning the IMF have also been given by Piskunov & Malkov (1987) and Zinnecker (1987).

Derivation of the IMF from LFs are complicated by the fact that observations of external galaxies suggest that star formation proceeds in bursts, rather than in a regular manner; in addition, many galaxies including our own are members of interacting systems and there is a strong possibility that a significant proportion of the stars in the larger galaxies were born in small satellite systems that were subsequently captured. Star formation itself is one of the least understood and complex areas of astrophysics. Shu et al (1987) recently reviewed star formation and suggests that the mass-to-magnetic flux ratio in a molecular cloud could introduce bimodality. Bimodal or biased star formation has often been invoked (e.g. Herbig 1962a,b; Eggen 1976, 1977; Sandage 1986; Larson 1987; Wyse & Silk 1987; Zinnecker 1987) to explain the apparent lack of low mass stars in some clusters and O-associations and the lack of massive stars in T-associations. It is also invoked to explain the higher O/Fe ratio in halo stars compared to the disk stars in the Galaxy and to predict that the dark halo material may consist of white dwarf remnants of an early epoch of massive star formation (Tamanaha et al 1990).

The complexities of galaxy evolution and stellar formation would appear to rule out the likelihood of a universal stellar LF and IMF. But the nature of the mass function in the solar neighborhood, namely the Salpeter power law and the Miller-Scalo log-normal law, strongly supports an underlying principle or mechanism operating in the disk.

7. CONCLUDING REMARKS

Recent advances in the measurement of the stellar luminosity function of low mass main sequence field stars and white dwarfs in the disk have yielded significant results and raised some important questions.

• There is no evidence for an increase in the slope of the LF or mass function for stars less massive than 0.1 $M_{\odot}$ in the solar neighborhood. That is, there is no evidence for a large population of "almost stars" with masses below 0.08 $M_{\odot}$, which supports suggestions that there is no local "missing mass."

• The scale height of M dwarfs with spectral types later than M6 is lower than for earlier spectral types, implying that they are on the average much younger stars. The apparent inconsistency between the young ages derived for the faintest stars and the theoretical behavior of the transition objects must be therefore reconciled or not.

• The different scale heights of the late M dwarfs and the high proportion of unresolved binary M dwarfs amongst the earlier spectral types probably account for the difference between the LFs of stars within 5 pc and the photometrically derived LFs of stars at larger distance.

• The search for brown dwarfs in young stellar associations has been inconclusive as yet, due partly to the lack of clear membership criteria but due also to uncertainties in the temperature scale of late M dwarfs.

• The white dwarfs near the Sun have a scale height of about 275 pc and an age deduced from WD cooling times of between 8 and 11 Gyr. Their birthrate function is comparable to that of planetary nebulae.

• The same cooling curves and the likelihood of bursts of massive star formation early in the history of the Galaxy suggest that there may be large numbers of faint cool white dwarfs in the halo.

• The slope of the globular cluster LFs for stars with $M < 0.3\ M_{\odot}$ correlates strongly with their dynamical interaction with the Galactic disk and bulge. Those clusters that have had little interaction show much steeper LFs than that of the disk stars (the halo LF is even steeper) which suggests that the slope of the IMF for globular clusters and halo field stars steepens at low mass and that a significant amount of mass could be in the form of low mass stars and brown dwarfs.

• The amount of this halo "missing mass" depends critically on the theoretical mass-luminosity relation adopted and on star counts at very faint magnitudes where incompleteness and confusion corrections are large.

Some of the questions raised can be answered by extending the magnitude limits of the surveys and by obtaining proper motion limits for the stars. There are good prospects to achieve this both with photographic plates and with CCDs. Second epoch plates of UK Schmidt fields are being taken, which will provide proper motions and colors for many more stars and to fainter limits than the LHS catalog (Luyten 1979) from which so much of our knowledge of faint M dwarfs and white dwarfs has come. There are also prospects for extending the range of limiting magnitude Schmidt plates. M. R. S. Hawkins (1992, personal communication) has combined up to 64 Schmidt I plates and extended his limiting magnitude by 2 mags. Just as importantly, CCDs are becoming readily available in sizes up to 2048 × 2048 pixels with possibilities of mosaicing them to cover even larger areas. With their much greater quantum efficiencies and higher S/N capabilities than photographic plates they will increasingly provide data on the faintest objects. The new IR area detectors will also be invaluable in extending the search for faint stars to IR wavelengths where the coolest brown dwarfs are expected to radiate most strongly. We expect that the remaining questions on the nature of the faint M dwarfs of the disk and halo, and on the existence of a vast population of faint halo white dwarfs, will be answered within the next few years from data that have started to be collected.

Other questions involve theoretical modeling. It is necessary to pursue the problems identified in the interior and atmospheric modeling of cool stars so that reliable mass-luminosity relations and reliable temperature-luminosity relations can be provided. Only when this is done, can more believable mass functions be derived from the luminosity functions, and brown dwarfs be more readily identified.

Finally, a better understanding of star formation is essential to explain the underlying mass functions of clusters and field and how these might change with space and time. One of the holy grails of astrophysics, an understanding of star formation, may be within reach using hydrodynamic modeling on the newer super-computers. Accretion disks and magnetic fields are currently being tackled with gratifying results and we anticipate rapid advances to follow.

Literature Cited

Allard, F. 1991. PhD thesis. ITA, Heidelberg

Bahcall, J. N. 1986. *Annu. Rev. Astron. Astrophys*. 24: 577–611

Bahcall, J. N., Schmidt, M., Soneira, R. M. 1983. *Ap. J*. 265: 730–47

Barsony, M., Burton, M. G., Russell, A. P. G., Carlstrom, J. E., Garden, R. 1989. *Ap. J*. 346: L93–96

Barsony, M., Schombert, J. M., Kis-Halas, K. 1991. *Ap. J*. 379: 221–31

Basri, G., Bertout, C. 1989. *Ap. J*. 341: 340–58

Berriman, G., Reid, I. N. 1987. *MNRAS* 227: 315–29

Berriman, G., Reid, I. N., Leggett, S. K. 1992. *Ap. J*. 392: L31–33

Bessell, M. S. 1982. *Proc. Astron. Soc. Aust*. 4: 417–19

Bessell, M. S. 1990a. *Astron. Astrophys. Suppl*. 83: 357–78

Bessell, M. S. 1990b. *Publ. Astron. Soc. Pac*. 102: 1181–99

Bessell, M. S. 1991. *Astron. J*. 101: 662–76

Bessell, M. S., Brett, J. M. 1988. *Publ. Astron. Soc. Pac*. 100: 1134–51

Bessell, M. S., Wickramasinghe, D. T. 1979. *Ap. J*. 227: 232–43

Bessell, M. S., Wood, P. R. 1984. *Publ. Astron. Soc. Pac*. 96: 247–49

Blackwell, D. E., Petford, A. D., Arribas, S., Haddock, D. J., Selby, M. J. 1990. *Astron. Astrophys*. 232: 396–410

Blackwell, D. E., Petford, A. D., Shallis, M. J. 1980. *Astron. Astrophys*. 82: 249–52

Boeshaar, P. C. 1976. PhD thesis. Ohio State Univ., Columbus

Boyle, B. J. 1989. *MNRAS* 240: 533–549

Brett, J. M., Plez, B. 1993. *Proc. Astron. Soc. Aust*. In press

Bryja, C., Jones, T. J., Humphreys, R. M., Lawrence, G., Pennington, R. L., Zumach, W. 1992. *Ap. J. Lett*. 388: L23–26

Burkert, A., Truran, J. W., Hensler, G. 1992. *Ap. J*. 391: 651–58

Burrows, A., Hubbard, W. B., Lunine J. J. 1989. *Ap. J*. 345: 939–59 (BHL)

Burrows, A., Hubbard, W. B., Saumon, D., Lunine, J. I. 1993. *Ap. J*. 406: 158–71

Burton, M. 1992. *Proc. Astron. Soc. Aust*. 10: 52–54

Dahn, C. C., Liebert, J. W., Harrington, R. S. 1986. *Astron. J*. 91: 621–25

D'Antona, F. 1987. *Ap. J*. 320: 653–62

D'Antona, F., Mazzitelli, I. 1985. *Ap. J*. 296: 502–13

D'Antona, F., Mazzitelli, I. 1990. *Annu. Rev. Astron. Astrophys*. 28: 139–81

Dorman, B., Nelson, L. A., Chau, W. Y. 1989. *Ap. J*. 342: 1003–18

Doyle, J. G., Butler, C. J. 1990. *Astron. Astrophys*. 235: 335–39

Drukier, G. A., Fahlman, G. G., Richer, H. B., VandenBerg, D. A. 1988. *Astron. J*. 95: 1415–21

Eggen, O. J. 1973. *Ap. J*. 182: 821–37

Eggen, O. J. 1976. *Q. J. R. Astron. Soc*. 17: 472–87

Eggen, O. J. 1977. *Publ. Astron. Soc. Pac*. 89: 187–204

Eggen, O. J. 1979. *Ap. J. Suppl*. 39: 89–101

Eggen, O. J. 1980. *Ap. J. Suppl*. 43: 457–68

Eggen, O. J. 1983. *Ap. J. Suppl*. 51: 183–202

Eggen, O. J. 1993. Preprint

Fahlman, G. G., Richer, H. B., Searle, L., Thompson, I. B. 1989. *Ap. J. Lett*. 343: L49–51

Fischer, D. A., Marcy, G. W. 1992. *Ap. J*. 396: 178–94

Forrest, W. J., Ninkov, Z., Garnett, J. D., Skrutskie, M. F., Shure, M. 1990. In *Strongly Coupled Plasma Physics*, ed. S. Ichimaru, pp. 33–42. Yamada: Elsevier

Gilmore, G., Reid, N. 1983. *MNRAS* 202: 1025–47

Gilmore, G., Roberts, M. S. 1988. *Comm. Astrophys*. 12: 123–29

Gliese, W. 1969. *Veröff. Astron. Rechen-Inst. Heidelberg No. 22*

Gliese, W., Jahreiss, H. 1979. *Astron. Astrophys. Suppl*.38: 423–48

Gliese, W., Jahreiss, H. 1992. *Astron. Astrophys. Suppl*. In press

Greene, T. P., Young, E. T. 1992. *Ap. J*. 395: 516–28

Greenstein, J. L. 1989a. *Publ. Astron. Soc. Pac*. 101: 787–810

Greenstein, J. L. 1989b. *Comm. Astrophys*. 13: 303–23

Greenstein, J. L, Neugebauer, G., Becklin, E. E. 1970. *Ap. J*. 161: 519–31

Gunn, J. E., Stryker, L. L. 1983. *Ap. J. Suppl*. 52: 121–53

Habets, G. M. H. J., Heintze, J. R. W. 1981. *Astron. Astrophys. Suppl*. 46: 193–237

Haisch, B. M., Strong, K. T., Rodonò, M.

1991. *Annu. Rev. Astron. Astrophys.* 29: 275–324
Hambly, N. C., Hawkins, M. R. S., Jameson, R. F. 1991. *MNRAS* 253: 1–8
Hambly, N. C., Jameson, R. F. 1991. *MNRAS* 249: 137–44
Harrington et al 1978. *Publ. USNO Vol XXIV Part IV*
Hartigan, P., Hartmann, L., Kenyon, S., Hewett., R., Stauffer, J. 1989. *Ap. J. Suppl.* 70: 899–914
Hartmann, L. W., Kenyon, S. J. 1990. *Ap. J.* 349: 190–96
Hartwick, F. D. A., Cowley, A. P., Mould, J. R. 1984. *Ap. J.* 286: 269–75
Hawkins, M. R. S. 1987. *MNRAS* 223: 845–58
Hawkins, M. R. S. 1988. *MNRAS* 234: 533–37
Hawkins, M. R. S., Bessell, M. S. 1988. *MNRAS* 234: 177–91
Heintz, W. D. 1989. *Astron. Astrophys.* 217: 145–45
Henry, T. J., Johnson, D. S., McCarthy, D. W. Jr., Kirkpatrick, J. D. 1992. *Astron. Astrophys.*254: 116–20
Henry, T. J., McCarthy, D. W. 1990. *Ap. J.* 350: 334–47
Herbig, G. H. 1956. *Publ. Astron. Soc. Pac.* 68: 531–33
Herbig, G. H. 1962a. *Ap. J.* 135: 736–47
Herbig, G. H. 1962b. *Adv. Astron. Astrophys.* 1: 47–103
Hubbard, W. B., Burrows, A., Lunine, J. I. 1990. *Ap. J.* 358: L53–55
Ianna, P. A. 1993. In *Developments in Astrometry and Their Impacts on Astrophysics and Geodynamics, IAU Symp. No. 156*, ed. I. I. Mueller. Dordrecht: Kluwer. In press
Iben, I., Laughlin, G. 1989. *Ap. J.* 341: 312–26
Jahreiss, H. 1987. In *Impacts des Surveys du visible sur notre Connaisance de La Galaxie*, ed. A. Fresneau, M. Hamon, pp. 73–83. Strasbourg: Comptes Rendus Journ. Strasbourg 9ème Reunion
Jameson, R. F., Skillen, I. 1989. *MNRAS* 239: 247–253
Jones, B. F., Stauffer, J. R. 1991. *Astron. J.* 102: 1080–90
Kirkpatrick, J. D., Henry, T. J., McCarthy, D. W. 1991. *Ap. J. Suppl.* 77: 417–40
Kirkpatrick, J. D., Kelly, D. M., Rieke, G. H., Liebert, J., Allard, F., Wehrse, R. 1992. *Ap. J.* 402: 643–54
Kroupa, P., Tout, C. A. 1991. *CTS Workshop No 1, Prague, Evolution of Interstellar Matter and Dynamics of Galaxies.* Cambridge: Cambridge Univ. Press
Kroupa, P., Tout, C. A., Gilmore, G. 1990. *MNRAS* 244: 76–85 (KTG)
Kroupa, P., Tout, C. A., Gilmore, G. 1991. *MNRAS* 251: 293–302
Kroupa, P., Tout, C. A., Gilmore, G. 1993. *MNRAS* In press
Kui, R. 1991. PhD thesis. Aust. Natl. Univ., Canberra
Kuijken, K., Gilmore, G. 1991. *Ap. J. Lett.* 367: L9–13
Kurucz, R. L. 1979. *Ap. J. Suppl.* 40: 1–340
Lada, C. J., Wilking, B. A. 1984. *Ap. J.* 287: 610–21
Larson, R. B. 1987. In *Stellar Populations*, ed. C. Norman, A. Renzini, M. Tosi, pp. 101–19. Cambridge: Cambridge Univ. Press
Laughlin, G. P., Bodenheimer, P. 1993. *Ap. J.* 403: 303–14
Leggett, S. K. 1989. *Astron. Astrophys.* 208: 141–45
Leggett, S. K. 1992. *Ap. J. Suppl.* 82: 351–94
Leggett, S., Hawkins, M. R. S. 1988. *MNRAS* 234: 1065–90
Leggett, S., Hawkins, M. R. S. 1989. *MNRAS* 238: 145–53
Liebert, J. 1980. *Annu. Rev. Astron. Astrophys.*18: 363–98
Liebert, J., Dahn, C., Monet, D. G. 1988. *Ap. J.* 332: 891–909
Liebert, J., Dahn, C., Monet, D. G. 1989. In *White Dwarfs, IAU Colloq. No. 114*, ed. G. Wegner, pp. 15–28. Berlin: Springer-Verlag
Liebert, J., Probst, R. G. 1987. *Annu. Rev. Astron. Astrophys.* 25: 473–519
Luyten, W. J. 1968. *MNRAS* 202: 1025–47
Luyten, W. J. 1979. *The LHS Catalogue.* Minneapolis: Univ. Minn.
Magazzu, A., Martin, E. L., Rebolo, R. 1993. *Ap. J.* 404: L17–20
Malkov, O. Y. 1987. *Astrophys.* 26: 288–94
Mestel, L. 1952. *MNRAS* 112: 583–97
Miller, G. E., Scalo J. M., 1979. *Ap. J. Suppl.* 41: 513–47
Mochkovitch, R., Garcia-Berro, E., Hernanz, M., Isern, J., Panis, J. F. 1990. *Astron. Astrophys.* 233: 456–61
Monet, D. G., Dahn, C. C., Vrba, F. J., Harris, H. C., Pier, J. R., et al. 1992. *Astron. J.* 103: 638–65
Mould, J. R. 1976. *Astron. Astrophys.* 48: 443–59
Noh H. R., Scalo, J. 1990. *Ap. J.* 352: 605–14
Norris, J. E. 1992. In *The Globular Cluster–Galaxy Connection*, ed. J. P. Brodie, G. H. Smith, ASP Conf. Ser. SanFrancisco: Astron. Soc. Pac. In press
Petterson, B. R. 1980. *Astron. Astrophys.*82: 53–60
Piskunov, A. E., Malkov, O. Yu. 1987. In *Evolution in Galaxies, Proc. 10th IAU*

Regional Meet., ed J. Palous, pp. 87–90. *Publ. Astron. Inst. Czech. Acad. Sci. No 69*

Popper, D. M. 1980. *Annu. Rev. Astron. Astrophys.* 18: 115–64

Pozio, F. 1991. *Mem. Soc. Astron. Ital.* 62: 171

Prosser, C. F., Stauffer, J., Kraft, R. P. 1991. *Astron. J.* 101: 1361–76

Rebolo, R., Martin, E. L., Magazzu, A. 1992. *Ap. J. Lett.* 389: L83–86

Reid, I. N. 1984. *MNRAS* 206: 1–17

Reid, I. N. 1987. *MNRAS* 225: 873–901

Reid, I. N. 1991. *Astron. J.* 102: 1428–38

Reid, I. N., Gilmore, R. F. 1984. *MNRAS* 206: 19–35

Richer, H. R., Fahlman, G. G. 1992. *Nature* 358: 383–86

Richer, H. B., Fahlman, G. G., Buonanno, R., Pecci, F. F. 1990. *Ap. J. Lett.* 359: L11–14

Richer, H. B., Fahlman, G. G., Buonanno, R., Pecci, F. F., Searle, L., Thompson, I. B. 1991. *Ap. J.* 381: 147–59

Rieke, G. H., Ashok, N. M., Boyle, R. P. 1989. *Ap. J.* 339: L71–74

Rieke, G. H., Rieke, M. J. 1990. *Ap. J. Lett.* 362: L21–24

Ruiz, M. T., Takamiya, M. Y., Roth, M. 1991. *Ap. J.* 367: L59–L61

Salpeter, E. E. 1955. *Ap. J.* 121: 161–67

Sandage, A. 1986. *Annu. Rev. Astron. Astrophys.* 24: 421–58

Scalo, J. M. 1986. *Fundam. Cosmic Phys.* 11: 1–278

Schmidt, M. 1959. *Ap. J.* 129: 243–58

Schmidt, M. 1975. *Ap. J.* 202: 22–29

Schmidt, M. 1983. In *The Nearby Stars and the Stellar Luminosity Function, IAU Colloq. No. 76*, ed. A. G. Davis Philip, A. R. Upgren, pp 155–61. Schenectady, NY: L. Davis

Shu, F. H., Adams, F. C., Lizano, S. 1987. *Annu. Rev. Astron. Astrophys.* 25: 23–81

Simons, D. A., Becklin, E. E. 1992. *Ap. J.* 390: 431–39

Sion, E. M. 1986. *Publ. Astron. Soc. Pac.* 98: 821–37

Skrutskie, M. F., Forrest W. J., Shure, M. 1989. *Astron. J.* 98: 1409–17

Skumanich, A. 1986. *Ap. J.* 309: 858–63

Smith, R. C. 1983. *Observatory* 103: 29–31

Stahler, S. W. 1985. *Ap. J.* 293: 207–15

Stauffer, J., Hamilton, D., Probst, R., Rieke, G., Mateo, M. 1989. *Ap. J. Lett.* 344: L21–24

Stauffer, J., Herter, T., Hamilton, D., Rieke, G. H., Rieke, M. J., et al. 1991a. *Ap. J. Lett.* 367: L23–26

Stauffer, J., Klemola, A., Prosser, C., Probst, R. 1991b. *Astron. J.* 101: 980–1005

Stevenson, D. J. 1991. *Annu. Rev. Astron. Astrophys.* 29: 163–93

Stobie, R. S., Ishida, K., Peacock, J. A. 1989. *MNRAS* 238: 709–27

Stringfellow, G. S. 1989. PhD thesis. Univ. Calif., Santa Cruz. Michigan: Univ. Microfilms

Stringfellow, G. S. 1991a. *Ap. J. Lett.* 375: L21–25

Stringfellow, G. S. 1991b. *Mem. Soc. Astron. Ital.* 62: 785–97

Tagliaferri, G., Doyle, J. G., Giommi, P. 1990. *Astron. Astrophys.* 231: 131–33

Tamanaha, C. M., Silk, J., Wood, M. A., Winget, D. E., 1990. *Ap. J.* 358: 164–69

Tinney, C. G. 1993. *Astron. J.* 105: 1169–78

Tinney, C. G., Mould, J. R., Reid, I. N. 1992. *Ap. J.* 396: 173–77 (TMR)

Tinney, C. G., Mould, J. R., Reid, I. N. 1993. *Astron. J.* 105: 1045–59

Tokovinin, A. A. 1992. *Astron. Astrophys.* 256: 121–32

Turnshek, D. E., Turnshek, D. A., Craine, E. J., Boeshaar P. C. 1985. *An Atlas of Digital Spectra of Cool Stars*. Astron. Astrophys. Ser. Vol. 1. Tucson: Western Res. Co.

van Altena, W. 1992. *The Yale Parallax Catalogue*. New Haven, Conn.: Yale Univ.

VandenBerg, D. A., Bell, R. A. 1985. *Ap. J. Suppl.* 58: 561–621

VandenBerg, D. A., Hartwick, F. D. A., Dawson, P., Alexander, D. R. 1983. *Ap. J.* 266: 747–54

Veeder, G. 1974. *Astron. J.* 79: 1056–72

Weidemann, V. 1988. In *A Decade of UV Astronomy with the IUE Satellite. ESA SP-821*, ed. E. Rolfe, pp. 1–17. Paris: ESA

Weidemann, V. 1991. In *White Dwarfs*, ed. G. Vauclair, E. Sion, pp. 67–73. Dordrecht: Kluwer

Weis, E. W. 1984. *Ap. J. Suppl.* 55: 289–99

Weis, E. W. 1988. *Astron. J.* 96: 1710–22

Wickramasinghe, D. T., Bessell, M. S., Cottrell, P. L. 1977. *Ap. J. Lett.* 217: L65–68

Wielen, R. 1974. In *Highlights of Astronomy 3*, ed. G. Contopoulos, pp. 489–99. Dordrecht: Reidel

Wielen, R., Jahreiss, H., Kröger, R. 1983. In *The Nearby Stars and the Stellar Luminosity Function, IAU Colloq. No. 76*, ed. A. G. Davis Philip, A. R. Upgren, pp 163–69. Schenectady, NY: L. Davis

Wilking, B. A., Lada, C. J., Young, E. T. 1989. *Ap. J.* 340: 823–52

Winget, D. E., Hansen, C. J., Liebert, J., Van Horn, H. M., Fontaine, G., et al. 1987. *Ap. J. Lett.* 315: L77–81

Wood, M. A. 1991. In *White Dwarfs*, ed. G. Vauclair, E. Sion, pp 89–97. Dordrecht: Kluwer

Wyse, R. F. G., Gilmore, G. 1988. *Astron. J.* 95: 1404–14
Wyse, R. F. G., Silk, J. 1987. *Ap. J. Lett.* 313: L11–14
Yuan, J. W. 1989. *Astron. Astrophys.* 224: 108–16
Zinnecker, H. 1987. In *Evolution in Galaxies, Proc 10th IAU Regional Meet.*, ed. J. Palous, pp. 77–85. *Publ. Astron. Inst. Czech. Acad. Sci. No. 69*
Zinnecker, H., McCaughrean, M. J., Wilking, B. A. 1993. In *Protostars and Planets III*, ed. E. H. Levy, J. I. Lunine, M. S. Matthews. Tucson: Univ. Ariz. Press. In press
Zuckerman, B., Becklin, E. E. 1987. *Ap. J.* 319: L99–L102