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Comparison of a 7000-year lunar ephemeris with analytical theory

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Abstract. A numerical integration on the JPL DE200 model of Newhall, Standish, and Williams is compared to a medium-accuracy version of the Chapronts' ELP 2000–85 Lunar theory from 5,000 B.C. to the present. According to this continuation of the DE200, the Chapronts' Moon is much closer to the LE200 Moon than their own comparison to the DE102/LE51 Moon suggested. (This does not imply, however, that anything is wrong with the LE51 or the theory.) Small drifts of order t^4 are noted in both the longitude and the perigee. After adjusting for the drifts and for four oscillatory terms not present in the theory, the r.m.s. difference near 5000 B.C. is 2.3″, compared to 1.1″in modern times. The fit at remote epochs is much improved by including secular phase shifts in the time-varying and planetary arguments of the theory, especially those that involve the Moon's mean anomaly.

Key words: moon – ephemerides

1. Introduction

In the past decade, improved semi-analytical planetary theories have been developed by P. Bretagnon and others (Bretagnon, 1988); these agree well with the DE102 numerical integration of Newhall et al (1983). Concurrently, Chapront-Touzé and Chapront (1988) produced a Lunar theory of high quality; but they found a substantial longitude drift, quadratic in time, from their comparison with the DE102. This report describes a direct numerical continuation of the DE200 model and shows that no quadratic or even cubic longitude adjustments are required to fit the Lunar theory.

The Chapront theory was adjusted to agree with JPL's DE200 ephemeris, a standard adopted by numerous almanac agencies. The DE200 was integrated back only to 1800 A.D., so comparisons at ancient times have to be made with the aid of projected correction formulas applied to the DE102 or by adjustment to the DE102's constants. The DE102 itself ends at about 1400 B.C., while the historical record of ancient eclipses now extends at least to 2100 B.C. For the present study a long term continuation of the DE200/LE200 model was computed, permitting a closer direct comparison between numerical and analytical results from 5000 B.C. to the present.

Comparisons reveal very little difference between the numerical result and the Chapront theory after 500 B.C. At 1500 B.C. the analytical theory is 10 times closer in mean longitude to this result than would be suggested by the Chapront's comparison to the LE51 Moon. Since the LE51 and LE200 used the same form of physics model and integrator, the discrepancies found by the Chapronts may simply be due to an imperfect adjustment to a perhaps imperfectly documented LE51 model.

The Chapront theory is said to be usable as far back as 4000 B.C. To compute an ephemeris from the analytical theory, the tables from the Chapronts' 1988 paper were augmented by including all terms from their book (Chapront-Touzé and Chapront 1991) that would affect the longitude or radius by 0.25″ or more at 4000 B.C. A few additional latitude terms were also included. The truncation level of the theory actually used was thus somewhat more precise than in the 1988 paper but not the "full precision" theory of the book. The latter is about ten times more accurate near J2000; it matches the DE200 to about 0.5″.

2. Analysis of the difference ephemeris

Figure 1 shows the result of subtracting the Moon's ecliptic longitude of date, according to the numerically integrated Lunar ephemeris, from the longitude given by the Chapront's ephemeris. In the interval 3500 to 4000 B.C. the longitude difference between the two ephemerides, without any corrections, is 32″ r.m.s., with a maximum of 82″ noted. After adjusting for several effects discussed below, the differences are reduced (see Fig. 2) to 1.5″ r.m.s. with a maximum of perhaps 8″.

Several major features of the difference ephemeris over time are apparent. One is an oscillation whose amplitude increases in the past. Its period seems to be approximately 270 years. This oscillation was noted by the Chapronts (1988) in their comparison to the DE102. One might think it should be possible to approximate it by a known periodic perturbation near that frequency. However, closer inspection reveals that the oscillation changes in frequency as well as amplitude. It is not cancelled by adding a simple function that returns to zero periodically.

To explore this difficulty, the dominant t^2 terms from the Chapront theory were studied by Fourier analysis of the difference between theory and numerical integration. Since these terms are present in the analytical theory, differences at the frequencies of these terms should have been small. However, near 4000 B.C. the actual Fourier amplitudes of differences corresponding to terms involving l, the Moon's mean anomaly, were as large as the amplitudes of the terms themselves. It would have been better to leave the terms out than to use them.

In the analytical theory, all the fundamental arguments are taken to be linear functions of time except in the t^0 Main Theory terms, whose arguments are polynomials in time of degree up to 4. From the polynomial expression, the actual phase of l at 4000

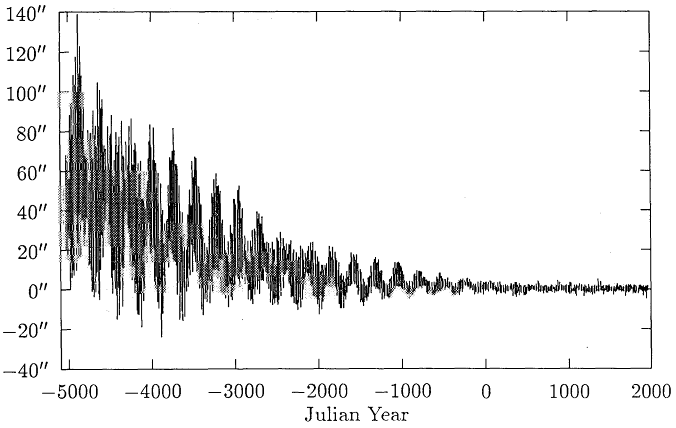

Fig. 1. Longitude difference, unadjusted analytical theory minus numerical integration.

B.C. amounts to some 25° compared to the linear function of time used in the theory for the t^2 perturbations. Comparisons at different epochs suggested that this time-varying phase shift could account for the poor fit at times in the distant past. Inclusion of the higher degree polynomial terms for l indeed removed all of the noted discrepancies. The amplitude coefficients seemed to need no change; only the arguments were modified.

As a consequence of this finding, it was decided to use the full nonlinear polynomials for all the theory arguments. The higher degree terms, in t^2, t^3, ..., of all the mean elements of the Sun, Moon, and planets were included in the arguments of all perturbations, including the time-varying Poisson terms of the Main Theory and the planetary perturbations. The higher degree coefficients for the planetary longitudes were taken from VSOP82 (Bretagnon 1988). It is understood that the theory was not constructed in this manner. It may be noted, however, that adjustments of this sort have been practiced before (see Newcomb, 1898) in order to move the epoch about which a theory was originally expanded.

With arguments thus phase shifted it was possible to fit the 270-year oscillation by adding the following two terms to the Moon's longitude:

$$-230.3 \sin(18\mathrm{Ve} - 16\mathrm{Te} - l + 0\,\mathrm{deg})\ 10^{-5}\ t^2\ ''$$

$$-17.7 \sin(8\mathrm{Ve} - 13\mathrm{Te} + 22.5\,\mathrm{deg})\ 10^{-5}\ t^2\ ''$$

Here Ve and Te are the mean longitudes of Venus and the Earth, referred to the J2000 ecliptic and equinox; l is the mean anomaly of the Moon referred to the ecliptic and equinox of date. Time t is measured in Julian centuries from J2000. The t^2 terms of these perturbations are not present in the Chapront theory.

A second major feature of the difference ephemeris is a smoothly varying long-term drift. This drift is approximated by a small term

$$d\mathrm{L} = -3.513 \cdot 10^{-6}\ t^4 - 1.69 \cdot 10^{-8}\ t^5\ ''$$

added to the mean longitude, L, of the Moon and to the Delaunay arguments D (the mean elongation of the Moon from the Sun), F (the mean longitude of the Moon minus the mean longitude of the Moon's node), and l.

After adjusting for the above differences, a third oscillatory component of about 2000 years' period became evident. This was fit by the expression

$$-30.1 \sin(10\mathrm{Ve} - 3\mathrm{Te} - l - 60.8\,\mathrm{deg})\ 10^{-5}\ t^2\ ''$$

Besides the long term differences there are fluctuating but generally increasing oscillations at higher frequencies. Much of this difference is concentrated at or near the frequency of the Moon's mean anomaly and is in quadrature to (90 degrees out of phase with) the mean anomaly. This quadrature signal may be analyzed as a drift in the Moon's perigee. The interpretation is seen easily from the Equation of the Center for a Keplerian orbit. To first order terms,

$$v = l + 2e \sin l\ ,$$

where v = true anomaly, l = mean anomaly = mean longitude minus perigee, e = eccentricity. Differentiating gives

$$dv = dl + 2e \cos l\ dl$$

showing that the true longitude advances by the direct change plus a cosine term in the mean anomaly. If l is in error by a small constant amount dl, the longitude will be in error by the same amount plus the periodic term. The presence of this cosine component in the difference ephemeris suggests that there is a drift dl in the mean anomaly that is not the same as the drift dL in mean longitude.

An adjustment to the mean anomaly was estimated by fitting c and s in the expression

$$[c \cos l + s \sin l]\ t^4\ .$$

The cosine amplitude ct^4 of the fitted term was multiplied by $1/2e$ (from the Equation of the Center) and added to the secular polynomial for the mean anomaly. This procedure was iterated to find the differential correction

$$dl = -8.733 \cdot 10^{-6}\ t^4\ ''$$

to be added the mean anomaly l for input to the analytical theory. The adjustment dL was then *not* added to the mean anomaly.

A quadrature component was also present at the frequency of l', the mean anomaly of the Sun. This was almost entirely eliminated merely by substituting an expression for l' derived from Laskar's (1986) tenth-degree polynomials, in place of Bretagnon's polynomials. A small in-phase residual at this frequency was removed by adding the term

$$-0.944 \sin l'\ \ 10^{-5}\ t^3\ ''$$

to the Moon's longitude.

No other adjustments were applied to the theory. Delaunay argument F received the same correction dL as the longitude on the assumption, mildly tested, that the Moon's node did not drift appreciably. The maximum latitude difference noted, after the adjustments, was 12″. The Moon's geocentric distance differed by a maximum of 10 km. Figure 2 shows that the described adjustments had the salutary effect of reducing the overall longitude difference between theory and integration by an order of magnitude.

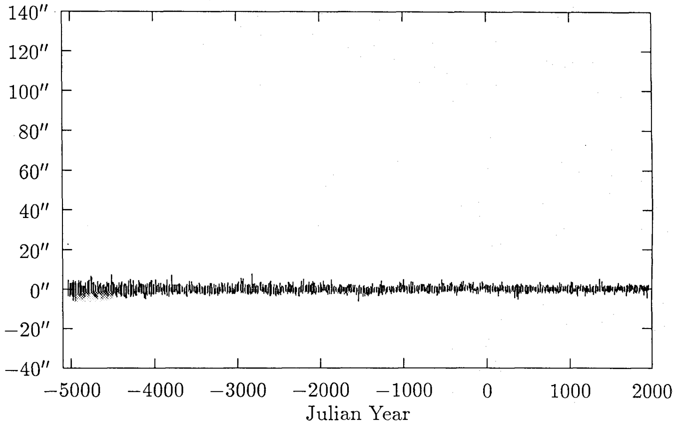

Fig. 2. Longitude difference, after adjustment of the analytical theory.

3. Discussion

The absolute accuracy of the adjustment coefficients given here is not high, partly because background noise from the myriad terms truncated from the theory affected the fitted estimates. The estimated coefficients do fit the data, but in addition to fitting the signal components for which models are explicitly provided, the least-squares procedure also attempts to fit components for which no models are given and which happen to correlate with the given models over the finite sample set. Deleting the earliest 240 years' data changes the coefficients of the fitted t^2 terms by $2 \cdot 10^{-5}$. Thus it is unlikely that the fitted terms have accuracy better than 0.1″ at 5,000 B.C.

The fit of the mean anomaly drift is affected by the fact that the Moon's eccentricity fluctuates widely due to the large perturbations by the Sun; this amplitude-modulates the mean anomaly error, and it introduces some noise into the least squares fit. Consequently it should be expected that the computed fit would be somewhat different if more terms were included or if the data set covered a larger span of time. Beyond the fitted interval, the post-fit longitude error grows to nearly four arc minutes by 9,000 B.C. and requires secular terms up to t^6.

The coarse sampling of this comparison study (400 days in most instances) implies that the observed maximum discrepancies have a statistical uncertainty, so the true maximum values are somewhat higher than indicated by the Figures. The least squares fit of terms having predetermined frequencies is not affected by undersampling, except for unlucky combinations of aliasing and signal frequencies. To guard against gross errors that might result from sampling effects, comparisons were also performed against two 20-year ephemerides that were sampled at 4-day intervals.

In their 1988 paper, the Chapronts compared their theory to JPL's LE51 Long Ephemeris. After adding a drift correction of $+0.0351t^2$ ″, they found a peak discrepancy of about 10″ at 1400 B.C. The spread found here is about half that, owing to the oscillatory corrections – in fact, the spread is not greatly different from the spread for the 20th century, with the chosen series truncation level of the theory. Also, the drift correction here is smaller — −4.4″ versus +43″ at 1500 B.C. — presumably because the present numerical integration was an attempt to reproduce the DE200, to which the Chapronts fitted their theory also. No drift correction in t^2 seems to be required.

The t^4 coefficients of the polynomials for L and l were the only numbers from analytical theory that were actually revised to obtain Fig. 2. Corrections of approximately the same amounts would be indicated even if the theory arguments were not phase shifted. It seems likely that there remains some discrepancy in parameters or approximating forms between the analytical theory and the numerical model. Specific questions about the numerical model are described in the next section.

4. Documentation of the numerical model

The numerical integration computer program used here follows completely the physics model by Newhall et al (1983) used for the Jet Propulsion Laboratory DE/LE series ephemerides. The actual ephemeris reconstructed was the DE118; a rotation to the DE200 coordinate system was accomplished by the transformation of Standish (1982). An early version of the computer program is generally available on computer networks under the name DE118.ARC. Some minor changes were made to that program, to increase the arithmetic precision from 64-bit double to 80-bit extended double precision format. To obtain nearly the same results from running the program on different computers, it was found necessary to provide explicit replacements for the trigonometric library functions; these vary somewhat from one computer system to another. Algorithms from Moshier (1989), adapted for extended precision, were used for this. Exact duplication of results is hampered by the fact that different software compilers may generate different sequences of arithmetic operations.

The attendant ephemerides of all of the planets agree, over the period covered by the DE200, to within JPL's estimates of their program's numerical error. Small model differences do exist in the oblateness computation. In particular, the exact method used by JPL to find the Earth's orientation (precession, nutation, and obliquity) was not available. For this integration, Laskar's (1986) 10,000-year precession and obliquity formulas were arbitrarily substituted into the oblateness calculations, without attempting to refit any of the other model parameters. At the initial 1969 epoch of the integration this Earth orientation produced a disagreement in the inertial acceleration of the Moon that is equivalent to tilting the Earth's axis by +0.21″ in longitude and −0.00083″ in obliquity. Upon actually integrating with a constant offset of this amount over 80,000 days, the longitude difference varied linearly with time and amounted to 0.001″. This is less than JPL's intended integrator drift ($2 \cdot 10^{-16}$ au/d in each axis) projected linearly in time, and two orders of magnitude less than JPL's measured estimate of their integrator drift due to step size.

For the nutation series, the IAU Theory of Nutation (I.A.U., 1980) was used, with only the leading constant coefficients of the first term involving the Moon's node. The full recommended polynomial for the mean node was used in the argument.

The final undocumented number used in this study is the Lunar moment of inertia ratio

$$C/MR^2 = 0.390689526131941$$

derived, with unwarranted precision, to fit the given initial conditions of the libration angles. All other parameter values were taken from 18-digit printouts of the actual DE118 kindly supplied by JPL.

Integrating with different obliquity and nutation formulas over the entire 7,000 year period produced a longitude drift that was linear for the first 2,000 years, then departed at a t^5 rate. It was possible to fit the analytical theory to that ephemeris without incorporating a t^5 adjustment to the Moon's mean longitude. Further computations will, it is hoped, provide useful estimates of the partial derivatives over the time period.

In this integration the arithmetic precision was 64 bits (80-bit IEEE type) throughout, and the step size was a constant 1/8 day. The order of the Adams-Bashforth-Moulton integrator was 12. Because of the smaller step size and higher precision arithmetic, the projection into the past may as much as three orders of magnitude more precise than the JPL computation. Although this level of precision is physically meaningless in itself, a high precision is needed to estimate the long-term effect of small parameter differences.

Figure 3 shows the trend of variation in the Moon's longitude with respect to the numerical integrator step size. It also depicts a direct comparison with the LE200 Moon. The curves correspond, from top to bottom, to step sizes of 7/32 day; the LE200; 3/16 day; and 1/8 day. Curves *a*, *c*, and *d* are for an 11th-degree integrator formula; the reference for all the curves is an integration with 1/16 day steps and a 12th-degree formula.

The progression of error with step size is predictable by numerical analysis of the integrator method. From such intercomparisons, a numerical drift of $10^{-6}t^{1.5}$ arcsec is estimated for the present computation. The LE200, curve *b* of Fig. 3, shows a fluctuating difference that is actually negative for the first 20 years, then changes sign and seems to approach a limit line that would be quite appropriate to its typical step size and integrator order. Although the effect of imperfectly matching model parameters is visible, the trend suggests that these differences are dominated by the larger effect of integrator step size after about 50 years.

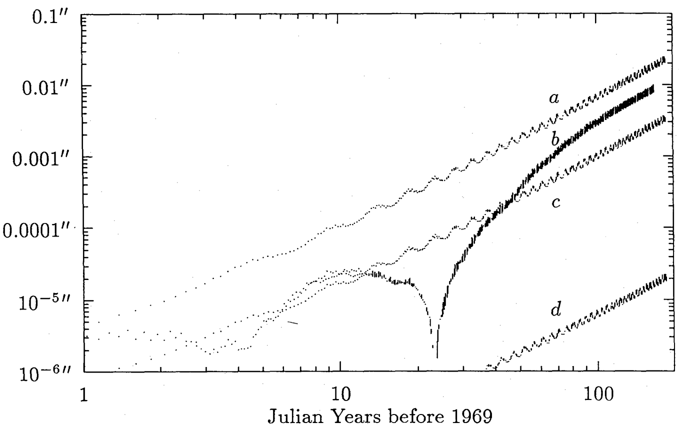

Fig. 3. Variations of the Moon's longitude arising from numerical integrator step size. Curve *b* represents the LE200.

5. Conclusion

The outcome of this study should be taken generally as in agreement with the analytical theory in the period from 5000 B.C. to the present. The result indicates that the Chapronts' longitude correction of $+0.0351t^2$ ″, derived from their comparison with the LE51, is better omitted than included. No reason was found to doubt either the Chapront theory or the JPL extrapolations; perhaps only the comparison itself was imperfect. A mean longitude discrepancy of order t^4 does remain, but the alternative adjustments discussed here amount to at most a few minutes of time in historical Lunar events.

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