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GLARE AND CELESTIAL VISIBILITY

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ABSTRACT

Glare is when a bright source of light hides a nearby fainter source and is caused by scattering of light within the atmosphere, the telescope, and the eyeball. In this paper I develop a model of the effects of glare on the visibility of astronomical sources as viewed with the human eye whether unaided or through a telescope. This model is tested and found to closely reproduce observations of lunar appulses, Galilean satellites, the Martian moons, well-known double stars, and lunar occultations. Glare calculations are then applied to a wide variety of situations of historical and astronomical interest. (1) Ancient Chinese lunar appulse reports have been used to determine the acceleration of the Earth's rotation. However, a detailed analysis shows that the results depend critically on the adopted weighting scheme and that the ancient Chinese reports contain too many errors to allow for a meaningful conclusion. (2) The Crucifixion eclipse of A.D. 33 April 3 (or any other partial lunar eclipse with less than $\approx$70% coverage by the umbra) would not have appeared "blood colored" because of the eclipse, since scattered white light from the penumbra will always mask the faint red light in the umbra. (3) Contrary to a statement by Aristotle, the visibility of stars from the bottom of deep wells or chimneys is worse than the visibility under an open sky. (4) Venus can be barely visible at inferior conjunction under optimal conditions with no optical aid. (5) An observing strategy is proposed which may lead to the discovery of several sungrazing comets per year by observations with small ground-based telescopes. (6) The effects of glare on the visibility of stars in loose open clusters is small. (7) The conditions for the detection of double stars are derived. (8) The model suggests various procedures for observers afflicted with glare, of which the most important is that the magnification should be pushed to the maximum usable power. (9) Various challenges are presented for observers, including the creation of a worldwide network to discover sungrazing comets.

Key words: visibility of celestial sources–scattered-light models

1. Introduction

Before the discovery of photography, all observations were made with the human eye. The visual process retained its importance for many studies well into the twentieth century and even now is used by large numbers of amateur observers. A knowledge of the capabilities of the human eye is needed for a good understanding of the history of astronomy, the older astronomical data, and modern science carried out by amateurs.

A variety of situations arise where the astronomical source of interest is close to a bright object such that the fainter source is hidden in the glare. The glare is caused by scattered light which forms an apparent bright background around the source of interest. In this paper I will quantitatively model the scattering and its effects on the visibility of point sources of light. This model will be applied to many situations of relevance to astrophysics, the history of astronomy, and amateur astronomy. The model predictions will be tested against data both new and in the literature.

2. Model

2.1 *Glare*

Glare is caused by scattered light from the nearby glare source creating an aparently brighter background against which the source of interest must be distinguished. The light can be scattered in several ways: by the atmosphere, by diffraction in the telescope, by the telescope mirror, and in the human eye.

2.1.1 *Atmospheric Scattering*

The atmosphere scatters light as it passes to the observer. The best-known effect is that the core of the image of a point source is broadened into a Gaussian shape with an angular size scale of typically 1 arc second. This is caused by many small angle scatterings which result in a Gaussian shape by the Central Limit Theorem. Less

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well-known is the aureole around all star images, where the surface brightness of the scattered light falls off as the inverse square of the distance from the glare source. The cause of the aureole is not known (King 1971).

The light in the aureole will suffer from the same extinction as the glare source itself because the optical path lengths will be the same for small angle scattering. Therefore, to a close approximation, the brightness of the aureole should be proportional to the transparency of the atmosphere along the line of sight to the glare source.

King 1971 gives a profile of the aureole which is a good fit to an inverse square law out to at least 5°. When he integrates this profile to infinity, he finds that roughly 5% of the light of the star is contained in the aureole. However, this must be only a lower limit because he does not have data out past 5° where multiple scattering is important. This multiple scattering will raise the profile substantially above the inverse square law. After all, the daytime sky far from the Sun is relatively uniform and certainly does not follow an inverse square law (cf. Rozenberg 1966). Therefore, much more light will be in the outer aureole as compared with King's calculation.

King's profile can be used to construct a composite profile of the scattered light from a point source. His surface brightness can be converted to nanoLamberts (nL) by the fact that a zero-magnitude star per square arc second is 3.41×10^{10} nL (Schaefer 1990b, Garstang 1989, or Allen 1976). I find

$$B_{\mathrm{atm}} = \{3.33 \times 10^{16}\,(0\overset{\circ}{.}000278/\theta_s)^2\,(I^*)\exp(-\theta^2/2\theta_s^2) + 1.14 \times 10^7\,(I^*)\,\theta^{-2}\}\,[(10^{-0.4k_vX} - 10^{-0.8k_vX})/0.18] \ . \quad (1)$$

In this equation and throughout this paper, I will always have surface brightnesses (B) in units of nanoLamberts and illuminances (I) in units of footcandles. B_{atm} is the surface brightness of the light scattered by the atmosphere. I^* is the illuminance of the glare source above the atmosphere and is related to the visual magnitude as

$$m_v = -16.57 - 2.5\log(I^*) \ . \quad (2)$$

θ is the angle in degrees between the glare source and the position of interest. θ_s is the size scale of the Gaussian seeing disk and is 1 arc second or $0\overset{\circ}{.}000278$ for King's data. The visual extinction coefficient is k_v, which might be 0.3 mag per air mass for a typical site. The optical path length through the atmosphere relative to the optical path length to the zenith is X in units of air masses and is merely the secant of the zenith distance for sources not near the horizon. In constructing equation (1), I have assumed k_vX is 0.3 mag for King's observations. Also, the second term in equation (1) is an underestimate for angles larger than 5° or so because of multiple scattering and is an overestimate for small angles with $\theta < \theta_s$ so that the divergence at zero is avoided.

2.1.2 *Scattering in the Eye*

Light is scattered when it passes through the human eye. This light will add to the apparent background brightness near a glare source. Holladay 1926 has performed the definitive study of the effects of scattering in the eye. His psychophysiological results and his physical interpretation have been confirmed by physical measurements of stray light in excised eyes of humans and animals (Boynton, Enoch & Bush 1954).

Holladay's equations (9) and (10) show that the surface brightness caused by light scattered in the eye is

$$B_{\mathrm{eye}} = 4.63 \times 10^7\,(I)\,(M\theta)^{-2} \ , \quad (3)$$

where I is the illuminance of the glare source as it enters the eye. The angle on the sky between the glare source and the direction of interest is θ in degrees, while M is the magnification of the image. For naked-eye observations M will equal unity. The combination $M\theta$ in equation (3) implies that the scattered light in the eye depends on the apparent angular separation as viewed by the eye. Equation (3) is valid for apparent angles larger than the resolution of the eye. I will be related to I^* as

$$I^* = IF_I \ . \quad (4)$$

The factor F_I is intended to account for effects like the absorption of light by the atmosphere or possibly by a telescope. For the case of naked-eye observations, F_I is given in equation (3) of Schaefer 1990b or equation (23) of this paper. For telescopic observations, F_I will be more complex as many correction factors are needed (see below or Schaefer 1990b).

2.1.3 *Diffraction*

When an observer is using a telescope, diffraction will cause a point source to spread out in a characteristic shape called the Airy diffraction pattern. A good description of the physics is given by Jackson 1962. His equation (9.113) is valid for optical telescopes pointing near to a glare source and gives the power per solid angle as

$$dP/d\Omega = P_i\,|J_1(\pi\theta D/\lambda)|^2\,(1/\pi)\,\theta^{-2} \ . \quad (5)$$

Here, P_i is the power incident on the telescope aperture of diameter D, λ is the wavelength of visual light, θ is the angle from the source in radians, and J_1 is a Bessel function of the first kind of order one. Equation (5) is normalized so that the integral of $dP/d\Omega$ over a hemisphere is P_i, which is to say that no power is lost.

$dP/d\Omega$ will be proportional to the surface brightness from diffraction (B_{dif}), while P_i will be proportional to I^*. Therefore, equation (5) can be rewritten with B_{dif}, I^*, and some yet-to-be determined proportionality constant. We know from equation (10) of Holladay 1926 that the illuminance from surface brightness of B over a solid angle $d\Omega$ in steradians will be

$$dI = 2.96 \times 10^{-7}\,B\,d\Omega \ . \quad (6)$$

The integral of dI over a hemisphere should be equal to I^*, and this fact sets the proportionality constant. Therefore, the surface brightness caused by diffraction will equal

$$B_{\mathrm{dif}} = 3.38 \times 10^{-6} (I^*) \, |J_1(\pi\theta D/\lambda)|^2 (1/\pi) \, \theta^{-2} \ , \quad (7)$$

with θ given in radians.

This idealized Airy diffraction pattern will be smeared out for two reasons. First, the light received by the telescope will not be a point source because of the atmosphere. Therefore, what the observer sees will be a convolution of equations (1) and (7). Second, the observer detects light from a range of wavelengths roughly from 4000 Å to 7000 Å. Therefore, the surface brightness will have to be integrated over wavelength, so that the Airy pattern will be smoothed out. Away from the core of the image, the Bessel function can be approximated as

$$|J_1(z)|^2 = (2/\pi z) \cos^2(f(z)) \ , \quad (8)$$

where $f(z)$ is some function of z (see eq. (9.2.1) of Abramowitz & Stegun 1964). The effect of the smearing will be to average over a range of z, with the average value of a cosine squared function being 0.5. Hence, the Bessel function term in equation (7) can be approximated as $\lambda/\pi^2\theta D$. With this approximation and by setting λ to 5500 Å, I get

$$B_{\mathrm{dif}} = 4.43 \times 10^{5} (I^*) \, D^{-1} \theta^{-3} \ , \quad (9)$$

where D is the diameter of the telescope aperture in inches and θ is in degrees.

2.1.4 *Scattering by the Mirror*

Telescope mirrors are imperfect reflectors in that dust particles and scratches on the mirror surface will scatter a small fraction of the light. Again, this scattered light will add an apparent background brightness that will reduce the visibility of stars near a bright glare source. The theoretical and experimental effects have been well discussed by Jean M. Bennett and Harold E. Bennett in a series of papers appearing in *Applied Optics* and the *Journal of the Optical Society of America* during the 1960s and 1970s.

For glass mirrors, Elson & Bennett 1979 distinguish the scattering as caused by long-range waviness and short-range random roughness. The short-range random roughness is caused by scratches and dust on the mirror. Since these imperfections have a size scale comparable to that of light, the resulting scattered light will have a very broad radiation pattern and a low surface brightness. The long-range waviness is caused by small imperfections in the shape of the mirror and will lead to the scattered light being concentrated near the glare source. This concentration implies that the long-range waviness will dominate the scattering for angles within several degrees of the glare source.

Bennett & Porteus 1961 present an equation for scattered power which reduces to

$$dP/d\Omega = P_i R_0 \, 16\,\pi^3 \sigma^2 a^2 \lambda^{-4} \exp(-[\pi a \theta/\lambda]^{-2}) \ , \quad (10)$$

for small angle scattering. R_0 is the reflectivity, which is typically 0.91 for a freshly aluminized mirror. The rms roughness of the surface is σ, with a typical value of 50 Å for an aluminized glass mirror. The standard deviation of a best-fit Gaussian to the autocovariance function of the mirror surface is a, with a typical value of 25 μm (Elson & Bennett 1979). The wavelength of visual light is taken as 5500 Å. The angle from the glare source is θ in radians. Elson & Bennett 1979 derive an equivalent equation.

The mirror will scatter some percentage of the incoming light. This fraction will be

$$f = \int (dP/d\Omega)/P_i \, d\Omega = R_0 (4\pi\sigma/\lambda)^2 \ . \quad (11)$$

For typical values of R_0 and σ, the fraction f will be 1%.

The surface brightness from light scattered by the mirror (B_{mir}) will be proportional to $dP/d\Omega$, while the illuminance of the star (I^*) is proportional to P_i. The constant of proportionality will be set by the requirement that the total scattered light must yield an illuminance that is the fraction f of the input illuminance;

$$\int dI_{\mathrm{mir}} = 2.96 \times 10^{-7} \int B_{\mathrm{mir}} \, d\Omega = fI^* \ . \quad (12)$$

Then the formula for B_{mir} will be

$$B_{\mathrm{mir}} = 3.38 \times 10^{6} (I^*) R_0 \, 16\,\pi^3 \sigma^2 a^2 \lambda^{-4} \times \exp(-[\pi a \theta/\lambda]^{-2}) \ . \quad (13)$$

On substitution of the typical values as given above, the formula simplifies to

$$B_{\mathrm{mir}} = 2.60 \times 10^{8} (I^*) \exp(-[\theta/0.^\circ 4]^{-2}) \ , \quad (14)$$

where θ is in degrees. This brightness should be added in once for every mirror in the optical train.

2.1.5 *Summary*

For the observer with unaided vision, the only sources of scattered light are from the sky and the eye. These two backgrounds are to be simply added to the normal sky background brightness (B_{sky}). So for naked-eye observers, the effective background has a brightness of

$$B_{\mathrm{eff}} = B_{\mathrm{sky}} + B_{\mathrm{atm}} + B_{\mathrm{eye}} \ . \quad (15)$$

The extra apparent background caused by the glare source will be roughly

$$B_{\mathrm{extra}} = 4.7 \times 10^{7} (I^*) \, \theta^{-2} \ . \quad (16)$$

The light scattered in the eye dominates over the light scattered in the atmosphere, so the apparent background can be reduced by a factor of four by simply occulting the glare source.

The case for telescopic observations is more complicated. The various backgrounds are to be added together to give the effective background, but the light scattered in the eye must be corrected because it is scattered after passing through the telescope. Thus,

$$B_{\mathrm{eff}} = B_{\mathrm{sky}} + B_{\mathrm{atm}} + B_{\mathrm{dif}} + B_{\mathrm{mir}} + F_B B_{\mathrm{eye}} \ . \tag{17}$$

The value of F_B is given in equation (26) below or see Schaefer 1990b. Generally, the contribution from the scattering by the mirror will be much smaller than other contributions and can be ignored. Then the extra effective background caused by the glare source is

$$B_{\mathrm{extra}} = B_{\mathrm{atm}} + B_{\mathrm{dif}} + F_B \, B_{\mathrm{eye}} \ . \tag{18}$$

which is about

$$B_{\mathrm{extra}} = (4.7 \times 10^7 + 4.4 \times 10^5/[\theta D])\,(I^*)\,\theta^{-2} \ . \tag{19}$$

This equation uses the fact that $F_B/(F_I M^2)$ is roughly 0.76 (see below). This equation is valid for θ between roughly $0\overset{\circ}{.}0015$ and 5°. For many practical situations, the diffraction component is small enough to be ignored.

The background sky brightness will be composed of contributions from many sources, with

$$B_{\mathrm{sky}} = B_{\mathrm{night}} + B_{\mathrm{twi}} + B_{\mathrm{city}} \ . \tag{20}$$

B_{night} is the natural sky brightness of the dark sky including airglow, unresolved stars, and zodiacal light. It can be calculated from the observed naked-eye limiting magnitude for stars near the zenith (Schaefer 1990b) or can be estimated by comparison with the data tabulated in Krisciunas 1990, Pilachowski et al. 1989, Garstang 1989, Walker 1970, or Walker 1973. A typical value is 136 nL for a normal dark sky, 65 nL for the best skies in the world, and 1400 nL for a full-Moon sky. B_{twi} is the sky brightness of the twilight sky and can be estimated by equation (1) from Schaefer 1987 or from the data tabulations of Koomen et al. 1952. B_{city} is the sky brightness associated with light pollution from man-made light sources and can be calculated as in Garstang 1989 or as estimated by a comparison with his tabulated data.

2.2 *Visual Detection Thresholds*

2.2.1 *Direct Vision*

The visibility of point sources of light is a topic that has been extensively studied. The basic equation for the threshold of visibility (Hecht 1947) is

$$I = C\,(+ [KB]^{0.5})^2 \ , \tag{21}$$

where I is the illuminance of the point source in footcandles and B is the background surface brightness in units of nanoLamberts (nL). The constants C and K will depend on whether day vision or night vision is being used:

$$C = 10^{-8.35} \ , \ K = 10^{-5.90} \quad \text{if } \log(B) > 3.17 \ , \tag{22a}$$

$$C = 10^{-9.80} \ , \ K = 10^{-1.90} \quad \text{if } \log(B) < 3.17 \ . \tag{22b}$$

The experimental conditions leading to these equations were binocular vision, natural pupils, observer's choice fixation, and no atmospheric absorption for young adults with average vision who were experienced at looking for the point sources of known position.

The relation between the apparent illuminance and the visual magnitude of the source is given by equations (2) and (4). For naked-eye observations,

$$F_I = F_e F_c F_s \ , \tag{23}$$

where the values for F_e and F_c are given by equations (28) and (34). F_s is a correction factor to allow for variations in the sensitivity for detecting point sources from observer to observer. Normally, F_s will equal unity, but a very sensitive observer might have F_s as low as 0.2 or even 0.1 (Schaefer 1990b). F_s can be measured for an observer if the sky brightness and extinction coefficient are known and the observer measures the zenith limiting magnitude for detecting a star (see eq. (18) of Schaefer 1990b). The correction for sensitivity in detecting a source near threshold should not be applied when calculating the apparent brightness of the glare source.

The relation between the effective background brightness and the brightness as perceived by the eye is

$$B_{\mathrm{eff}} = B\,F_B \ . \tag{24}$$

The correction factor F_B for naked-eye observations will merely be the factor F_c, as given in equation (34) below. This color factor accounts for the fact that the scotopic vision of the eye has a different color sensitivity than the photopic vision. For a normal daytime or nighttime sky, the F_c factor will be roughly 0.5. For telescopic observations, the F_B factor is more complicated with other correction terms.

For naked-eye observations, the procedure to calculate the limiting magnitude near a bright source is simple. First, estimate or calculate B_{night}, B_{twi}, and B_{city} as described in the previous section. These values should be summed to get B_{sky}. Second, use equations (2), (4), (23), (28), and (34) to determine the I and I^* for the bright glare source. Third, calculate B_{atm} with equation (1) and B_{eye} with equation (3). Fourth, add all the brightnesses together (eq. (15)) to get the effective background brightness. Fifth, use equations (21), (22), (24), and (34) to calculate the perceived illuminance threshold. Lastly, use equations (2), (4), (23), (28), and (34) to get the limiting magnitude.

The required input for this calculation is m_v, θ, B_{sky}, k_v, X, and F_s. In general, the variations in the last four inputs are unimportant and the following typical values may be selected: $B_{\mathrm{sky}} \simeq 136$ nL, $k_v \simeq 0.3$ mag per air mass, $X \simeq 1$, and $F_s \simeq 1$.

2.2.2 *Telescopic Observations*

When a telescope or binoculars are used, the F_I and F_B will be different from the case for unaided vision. The necessary equations are presented in Schaefer 1990b. In summary,

$$I^* = I F_I \ , \tag{4}$$
$$B_{\rm eff} = B F_B \ , \tag{24}$$
$$F_I = F_{\rm b} F_{\rm e} F_{\rm t} F_{\rm p} F_{\rm a} F_{\rm r} F_{\rm SC} F_c F_{\rm s} \ , \tag{25}$$
$$F_B = F_{\rm b} F_{\rm t} F_{\rm p} F_{\rm a} F_{\rm m} F_{\rm SC} F_c \ . \tag{26}$$

The correction factors are

$$F_b = 1.41 \ , \tag{27}$$
$$F_{\rm e} = 10^{0.4} k_v X \qquad \text{if } \log(B) > 3.17 \ , \tag{28a}$$
$$F_{\rm e} = 10^{0.48} k_v X \qquad \text{if } \log(B) < 3.17 \ . \tag{28b}$$
$$F_{\rm t} = 1/T_{\rm tel} \ , \tag{29}$$
$$F_{\rm p} = \max(1, [D/MD_{\rm e}]^2) \ , \tag{30}$$
$$F_{\rm a} = (D_{\rm e}/D)^2 \ , \tag{31}$$
$$F_{\rm r} = \max(1, 2M\theta_{\rm s}/\theta_{\rm CVA}), \tag{32}$$
$$F_{\rm SC} = \min(1, (D_{\rm e}M/D)[1 - \exp(-0.026\{D/M\}^2)]/[1 - \exp(-0.026 D_{\rm e}^2)]) \qquad \text{if } \log(B) > 3.17 \ , \tag{33a}$$
$$F_{\rm SC} = \min(1, [1 - (D/12.4M)^4]/[1 - (D_{\rm e}/12.4)^4]) \qquad \text{if } \log(B) < 3.17 \ , \tag{33b}$$
$$F_c = 1 \qquad \text{if } \log(B) > 3.17 \ , \tag{34a}$$
$$F_c = 10^{-0.4}(1 - [B - V]/2) \qquad \text{if } \log(b) < 3.17 \ , \tag{34b}$$
$$F_{\rm m} = M^2 \ . \tag{35}$$

The value for $F_{\rm s}$ is given in the text following equation (23) and is unity for an average observer. As before, X is the optical path length in air masses and k_v is the visual extinction coefficient in units of magnitudes per air mass. $T_{\rm tel}$ is the transmission of the telescope while considering effects like the reflectivity of the mirrors and eyepiece and the obstruction by the secondary mirror. D is the diameter of the aperture of the telescope in millimeters. M is the magnification of the telescope. $D_{\rm e}$ is the diameter of the pupil of the observer's eye in millimeters as given in Schaefer 1990b or Holladay 1926. $\theta_{\rm s}$ is the Gaussian width of the seeing disk. $\theta_{\rm CVA}$ is the critical visual angle, a value similar to the resolution for the eye. The resolution will vary with the brightness of the background as given by Blackwell 1946, but note that the glare source is well above threshold so that the relevant $\theta_{\rm CVA}$ for this source will be near the minimum. $\theta_{\rm CVA}$ varies from 42″ for $B > 10^7$ nL to around 600″ to 900″ for very dim backgrounds. $\theta_{\rm CVA}$ will also vary inversely with the observer's Snellen ratio, so that an observer with a Snellen ratio of 20/10 will have the minimum $\theta_{\rm CVA}$ equal to 21″. The $(B - V)$ value will be equal to the color index and is tabulated for many stars in standard references. The color index of the background should be used for calculating F_c in equation (24) and this will be roughly 0.7 for the day and night sky or equal to the $(B - V)$ of the glare source if glare dominates.

Equation (32) has been generalized and modified from equation (7) of Schaefer 1990b. The inclusion of the $\theta_{\rm CVA}$ term allows the equation to work at any light level. The change to a direct proportionality constant is indicated because many cases of interest in this paper involve a greatly overresolved star disk. (In virtually all of the observations examined in Schaefer 1990b, the seeing disk is either unresolved or marginally resolved so that the square-root law is valid for the narrow transition region involved.)

For telescopic observations, the procedure to calculate the limiting magnitude near a bright source is similar to that used for direct-vision observations. First, estimate or calculate $B_{\rm night}$, $B_{\rm twi}$, and $B_{\rm city}$ as described in an earlier section. These values should be summed to obtain $B_{\rm sky}$. Second, use equations (2) and (4) to determine the I and I^* for the bright glare source. The value for F_I must be determined from equations (25) and (27)–(34). Remember that the relevant value of $\theta_{\rm CVA}$ in equation (32) will be for the brighter glare source. Third, calculate $B_{\rm atm}$ with equation (1), $B_{\rm eye}$ with equation (3), $B_{\rm dif}$ with either equation (7) or (9), and $B_{\rm mir}$ with either equation (13) or (14). Fourth, add all the brightnesses together (eq. (17)) to get the effective background brightness. The value for F_B must be determined from equations (26)–(27), (29)–(31), and (33)–(35). Fifth, use equations (21) and (22) to calculate the perceived illuminance threshold. Note that the F_I value must be recalculated because the relevant $\theta_{\rm CVA}$ has changed. Finally, use equations (2) and (4) to get the limiting magnitude.

The required input for this calculation is m_v, θ, D, M, $B_{\rm sky}$, k_v, X, $F_{\rm s}$, $T_{\rm tel}$, $D_{\rm e}$, $\theta_{\rm s}$, $\theta_{\rm CVA}$, and $(B - V)$. In general, the variations in the last eight inputs are not critical and the following typical values may be selected; $B_{\rm sky} \simeq 136$ nL, $k_v \simeq 0.3$ mag per air mass, $X \simeq 1$, $F_{\rm s} \simeq 1$, $T_{\rm tel} \simeq 0.8$, $D_{\rm e} \simeq 7$ mm, $\theta_{\rm s} \simeq 1$ arc second, $\theta_{\rm CVA} \simeq 42$ arc seconds for bright backgrounds and $\theta_{\rm CVA} \simeq 600$ arc seconds for dark backgrounds, and $(B - V) \simeq 0.7$.

3. Comparison of Model with Data

Any complicated theoretical model must be compared with suitable data for validity before the model should be accepted. This section will compare a wide variety of data with the glare model. Some data have been obtained for this paper while others have been taken from the literature.

Many aspects of the model have already been exhaustively tested with astronomical data. The equations relating to visibility of point sources have been tested at both low and high brightness levels (Schaefer 1986, 1987, 1991; Tousey & Hulbert 1948). Similar equations for the visibility of an extended source have been tested in Schaefer 1988 and Doggett & Schaefer 1989, 1991. The visibility equations and the telescope equations were tested in Schaefer 1990b. In all cases the observations were closely predicted by the model. The vast amounts of analyzed data, the large number of observers, and the wide variety of the experiments all go to show that the equations are both accurate and of wide validity. This good agreement is a great relief to me since the equations have no "fudge factors" that can be varied to improve the fit.

Nevertheless, the validity of the glare equations should be checked against data.

3.1 *Lunar Appulses*

A lunar appulse is when the Moon passes close to an object without an occultation. Appulses are regularly noted in such periodical publications as *The Astronomical Almanac*, *Sky and Telescope* magazine, and the *Observer's Handbook*. An appulse of the crescent Moon and Venus has been attributed as the origin of the Islamic star and crescent symbol (Schaefer 1990c).

If a planet or star passes close to the Moon, then glare might render the object invisible to the unaided eye. The most important factors in determining the visibility are the magnitude of the star, the phase of the Moon, and the distance from the Moon. Other factors are generally of lesser importance. The procedure presented in Section 2.2.1 can be used to predict the limiting magnitude for detecting stars near the Moon. The total visual magnitude of the Moon is given by

$$m_v(\text{Moon}) = 0.026\alpha + 4 \times 10^{-9}\,\alpha^4 - 12.73 \ , \quad (36)$$

where α is the phase angle of the Moon (equal to the apparent angular distance of the Moon from the antisolar point) in degrees (Allen 1976). I will take the moonlight as if it all comes from a point source at the Moon's center. Typical results are presented in Figure 1. For an object close to the full Moon, the threshold is about zeroth magnitude, while for the quarter Moon, the threshold is about second magnitude.

I have collected 161 observations from nine nights of the visibility of stars and planets near the Moon. The observers were James Lochner and Barbara Lochner (in Los Alamos, New Mexico) and Martha W. Schaefer and myself (in Bowie, Maryland). The procedure was to choose a clear night when the Moon was well away from the horizon, to fully dark adapt the eyes to the ambient lighting, and to record all stars within ten degrees of the Moon that could be seen. The identities of the visible stars were taken from an examination of a star chart. At this time, stars not detected within the search radius were also identified. The visual magnitudes and distances from the Moon's center for all identified stars were determined from *The Astronomical Almanac,* as were the phase and altitude of the Moon. The observer's F_s was taken from the zenith limiting magnitude on a clear moonless night. The seasonal extinction coefficients for Washington, D.C., and New Mexico were taken from Flowers, McCormick & Kurfis 1969, Abbott et al. (1908–54), Yamamoto, Tanaka & Arao 1968, Joseph & Manes 1971, Angstrom 1961, and Husar & Holloway 1984.

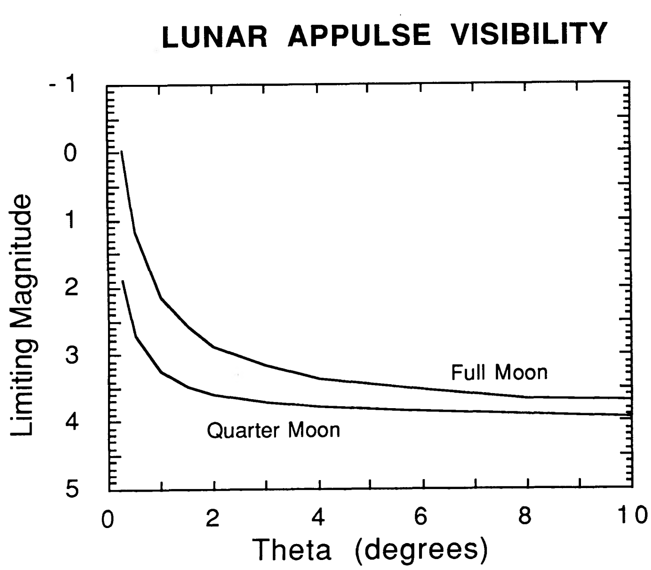

FIG. 1–The visibility of stars and planets near the Moon. The two curves display the limiting magnitude for the detection of point sources with the unaided eye of an average observer as a function of the sources distance from the Moon's center for a full Moon and for a quarter Moon.

I used the model to predict the limiting magnitude for the appropriate positions and conditions of all 161 observations. The magnitude of each star was then differenced with the prediction, and a histogram of the differences was constructed both for stars seen and not seen (see Fig. 2). Ideally, the histograms should show a sharp drop at a magnitude difference of zero; that is, that no observations will disagree with the model. And indeed, both histograms show sharp drops at zero, which proves that my glare model has no significant biases for this situation. That is to say, my model has no significant systematic errors which yield predictions that are too optimistic or pessimistic.

However, a small fraction of the data violates the model prediction by a small amount. These violations are distributed like a half-Gaussian with a standard deviation of roughly 0.35 mag. I interpret this standard deviation to be the uncertainty in my model results. In other words, my predicted thresholds have a typical error of a third of a magnitude. (The observations of telescopic limiting magnitudes in Schaefer 1990b yield a similar uncertainty.)

3.2 *Galilean Satellites*

The four Galilean satellites around Jupiter are all naked-eye brightness at opposition, with visual magnitudes ranging from 4.6 to 5.6. As such, they should be readily visible to the naked eye if it were not for the glare from Jupiter. Normal people under optimal conditions are unable to see any of the four satellites.

However, the literature contains frequent reports of especially sharp-eyed observers being able to spot from one to four satellites. For example, Xi 1981 found a report from 364 B.C. that Gan De reported a companion star following Jupiter. Hogg 1976 reports that "One well-

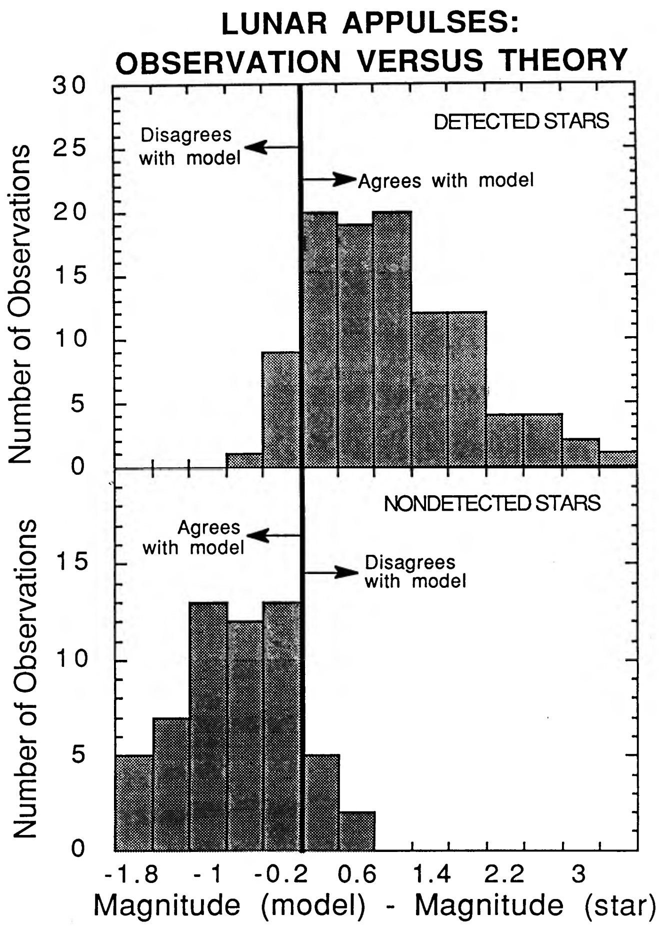

FIG. 2–A comparison of theory and observations for the 161 observations of star visibility near the Moon. For each datum, the glare model was used to predict the limiting magnitude. The figure shows histograms for the difference between the magnitude of the star and the theoretical threshold for the two cases where the star was and was not detected. Both histograms show sharp drops at magnitude differences of zero, which shows that the glare model does not have significant systematic errors. The few observations that disagree with the predictions are distributed as a Gaussian with a standard deviation of roughly a third of a magnitude. I take this to be a measure of the accuracy of my model. The falloff of the histogram far from zero on the side of agreement is because the Moon encounters few bright stars (compared to fainter ones) and because the observers did not bother to record the many very faint stars that it was hopeless to look for.

documented claim is that of Lieutenant Elliot Brownlow . . . [who had] exceptional vision, he was able with the unaided eye to make a sketch of the positions of the moons of Jupiter which was immediately verified by another observer using a telescope". Bobrovnikoff 1989 reports that "The 'eagle-eyed' [Harold] Dawson, a well known astronomer of the last century, could easily resolve the disk of Jupiter (40 arc-seconds in diameter) as well as that planet's Galilean satellites". Stephen J. O'Meara 1991 (private communication) reports several occasions when he was able to spot Ganymede and Callisto, and I have confirmed from *The Astronomical Almanac* that the moons were at elongation (on the stated side) at his times of observation. O'Meara is well-known to be an exceptionally keen-eyed observer with F_s equaling 0.22 (Green 1985; Schaefer 1990b).

So my model of glare should predict that normal observers (with $F_s = 1$) cannot see the Galilean satellites under optimal conditions, while the very best observers (say with $F_s = 0.15$) can see them all. At opposition, the m_v of Jupiter is -2.6 and I will adopt $B_{sky} = 136$ nL and $k_v X = 0.2$ mag. The calculated limiting magnitudes for a normal and keen-eyed observer are presented as a function of θ in Figure 3. The opposition visual magnitudes and range of θ for each satellite are presented as horizontal lines. My model predicts that a satellite should be visible at times if its horizontal line protrudes into the region to the upper right of the model limiting magnitude curve.

The model does indeed predict that normal observers cannot hope to see the Galilean satellites under even the best of conditions. Also, keen-eyed observers should be able to spot all four moons at elongation under optimal conditions. So, my model of glare passes this observational test.

3.3 *Phobos and Deimos*

Phobos and Deimos are the two faint moons around Mars. Their detection with medium-sized telescopes is a notorious problem relating to glare. During the favorable opposition of 1988, *Sky and Telescope* ran an appeal for

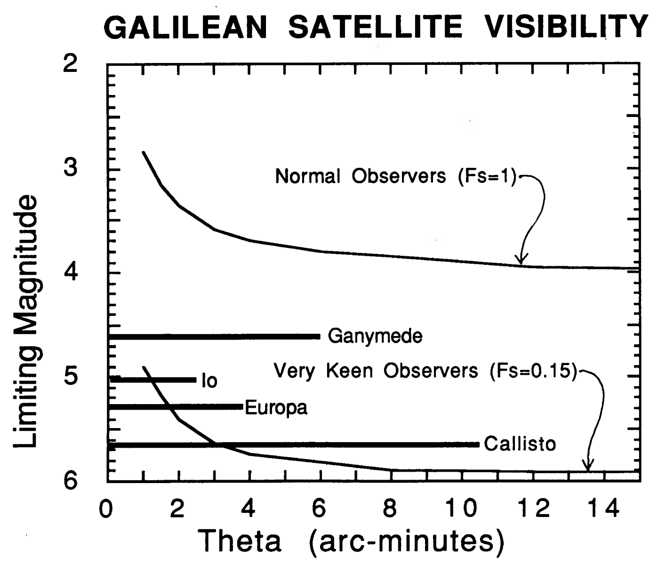

FIG. 3–The visibility of the Galilean satellites around Jupiter for a naked-eye observer. The calculated magnitude limit of the model for optimal conditions as a function of angular distance from Jupiter is given as a curve for an observer of normal sensitivity (with $F_s = 1$) and for an exceptionally keen-eyed observer (with $F_s = 0.15$). Each of the four moons will have a constant magnitude as it moves to elongation, as shown by the labeled horizontal lines. A moon should be visible when it is in the region to the upper right of the threshold curve. It comes as a surprise that Io is as easily visible as Callisto at elongation. The model accurately shows that normal people cannot see the moons, whereas the very best observers can see all four Galilean satellites.

amateur observers to try to hunt for the moons of Mars (MacRobert 1988). He summarized historical observations in which Deimos was visible in a telescope as small as 7.3 inches of aperture. MacRobert 1990 summarized the reports from the 1988 opposition which had Deimos being glimpsed in a 6-inch (15-cm) telescope, while Phobos needed an 8-inch (20-cm) telescope. However, in general, the telescope had to have an aperture of at least 10 (25 cm) inches for most observers.

These reports of the smallest aperture required to view each Martian moon provide a test for my model of glare. For the 1988 opposition, the visual magnitude of Mars was -2.7, while Phobos and Deimos had magnitudes of 10.5 and 11.5. At elongation, the angular distance from the center of Mars was 34 arc seconds and 84 arc seconds, respectively. I will assume that the observer uses the highest usable magnification, which is traditionally taken to be 50 power per inch of aperture. I will assume B_{sky} = 136 nL, $k_v X = 0.3$ mag, $T_{tel} = 0.8$, $D_e = 7$ mm, and $\theta_s = 1$ arc second. The effective background brightness implies θ_{CVA} = 100 arc seconds for Phobos and θ_{CVA} = 200 arc seconds for Deimos.

With these inputs I can calculate the smallest aperture required to detect the moons for a normal observer (F_s = 1). For Phobos, the minimum D is 10.0 inches. For Deimos, the minimum D is 10.5 inches. For Phobos to be seen in an 8-inch telescope, F_s must be 0.8. For Deimos to be seen in a 6-inch telescope, F_s must be 0.6.

The model predictions of the visibility of the Martian moons is in good agreement with observation for the 1988 opposition. That is, both data and theory show both moons to be generally visible only in telescopes 10 inches or larger, while keen-eyed observers can spot the moons in telescopes as small as 6 inches in aperture.

3.4 *Sirius, Procyon, and Mizar*

The visibility of a faint member of a double star can be strongly affected by glare. That is, a bright primary star can mask a faint secondary star in its glare. My model for glare can make predictions of whether the secondary star can be detected. (Note that for very close double stars the additional issue of resolution must be considered.) Unfortunately, little data have been published concerning systematic tests toward determining the visibility of faint secondaries as a function of either aperture or magnification. The best that I could find is limits on the required apertures for detecting the companion star to Sirius, Procyon, and Mizar.

Sirius and its famous white-dwarf companion star provide one of the most notorious cases of glare in astronomy. Burnham 1978 describes a systematic experiment in 1962 where he could stop down a large refractor to an aperture of 6 inches and still see the Puppis star. At the time, the Pup star was 9.7 arc seconds away from Sirius. In the middle 1970s, *Sky and Telescope* ran a series of letters and reports from observers concerning the minimum required aperture for the time of elongation (with a separation of 11.3 arc seconds). The conclusion was that the companion could be detected in a telescope as small as 6 inches in aperture (Ashbrook 1975). In this observation, the star was detected at a power of 225× (Chauvin 1973). D. Gellera is reported to have detected the companion with a telescope of 4.3-inch (11.0-cm) aperture in 1983 when the separation was 9.5 arc seconds (Argyle 1986).

My theory of glare can be used to predict the minimum required aperture for each of the dates with actual measurements. I have assumed all of the typical values suggested at the end of Section 2. (Note that the diameter of the pupil is not significantly contracted by even the light of Sirius, as shown from the formulae in Holladay 1926.) I further assume that the observer employs the maximum usable magnification of 50 power per inch of aperture unless the power is actually stated. The visual magnitude of the Pup star is 8.8 mag. The predicted minimum aperture for Burnham's 1962 observation is 4.7 (11.9 cm) inches as compared to his experimental value of 6 inches. The predicted minimum aperture for Chauvin's 1973 observation is 5.0 (12.7 cm) inches as compared to his experimental value of 6 inches. The predicted minimum aperture for Gellera's 1983 observation is 4.9 (12 cm) inches as compared to his experimental value of 4.3 inches. In all cases, the difference between theory and observation corresponds to less than 0.2 mag in the threshold.

Procyon is another double star similar to Sirius in that a faint white-dwarf companion is hidden near to a bright primary. The visibility of the companion is another notorious problem caused by glare, such that only large telescopes have ever detected Procyon B (Burnham 1978; Hartung 1968). Aitken 1932 presents a list of positive and negative detections for times when the separation was 4.6 arc seconds on average. The companion was visible in the Yerkes 40 inch (1 m), the Lick 36 inch (91 cm), the Greenwich 28 inch (71 cm), the U.S. Naval Observatory 26 inch (66 cm), and the Berlin 25.6 inch (65 cm), while on other occasions it was not seen with the Yerkes 40 inch and the Lick 36 inch. Apparently, the minimum aperture required for detection under good conditions is roughly 25.6 inches.

The visual magnitude of Procyon B is 10.8 and the other observational parameters are as assumed above. The model of glare predicts that the smallest aperture will be 25.0 inches, in good agreement with the observations.

Mizar and Alcor are a bright pair of stars in the handle of the Big Dipper that form a well-known naked-eye double star. The two stars have visual magnitudes of 2.1 and 4.0 with a separation of 11.8 arc minutes. The visibility of Alcor has traditionally been a test for eyesight, while medieval Islamic sources said that "people tested their eyesight with this star" and gave the name of "The Test" to

Alcor (Burnham 1978). In modern times Alcor is sometimes considered to be easily visible with the unaided eye (Consolmagno & Davis 1989). Apparently, Alcor is marginally visible for the normal observer with no optical aid.

For an assumed $F_s = 1$, $B_{sky} = 136$ nL, and $k_v X = 0.3$ mag, I calculate that the limiting magnitude for the naked eye at the position of Alcor is 3.99 mag. Thus, Alcor is almost exactly at the predicted threshold, just as indicated by observations.

3.5 *Lunar Occultations*

The visibility of a star at the time of occultation by the moon is strongly affected by the glare of moonlight. I have collected 1621 observations of lunar occultation from observers around the world. The analysis of this database will be reported in another paper (Schaefer, Bulder & Bourgeois 1991). The conclusion from this study is that the model for glare accurately reproduces the observations.

3.6 *Summary*

The model of glare has been tested with visibility data for stars near the Moon; the Galilean satellites; Phobos and Deimos; Sirius, Procyon, and Mizar; and lunar occultations. In all cases, the model predictions closely matched the observations. The wide variety of situations and the large number of observers show that the glare equations are of general validity. The model predictions for the threshold are found to have a typical uncertainty of a third of a magnitude.

4. Applications

4.1 *Ancient Chinese Appulse Data*

The visibility of planets near the Moon is of astrophysical interest. The application is to modern analyses on ancient Chinese appulse data which are seeking to measure the rate of acceleration of the Earth's rotation over long time scales. Hilton, Seidelmann & Liu (1989, 1991) analyze a selection of 58 naked-eye records of occultations of the planets by the Moon from Chinese dynastic histories from 68 B.C. to A.D. 575. Their motivation is to measure the rate of change of the Earth's rotation in ancient times. Their procedure was to calculate the closest approach distance for each event as a function of the change of the rotation rate. They then sought the acceleration that minimized the weighted sum of the distances.

A worrisome feature of their analysis is that their deduced acceleration depends grossly on the details of the data-weighting scheme chosen. That is, for their six weighting schemes, the deduced value ranges from 12.6 to 49.9 s/cy^2. The claimed uncertainty for each weighting scheme ranges from 0.6 to 3.3 s/cy^2. They present plausible reasons for selecting one particular value, but this happens to be close to the modern measures so that their a posteriori choice can be questioned.

For 39 of the ancient reports, the minimum separation between the Moon's center and the planet was less than the Moon's radius (i.e., an occultation occurred just as reported) for any reasonable acceleration. These reports do not contain useful information about the Earth's rotation because an occultation is expected in all cases. The remaining 19 events might have been either an appulse or occultation at closest approach.

Thirteen of these 19 relevant events are found to not be occultations (as reported by the ancient Chinese) for any reasonable acceleration parameter. Hilton, Seidelmann, and Liu hypothesize that in all these cases the planet was lost in the glare of the Moon so that it would have appeared as if an occultation had occurred. As support for this, they point out that most of the appulse events involve the faint planets (Mercury, Mars, and Saturn).

However, this explanation is not satisfactory because all the planets of antiquity should remain visible at all distances from the Moon. Venus and Jupiter are always much brighter than 0 mag, so they will always be easily visible during an appulse. At opposition, Mars is roughly -2 mag and so is visible next to a full Moon, while closer to conjunction Mars can be as faint as 2 mag, which is still adequate to spot against the crescent Moon. Mercury is always fainter than -2 mag but can only be seen against a thin crescent which has little effect, especially as compared to the inevitable twilight. Saturn varies little with elongation and is as bright as a first-magnitude star, so that it should easily be visible at phases away from full and should be a difficult object next to a full Moon. The point is that the theory of glare for lunar appulses (see Section 3.1) shows all naked-eye planets (with the marginal exception of Saturn near opposition) to be easily visible right up to the lunar limb. Detailed calculations for all 19 appulse/occultation events from the Chinese reports show that the planet was at least 0.7 mag above threshold. Fifteen of these 19 events have the planet more than 2.0 mag brighter than threshold. Hence, the explanation for the appulse events being reported as occultations because of lunar glare is wrong.

So other explanations have to be invoked, including the possibility of thin clouds, an observing site away from the capital, or an observation of an appulse wrongly recorded as an occultation. Another explanation could be that the ancient Chinese considered a close appulse to be the same thing as an occultation or that their choice of words did not make the correct distinction. Finally, the ancient reports all were transcribed by court historians who might have been ignorant of or uncaring for the distinction. For all of these explanations, 13 out of 13 observations of definite appulse events must have suffered from some such trouble. Thus, these observations should not be used for any analysis because they are known to have large systematic errors of unknown cause.

This leaves only six critical observations on which to base an acceleration of the Earth's rotation. These six critical reports are internally inconsistent in that some require a high acceleration while others require low acceleration. There is no reason to think that these six might not have suffered from the same systematic errors that the 13 appulses suffered from.

The analysis of the ancient Chinese lunar appulse data is fraught with many difficulties: (1) The answer is greatly sensitive to the data-weighting scheme chosen. (2) Thirteen reports must have suffered from some large systematic error such that the report is useless for modern calculations. (3) The entire analysis comes down to using only six reports. (4) These six critical events are internally inconsistent in the deduced limits on the Earth's rotation. Therefore, I conclude that the ancient Chinese lunar/planetary occultation lists are not reliable sources of information for calculations of the Earth's rotation in ancient times.

4.2 *The Crucifixion Eclipse*

The chronology of the New Testament as well as some theological points hinge on the date of the Crucifixion of Jesus Christ. Among modern researchers (Maunder 1911; Fotheringham 1924; Ogg 1962; Maclean & Grant 1963; Finegan 1964; Doggett 1976; Hoehner 1977; Humphreys & Waddington 1983), there is virtually universal agreement that Christ died on a Friday afternoon that was either the fourteenth or fifteenth day of the Jewish lunar month of Nisan probably in the years A.D. 28 to 33 and certainly in the years A.D. 26 to 36. Astronomical calculations of the visibility of the thin crescent Moon then show that only two candidate dates (A.D. 30 April 7 and A.D. 33 April 3) meet the requirements. Recently, Humphreys & Waddington (1983, 1989) have revived the old claim (dating back at least to the sixteenth century, see Kokkinos 1989) that a partial lunar eclipse on A.D. 33 April 3 might be used to argue that this candidate date was the actual date of the Crucifixion. This old claim has been thoroughly refuted (Fotheringham 1934; Kokkinos 1989; Schaefer 1990a). One of the main points of the refutation is that the partially eclipsed Moon would not appear "blood colored" as suggested by the proponents.

As an observational fact, it is well established (see Schaefer 1990a for references) that a partially eclipsed Moon with a significant portion of the penumbra showing will display no red coloration. However, the theoretical reason for this observational fact has never been given. The closest explanation is the calculations in Schaefer 1990a where the visibility of the coloration "is like trying to spot an 8 watt red light bulb next to a megawatt searchlight". The idea is that glare must somehow be involved, but the mechanism was not known.

Glare is indeed the reason the reddish color of a partial eclipse is hidden. The mechanism is that scattered white light from the bright penumbral regions will appear superposed on the faint red light from the umbral regions so that the resulting color has an undetectably small red tinge. The model of glare will allow for this statement to be quantified.

The brightness can be calculated by integrating the contribution from all regions of the penumbra for any particular point in the umbra. I have numerically performed the integral for the case where the edge of the umbra crosses the center of the Moon. At the deepest part of the umbra (on the limb of the Moon), the scattered light has a surface brightness of 8.2×10^5 nL. Halfway between this position and the center of the Moon, the scattered brightness is 1.6×10^6 nL. Presumably, it is possible to hide the penumbral regions from direct view (although this is usually not done) so that these brightnesses will be a third as bright from the atmospheric scattering contribution alone.

The circumstances for the Crucifixion eclipse are that it had a maximum coverage of 59% while roughly 20% was covered at the time of moonrise in Jerusalem over an hour later. For the shadow coverage as at moonrise for Jerusalem, the brightness will always be brighter than 3.0×10^6 nL for everywhere in the umbra.

The umbral surface brightness can be estimated from the fact that a typical visual magnitude of the totally eclipsed Moon is zero (Keen 1983). Therefore, the Moon is dimmed by roughly a factor of 10^5 when compared to the full Moon. The full Moon has a surface brightness of 1.6×10^9 nL, so that the umbral surface brightness will be of order 1.3×10^4 nL with normal extinction included.

So now we have an easy explanation for why the umbral regions lose their color soon after totality ends, despite the fact that the same portions of the umbral shadow are reflecting red light back to Earth as during totality. The answer is that the white glare from the bright penumbral regions totally swamps the faint red umbral light. For 50% coverage, the white scattered light is a factor of 60 times brighter at minimum. This would correspond to a change in the $(B-V)$ of less than 0.02 mag. I estimate that the red umbra will be recognized only for times when the umbra covers more than about 70% of the Moon's disk.

At the time of moonrise in Jerusalem, the red umbral light is 1.3×10^4 nL in brightness while the white scattered light is 3.0×10^6 nL at its faintest. Hence, the white light is more than 230 times brighter than the red light. The detection of any reddish tinge would be equivalent to measuring a change in the color index of 0.005 mag—a feat that any photoelectric photometrist would be proud of. In summary, the glare of penumbral light completely masked any reddish color from the Crucifixion eclipse.

4.3 *Aristotle's Well*

In *Generation of Animals*, Aristotle claims that "The man who shades his eye with his hands or looks through a tube will not distinguish any more or any less the differences of colours, but he will see further; at any rate, people in pits and wells sometimes see the stars" (Hughes 1983). Pliny, Cleomedes, Gregory of Tours, Kipling, Dickens, and Herschel all imply that stars can be seen in the daytime from the bottom of wells, cisterns, gorges, pits, or chimneys (Hughes 1983). The British astronomer Richard A. Proctor proposed that the ancient Egyptians used the passages in the Great Pyramid of Cheops to monitor the motion of the polestar Thuban and α Centauri even during the daytime (Tompkins 1971). Could it be that Aristotle's well can allow stars to be visible to the unaided eye during the daytime?

One possible mechanism for improving the visibility of stars from the bottom of a well is that the eye will no longer have the glare from much of the sky masking the star. In other words, perhaps the summation of $B_{\rm eye}$ over the sky will cause a significant brightening of the apparent background. Equation (11) from Holladay 1926 gives the appropriate integral as

$$B_{\rm eye} = (43/\pi) \int d\Omega \, B_{\rm sky} \, \theta^{-2} \ . \qquad (37)$$

The integration should be over θ from perhaps 1° to 90° and must be done numerically. The result is that $B_{\rm eye}$ will be 11.3% of $B_{\rm sky}$, with a variation of several percent as the range of integration is varied over reasonable values. The point is that Aristotle's well will indeed lead to a lowering of the apparent background by roughly 11%. Unfortunately, equation (21) shows that an 11% reduction in B (when B is over 10^8 nL, see Weaver 1947) will yield an improvement of only 0.12 mag in the threshold. Therefore, the reduction of glare from the sky is an insignificant effect for the visibility of stars from wells.

At the same time, physiological effects will more than offset the gain from glare. Martin 1923 and Emerson & Martin 1925 performed a series of experiments where the visibility of a small target in a small field was measured while the background around the small field was varied. Such an experiment will include effects due to glare as well as physiological effects. They found that the threshold for visibility of the small target depended on the brightness of the large background. Generally, the threshold was raised when the background was made dimmer than the small field that contained the target. The reduction in sensitivity equals from 0.2 to 1.0 mag depending on the conditions. Their tests did not include conditions with angular sizes or brightnesses relevant for the problem of Aristotle's well, so the exact amount of reduction in sensitivity is not known. However, it appears likely that the threshold will be worse for Aristotle's well relative to the threshold for an open sky.

Actual tests of the visibility of stars in chimneys and mines have been carried out by Hynek 1951 and Smith 1955, while Hughes 1983 summarizes the results of A. von Humbold. All tests show that bright stars are not visible during the daytime from inside chimneys. In summary, both theory and observation show that Aristotle's well is just "an old philosopher's tale".

4.4 *Venus at Conjunction*

At inferior conjunction, Venus has a visual magnitude of −4.1 and can have a distance from the Sun as large as 8°.7. Therefore, an observer far away from the equator will have Venus appearing high above the horizon at sunset at the time of conjunction. For optimal conditions, the altitude of Venus at the time of sunset (the so-called *arcus visionis*) will be 8°.7. Schaefer 1987 has presented a detailed model of the visibility of point sources during twilight and finds that Venus should be visible if the *arcus visionis* is larger than 7° or so for average observing conditions. Therefore, it is possible to spot Venus at conjunction both near sunrise and sunset on the same day.

However, Venus is also bright enough to be seen at noon by a keen-eyed observer. A necessary precaution is to view with the eye in shadow to avoid the painful direct sunlight and so that $B_{\rm eye}$ will be zero. The limiting magnitude for detecting point sources near the Sun is

$$m_v = -9.28 + 5\log(\theta) - 2.5\log(F_s) \ , \qquad (38)$$

for $k_v X = 0.3$ mag and $m_v = -26.7$ for the Sun. Therefore, if Venus is to be visible at an inferior conjunction with $\theta = 8^\circ\!.7$ and $m_v = -4.1$, then the observer must have F_s less than 0.6. Such acuity is not rare, so it is not surprising that there are occasional reports in the literature of viewing Venus at conjunction at noon with the unaided eye.

For telescopic observations of Venus near the Sun, a similar equation can be derived (cf. eq. (39)). The model predicts that Venus at superior conjunction can be seen near the solar limb with a telescope of about 8-inch aperture, and indeed the Reverend W. R. Dawes did follow Venus to within an arc minute of the solar limb with his 7.25-inch (18-cm) Clark refractor (Bortle 1985).

4.5 *Sungrazing Comets*

The Kreutz group of sungrazing comets (Marsden 1967, 1989) comprises roughly 30 comets with similar orbits that have perihelion distances of less than 0.02 AU. The most famous sungrazer was the great comet Ikeya-Seki which was viewed by many naked-eye observers during the daytime when it was as close as one degree from the Sun. At times soon before perihelion, the head of the comet was estimated to have a visual magnitude ranging from −10 to −15 (Marsden 1965). The analysis from equation (36) shows that a point source should be visible if it is brighter than $m_v = -9.3$ (for $\theta = 1°$) or $m_v = -7.8$ (for $\theta = 2°$). Therefore, comet Ikeya-Seki should have been (and was) an easily visible object with the unaided eye. Bortle 1985 summarizes reports of 13 comets visible in the daytime near the Sun. In one case (Elkin 1882; Finlay 1882) the comet was even seen to touch the solar limb!

Various observers have used various schemes to search for sungrazing comets. (1) Robert McNaught has used Marsden's search ephemeris (Marsden 1967) to look for comets immediately after sunset from the Southern Hemisphere. However, such a program will succeed only for a comet similar in brightness to comet Ikeya-Seki because of atmospheric absorption. Low on the horizon, the path length through the air will be roughly 20 air masses or greater (for optimal conditions with the comet within 2°.5 of the Sun, see, for example, Allen 1976) so that $k_v X$ is likely to be worse than 6 mag. The twilight brightness near the sunset will be roughly 3×10^9 nL (Koomen et al. 1952), so that the threshold m_v will be -10.7. (2) Michael Mattei has used a binocular occulting device and a neutral density filter (two pairs of sun glasses) to visually examine the region near the Sun. For such a program equation (38) is applicable, so that a sungrazer must be brighter than -9.3 to be detected one degree from the Sun's center.

I would like to propose an alternate sungrazer search strategy that is simple and cheap, yet will detect comets greatly fainter than the other strategies. The idea is to use a small telescope at high power to scan along Marsden's orbital tracks. (Any potential observers are cautioned to take all necessary safety measures to ensure that the disk of the Sun is never directly viewed.) The threshold for comet visibility near the Sun can be calculated as above for telescopic observations. The coma of the comet is not a point source but is extended so that its effective radius (i.e., roughly half of the FWHM of the image profile) is θ_c. The size of the coma will have a similar effect as that of a point source viewed through bad seeing; that is, I model the comet as a point source viewed through seeing conditions with $\theta_s = \theta_c$. The limiting magnitude for a comet will be roughly

$$m_v = -1.7 + 5\log(\theta) + 2.5\log(D) - 2.5\log(\theta_c) \ , \quad (39)$$

where θ is the distance from the center of the Sun in degrees, D is the telescope diameter in inches, and θ_c is the effective radius of the coma in arc seconds. The value of θ_c for comets near the Sun is given variously as 6″ (Bortle 1985), 2″ (Finlay 1882), 2″.5 (Elkin 1882), and 4″ (Hind 1847). For deriving this equation I have assumed that the maximum usable power of 50× per inch of aperture was used, that the observer is of average acuity with pupil diameters of 0.1 inch, and the other parameters are as given in Section 2.2.2. So a typical amateur-sized telescope (with $D = 8$ inches) could detect a comet with roughly $m_v = 0$ and $\theta_c = 6''$ at a distance of $\theta = 2°$. For a pair of 7×50 binoculars, the constant term in equation (37) is -5.4.

Recently, coronagraphs on satellites (SOLWIND and SMM) in Earth orbit have discovered 16 sungrazing comets (Marsden 1989; MacQueen & St. Cyr 1991). The comets were discovered and observed in a range of θ from 1° to 2°. Their FWHM coma radius (θ_c) is roughly 10″ or smaller. Three of the eleven comets detected by SMM were of visual magnitude 0 or brighter for $\theta = 2°$ and, hence, would have been discoverable by an amateur from the ground. For a two-year period in the late 1980s, the frequency of comets discoverable by amateurs was perhaps two per year.

4.6 *Open Clusters*

Open clusters are compact groupings of stars. Many magnitude sequences for calibration and testing have been constructed in open clusters because of the close proximity of easily located stars spanning a wide range of brightnesses (Sinnott 1973; Everhart 1984; Clark 1989; Schaefer 1989). However, for many applications, the presence of glare from other stars in the cluster might significantly raise the background. Certainly the glare is important in crowded clusters, but its effect on loose clusters must be determined by direct calculation.

To take a specific case, let us examine whether glare has any significant effects on the study of limiting magnitudes of stars in M 67 (Schaefer 1990b). The procedure will be to calculate the sum of B_{extra} (as given by eq. (19)) for all stars in the cluster. In general, the summation will be dominated only by close stars or by distant bright stars. The aperture (D) appropriate for each star is that aperture which will just barely detect the star. Of all stars in the sequence in Schaefer 1989, the star with the worst glare problem is star G, and this is solely because of an eleventh-magnitude star 30 arc seconds away to the southeast. For typical conditions and maximum power, star G requires at least a 5-inch telescope. The contribution to B_{extra} from this one star is 7.8 nL. Similarly, the nineteenth-magnitude star 9 arc seconds to the northwest adds 0.078 nL. Star E (with $\theta = 0°.012$ and $m_v = 12.26$) contributes 1.1 nL. The bright star A ($m_v = 10.6$) is 0°.063 distant for a contribution of 0.16 nL, and there are a half-dozen stars of comparable brightness and distance. When I add up the contribution from all stars in the cluster, I find a total extra background of 16 nL. This B_{extra} is to be compared to the typical B_{sky} value of 136 nL. From equation (21) we see that the effect of the glare on the threshold is as if the whole sky had brightened so that the zenith-limiting magnitude had changed by 0.07 mag. For observations with a 5-inch telescope at maximum power, the glare will result in a loss of 0.02 in the limiting magnitude. All other stars in the sequence of M 67 (selected by D. DiCicco and presented in Schaefer 1989) have a smaller effect due to glare from cluster members. Therefore, we see that glare has no significant effects on the study of telescopic limiting magnitudes by Schaefer 1990b.

Similar analyses show that the sequences of Everhart 1984 and Clark 1989 are not significantly contaminated by glare from member stars in the open clusters. However, the sequence of Sinnott 1973 in the Pleiades has a $B_{\rm extra}$ of 70 nL for many of the stars near Electra, so that the limiting magnitudes will be affected by roughly a quarter of a magnitude. Even in this extreme case, the effect of glare is smaller than the observed uncertainty of half a magnitude (Schaefer 1990b).

4.7 *Double Stars*

Double stars with components of greatly unequal brightness are affected by glare, in that the scattered light from the primary hides the fainter secondary. If the magnitude difference is greater than several magnitudes, then the glare will be the most important constraint on the visibility of the secondary. Sirius and Procyon are the most infamous cases (see Section 3.4), although many other well-known double stars are similarly affected (for example, Rigel, Antares, Polaris, and Adhara).

The predicted visibility will depend on many parameters, so that it is difficult to give any simple rules. Instead, I have calculated the threshold for a standard case and for cases different from the standard by only one parameter. This will allow for the easy separation of the effects of each parameter. The standard case chosen was for a double star with separations between 2 and 32 arc seconds, with a second-magnitude primary star, as viewed through a 6-inch telescope operating at 50 power per inch of aperture, with θ_s equal to one arc second, and all other parameters set to the defaults listed at the end of Section 2.2.2. The limiting magnitudes are also calculated for other cases which are different from the standard case only in that (1) $D = 2$ inch and $M = 100$ power, (2) $D = 20$ inch and $M = 1000$ power, (3) m_v (primary) $= 6$ mag, (4) $\theta_s = 2$ arc seconds, and (5) $M = 150$. The results are plotted in Figure 4.

4.8 *Observing Tips*

The threshold changes rapidly with magnification, and always in the sense that the limits improve with high power. So for the deepest limit, the observer should always put on the highest usable power, that is 50× for every inch of aperture. In my review of the literature I frequently see cases of observers who do not get the most out of their telescopes when they are trying to go faint or are battling against glare. It seems that the use of maximum power is the one biggest improvement that most observers can get out of their telescope.

The size of the seeing disk is an important determinant of limiting magnitude. The reason is that the deepest limits come with the highest magnification, which often implies that the seeing disk is magnified enough to be resolved. The human eye is much less sensitive at detecting some number of photons spread out over a broad angle than if the same number of photons were concentrated in an angle less than $\theta_{\rm CVA}$. So in good seeing, the image is smaller and easier to detect than in bad seeing.

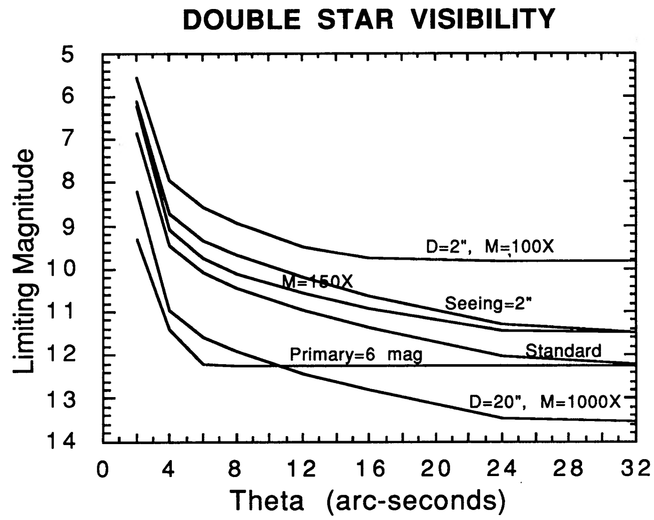

FIG. 4–The visibility of double stars for a variety of cases. The graph plots the visual magnitude of the faintest secondary star that is detectable as a function of angular distance from the primary. The standard case is for a normal observer ($F_s = 1$) with good seeing ($\theta_s = 1$ arc second) observing with a 6-inch telescope at maximum usable magnification looking at a double whose primary is second magnitude. Also plotted are the results for several other cases where only one parameter has been varied, so as to easily see the importance of each.

An additional effect comes in when the glare source is within 5 arc seconds or so of the object of interest. For such small values of θ, the contribution to $B_{\rm atm}$ coming from the seeing disk will dominate over the aureole so that changes in θ_s will greatly change $B_{\rm atm}$. The moral is that good seeing can improve your limit by a magnitude or more.

When observing near a bright source, it is natural to try to shield the eye from directly viewing the glare source, just as it is natural to shadow the eyes with an outstretched hand when looking toward the Sun. This occultation technique can be used in naked-eye observations by getting in the shadow of a building corner or a telephone pole, while telescopic observers can place the bright source just outside the field of view. Young & Young 1972 describe the requirements for constructing external occulting disks. A similar trick is to use an occulting bar placed in the focal plane of the eyepiece. (See MacRobert 1990 for a description of making an occulting bar.) The gain from these tricks is that the apparent sky brightness will no longer have a $B_{\rm eye}$ component. For naked-eye observations, $B_{\rm eye}$ is the dominant background, so substantial gains in threshold can be made. For telescopic observations, the $B_{\rm eye}$ component is of varying importance, but it can reach even half of the background for reasonable conditions.

Another trick is to place an aperture mask on the front of the telescope. The idea is to change the diffraction pattern so that the light is concentrated in certain directions from the primary, and the secondary can then be placed at a position with lower background. In effect, the circular symmetry assumed for equation (7) is modified so that some position angles are brighter and some are fainter. This trick is effective only when the B_{dif} has a large contribution to the total effective background. Since B_{dif} falls off as θ^{-3} while B_{atm} and B_{eye} fall off as θ^{-2}, the diffraction will dominate at small θ. Therefore, the aperture mask will only be effective for close doubles.

Occasionally, commentators will mention that glare can be minimized by using clean optics. While this is true, the importance of the assertion is minimal. For all the situations discussed in this paper, B_{mir} is at least several orders of magnitude fainter than the effective background. The primary effect of dirty optics is not related to the contributed background light but is related to the loss of telescope throughput. So, for example, the difference between a clean mirror ($f = 1\%$) and a horribly dirty mirror (say, $f = 10\%$) will only be under 0.1 mag in the detection threshold.

Various observers report observations where a neutral density filter was used. This will indeed reduce the background but will also reduce the source intensity by the same amount. Examination of equation (21) shows that such a procedure can never improve the threshold and it usually greatly degrades the threshold. However, for very bright background conditions, a neutral density filter will ease the pain from the light yet will not greatly lower the detection threshold (cf. Hecht, Ross & Mueller 1947). Should the background be significantly polarized, then the use of a polarizing filter may improve the visibility by selectively reducing the background as compared to the source. In theory, color filters may be chosen that will selectively block the background light, but I and others have observed no significant help for sources without strong emission lines.

4.9 *Challenges for Observers*

4.9.1 *Galilean Satellites*

In Section 2.2 I showed that all four Galilean satellites around Jupiter should be visible to the keenest observers. For three of the moons, F_s must be smaller than 0.2, so that an observer is sufficiently keen only if he can spot a 7.7-mag star without any aid. Few people are this sharp-eyed, but those who are may enjoy an experiment of regularly monitoring the moons on many nights. It would be interesting if such data could reveal the number and periodicities of the satellites. It is vital for such a program that the observer have no knowledge at any time of the actual positions. Less keen observers might try the same experiment with the aid of some distant occulter.

4.9.2 *Double Stars*

It would be a useful test of my glare model if experienced double-star observers measured the minimum required aperture for detecting many double stars. Ideally, the tests should be performed all on one night for each star with a single telescope that has circular aperture stops of various diameters. It might also be interesting to experiment with magnifications including those larger than the traditional maximum usable power of 50× per inch of aperture.

4.9.3 *Sungrazing Comets*

In Section 4.5, I showed that an amateur telescope could detect sungrazing comets as faint as zero magnitude near the Sun. Roughly a quarter of the sungrazing comets discovered by the SOLWIND and SMM satellites were sufficiently bright (MacQueen & St. Cyr 1991) so that they could have been discovered by any dedicated amateur in his own backyard around noontime. The dedication comes in because the area near the Sun will have to be monitored daily since the comets move so fast. Fortunately, only the area of the sky near the orbital path of the sungrazers (see Marsden 1967, 1989) needs to be examined. Some skill will be required to develop a search pattern that completely covers the sungrazer track (despite the small fields of view forced by the high magnification) yet which affords absolute safety from direct viewing of the Sun.

Great care must be taken so that the disk of the Sun is not accidentally looked at. It is well-known that a direct view of the Sun can quickly damage the eye permanently. A telescope will form a magnified image so that large portions of the retina can be damaged, with resulting blindness. This horrible fate must be guarded against by all observers. A possible procedure to avoid direct views of the Sun is to observe only with the entire telescope in the umbral region of the shadow of a distant and stable occulter. Never observe when the Sun's sidereal motion will cause the solar disk to appear from behind the occulter. So, for example, observations near the setting Sun behind a building top might be a good idea. A good safety idea is to always have a companion observer whose sole job is to ensure that the telescope is completely in shadow during observations. Remember that your eyesight is infinitely more valuable than any possible observation.

A lesser worry relates to the possibility of false alarms for comet discoveries. That is, there are many things which could be confused for a comet. These include ghost images in the optics and particles in the air such as floating seeds, planets, and stars. Before any discovery is reported, (1) the object must be viewed sufficiently long to prove that its motion is roughly sidereal so as to eliminate objects in the atmosphere, (2) the position of the object on the sky must not depend on the position in the field of view so as to rule out ghost images, (3) the observer must check that no planets or bright stars are at the position of the object, and (4) the object must move significantly with respect to the Sun due to the motion of the comet. In addition, the discoverer should obtain independent confirmation if this is speedily possible. These rules should be followed before any report of a comet discovery, since otherwise much time will be wasted when telescopes around the world are mobilized for each false alarm.

The search will require a skilled observer, not only for establishing a safe search pattern but also for establishing the correct focus. The telescope can be brought to proper focus while looking at Venus in the daytime or by various other obvious tricks. Unfortunately, it may be difficult for the eye to know how to focus because nothing will usually be in the field of view. Bortle 1985 reports that this is a significant problem, as he has frequently overlooked a source at a known position for a substantial length of time until the right focus was used. A contributing factor to this trouble is that the eye will tend to focus for dust on the eyepiece lenses. A possible solution would be to use an eyepiece with a reticle in the focal plane so that the eye has a cue for the correct focus.

The frequency of sungrazing comets is highly episodic (MacQueen & St. Cyr 1991), so even a network of dedicated amateurs may discover no comets. Alternatively, a network of comet observers might make many exciting discoveries. For example, if a worldwide network had been operating in the late 1980s, then the SMM data suggest that two comets per year could have been discovered from the ground. Many frequent observers would be required so that any discovery would have independent confirmation. Such a program would be scientifically useful, since both SOLWIND and SMM have stopped sending data, so new information on frequencies, magnitudes, and orbits of sungrazers must come from the ground. Such a program would also be exciting, since it offers the possibility of frequent comet discoveries by exotic means.

4.9.4 *Lunar Eclipses from Space*

During a partial lunar eclipse, the reason that the red light in the umbra is invisible is because of the glare from the brilliant white light of the penumbra. A ground-based observer can always hide the bright portion of the Moon to eliminate $B_{\rm eye}$, yet the $B_{\rm atm}$ contribution alone is enough to hide the red color. However, an astronaut has no atmosphere to scatter light and can hide the penumbral parts. A potential trouble is that the necessary window which separates the man from the vacuum might introduce enough scattered light to mask the color. Nevertheless, I predict that an astronaut can detect red coloration during a partial lunar eclipse.

4.9.5 *The Pup Star and the Martian Moons*

The Pup star is visible in a 4.3-inch telescope when at elongation, yet for the next decade it will be within 5 arc seconds of Sirius. Phobos and Deimos are visible in small amateur telescopes during favorable oppositions of Mars, yet the next reasonable opposition will be in 2001. In both cases I calculate that the visibility will be very difficult even in medium-sized telescopes. So my final challenge to observers is to detect the Pup star near closest approach or to detect the Martian moons at an unfavorable opposition. This will take a highly skilled observer and superb seeing conditions. Presumably any such detection would need an occulting bar and possibly some well-designed aperture mask.

I hope and anticipate that observers will take up these challenges.

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