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Big Bounce in the very early universe

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Abstract. A cosmological model is discussed which replaces the Friedmann singularity in the very early universe by a de Sitter solution with a closed spherical space metric stretched by a quantum vacuum. This model contracts from past infinity to a minimum radius and then re-expands (Big Bounce).

After passing the minimum, a phase transition is required which converts the quantum vacuum into the primordial relativistic matter. During this process the exponential de Sitter expansion turns into a Friedmann–Lemaître model.

This model is compared with a similar singularity-free model proposed by Israelit & Rosen (1989), which starts from a cosmic egg at the Planck time with "prematter" of Planck density.

Key words: cosmology

1. Introduction

According to the singularity theorem of Hawking & Ellis (1968) and Penrose (1965), see also Kundt (1968), the Friedmann–Lemaître models lead to a singularity in the very early universe. However, the energy condition assumed by the theorem

$$\varepsilon + 3p \geq 0$$

need not be satisfied by all forms of matter fields and depends on the equation of state. Moreover, the usual concept of separating space–time and matter may be violated during the very early universe (Einstein 1954), when the curvature of space varies very significantly on the scale of a Planck time. At this time quantum-gravitational effects become dominant. The typical size of a mini universe emerging from the space–time foam (Misner et al. 1973) is assumed to be of the order of the Planck length $L_{\rm PL}$. The energy density should then be of the order of the Planck density $\varepsilon_{\rm PL} = \rho_{\rm PL} c^2$. Attempts to describe the quantum creation of the universe as quantum tunnelling from "nothing" (i.e. no classical space-time and matter) have been made e.g. by Starobinsky (1980), Atkatz & Pagels (1982), Vilenkin (1988).

If we extrapolate the Friedmann models back to the Planck time $t_{\rm PL}$ – the assumed threshold of classical cosmology – we are faced with an enormous discrepancy with respect to the initial dimensions assumed in the concept of quantum creation.

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On the one hand, the density of the Friedmann models at the Planck time differs only slightly from the Planck density while, on the other hand, the size $R_{\rm F}$ of the Friedmann models measured by the worldline of the ($z = 5$)-quasar horizon at $t_{\rm PL}$ shows an enormous excess compared to the Planck length $L_{\rm PL}$ of the mini universe:

$$R_{\rm F}(t_{\rm PL}) \approx 10^{30}\, L_{\rm PL},$$

with $L_{\rm PL} = 1.6\ 10^{-33}$ cm and $t_{\rm PL} = 5.4\ 10^{-44}$ s.

To eliminate this discrepancy, a mechanism of exponential expansion is required to reconcile the classical model with its possible quantum origin. An exponential expansion can be realized when the universe is dominated by a substrate with positive energy density ε but negative pressure $p = -\varepsilon$ (Gliner 1966).

In order to achieve the rapid expansion, the inflationary scenario (Guth 1981; Linde 1983) uses the scalar Higgs field of the Grand Unified Theories. When the energy density of the Higgs field $\varepsilon_{\rm Hi}$ (with $\varepsilon_{\rm Hi} = \rho_{\rm Hi} c^2$ and $p_{\rm Hi} = -\rho_{\rm Hi} c^2$ with $\rho_{\rm Hi} \approx 10^{76}$ g cm^{-3}) dominates the matter density at $t \approx 10^{-35}$ s, an exponential inflation takes place. Then the required phase transition transforms the Higgs energy into the primordial elementary particles at about $t \approx 10^{-33}$ s. For a critical review see Börner (1988). The inflationary scenario does not imply a solution to the conundrum of the origin of matter in a Big Bang or at the Planck time. Furthermore, it does not explain the required exceedingly large expansion rate in the beginning. Big Bang models assume both infinitely large energy density and expansion rate as unexplained initial conditions.

2. Big Bounce

Our approach is based on the assumption of a primordially homogeneous and isotropic space–time, in which all fields exist in their ground state.

The lowest energy state of the quantized fields defines the energy level of the quantum vacuum. This picture is supported by quantum field theory, in which real matter (elementary particles) represents the excited states of the space-filling matter fields. Thus, the vacuum is the more fundamental entity, which can exist independent of real particles. Therefore, it seems a natural hypothesis that the quantum vacuum was the primordial stage before elementary particles came into being (Priester & Blome 1987).

This hypothesis implies the decoupling of the origin of "ordinary" matter (quarks, leptons, photons, etc.) from the creation of space–time. A phenomenological description of the quantum vacuum by an equation of state for the pressure $p_v = -\rho_v c^2$ was given by Zel'dovich (1968). With this assumption the de Sitter solutions follow as possible cosmological models from the Einstein equations.

For a homogeneous and isotropic universe the Einstein equations reduce to the Einstein–Friedmann equations:

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{1}{3}\Lambda c^2 - \frac{kc^2}{R^2(t)} \tag{1}$$

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3}\left(\rho(t) + \frac{3p(t)}{c^2}\right) + \frac{1}{3}\Lambda c^2 \tag{2}$$

Here $R(t)$ is the time-dependent scale factor. The curvature index is $k = +1$ for a spherical space metric and 0 or -1 for Euclidean or hyperbolic metrics, respectively.

The cosmological constant Λ occurs in the Einstein equations if derived in a general way using the Hamilton variational principle. It is convenient for comparison with the density to replace Λ by its equivalent density $\rho_\Lambda = \Lambda c^2/(8\pi G)$.

The density $\rho(t)$ and the pressure $p(t)$ are combinations of several components, which represent the (nonrelativistic) matter (index M), the radiation (relativistic matter) (index R) and the quantum vacuum (index V).

Except for the short time intervals of phase transitions the interaction between the components can be neglected. Thus, the density and pressure can in general be expressed as a sum of the components. Only the time intervals of phase transitions require special considerations. These are:

(a) The assumed phase transition between the quantum vacuum and the radiation (relativistic particles) at a very short time interval, somewhere between $t = 10^{-42}$ and 10^{-32} s (Friedmann time).

(b) The quark–hadron phase transition with subsequent annihilation of the antimatter at $t \approx 10^{-6}$ s.

(c) The transition from the radiation cosmos to the era dominated by incoherent matter ("dust" model, galaxies) at $t \approx 10^5$ to 10^6 yr. With these restrictions we have

$$\rho(t) = \rho_M(t) + \rho_R(t) + \rho_v(t),$$
$$p(t) = p_M(t) + p_R(t) + p_v(t), \tag{3}$$

with the equations of state

$$p = (\gamma - 1)\rho c^2: \tag{4}$$

$p_M = 0$	for incoherent matter ($\gamma = 1$),
$p_R = (1/3)\rho_R c^2$	for relativistic particles (e.g. photons, neutrinos and all particles in the early universe) ($\gamma = 4/3$),
$p_v = -\rho_v c^2$	for the quantum vacuum ($\gamma = 0$).

The local energy balance

$$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{p}{c^2}\right) = 0 \tag{5}$$

allows one to calculate the dependence of the density on the scale factor R, resulting in

$$\rho = \rho_{PL}\left(\frac{L_{PL}}{R}\right)^{3\gamma} \tag{6}$$

for the various equations of state (with L_{PL} = Planck length and ρ_{PL} = Planck density).

The energy density of the quantum vacuum is assumed to be constant from the inifinite past until the phase transition begins. At that time the quantum vacuum energy is converted into relativistic particles and is thereby reduced to zero or to an insignificant amount.

The concept of the Big Bounce implies that there is no matter and no relativistic matter and radiation ($\rho_v \gg \rho_R = 0$, $\rho_M = 0$) before the phase transition. Thus, the dynamics of the universe are driven solely by the stress of the quantum vacuum.

Under these circumstances Eqs. (1) and (2) have as solutions the de Sitter space–time:

$$R(t) = \frac{c}{H}\sinh(Ht) \qquad \text{for } k = -1, \tag{7a}$$

$$R(t) = \frac{c}{2H}\exp(Ht) \qquad \text{for } k = -0 \tag{7b}$$

$$R(t) = \frac{c}{H}\cosh(Ht) \qquad \text{for } k = +1, \tag{7c}$$

with the expansion rate

$$H = \frac{\dot{R}(t)}{R(t)} = \sqrt{\frac{8\pi G}{3}\rho_v} \tag{8}$$

for ρ_v = const. The characteristic time is $t_H = 1/H$.

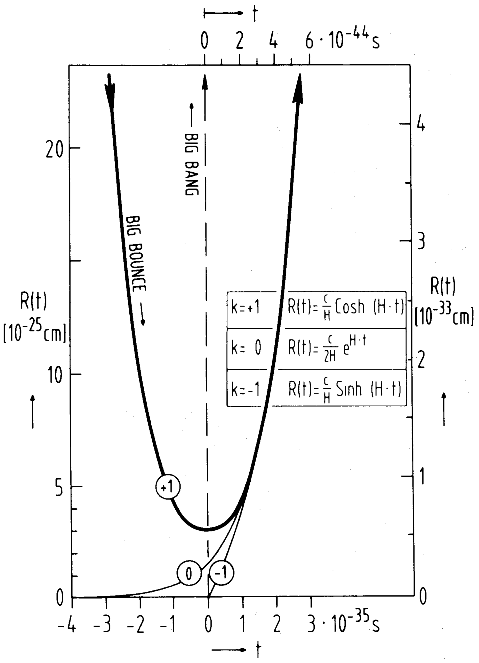

Fig. 1. Three evolutionary scenarios of the very early universe: The curvature radius $R(t)$ as function of time. (a) Big Bang and Big Bounce ($k = +1$): The scale of $R(t)$ is given on the left-hand ordinate and the time scale in the lower abscissa. (b) Israelit–Rosen model ($k = +1$): $R(t)$ on the right ordinate and time scale in the upper abscissa. This model begins its expansion from a homogeneous "cosmic egg" with almost Planck density and a radius of 0.56 10^{-33} cm. The de Sitter models for open space [$k = 0$ (Euclidean metric) and $k = -1$ (hyperbolic metric)] are given for comparison

Figure 1 shows appropriate examples of the de Sitter solutions for $\rho_v = 2\,10^{76}\,\mathrm{g\,cm^{-3}}$ and for $\rho_v = 5.2\,10^{93}\,\mathrm{g\,cm^{-3}}$ (= Planck density).

The solution (7c) for a spherical space metric avoids the singularity and represents a closed universe, which contracts from infinity and re-expands after the Big Bounce.

The selection of a de Sitter solution with spherical metric as a mathematical model of the very early universe is quite natural within the context of quantum transitions from a pre-Planck stage at the threshold of classical cosmology (see eg. Vilenkin 1988).

Moreover, observations of the present universe are compatible with Friedmann–Lemaître models with spherical space metric ($k = +1$), in agreement with the limits of the observed boundary conditions (H_0, $\rho_{\mathrm{M,0}}$) (Blome & Priester 1985). The curvature index k is a topological constant which cannot change in the course of the evolution of the universe. Therefore, the space of the precursor stage modelled as a de Sitter solution should also be closed ($k = +1$).

The transition from the de Sitter stage into the Friedmann–Lemaître radiation cosmos is characterized by a very brief phase transition during which matter is created from the energy of the quantum vacuum.

As the vacuum energy density decays, the universe experiences a rapid transition from the vacuum state with $p_v = -\varepsilon_v$ into a cosmic epoch dominated by the energy density ε_R of relativistic matter with pressure $p_R = (1/3)\,\varepsilon_R$. It is worth mentioning that the matter production in quantum field theories [see Parker (1988) for a review on these problems] can be treated by classical general relativity if pressure is allowed to be negative (Gunzig & Nadone 1987). This can be seen directly by the local energy balance equation in the form $(\rho R^3)^{\cdot} + p(R^3)^{\cdot} = 0$, or $\dot{m} + p\dot{V} = 0$, where $m \sim \rho R^3$ is the mass and $V \sim R^3$ is the volume. Thus, $\dot{m} > 0$ requires $p < 0$ in an expanding cosmos ($\dot{V} > 0$).

During the process of matter creation, the exponential growth of the scale factor $R \sim \exp(t/t_H)$ slows down and approaches the usual Friedmann relation, $R \sim t^{1/2}$, for a radiation dominated cosmos. This phase transition must occur within a very short time interval.

The exponential expansion explains the initially very large expansion rate of the subsequent Friedmann radiation cosmos. This is another advantage over the conventional Big Bang model which simply has to assume the unexplained boundary conditions at the Planck time t_{PL}:

(a) the energy density corresponding to the Planck density $\rho_{\mathrm{PL}} = 5\,10^{93}\,\mathrm{g\,cm^{-3}}$,

(b) the extreme expansion rate $H = 1/(2t_{\mathrm{PL}}) = 10^{43}\,\mathrm{s^{-1}}$.

The boundary conditions for the closed Friedmann radiation model ($k = +1$) can be derived from the range of possible cosmological models at the present time (Blome & Priester 1985).

3. The present universe as consequence of an early Big Bounce

The allowed range of realistic closed Friedmann–Lemaître models can be divided into two model classes which are compatible with the presently observed boundary values (Hubble parameter $44 < H_0 < 110\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, a lower limit of the age of the universe of $t_0 \approx 14\,10^9$ yr):

Class A ($\Lambda = 0$): High-density models with $44 < H_0 < 47$ km $\mathrm{s^{-1}\,Mpc^{-1}}$ and ages $9\,10^9\,\mathrm{yr} < t_0 < 14\,10^9$ yr. These $k = +1$ models require a contribution of unobserved, non-baryonic matter which amounts to 90% or more of the average density.

It should be noted here that at present there is no tangible evidence for such a high amount of dark matter, neither from the flat rotation curves of galaxies nor from the overdensities in clusters of galaxies.

Class B ($\Lambda > 0$): Low-density models with $47 < H_0 < 110\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and ages $t_0 > 14\,10^9$ yr. These $k = +1$ models are based on a density $\rho_{\mathrm{M,0}}$ which has been derived from the analysis of the Helium–deuterium production in the early universe in comparison with the observed helium content (see Fig. 2). We obtain for the baryonic density a range of $0.1\,10^{-30}\,\mathrm{g\,cm^{-3}} < \rho_{\mathrm{M,0}} < 0.9\,10^{-30}\,\mathrm{g\,cm^{-3}}$. Here we shall use $\rho_{\mathrm{M,0}} = 0.5\,10^{-30}\,\mathrm{g\,cm^{-3}}$. This figure accounts also for the possible mass contribution by brown dwarfs and neutron stars in the galactic halos. There is no significant amount of non-baryonic matter required in these models.

In Fig. 3 we have shown 4 examples of $k = +1$ models. The model of class A with $H_0 = 46.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is favoured by Israelit & Rosen. We further present three models of class B with $H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and ages between 19.8 and $24\,10^9$ yr which are classified by the general density parameter $\Omega_0^{\blacktriangle}$, defined by

$$\Omega_0^{\blacktriangle} = \frac{\rho_{\mathrm{M,0}} + \rho_{\mathrm{R,0}} + \rho_\Lambda}{\rho_{\mathrm{c,0}}} = \Omega_0 + \omega_0 + \lambda_0 \begin{cases} > 1 & \text{for } k = +1 \\ = 1 & \text{for } k = 0 \\ < 1 & \text{for } k = -1. \end{cases} \tag{9}$$

Here $\rho_{\mathrm{M,0}}$ is the baryonic matter density, $\rho_{\mathrm{c,0}} = 3H_0^2/(8\pi G)$ is the critical density, $\rho_\Lambda = \Lambda c^2/(8\pi G)$ is the Λ-equivalent density and $\rho_{\mathrm{R,0}}$ the equivalent density of the background radiation. Ω_0 is the

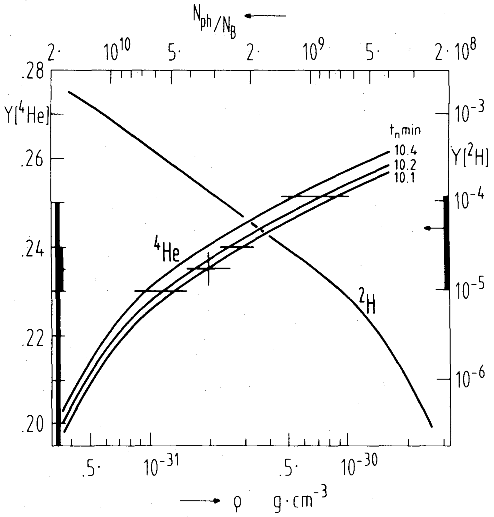

Fig. 2. The mass ratio of helium (^{4}He) and deuterium (^{2}H) relative to the total mass (H + ^{4}He) produced by primordial synthesis. The curves are calculated using the formulae given by Olive et al. (1981) and (1989). Nowadays, the number of neutrino species is restricted to three. The most recent experimental value of the neutron lifetime t_n is 10.13 ± 0.12 min (Paul et al. 1989). A comparison with the observed ratios for ^{4}He (23 to 25%) and ^{2}H (10^{-4} to 10^{-5}) yields the present baryonic matter density (see also Blome & Priester 1984)

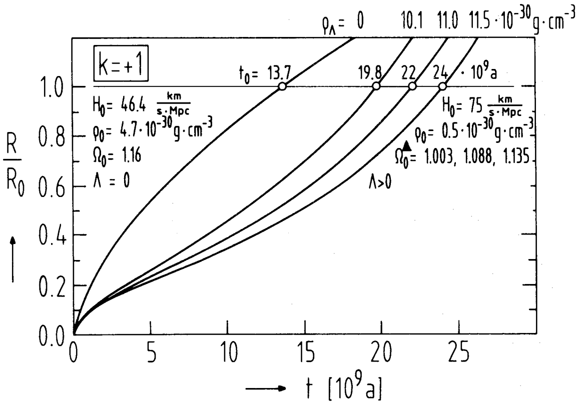

Fig. 3. Friedmann–Lemaître models with spherical space metrics result from both scenarios of the very early universe. The normalized curvature radius (scale factor) R/R_0 is given as function of time. Israelit & Rosen (1989) favour $\Lambda=0$, a present Hubble parameter $H_0=46.4\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and age $t_0=13.7\ 10^9$ yr. This model requires a present matter density of $4.7\ 10^{-30}\ \mathrm{g\,cm^{-3}}$, containing a high percentage of "dark matter". The three models on the right-hand side are examples of the solutions with positive values of the cosmological constant. They are based on the observed matter density and an age of the universe above $14\ 10^9$ yr

present matter density parameter. λ_0 is the normalized cosmological term:

$$\lambda_0=\frac{\Lambda c^2}{3H_0^2}=\frac{\rho_\Lambda}{\rho_{c,0}}.$$

ω_0 is the present density parameter for photons and other relativistic particles. Since $\omega_0 \ll \Omega_0$, it can often be neglected. $\Omega_0^\blacktriangle$ is the appropriate parameter which represents the type of space metric in Friedmann–Lemaître models. With this parameter the curvature radius for $k=+1$ and $k=-1$ models retains its well-known form:

$$R_0=\frac{c}{H_0}\sqrt{\frac{k}{\Omega_0^\blacktriangle-1}}. \tag{10}$$

All these models assume that the quantum vacuum was completely converted into the primordial matter. The cosmological constant has often been identified as a residual of the primordial vacuum density. In the original interpretation, however, Λ is a quantity which contributes inherently to the curvature of space–time. Therefore, it must be regarded as a fundamental constant of nature, which by no means must be assumed to be zero as done in the standard cosmological models [see recent discussions by Weinberg (1989) and Priester et al. 1989]. Here, for instance, a positive Λ assures a spherical space metric even in a low-density universe, while in standard cosmology with $\Lambda=0$ the low densities imply a hyperbolic metric, exclusively. The term "low density" is, of course, meant in comparison to the critical density $\rho_{c,0}=3H_0^2/8\pi G$.

4. Models without singularity

A singularity-free cosmological model has been proposed by Israelit & Rosen (1989). According to their assumptions, the universe emerges from a small bubble ("cosmic egg") at the bounce point of a de Sitter model filled with a cosmic substrate ("prematter") characterized by an equation of state $p=-\rho c^2$ (see Fig. 1).

The cosmos springs forth from the quantum epoch and starts at the Planck time t_{PL} modelled by a de Sitter solution for $k=+1$, with a prematter density $\rho_{\mathrm{I}}=\rho_{\mathrm{v}}=\rho_{\mathrm{PL}}=5.2\ 10^{93}\ \mathrm{g\,cm^{-3}}$ and a constant expansion parameter

$$H=\sqrt{\frac{8\pi G}{3}\rho_{\mathrm{PL}}}=5.37\ 10^{43}\ \mathrm{s^{-1}}. \tag{11}$$

The model starts from a finite size $R_{\mathrm{I}}=5.58\ 10^{-34}$ cm, with a zero expansion rate, $\dot{R}(t_{\mathrm{I}})=0$.

With these initial conditions the Israelit–Rosen model undergoes a very rapid expansion (Figs. 1 and 4) in the course of which a transition takes place from the prematter period with exponential inflation to the radiation-dominated period. The phase transition is modelled by an ad hoc assumed interpolation formula for the equation of state:

$$p=\frac{1}{3}\rho c^2-\frac{4}{3\rho_{\mathrm{PL}}}\rho^2 c^2. \tag{12}$$

According to the proposed model, the parameters characterizing the present stage of the universe are $t_0=13.7\ 10^9$ yr, $H_0=46.5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_0=1.16$ and $\Lambda=0$.

This is the signature of a cosmos with spherical space metric. It will reach its maximum extension of about $R_{\mathrm{MAX}}=3.6\ 10^{29}$ cm in the future at $t_{\mathrm{MAX}}=6\ 10^{11}$ yr. From then on it will recollapse.

In Fig. 4 the model I on the left side is the one given by Israelit & Rosen (1989) and the model II describes our model. The corresponding densities can be considered as the maximum possible requirement in case I (Planck density) and the minimum requirement in case II. The latter density of $2\ 10^{76}\ \mathrm{g\,cm^{-3}}$ corresponding to energies of 10^{14} GeV is necessary for the production of X-bosons which are assumed to transform into quarks and leptons and, thus, provide the matter of the universe. According to the assumptions by Israelit and Rosen, the universe begins its

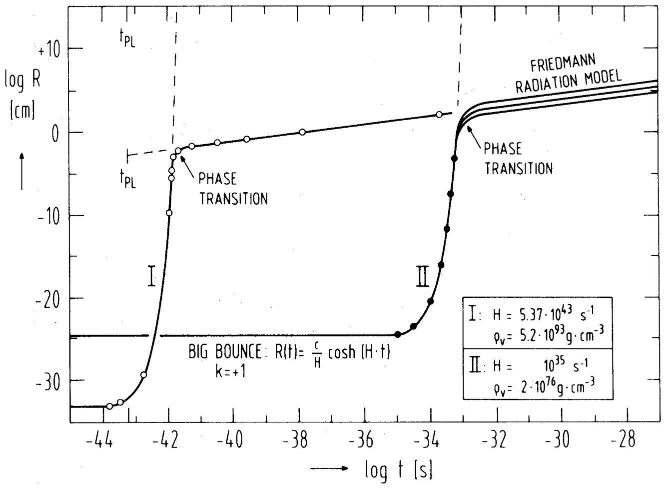

Fig. 4. Curvature radius $R(t)$ as function of time for the Israelit–Rosen model (I) (marked by open circles) and the Big Bounce model (II) (marked by black dots). The models cover the allowed range for the energy density of the quantum vacuum in the very early universe: $\rho_{\mathrm{PL}} \geq \rho_{\mathrm{v}} \geq 10^{76}\ \mathrm{g\,cm^{-3}}$. After phase transition (particle creation) both models merge into Friedmann models

expansion from a homogeneous and isotropic "cosmic egg", with the Planck density, in the form of a closed 3-dimensional space without a singularity. The expansion is driven by a prematter substrate with $p=-\rho c^2$ (see Figs. 1 and 4).

In our model the universe is described by a singularity-free, closed de Sitter solution of the Einstein equation (Big Bounce). This implies a space–time dominated by a vacuum energy density $\varepsilon_v=\rho_v c^2$, with $\rho_v=2\ 10^{76}$ g cm^{-3} and $p_v=-\varepsilon_v$ in the infinite past. The solution reaches a maximum of curvature at a bounce point the time of which is taken corresponding to the zero point of the Friedmann time. The curvature radius R_{MIN} at the bounce is

$$R_{\mathrm{MIN}}=\sqrt{\frac{3c^2}{8\pi G\rho_v}}=3.10^{-25}\ \mathrm{cm}=2\ 10^8\ L_{\mathrm{PL}}. \tag{13}$$

Here we have used for ρ_v the minimum requirement discussed above.

One important and unsolved problem remains in the model of Israelit and Rosen. It concerns the production of magnetic monopoles at energies of 10^{16} GeV, corresponding to densities of about 10^{83} g cm^{-3}. Since at these densities this model is already in the Friedmann radiation stage, there is not enough dilution to diminish the huge number density of magnetic monopoles which are expected to be produced at these energies. This problem does not exist in our model as the maximum energy density remains well below the values required for the formation of magnetic monopoles.

After the phase transition at about $t=2\,10^{-42}$ s, the Israelit–Rosen model merges into a closed Friedmann cosmos the present density parameter of which is $\Omega_0^{\blacktriangle}=\Omega_0=1.16$. It was assumed that the cosmological constant is zero. This assumption, together with a Hubble constant of $H_0=46.4$ km s^{-1} Mpc^{-1}, results in a cosmic age of $t_0=13.7\ 10^9$ yr.

In our solution we do not assume an a priori fixed value for the cosmological constant Λ. We leave open the question whether the effective cosmological constant for the present universe contains still a part which accounts for the possible incomplete cancelling of the zero-point energies from fermionic and bosonic fields, which contribute to the vacuum energy with different signs (Zel'dovich 1968). In any case, this remaining part must be extremely small as compared to the primordial energy density.

In Fig. 4 we give three examples for models with positive values of the cosmological constant. They cover a range of possible solutions for the Friedmann radiation cosmos after the phase transition, which in our case takes place at $t=10^{-33}$ s. These models do not recollapse but expand into infinity in the future. The uppermost of the three lines of the radiation models in Fig. 4 corresponds to a present general density parameter $\Omega_0^{\blacktriangle}=1.003$ and an age of $t_0=19.8\ 10^9$ yr. This value is close to the Euclidean case, with $\Omega_0^{\blacktriangle}=1.000$ and an age of $t_0=19.7\ 10^9$ yr. These models are based on a Hubble parameter $H_0=75$ km s^{-1} Mpc^{-1} and a matter density of $\rho_{\mathrm{M,0}}=0.5\ 10^{-30}$ g cm^{-3}.

All the four models given in Fig. 3 are possible examples for a present universe which resulted either from the Israelit–Rosen model or from our Big Bounce model.

5. Conclusions

The aforementioned Big Bounce model should not be confused with the bouncing Eddington–Lemaître models discussed by Ehlers & Rindler (1989) and Börner & Ehlers (1988). They have shown that the Eddigton–Lemaître models can be excluded from the range of models which are compatible with the recently observed maximum redshifts ($z>4$) and a lower limit of the matter density ($\Omega_0(\mathrm{MIN})=0.02$). The models can be excluded if

$$\frac{1}{1+z(\mathrm{MAX})}<\sqrt[3]{\frac{\Omega_0}{2\lambda_0(\mathrm{MAX})}}, \tag{14}$$

with

$$\lambda_0(\mathrm{MAX})=1-\Omega_0+\frac{3}{2}\sqrt[3]{\Omega_0^2(1-\Omega_0+\sqrt{1-2\Omega_0})}$$
$$+\frac{3}{2}\sqrt[3]{\Omega_0^2(1-\Omega_0-\sqrt{1-2\Omega_0})}. \tag{15}$$

This equation was derived by Blome & Priester (1985) for $\Omega_0<0.5$. A more general formula was given by Felten & Isaacman (1986).

Figure 5 summarizes the classification of Friedmann–Lemaître models. The upper left corner contains the Eddington–Lemaître models which nowadays can be excluded. The permitted solutions lie below the $\lambda_0(\mathrm{MAX})$ curve given in Eq. (15), which forms the upper limit for Friedmann–Lemaître models with a point of inflection. These points are designated by * in the insets in Fig. 5. All recollapsing models lie below the $\lambda_0(\mathrm{MIN})$ curve or below $\lambda_0=0$. The so-called standard models are restricted to the one line with $\lambda_0=0$.

A realistic value of the present density parameter Ω_0 can be derived from (a) the primordial nucleosynthesis (see Fig. 2) and (b) the observed luminosity function of galaxies. One obtains $\Omega_0=0.02\ (+0.04-0.01)$ based either on the observed helium mass ratio or on the observed luminosity density of galaxies (Schuecker et al. 1989), with a mass to luminosity ratio $M/L=25\ h$ in solar units with the limiting values of 75 and 12, where $h=H_0/(100$ km s^{-1} Mpc$^{-1})$. These values contain an appropriate mass contribution by the galactic halos and, furthermore, account for the virial mass of the clusters of galaxies. Observed infrared luminosity densities suggest a somewhat larger value for the density parameter Ω_0.

There is no tangible observational evidence for the often-quoted large percentage (>95%) of dark matter. A positive value of the cosmological constant can easily compensate any amount of the so-called "missing mass". The earlier often-publicized statement that the inflationary scenario could "predict" $\Omega_0=1$ can no longer be maintained. This was shown by Madsen & Ellis (1988). This implies that all models which are based on the observed boundary values are restricted to the small range of $0.01<\Omega_0<0.06$ as indicated by the dotted lines in Fig. 5.

This limited range of solutions can be further restricted if the universe had developed from a Big Bounce. Inevitably, this leads to a universe with a spherical space metric ($k=+1$) and a very limited range for the cosmological constant. Its permitted range is given by

$$1-\Omega_0<\lambda_0<\lambda_0(\mathrm{MAX}), \tag{15}$$

with $\lambda_0(\mathrm{MAX})$ following from Eq. (15). For the observed optimum value of $\Omega_0=0.02$ the limits are $0.98<\lambda_0<1.12$. Models within this highly limited range show a point of inflection with a low expansion rate corresponding to redshifts of about $z\approx4$. This could be envisaged as an epoch of galaxy formation.

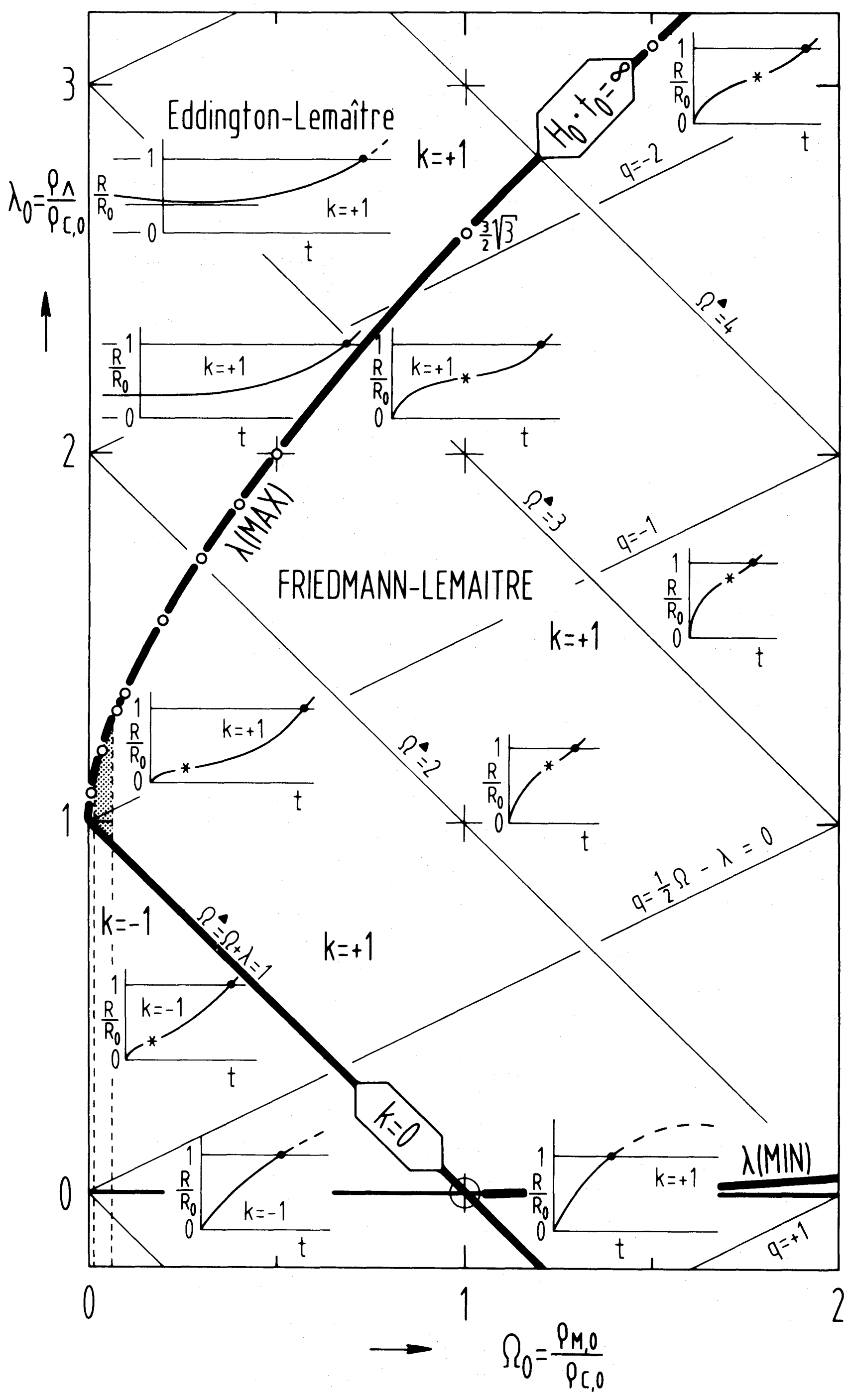

Fig. 5. Classification of cosmological models depending upon the present density parameter and the normalized cosmological term λ_0. The Eddington–Lemaître models (upper left corner) can be excluded. The permitted solutions lie below the λ(MAX) curve, the upper limit for Friedmann–Lemaître models with a point of inflection (see the $*$ in the insets). The observed matter densities $0.01 < \Omega_0 < 0.06$ restrict the models to the small range within the dashed lines on the left. The Big Bounce scenario limits the models further to the dotted area with $k = +1$

At the beginning of the Friedmann expansion two important boundary conditions must have been provided for a cosmos which was able to develop into our present universe: the creation of primordial matter with an extreme energy density and an extremely large expansion rate at that time. These two fundamental requirements can be fulfilled by a Big Bounce scenario. It provides the extreme expansion rate from a de Sitter solution of the Einstein equations at the moment of the phase transition by which the primordial elementary particles orginate. In this scenario the principle of causality is being maintained. Space and time extend infinitely into the past.

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