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THE CRUCIAL ROLE OF COOLING IN THE MAKING OF MOLECULAR CLOUDS AND STARS[1]

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ABSTRACT

We examine the role that velocity or pressure fluctuations in the interstellar medium can play in initiating compression of sub-Jeans mass diffuse clouds, paying special attention to the cooling properties of the medium. *Non*equilibrium energy arguments are used to determine the fluctuation amplitudes that are required to initiate gravitational collapse of low-mass cloud clumps. Clouds that cool under compression—i.e., that have effective adiabatic exponents $0 < \Gamma < 1$—can be induced to collapse by pressure fluctuations $\delta P_e/P_e \gtrsim \Gamma/(1-\Gamma)$ or by implosion velocities of Mach number $\mathscr{M}_0 \gtrsim [10/(3-3\Gamma)]^{1/2}$. This result is *independent of cloud mass* and is markedly different from the behavior found for isothermal gases. Since the interstellar medium has an effective $\Gamma \sim \frac{3}{4}$ at densities $10\ \mathrm{cm}^{-3} \lesssim n \lesssim 10^5\ \mathrm{cm}^{-3}$, we deduce that comparatively mild pressure fluctuations ($\delta P_e/P_e \sim 3$) and velocity disturbances ($|v| \sim 2\ \mathrm{km\ s}^{-1}$) can effectively compress H I gas to molecular cloud densities over a wide range of cloud masses ($1\ M_\odot \lesssim M \lesssim 10^3\ M_\odot$). We conclude, therefore, that the cooling properties of the ISM play a crucial role in the star formation process.

Subject headings: hydrodynamics — interstellar: molecules — stars: formation

I. INTRODUCTION

A primary hindrance to our understanding of the process of star formation in the interstellar medium is our lack of understanding of how molecular clouds form. Cool H I clouds having densities $n \sim 40\ \mathrm{cm}^{-3}$ and temperatures $T \sim 100$ K can exist in pressure equilibrium with a warmer, diffuse medium, but molecular clouds ($n_{H_2} \gtrsim 10^4\ \mathrm{cm}^{-3}$, $T \sim 10$–30 K) have internal pressures that are typically much larger than the pressure in H I clouds so they cannot be understood as a dense component of a multiphased, pressure-balanced medium.

Gravity almost certainly plays a role in defining the internal properties of molecular clouds since clumps $\lesssim 10\ M_\odot$ are often observed to have temperatures and sizes that indicate they are gravitationally bound, but it is not obvious how clumps of this size actually form. We are in an uncomfortable position if we ask gravity to be solely responsible for the transformation of diffuse H I clouds into self-bound molecular clumps because, at their typical temperature and density, H I clouds must acquire a mass $M \gtrsim M_J \sim 10^3\ M_\odot$ before they can become Jeans unstable. If we are to completely understand the process of star formation, we must understand how H I clouds transform into a clumpy molecular medium.

In this paper, we briefly outline the role that velocity or pressure fluctuations in the H I cloud medium can play in initiating compression of sub-Jeans mass diffuse clouds. In § II, we review the frequently discussed idea that substantial overpressures arising in the warm medium and/or highly supersonic compressions of H I clouds can push sub-Jeans mass clumps to densities where gravity can take over and complete the star formation process (Öpik 1953; Shu *et al.* 1972; Loren 1976; Woodward 1976; Elmegreen and Lada 1977; Herbst and Assousa 1977; Sandford, Whitaker, and Klein 1982; Klein, Sandford, and Whitaker 1983; Krebs and Hillebrandt 1983; Hausman and Roberts 1984; Oettl, Hillebrandt, and Mueller 1985). One difficulty with this picture is that it is not at all clear that such strongly nonlinear disturbances in an H I medium can efficiently promote star formation—they are just as likely to disrupt a cloud as actually to lead to ordered compression (Woodward 1976; Hausman 1981; Krebs and Hillebrandt 1983; Lattanzio *et al.* 1985). What we show here is that highly nonlinear disturbances are *not* always required to initiate substantial compression of sub-Jeans mass clouds in the interstellar medium.

In the past, *equilibrium* arguments have usually been used to quantitatively estimate the (nonlinear) amplitude that a given external disturbance must have if it is to successfully initiate the collapse of a pressure-supported gas cloud. When a disturbance occurs on a time scale short compared to the sound-crossing time of a cloud, however, equilibrium arguments are inappropriate. In § III, a nonequilibrium model is used to analyze the impact of external disturbances on gas clouds. As a result, estimates of the required disturbance amplitudes are substantially reduced from the previous estimates that were based on equilibrium arguments. Similar points have been made recently by Hunter (1979), Whitworth (1981), and Hunter and Fleck (1982).

Our nonequilibrium analysis reveals that a cloud which cools under compression—i.e., a cloud of gas for which the effective adiabatic exponent Γ is $0 < \Gamma < 1$—is particularly sensitive to mild disturbances from its environment. Further-

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more, the specific energy required to trigger effective compressions in a cooling medium is nearly independent of the cloud mass. This surprising result suggests that, for a given size disturbance in the H I medium, a wide spectrum of cloud masses below the canonical Jeans mass will condense into self-gravitating clumps. Since diffuse clouds which are sufficiently large that they are self-shielded against outside heating do cool under compression, we propose that mildly nonlinear disturbances play a primary role in the formation of molecular clouds and, in turn, stars.

II. A GLOBAL ENERGY FUNCTION

If we ignore the influence of magnetic fields—which we will do throughout this section in order to illustrate our main point—a gas cloud of mass M, internal sound speed a_0, and total angular momentum J that is embedded in a warmer, low-density medium of pressure P_e has four global energy quantities that are important in defining its equilibrium structure: $S \equiv$ internal thermal energy, $W \equiv$ gravitational potential energy, $T_R \equiv$ rotational kinetic energy, and $P_e V$, where V is the cloud volume. To within factors of order unity, for a spherical cloud these global quantities are

$$S = \frac{3}{2} \frac{Ma_0^2}{\Gamma} \left(\frac{r}{r_0}\right)^{3(1-\Gamma)} , \tag{1}$$

$$T_R = \frac{5}{4} \frac{J^2}{Mr^2} , \tag{2}$$

$$W = -\frac{3}{5} \frac{GM^2}{r} , \tag{3}$$

$$V = \frac{4\pi}{3} r^3 , \tag{4}$$

where r is the cloud radius. The variation of the internal sound speed with cloud radius given by equation (1) comes from adopting the relation $d \ln T/d \ln \rho = \Gamma - 1$; here a_0 is the adiabatic sound speed established at some initial cloud size r_0. We define a global energy function of the form

$$G(r) = \tfrac{2}{3} b(\Gamma) S + W + T_R + P_e V , \tag{5}$$

where $b(\Gamma) \equiv (1 - \delta_{1\Gamma})(\Gamma - 1)^{-1} + \delta_{1\Gamma} \ln \rho$ is introduced as a coefficient to S in order to allow generalization to the isothermal case ($\Gamma = 1$) and $\delta_{1\Gamma}$ is the Kronecker delta function. This single function G houses a great deal of dynamical information about the cloud system, as has been previously pointed out in various contexts by Whitworth (1981), Stahler (1983), Tohline (1985), and Tohline and Christodoulou (1988; hereafter TC). For fixed M, J, P_e, a_0, and Γ, cloud sizes that permit a virial equilibrium are given by the condition $dG/dr = 0$; the relative radial stability of equilibrium states is determined by whether d^2G/dr^2 is positive or negative (the latter indicating stability); and, as Whitworth (1981) points out, the equation of motion for homologous contraction or expansion of the cloud can be derived from the Lagrangian

$$L = T_{\rm KE} - G , \tag{6}$$

where $T_{\rm KE} = 3M\dot{r}^2/10$ is the kinetic energy in radial motion (see also Weber 1976). Clearly the function G contains an enormous amount of useful information about the dynamical properties of our gas-cloud system.

Figure 1 shows qualitatively how the function $G(r)$ typically behaves for a cloud system in which $\Gamma < 4/3$. The three curves

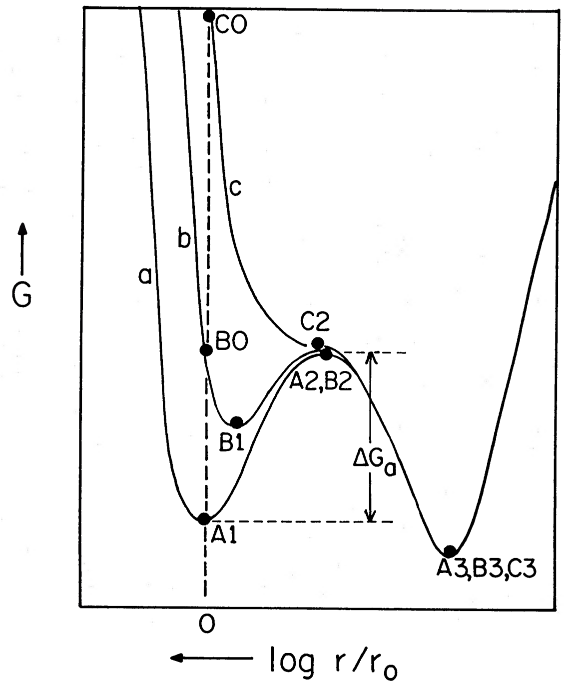

FIG. 1.—Illustration of the typical behavior of the energy function $G(r)$ for a cloud system in which $\Gamma < 4/3$. Curve a represents the situation for a sub-Jeans mass cloud of radius r_0 in pressure equilibrium with the lower density ISM whose ambient pressure is $P_e|_a$; a potential energy barrier of height ΔG_a separates the pressure-supported cloud at point A1 from its compact, gravitationally bound state at point A3. Curve c represents the situation for the same cloud (now positioned at point C0) after the ISM pressure has been rapidly increased to a pressure $P_e|_c$ sufficiently large that no local pressure-supported equilibrium state exists at a cloud radius r_0; the cloud is dynamically unstable at C0 and must collapse to the gravitationally bound state at point C3. Curve b represents the situation at an intermediate ISM pressure $P_e|_c > P_e|_b > P_e|_a$; a pressure-supported equilibrium state does exist at point B1, but the cloud positioned at B0 is in a "preinstability" state with enough potential energy available to allow it to collapse through state B1 and over the energy barrier at point B2.

illustrate, for a cloud of fixed M, J, a_0^2, and Γ, how $G(r)$ changes when the external pressure P_e adopts three different values—curve a is for the lowest of the three pressures and curve c is for the highest. [The same curves can also be used to illustrate how $G(r)$ changes, at fixed P_e, J, a_0^2, and Γ, for three different cloud masses. In this case, curve a represents the least massive cloud and c the most massive.] Curve a is typical for a cloud whose mass is too small to promote a Jeans instability at the normal external pressure $P_e|_a$. The cloud sits at the stable potential minimum marked by point A1, its structure being determined primarily by a balance between its internal thermal energy and the $P_e V$ energy of its warmer surroundings (virial equilibrium demands that $2S = 3P_e V$ at point A1). If the cloud could somehow climb out of its potential well to point A2 where an unstable virial equilibrium is established by the balance between internal thermal energy and gravity (i.e., $2S = -W$), it would experience a gravitational instability and collapse into the more compact state at the potential minimum A3. In this particular example, the stable minimum at A3 is governed by a balance between rotational energy and gravity ($2T_R = -W$), but even without rotation, in any real situation a stable minimum must exist to the right of point A2 in Figure 1 as collapse ultimately leads to a pressure-supported ($\Gamma > 4/3$) stellar state. There is generally a nontrivial energy barrier of size ΔG separating a sub-Jeans mass cloud at A1 from its compact state at A3. In order to allow the gravitationally assisted collapse of sub-Jeans mass diffuse clouds to occur, one must, in effect, devise a reasonable way for clouds to climb from point A1 to point A2. We will return to a discussion of this energy barrier problem in § III.

If the pressure of the warmer medium surrounding the cloud increases to a value $P_e|_c$ so that the relevant free energy function is curve c in Figure 1, the cloud will collapse dynamically from its initial size r_0 toward its compact equilibrium state at point C3. It will do so, because there is no longer a local minimum in the free energy function near r_0. Point C2 marks an extremum in the function $G(r)$ that is analogous to the point A2 on curve a, but for the critical curve c, it is an inflection point rather than a local maximum of the function. Collapse will ensue, in fact, if the external pressure is raised to any value $\geq P_e|_c$. For a given adiabat and cloud mass M, the critical pressure $P_e|_c$ is given by the expression

$$P_e\bigg|_c = [\theta^4 G^{-3\Gamma}(f/M)^{2\Gamma}]^{1/(4-3\Gamma)}\,, \tag{7a}$$

where $\theta \equiv \Gamma^{-1} a_0^2 \rho_0^{(1-\Gamma)}$ specifies the adiabat, ρ_0 is the cloud density at its initial radius r_0, and f is a dimensionless parameter of order unity that is a function of Γ. (According to TC, $f^2 \approx [375/(32\pi)](3\Gamma/2)^3[(4-3\Gamma)/4]^{(4-3\Gamma)/\Gamma}$; the exact value of f can be gotten from Kimura 1981 for a wide range of discrete values of Γ.) Equation (7a) can be inverted to also give the critical mass M_J above which no pressure-supported equilibrium exists for a given choice of P_e and θ:

$$M_J = f[\theta^{2/\Gamma} G^{-3/2} P_e^{(3\Gamma-4)/2\Gamma}]\,. \tag{7b}$$

The quantity M_J is, effectively, the familiar Jeans mass defined for a cloud in pressure equilibrium with its surroundings.

It is the effect of increasing the external pressure to a value $P_e|_c$ that, as mentioned in § I, has often been discussed as a viable mechanism for initiating the collapse of sub-Jeans mass H I clouds. Under isothermal ($\Gamma = 1$) compressions, relation (7b) shows that a factor of 5 increase in the external pressure should lead to the self-gravitating collapse of clouds having masses $5^{1/2}$ times smaller than M_J in the unperturbed medium. In a medium for which the effective Γ is $\frac{4}{7}$, say, relation (7b) shows that a factor of 5 increase in P_e will drop the unstable mass limit by a factor of 5^2. This is a substantial improvement over the meager factor of $5^{1/2}$ obtained for an isothermal environment. From this point of view, Shu *et al.* (1972) stressed the importance of a cooling (i.e., $\Gamma < 1$) medium. If the effective Γ actually drops as P_e increases, an additional drop in the unstable mass limit will occur because the normalization parameter f decreases with decreasing Γ.

III. SURMOUNTING THE ENERGY BARRIER

The first point we want to emphasize in this paper is that a sub-Jeans mass cloud initially at point A1 in Figure 1 need not be subjected to a pressure as large as $P_e|_c$ in order to initiate its collapse to point A3. The pressure $P_e|_c$—derived assuming *equilibrium* conditions—is needed only in a situation where the external pressure is increased slowly (on a time long compared to the sound crossing time of the cloud at A1). A rapid increase in P_e will find the cloud of radius r_0 (point C0 in Fig. 1) far from an equilibrium state. From Figure 1 it is clear that if the change occurs rapidly, a considerably milder increase in P_e, up to a value $P_e|_b$ represented by curve b, is sufficient to give the cloud of radius r_0 the potential to climb over the potential energy barrier that separates it from the desired equilibrium state A3 or, more correctly, B3. The truth actually lies somewhere in between. Inevitable dissipation during collapse from point B0 through B1 and toward B2 will probably prevent complete collapse when P_e is increased only to a value $P_e|_b$, but surely a value as large as $P_e|_c$ is not demanded.

Whitworth (1981) has discussed the important role of this type of "preinstability" that exists for a cloud at a point like B0 in Figure 1. He realized that for a given increase in P_e, or a sufficient inwardly directed radial kinetic energy, a cloud that would otherwise be stable could be induced into gravitationally assisted collapse. Hunter (1979) and Hunter and Fleck (1982) quantified the benefits of a nonequilibrium analysis of this problem in a slightly different way and came to the same conclusion. For the regime $1 < \Gamma < 4/3$, they determined the minimum mass cloud that could be forced into gravitational collapse by a given amount of kinetic energy, in the form of ordered radial implosion. They found that the cloud mass can be substantially reduced from the values predicted by equilibrium arguments and relation (7b). Ignoring the internal turbulent motions, which were part of the Hunter and Fleck discussion, we can rederive their basic result from our energy function G. In what follows, we present a simple analytic model which reproduces the results obtained by Whitworth (1981), Hunter (1979), and Hunter and Fleck (1982) in the range $1 < \Gamma < 4/3$, including both the effect of a rapid increase in external pressure and that of an imposed radial velocity field. The model is general in the sense that it includes the important regime $\Gamma < 1$; the minimum mass that can be forced into collapse is dramatically smaller in this regime than in the isothermal case. The conclusions from the analytic model are then substantiated through the results of numerical simulations of the spherically symmetric hydrodynamic collapse of a cooling gas.

a) An Analytic Model

At point A1 in Figure 1, the energy function G is dominated by the $P_e V$ and the $2b(\Gamma)S/3$ terms, while, at point A2, it is dominated by the terms $2b(\Gamma)S/3$ and W. Taking into account the virial demands on these various terms at the points A1 and A2, as mentioned above, the size of the energy barrier ΔG can immediately be written as

$$\Delta G = \left(\frac{2}{3}\,b_2 S_2 - 2S_2\right) - \left(\frac{2}{3}\,b_1 S_1 + \frac{2}{3}\,S_1\right)$$
$$= \frac{M}{\Gamma}\,a_0^2 \left|(b_2 - 3)\left(\frac{r_2}{r_0}\right)^{3(1-\Gamma)} - (1 + b_1)\right|\,, \tag{8}$$

where the subscripts 1 and 2 refer to states A1 and A2, respectively. Furthermore, the statement of virial balance at points A1 and A2 allows us to cast the ratio r_2/r_0 in terms of the ratio of the cloud mass to its critical mass as given by equation (7b). For $m \equiv M/M_J \ll 1$ (i.e., for $r_2/r_0 \ll 1$),

$$(r_2/r_0)^3 \approx g m^{2/(4-3\Gamma)}\,, \tag{9}$$

where $g \equiv [(4-3\Gamma)/4]^{1/\Gamma}(3\Gamma/4)^{3/(4-3\Gamma)}$. Therefore, equation (8) can be rewritten as

$$\Delta G \approx \frac{1}{\Gamma}\,M a_0^2\{(b_2 - 3)[g m^{2/(4-3\Gamma)}]^{(1-\Gamma)} - (1 + b_1)\}\,. \tag{10}$$

Three interesting regimes arise:

1. For $4/3 > \Gamma > 1$, relation (10) reduces to

$$\Delta G/M \approx a_0^2[4^{(\Gamma-1)/\Gamma}\Gamma^{1/(3\Gamma-4)}(4-3\Gamma)^{1/\Gamma}(\Gamma-1)^{-1}] \times [(\tfrac{3}{4})^3 m^2]^{-(\Gamma-1)/(4-3\Gamma)} \tag{11a}$$

because for $m \ll 1$ the second term inside the braces of relation (10) is negligibly small compared to the first term.

2. For $\Gamma = 1$, relation (10) becomes

$$\Delta G/M \approx -2a_0^2(\ln m + 0.875)\,, \tag{11b}$$

because $b_2 - b_1 = \ln(\rho_2/\rho_0) = -\ln(27m^2/256)$.

3. For $1 > \Gamma > 0$, relation (10) reduces to

$$\frac{\Delta G}{M} \approx \frac{a_0^2}{(1-\Gamma)}\,, \tag{11c}$$

because for $m \ll 1$ the mass-dependent term inside the braces becomes negligible compared to the $1 + b_1$ term.

If we ask ordered kinetic energy in radial motion to provide the needed boost over the energy barrier from point A1, then the required kinetic energy is $T_{\mathrm{KE}} \geq \Delta G$, that is,

$$\dot{r}_0^2 \geq \tfrac{10}{3}(\Delta G/M)\,. \tag{12}$$

As Hunter and Fleck (1982) determined, for $4/3 > \Gamma > 1$ the required implosion velocity $\dot{r}_0$ goes as a power law of the mass ratio m. Writing $\dot{r}_0 \propto m^{-x}$, we get $x = (\Gamma - 1)/(4 - 3\Gamma)$. In an isothermal regime, the required $\dot{r}_0^2$ is proportional to $\ln m$, in agreement with Hunter's (1979) conclusion. Hunter was struck by the logarithmic dependence of $\dot{r}_0^2$ on the mass ratio m in the isothermal case or, more specifically, by the fact that inverting the relation leads to an *exponential* dependence of m on the Mach number $\mathscr{M}_0 = \dot{r}_0/a_0$ of the applied implosion (see also Hunter 1969). The exponential behavior indicates that only a moderate increase in the level of external disturbances can drastically affect the (initially sub-Jeans mass) spectrum of cloud masses that are effectively pushed into gravitational collapse. It should be realized, however, that when the coefficients in relations (12) and (11b) are properly accounted for, the magnitude of $\mathscr{M}_0$ is not small unless m is very close to unity. In column (7) of Table 1, we have tabulated $\mathscr{M}_0$ as a function of m for isothermal compressions. (For $m \sim 1$ we have used the general expression for G—eq. [5]—to determine the barrier height instead of the approximate relation [11b].) For $m = \frac{1}{2}$, the required implosion is already supersonic, and a Mach 3.25 implosion is needed, for example, to push clouds with masses $M_J/10$ into collapse. We conclude, therefore, that substantially supersonic disturbances are generally required to initiate the collapse of sub-Jeans mass clouds when $4/3 > \Gamma \geq 1$.

We can also use expressions (11a)–(11c) to quantify the pressure enhancement $\delta P_e/P_e$ that must be realized in the confining, external medium in order to bring a given cloud to the "preinstability" state envisioned by Whitworth (1981). The energy *per unit volume* $V_0 = 4\pi r_0^3/3$ of the unperturbed cloud that is required to boost the cloud over its energy barrier is just $\Delta G/V_0 = (\Delta G/M)\rho_0 \approx (\Delta G/M)(\Gamma P_e/a_0^2)$. Therefore,

$$\frac{\delta P_e}{P_e} \gtrsim \frac{\Gamma}{a_0^2}\left(\frac{\Delta G}{M}\right). \tag{13}$$

We see that, for $4/3 > \Gamma \geq 1$, the required perturbation in pressure scales with cloud mass m in exactly the same way that $\dot{r}_0^2$ does. Again, the logarithmic dependence of $\delta P_e/P_e$ on m in the isothermal case looks enticing. What about the required magnitude of the fluctuations in P_e? In column (6) of Table 1 we have tabulated $\delta P_e/P_e$ as a function of m for $\Gamma = 1$. While relatively small pressure fluctuations ($\delta P_e/P_e < 1$) are sufficient to initiate collapse of clouds with masses $M \lesssim M_J$, a substantial nonlinear fluctuation in P_e is required to drive clouds with masses $M \ll M_J$ into self-gravitating collapse.

An important point not specifically addressed by either Whitworth (1981) or Hunter and Fleck (1982) is the behavior of $\Delta G/M$ for a cooling medium. As expression (11c) shows, for $1 > \Gamma > 0$ the barrier height, expressed in ergs per gram *is independent of the cloud mass*! Combining equation (11c) with equation (12), we see that an implosion with a Mach number

$$\mathscr{M}_0 \gtrsim \left[\frac{10}{3(1-\Gamma)}\right]^{1/2} \tag{14a}$$

will push all clouds into gravitationally assisted collapse. Or, using equation (13), we see that a rapid pressure enhancement of magnitude

$$\frac{\delta P_e}{P_e} \gtrsim \frac{\Gamma}{(1-\Gamma)} \tag{14b}$$

will do the same thing. These are, of course, the perturbations required in the limit $m \ll 1$. A more complete treatment shows that as m approaches 1, the required $\mathscr{M}_0$ or $\delta P_e/P_e$ is even less than the values set by the limiting case. We have used equation (5) to determine the exact height of the energy barrier as a function of cloud mass m for $\Gamma = \frac{3}{4}$ and for $\Gamma = \frac{1}{2}$ and have tabulated the results in Table 1. Again, we have used relations (13) and (12) to convert barrier height into the required $\delta P_e/P_e$ and $\mathscr{M}_0$. For comparison purposes, all the data from Table 1 has been plotted in Figures 2 and 3. Figure 2 shows $\delta P_e/P_e$ versus log m, and Figure 3 shows $\mathscr{M}_0$ versus log m for $\Gamma = \frac{1}{2}$, $\Gamma = \frac{3}{4}$, and $\Gamma = 1$ gases.

As the data in Table 1 and as Figures 2 and 3 illustrate, the energy barrier that separates a cloud of a given mass from its self-gravitating, collapsed state gets smaller for smaller values of Γ. Therefore, by comparison with clouds that compress isothermally, clouds that evolve along $\Gamma < 1$ adiabats can be triggered into collapse by relatively milder external disturbances. Also, as Γ decreases, the spectrum of cloud masses that can be pushed into collapse by *subsonic* disturbances, or by fluctua-

TABLE 1

REQUIRED DISTURBANCE AMPLITUDES

	$\Gamma = \frac{1}{2}$		$\Gamma = \frac{3}{4}$		$\Gamma = 1$	
m[a] (1)	$\delta P_e/P_e$ (2)	$\mathscr{M}_0$ (3)	$\delta P_e/P_e$ (4)	$\mathscr{M}_0$ (5)	$\delta P_e/P_e$ (6)	$\mathscr{M}_0$ (7)
1	0.0	0.0	0.0	0.0	0.0	0.0
0.99	0.0002	0.039	0.0005	0.047	0.001	0.062
0.50	0.109	0.854	0.245	1.043	0.613	1.430
0.10	0.442	1.718	1.052	2.163	3.166	3.249
10^{-2}	0.752	2.239	1.928	2.927	7.526	5.009
10^{-3}	0.896	2.444	2.432	3.288	12.079	6.345
10^{-5}	0.983	2.560	2.846	3.556	21.276	8.421
10^{-7}	0.997	2.578	2.958	3.626	30.486	10.081
10^{-9}	0.9996	2.581	2.989	3.645	39.696	11.503
0	1.0000	2.582	3.0000	3.651	∞	∞

[a] Using boundary conditions on an homoentropic, spherically symmetric gas cloud of $\rho_b = 2 \times 10^{-23}$ g cm^{-3}, $T_b = 80$ K, and $\mu = 1.3$ (corresponding to $P_e = 1.0 \times 10^{-13}$ ergs cm^{-3}), the exact critical mass M_J is (Shu *et al.* 1972; Kimura 1981): 516 $M_\odot$ for $\Gamma = \frac{1}{2}$, 1242 $M_\odot$ for $\Gamma = \frac{3}{4}$, and 2708 $M_\odot$ for $\Gamma = 1$.

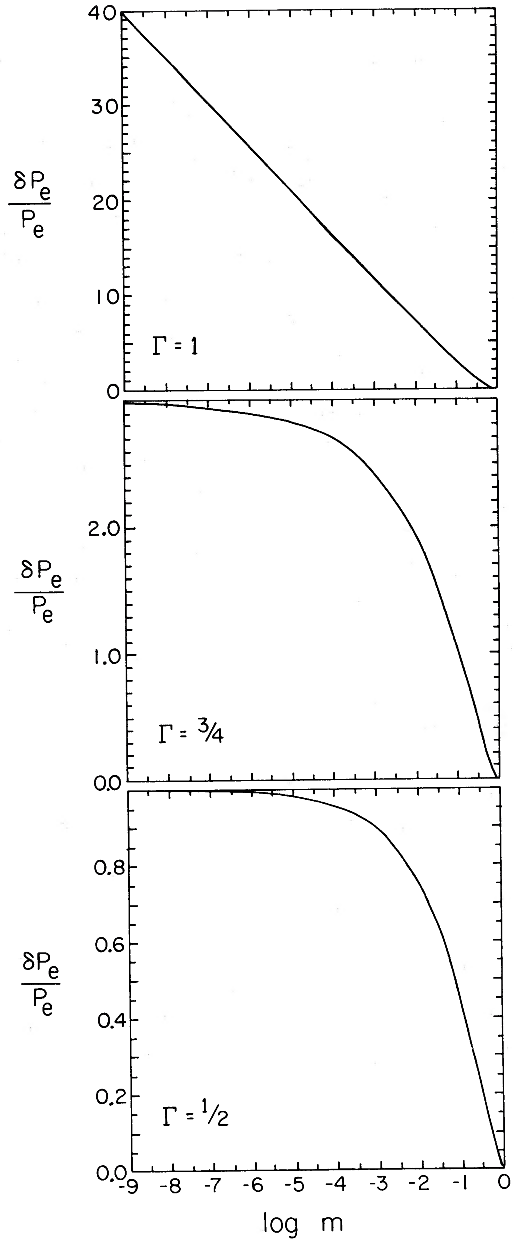

FIG. 2.—Fractional pressure-enhancement $\delta P_e/P_e$ required to push a cloud of mass M into gravitationally assisted collapse is plotted vs. log m ($m \equiv M/M_J$) for three different adiabatic indices: $\Gamma = 1, \frac{3}{4}, \frac{1}{2}$. For $m \ll 1$, the curves asymptotically approach the limiting behavior described by combining relation (13) with relations (11b) or (11c) in the text.

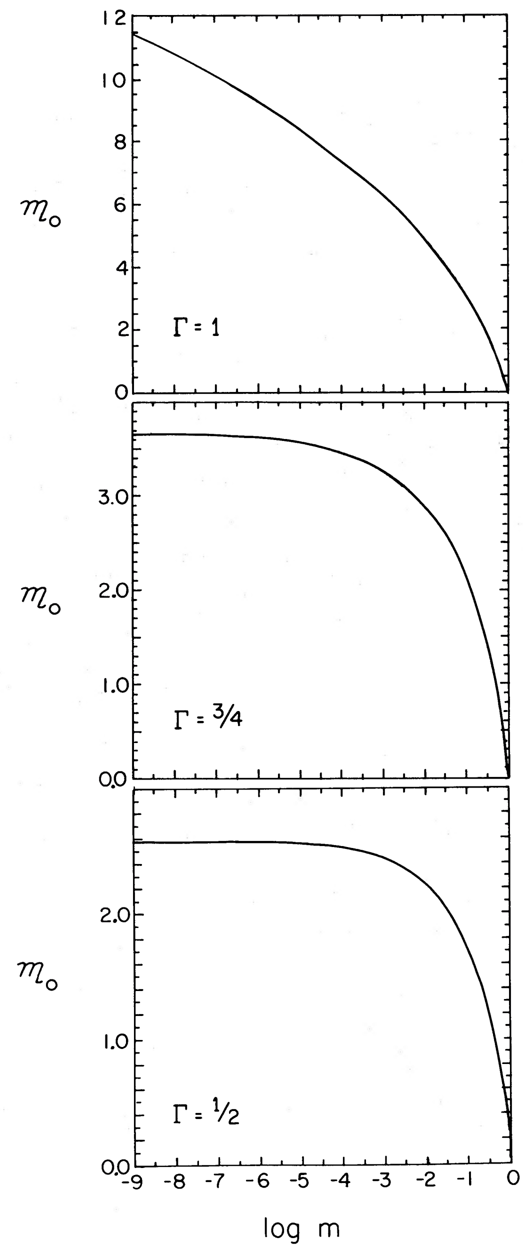

FIG. 3.—The Mach number $\mathcal{M}_0$ of the surface implosion velocity that is required to push a cloud of mass M into gravitationally assisted collapse is plotted vs. log m ($m \equiv M/M_J$) for the same three adiabatic indices used in Fig. 2. For $m \ll 1$, the curves approach the limiting behavior described by combining relation (12) with relation (11b) or (11c) in the text.

tions in P_e of amplitude $\delta P_e/P_e < 1$, noticeably broadens. Clearly, though, the most striking thing about $\Gamma < 1$ gases is the behavior of the energy barrier in the limit $m \to 0$. In this limit the barrier asymptotically approaches the finite values given by relations (14a) and (14b). The fact that $\Delta G/M$ does not approach infinity but, indeed, stays at a fairly low value for very small mass clouds clearly identifies the $\Gamma < 1$ regime as an important one for problems of star formation.

b) *Hydrodynamic Simulations*

In order to demonstrate both the correctness and the significance of the analytic results just stated, we modeled the response of cooling clouds to fluctuations in P_e and to imposed implosion velocities using the one-dimensional (spherically symmetric) implicit, Lagrangian hydrodynamic computer program developed by Bodenheimer (1968). Specifically, we constructed two clouds of different masses in hydrostatic equilibrium with an external, lower density medium of pressure $P_e = 1.0 \times 10^{-13}$ ergs cm^{-3}—a pressure typical of conditions thought to pertain in the interstellar medium. Both clouds were initially homoentropic models constructed with $\Gamma = \frac{1}{2}$ (i.e., their structures were initially like a truncated polytrope with index $n = -2$), having a boundary temperature $T_b = 80$ K, boundary density $\rho_b = 2 \times 10^{-23}$ g cm^{-3}, and mean molecular weight $\mu = 1.3$. Under these conditions, the adiabatic sound speed at the cloud boundary is $a_0 = 0.50$ km s^{-1}, and, by equation (7b), our analytic estimate of the critical mass is $M_J \approx 889\ M_\odot$. As is indicated in the footnote to Table 1, the exact value for this $n = -2$ polytrope is 516 $M_\odot$ (Shu *et al.* 1972; Kimura 1981). With this in mind, we chose cloud masses of 200 $M_\odot$ ($m_1 = 0.39$; model 1) and 0.02 $M_\odot$ ($m_2 = 3.9 \times 10^{-5}$; model 2).

In our first experiment, we imposed an initial velocity profile onto both clouds of a form $v(r) = v_b(r/r_0)$, where $v_b = -1.5$ km s^{-1}. This amounted to introducing an implosion of Mach number $\mathscr{M}_b = 3.0$ at each cloud surface and a velocity profile that would ensure a nearly homologous contraction, at least initially. The chosen $\mathscr{M}_b$ was only slightly larger than the critical value set by the above discussion: $\mathscr{M}_0 > (20/3)^{1/2} = 2.582$. During the subsequent collapse, we constrained both clouds to follow a $\Gamma = \frac{1}{2}$ adiabat and, hence, remain homoentropic. The result was, as predicted, an indefinite collapse for both clouds. The evolution of model 2 was particularly interesting because the $\mathscr{M}_b = 3.0$ implosion, by itself, was able to compress the cloud by five orders of magnitude in central density before the collapse received any substantial assistance from gravity. The calculation was run up to a central density of 2×10^{-18} g cm^{-3}, at which point the temperature was 0.3 K. The density profile was fairly constant in the inner region, then dropped rapidly outward; a plot of the density, velocity, and pressure profiles for this model is presented in Figure 4 at the time when $\rho_c = 10^{-18}$ g cm^{-3}. At the end of the calculation, the mass in the central peak was $3.5 \times 10^{-3}\ M_\odot$, while the Jeans mass corresponding to this region's ρ, T, and Γ was $3.9 \times 10^{-4}\ M_\odot$. As the region was still infalling, it is clear that gravitational instability had been induced.

For comparison, we also imposed the same initial velocity structure with $\mathscr{M}_b = 3.0$ onto an isothermal gas cloud of mass 0.02 $M_\odot$ ($m_3 = 7.4 \times 10^{-6}$; model 3) that initially had the same P_e, T_b, ρ_b, and μ as models 1 and 2. The ensuing isothermal collapse proceeded only to a maximum central density of 6.5×10^{-23} g cm^{-3}. The internal pressure gradient of the cloud first slowed the implosion, then reversed the motion—causing the cloud to reexpand—long before the cloud became dense enough for gravity to take control. This result is consistent with the analysis presented above which predicts that an $\mathscr{M}_0 \geq 8.54$ is required before collapse can proceed indefinitely in an isothermal cloud with $m = m_3$. For $\mathscr{M}_0 = 3.0$, in fact the minimum mass which could be induced to collapse is only $m \approx 0.1$, i.e., $M \approx 270\ M_\odot$ (see Fig. 3*a* and col. [7] of Table 1).

Finally, we constructed two models identical to models 1 and 2 at rest, in equilibrium with the external medium of pressure $P_e = 1.0 \times 10^{-13}$ ergs cm^{-3}, then introduced a rapid increase in the bounding pressure to a value $2.4P_e$ over a time somewhat less than the crossing time. Following a $\Gamma = \frac{1}{2}$ adiabat, both clouds responded to this disturbance by going into an indefinite collapse. In the case of the cloud of 0.02 $M_\odot$, the calculation was run until the central density reached 2×10^{-15} g cm^{-3}, eight orders of magnitude above the original value. At this time, the density distribution was extremely centrally peaked, and a small central region was clearly approaching free-fall collapse. This result is in agreement with the prediction that, for $\Gamma = \frac{1}{2}$, a $\delta P_e/P_e \gtrsim 1$ is sufficient to assist H I clouds of any mass over the potential energy barrier that separates them from a dense, self-gravitating equilibrium state.

IV. DISCUSSION

We have demonstrated that conditions in the interstellar medium that produce cooling upon compression (i.e., an effective adiabatic exponent $1 > \Gamma > 0$) are extremely favorable conditions for promoting the collapse of sub-Jeans mass gas clouds. Infinitesimal perturbations in the medium will not produce collapse, but finite disturbances of not an unreasonable magnitude can induce collapse of clouds over a wide range in mass. For a given $1 > \Gamma > 0$, a radially directed implosion of Mach number $\mathscr{M}_0 \geq [10/(3 - 3\Gamma)]^{1/2}$ or a rapid enhancement in the confining pressure of the external medium of magnitude $\delta P_e/P_e \gtrsim \Gamma/(1 - \Gamma)$ is sufficient to send gas clouds of all masses into a self-gravitating collapse. In the solar

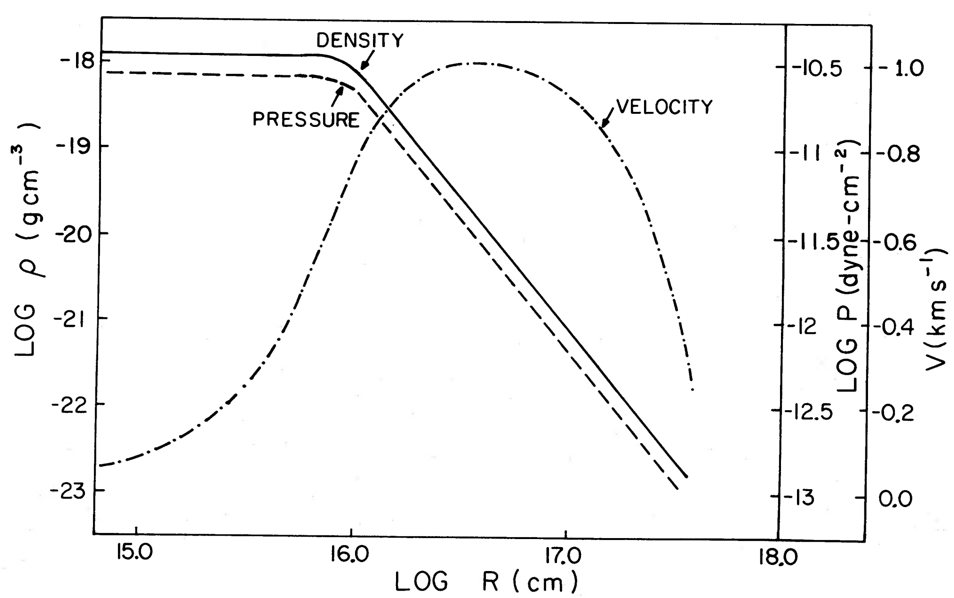

FIG. 4.—Distributions of density, pressure, and collapse velocity for model 2 are shown at a time 5×10^{12} s after the introduction of a velocity field with a maximum inward velocity of 1.5 km s^{-1}. The half-mass point lies at log $r = 16.7$. About 1/10 of the mass lies in the central density peak.

neighborhood, at least, conditions are such that cooling of gas does occur effectively between densities of $n \sim 10\ \mathrm{cm}^{-3}$ (a typical density of diffuse H I clouds) and $n \sim 10^5\ \mathrm{cm}^{-3}$. The effective adiabatic exponent in this regime is $\Gamma \approx \frac{3}{4}$ (Larson 1985). Therefore $\mathcal{M}_0 \gtrsim 3.7$ and $\delta P_e/P_e \gtrsim 3$ should induce the self-gravitating collapse of a wide spectrum of cloud masses.

There are limitations to this scenario that should be clarified. First, mass-independent response to these finite disturbances only pertains as long as the effective adiabatic exponent remains less than 1 (i.e., as long as the medium continues to cool) all the way up to the density where gravity is able to take hold and assist further collapse. Since the ISM is apparently unable to cool below ~ 10 K at densities above $n \sim 10^5\ \mathrm{cm}^{-3}$, the spectrum of masses induced to collapse in our scenario should realize a cutoff around 1 $M_\odot$ (the isothermal Jeans mass at $n = 10^5\ \mathrm{cm}^{-3}$). In order to investigate this possibility, we have generalized the expression for internal energy (eq. [1]) in a straightforward way to allow for a Γ that varies with gas density and have calculated ΔG as a function of M for the following $\Gamma(\rho)$ function:

$$\Gamma = \begin{cases} \frac{3}{4}, & \text{for } 2 \times 10^{-23} \leq \rho < 10^{-18}\ \mathrm{g\ cm^{-3}}, \\ 1, & \text{for } 10^{-18} \leq \rho \leq 10^{-14}\ \mathrm{g\ cm^{-3}}. \end{cases}$$

This two-branch function has been chosen in order to mimick the temperature-density behavior of the ISM (see, for example, Larson 1985). Figure 5 shows the resultant behavior of $\Delta G/M$—expressed, as before, in terms of both $\mathcal{M}_0$ and $\delta P_e/P_e$—as a function of m. Point A ($\log m = -3.1$), marked in both parts of the figure, identifies the density $\rho = 10^{-18}\ \mathrm{g\ cm^{-3}}$ at which the adopted $\Gamma(\rho)$ function changes its value from $\Gamma = \frac{3}{4}$ to $\Gamma = 1$. Clouds having $\log m < -3.1$ (i.e., having masses $M \lesssim 1\ M_\odot$) must be compressed to densities higher than 10^{-18} g cm^{-3} if gravity is to dominate over thermal pressure, hence, starting at diffuse cloud densities, they must receive initial external "kicks" of amplitude $\delta P_e/P_e > 2.5$ or $\mathcal{M}_0 > 3.3$ if gravitationally assisted collapse is to be achieved. Point B ($\log m = -5.3$) in both parts of the figure identifies the density $\rho = 10^{-14}\ \mathrm{g\ cm^{-3}}$ above which our "realistic" $\Gamma(\rho)$ function has not been defined. Above this density (approximately), clouds are believed to become opaque to their primary cooling radiation and their effective Γ becomes $>4/3$. As a result, thermal energy dominates above this density and clouds with $\log m < -5.3$ ($M \lesssim 0.01\ M_\odot$) cannot achieve a state of self-gravitational collapse. In Figure 5, a 1 $M_\odot$ cloud ($\log m = -3.1$) is not singled out as a natural mass scale, but the shift to a $\Gamma = 1$ behavior at densities above 10^{-18} g cm^{-3} does change the plotted functions noticeably from the strict $\Gamma = \frac{3}{4}$ behavior illustrated in Figures 2*b* and 3*b*.

Second, since finite and, indeed, supersonic disturbances are required to initiate collapse, the limits quoted here for $\mathcal{M}_0$ and $\delta P_e/P_e$ should not be taken literally. Our estimated limits are derived assuming 100% of the disturbing energy gets funneled into ordered radial collapse. A more reasonable, lower efficiency rate will increase the required $\mathcal{M}_0$ and $\delta P_e/P_e$ appropriately.

Third, our entire discussion has centered around idealized, spherically symmetric cloud implosions. The quantitative limits set on $\mathcal{M}_0$ and $\delta P_e/P_e$ will certainly change when other less ordered disturbances are studied. Indeed, in the real ISM, dynamical gas flows will generally be quite complicated since cloud collisions or shock-driven implosions may lead to Rayleigh-Taylor instabilities, Kelvin-Helmholtz instabilities,

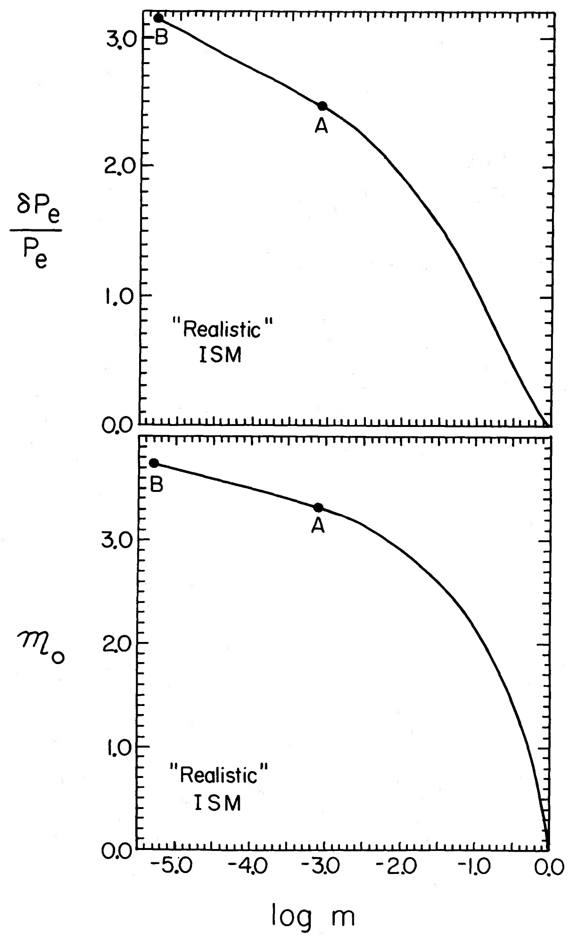

FIG. 5.—The pressure-enhancement $\delta P_e/P_e$ and the implosion Mach number $\mathcal{M}_0$ required to push a cloud of mass M into gravitationally assisted collapse are shown vs. $\log m$ ($m \equiv M/M_J$) as determined from a "realistic" model of the interstellar medium. Specifically, the two-branch $\Gamma(\rho)$ function defined in the text has been used to mimick the $\rho - T$ behavior of the ISM. From $\log m = 0$ up to point A ($\log m = -3.1$, corresponding to $\rho = 10^{-18}$ g cm^{-3}) a $\Gamma = \frac{3}{4}$ adiabat has been adopted; hence, in this region, direct overlap exists between the top diagram and Fig. 2*b* and between the bottom diagram and Fig. 3*b*. From point A to point B ($\log m = -5.3$, corresponding to $\rho = 10^{-14}\ \mathrm{g\ cm^{-3}}$), isothermal ($\Gamma = 1$) behavior has been adopted.

and large-scale vorticity (see the references to cloud disruptions cited in § I). However, an examination of the Hunter and Fleck (1982) analysis, which was done in a general fashion for planar and cylindrical disturbances as well as for spherical ones, indicates that insensitivity of the required disturbance energy to cloud mass will prevail for any type of adopted geometry of the disturbance. In this regard, we can comment on how the ISM's magnetic field might modify our general conclusions. If the energy density in the magnetic field that threads through a diffuse gas cloud is nonnegligible, then dynamical disturbances will tend to compress the cloud preferentially along the field lines—resulting in essentially planar implosions rather than spherically symmetric ones. Qualitatively, though, our basic conclusion remains unchanged: mild disturbances should effect substantial dynamical compressions of the gas because of its ability to cool. If the energy density in the magnetic field is initially large compared to the energy density in the cloud's gravitational field, dynamical compression of the gas to high density will probably not directly induce star formation because a planar compression will not significantly alter the relative energy density in the two fields. Dynamical expansion of the gas will immediately follow the phase of rapid compression. Nevertheless, the *cooling* property of the medium should, itself, permit relatively mild disturbances in the ISM to repeatedly produce significantly compressed states of the gas.

Tarafdar *et al.* (1985) noticed that when the effects of chemistry and its associated cooling are included in a model of a diffuse gas cloud, the evolution of the cloud can be substantially altered from that which would be derived in an isothermal treatment. They noted that even a relatively low mass cloud, starting from uniform density, goes into gravitational collapse in the presence of a constant P_e medium and attributed the effect largely to the "inward pressure gradient force" that arose due to their prescription for cooling. This force was simply a result of a somewhat artificial initial condition, and it would not have been developed if they had started with a proper equilibrium configuration and not a model of uniform density. We believe, however, that their result clearly illustrates the major point being made here. In adjusting *toward* its pressure-supported equilibrium state, their cloud with an effective $1 > \Gamma > 0$ generated enough energy in macroscopic motions simply to carry itself over the relatively low energy barrier that separated its diffuse state from a gravitationally bound one. A fairly mild disturbance was all that was required to send the cloud into gravitational collapse because its volume initially enclosed close to one Jeans mass of material. In a separate study, Hunter *et al.* (1986) have reported on the extremely compressible nature of a cooling medium taking part in a supersonic, planar cloud collision. These studies are good illustrations of the general phenomenon we have outlined in §§ II and III of this paper.

In his analysis of cloud fragmentation, Larson (1985) also noticed some interesting properties of systems having effective adiabatic exponents $1 > \Gamma > 0$. For example, once gravity becomes important in defining the structure of a flattened gas sheet, Larson states that "in contrast to the isothermal case, continuing fragmentation cannot be prevented as long as the temperature continues to decrease with increasing density."

We suggest that the clumpy structure observed on scales down to $\sim 1\ M_\odot$ in molecular cloud complexes is due in large part to the cooling properties of the ISM between densities of $\sim 10\ \mathrm{cm}^{-3}$ and $\sim 10^5\ \mathrm{cm}^{-3}$. Disturbances in the general ISM with velocities as small as $\sim 2\ \mathrm{km\ s}^{-1}$ or with fluctuations in the ambient pressure by a factor ~ 3 can easily compress cool H I gas up to $n \sim 10^5\ \mathrm{cm}^{-3}$ and on scales as small as one solar mass. In regions of our (or any other) galaxy where magnetic fields may not exert a dominating influence, the same types of fairly mild disturbances afflicting cool H I clouds could directly promote the self-gravitating collapse of sub-Jeans mass clumps over a wide mass spectrum. We conclude, therefore, that a cooling medium can play a crucial role in assisting the formation of both low-mass molecular cloud clumps and low-mass stars in the H I gas disks of galaxies.

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