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# GENERAL-RELATIVISTIC PERIASTRON ADVANCES IN ECLIPSING BINARY SYSTEMS

ALVARO GIMÉNEZ
Departamento de Astrofísica, Universidad Complutense, Madrid
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## ABSTRACT

The possibility of observing general-relativistic apsidal motions outside the solar system in well-detached eclipsing binaries is reviewed, taking into account the expected influence of the various parameters involved. A procedure to select candidate systems among eccentric eclipsing binaries is presented together with a list of suggested targets. A simplified method to obtain the apsidal motion rates from the observation of times of minimum light is proposed.

*Subject headings:* relativity — stars: eclipsing binaries

## I. INTRODUCTION

The observation of a secular displacement of the periastron in eccentric orbits has been pursued for long time as one of the best ways to test the equations of motion given by general relativity. A significant check of this theory was actually provided when the new equations were successfully applied to the observed perihelion advance of the planet Mercury. The same relativistic effect could be also detected later in some less favorable cases within the solar system (e.g., see Weinberg 1972, p. 198). The observation of general-relativistic apsidal motions exhibited by stellar-size objects outside the solar system has been shown to be possible in some close binaries. The present paper intends to contribute to the study of the observational and physical conditions that enhance such possibilities.

The observable apsidal motions are known to be due to the contribution of a classical term (caused by the gravitational quadrupole moment induced by rotation and tides) as well as the general-relativistic term. Up to the level permitted by the uncertainties of the observations, both contributions are separable, and the observational apsidal motion rate is expressed by

$$\dot{\omega} = \dot{\omega}_{\rm N} + \dot{\omega}_R \,, \tag{1}$$

where $\dot{\omega}_{\rm N}$ denotes the classical or Newtonian term and $\dot{\omega}_R$ is the relativistic contribution. The period of the periastron rotation, $U$, is furthermore given in days by

$$U = 360P/\dot{\omega} \,, \tag{2}$$

where $\dot{\omega}$ is expressed in degrees per cycle and $P$ is the anomalistic period expressed in days.

From a theoretical point of view, Levi-Civita (1937) showed that the equation originally found for the apsidal motion period of a test particle in the field of a mass point is also valid in the case of a binary system with two components of similar masses if expressed in terms of the relative orbit. The general conditions under which this is true are currently fulfilled in well-detached binaries to the required level of accuracy. This result was later confirmed and extended by Robertson (1938) and Einstein, Infeld, and Hoffman (1938), but it was, no doubt, the discovery of the binary pulsar by Hulse and Taylor (1975) that increased the interest in the study of two-body effects in general relativity. In particular, Barker and O'Connell (1978) carried out the extension of the equations to include axial rotation of the components.

From the observational point of view, the application of theoretical equations to actual cases of eclipsing binaries goes back to the work by Rudkjøbing (1959), who pointed out DI Her as a good candidate for the measurement of relativistic effects in its orbit. Koch (1973) proposed additional binary systems some years later. The survey, nevertheless, was far from complete, as noted by Koch, since only five candidates were included. In another paper, Koch (1977) analyzed the apparent period variations using the available times of minimum for those binary systems. In general, the periods of periastron revolution were found to be very long compared to the time intervals covered by the observations. At that time, the required accuracy to make a confident comparison between theory and observations could not be reached. Only for two systems of the sample the period variations were found to agree in sign, although not in amplitude, at the 1 $\sigma$ level.

Notwithstanding, the picture today is found to be quite different because of the much more accurate data provided for some detached eclipsing binaries (Andersen, Clausen, and Nordström 1984), the improvement of the methods for the apsidal motion determination (Giménez and Garcia-Pelayo 1983), and the present knowledge of the behavior of the internal density distribution for stars along the main sequence (Giménez and García-Pelayo 1982; Jeffery 1984). Concerning this latter point, needed to correct the observed apsidal motion periods for the expected classical contribution, new computations throughout the H-R diagram of the internal structure constants $k_j$ have been recently accomplished by Hejlesen (1982) from his own evolutionary tracks (Hejlesen 1980). Finally, the importance of enlarging the number of available minima for candidate systems makes worthwhile a rediscussion of the present status of relativistic periastron advances in eclipsing binaries so that a reliable list of targets can be obtained.

In this paper we present the procedure and equations to identify eclipsing binaries suitable for the study of relativistic apsidal motions and an improved method to analyze the observations. The first positive detections which support the work summarized in this paper have been published as separate contributions, but some details will be given in § VII.

## II. THE RELATIVISTIC CONTRIBUTION

The secular displacement of the periastron in a binary star was first derived by Levi-Civita (1937) in terms of the total mass of the system and the "semilatus rectum" of the orbit. For this purpose, the equations of motion obtained within the frame of general relativity under the first post-Newtonian approximation were adopted. Introducing directly observable parameters such as the orbital period, $P$, and the eccentricity, $e$, the relativistic apsidal motion, expressed in degrees per cycle, is given by

$$\dot{\omega}_R = 5.45 \times 10^{-4} \frac{1}{(1-e^2)} \left(\frac{m_1 + m_2}{P}\right)^{2/3} , \qquad (3)$$

where $m_i$ denotes the individual masses of the components in solar units. This equation becomes a convenient expression to compare theoretical predictions with the observations since $e$, $P$, and the total mass $M = m_1 + m_2$ can be accurately determined in double-lined eclipsing binaries. Nevertheless, it may be useful to take the semiamplitude of the radial velocity curve, $K = K_1 + K_2$, since this permits us to write

$$\dot{\omega}_R = 1.2 \times 10^{-8} K^2 \sin^{-2} i , \qquad (4)$$

if $K$ is expressed in km s$^{-1}$.

It is easily noticed thus that, in principle, the rate of periastron revolution will be faster for more massive systems with shorter periods. For nondegenerate stars, these conditions imply close components, but increasing proximity of the stars is also the origin of larger tidal distortions as well as faster axial rotation. In such cases, the term $\dot{\omega}_N$ in equation (1) becomes dominant as a result of the perturbations caused by polar flattening and tidal distortion, making less determinate the relativistic effect. Furthermore, from a purely relativistic point of view, the validity of the equations of motion leading to the simple result expressed by equation (3) would be severely restricted. Finally, observational selection indicates that only well-detached binaries with moderate masses can be expected to show a slow rotation and a mass pointlike behavior.

In order to have a more detailed idea of the existing relation between the theoretical log $\dot{\omega}_R$ and the observable ratio $M/P$ as indicated by equation (3), we have plotted values of this latter ratio from 0 to 1 in Figure 1. Different assumptions on the orbital eccentricity, namely, 0.05, 0.25, and 0.50, have been taken to show the slight increase of the apsidal motion rate with $e$ which only becomes relevant for very high eccentricities.

It may be assumed that the apsidal motion becomes undetectable for values below 0.0001 degrees per cycle. Figure 1 shows that systems suitable for relativistic studies should have orbital periods $P < 10M$. For the binaries selected in § V, the elements ($e$, $P$, and $M$) involved in the evaluation of equation (3) and their expected errors according to results for normal detached binaries (Popper 1980) suggest that the most accurate estimations will not be far from $\sigma(\dot{\omega}_R) = 0.0001$ degrees per cycle. This is also around the limit of the observational precision of periastron advances derived from the analysis of times of minimum as shown in § VI. Finally, the values quoted in Table 4 (§ VII) are good examples of these estimations.

Because of observational selection effects, most eclipsing binaries are known to verify this condition for the orbital period and the total mass, suggesting that the relativistic apsidal motion rate will be generally larger than the observational threshold value. A compromise is thus implied between periods long enough to avoid important classical contributions and short enough to verify the above given condition of detectability.

Let us now consider some deviations from the assumptions used to derive equation (3). Adopt the second-order post-Newtonian approximation for the relativistic equation of motion. The additional contribution to the relativistic apsidal motion is much smaller than the expected mean errors involved in the evaluation of equation (3), remaining com-

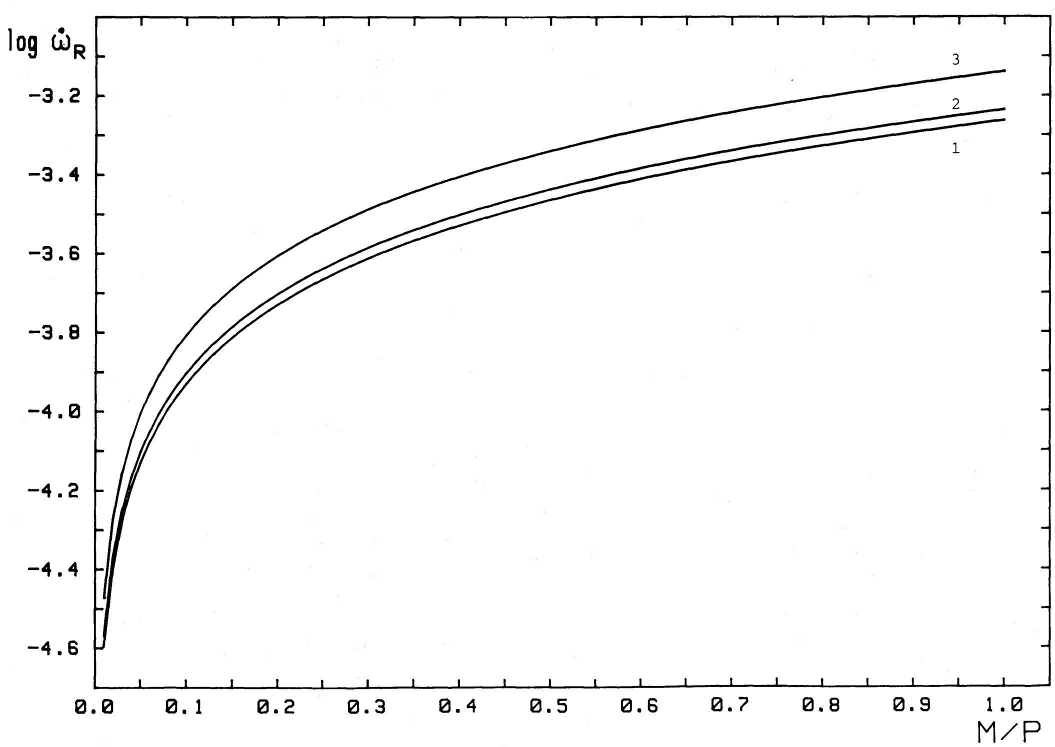


FIG. 1.—Relativistic apsidal motion rates for orbital eccentricities 0.05 (1), 0.25 (2), and 0.50 (3)

pletely negligible for normal nondegenerate stars (Giménez and Costa 1981). To estimate orders of magnitude, the equations obtained by Antonacopoulos and Tsoupakis (1979) have been used which only neglect terms of the order of $c^{-5}$ in the construction of the Hamiltonian (in order to avoid the loss of energy in the form of gravitational waves).

Concerning the possible perturbations of the relativistic orbit due to axial rotation, the equations given by Barker and O'Connell (1974) and expressed in a simpler form by Esposito and Harrison (1975) can be rewritten in terms of the angular momenta of the components, $J_i$, as

$$\sim 20 \frac{\pi^3}{c^2 PM} (J_1 + J_2) . \tag{5}$$

This permits us to predict that the spin-orbit relativistic contribution is proportional to $(v/c)^2$, where $v$ denotes the equatorial rotational velocity. The term will thus remain very small even in fast rotating pulsars, and become negligible in slow rotators like the components of detached binaries belonging to the main sequence.

## III. THE NEWTONIAN CONTRIBUTION

It is well known (Cowling 1938; Sterne 1939) that when the components of a binary system do not behave like mass points, the line of the apses has a secular rotation whose period, $U$, is given by

$$\frac{P}{U} = \frac{\dot{\omega}_N}{360} = c_1 k_{21} + c_2 k_{22} , \tag{6}$$

under the assumptions of coplanarity of the orbital and equatorial planes as well as no tidal lag (Kopal 1978). The $k_{2i}$ are known as the second-order internal structure constants for each component which can be evaluated from theoretical models by numerical integration of the Radau equation for any particular distribution of the internal density. Moreover, a weighted mean value of both components can be estimated by observing fast apsidal motions in eclipsing binaries with negligible relativistic effects (Giménez and García-Pelayo 1982). The computer-constructed stellar models could thus be tested satisfactorily against observational evidence within the main sequence. The $c_i$ coefficients are known functions of the mass ratio, the orbital eccentricity, and the relative radii $r_i$ (with respect to the semimajor axis of the orbit) and are expressed as

$$c_i = r_i^5 \left\{ \frac{m_{3-i}}{m_i} [15f(e) + \gamma_i^2 g(e)] + \gamma_i^2 g(e) \right\} , \tag{7}$$

where $g(e)$ and $f(e)$ are functions of the eccentricity alone given by Kopal (1978, eq. [3-53]) and $\gamma_i$ denote the ratio of the angular velocity of axial rotation to that of orbital motion.

Adopt a pseudo-synchronization of the rotational velocity for eccentric orbits, as suggested by tidal theories (Hut 1981) as well as observational evidences (Giménez and Andersen 1983), corresponding to maximum angular velocity being achieved at periastron. We can write as a good approximation to the actual ratio,

$$\gamma_i^2 = \frac{1 + e}{(1 - e)^3} , \tag{8}$$

and thus

$$c_1 = r_1^5 f(e, q) , \tag{9}$$

where $q$ is the mass ratio $m_2/m_1$ and $f(e, q)$ is a new function given by equations (7) and (8) which can be easily evaluated from observable parameters. In Figure 2, we have plotted this function for three values of the mass ratio against the orbital eccentricity.

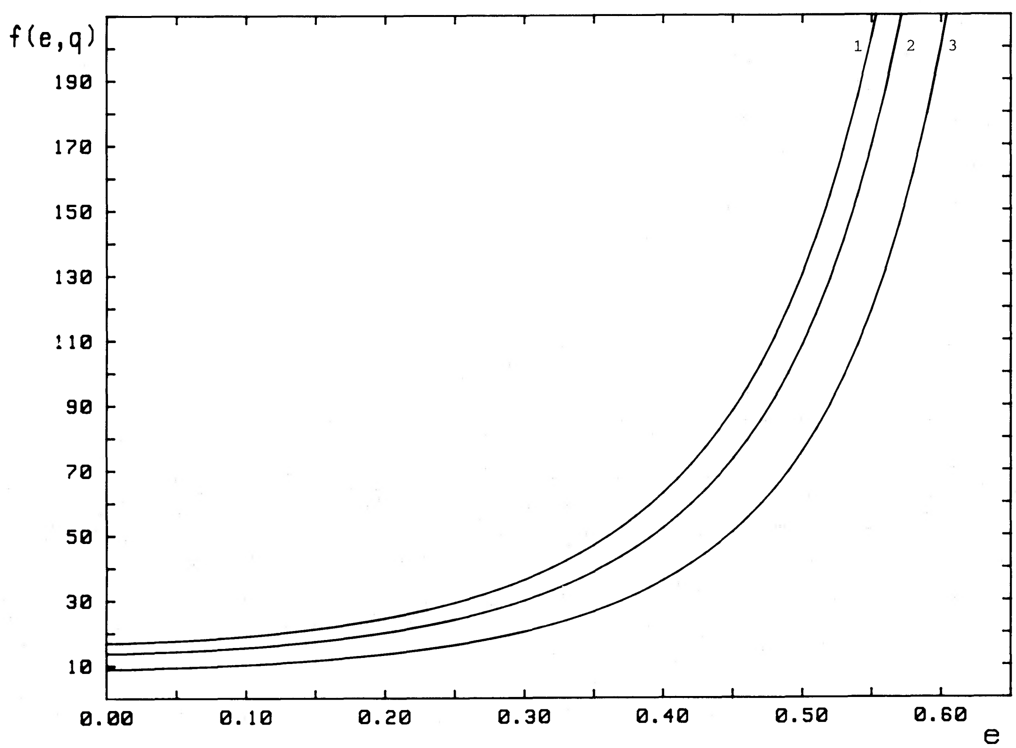


FIG. 2.—The $f(e, q)$ function for mass ratios 1.0 (1), 0.8 (2), and 0.5 (3)

It should be noted that in equation (6) we have only considered the second-order surface harmonics in the tidal distortion. For well-detached binaries, $r_i$ is always small and higher order terms (e.g., $j = 3$, 4) become irrelevant because of the relation $c_{ji} \propto r_i^{2j+1}$, while the corresponding constants, $k_{ji}$, also decrease very rapidly for increasing values of $j$ (Cisneros-Parra 1970).

These equations are important for isolating the relativistic effects in the observed apsidal motion rates according to equation (1). To have a better idea of the classical or Newtonian contribution, predicted values of log $\dot{\omega}_N$ are plotted versus relative radius in Figure 3 under the assumption of identical components. Three values of the internal structure constant have been taken (0.005, 0.0075, and 0.010), corresponding approximately to main-sequence models of 1.8, 3.2, and 5 solar masses, respectively, following the calculations by Hejlesen (1982) for a chemical composition $(X, Z) = (0.70, 0.02)$. The orbital eccentricity has been taken to be 0.05, but, in order to show also the dependence on $e$, the loci corresponding to the values 0.25 and 0.50 have been plotted for the internal structure constant 0.0075 only.

Equation (9) indicates the importance of using eclipsing binaries in any study of relativistic apsidal motions where it is necessary to correct for classical terms. The relative radii should be accurately known from good photoelectric light curves since statistical estimations of the radii in terms of the spectral type, even for main-sequence stars, are of low quality (Shallis and Blackwell 1980). Unfortunately, the probability of finding eclipsing systems decreases for increasing values of the orbital period, thus reducing the sample of probable candidates for the study of relativistic apsidal motions.

If synchronized axial rotation or coplanarity of the orbital and equatorial planes has not been achieved, the dynamical behavior of the system may be very different, as discussed by Kopal (1978), and the apparent period variations will not be caused only by the periastron displacement expressed as equation (6). The time scales for tidal evolution will depend on the masses and evolutionary stage of the components (Zahn 1977). In our sample of candidates, moderate orbital periods correspond necessarily to later type systems in order to keep the relative radii small enough. These binaries, moreover, present larger tidal dissipation and remain a longer time within the main sequence, thus making unimportant the above mentioned dynamical effects. Furthermore, when the orbital period is relatively long, tidal and rotational distortions are expected to be very small. Thence only the relativistic contribution, if any, will be detectable, and the asymmetries in the stellar configurations would not have any observable effect.

On the other hand, Papaloizou and Pringle (1980) suggested that resonances of the tidal perturbation with stellar oscillation modes may have considerable influence on the rate of periastron advance. In our cases, important relativistic contributions (with respect to the total apsidal motion) require small relative radii, always $\lesssim 0.1$, while the mentioned effect would only be relevant for $r > 0.2$ (Pringle 1983).

## IV. COMPARISON OF BOTH CONTRIBUTIONS

According to equation (6) and using Kepler's law, we have that $\dot{\omega}_N \propto P^{-10/3}$; i.e., the Newtonian apsidal motion decreases much faster than the relativistic term given by equation (3) and shows a dependence $\dot{\omega}_R \propto P^{-2/3}$ with the orbital period. As a consequence, we can expect to have a wide range of periods for which $\dot{\omega}$ remains detectable and is dominated by the relativistic contribution.

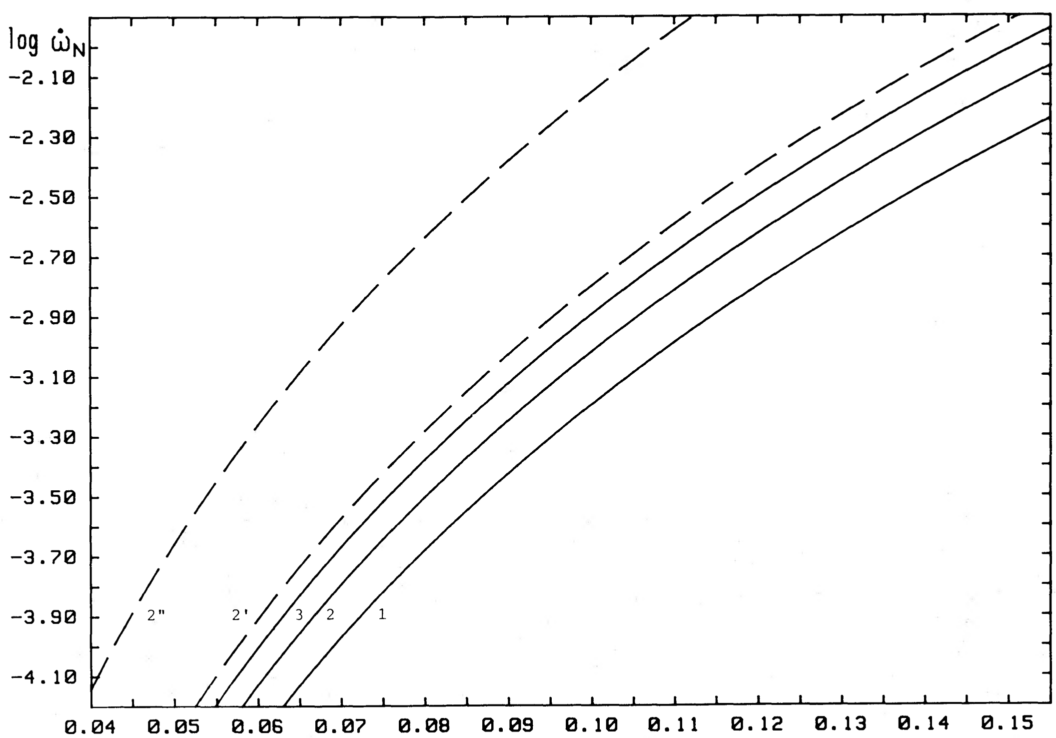


FIG. 3.—Newtonian apsidal motion rate for equal components, $e = 0.05$, and internal structure constants 0.0050 (1), 0.0075 (2), and 0.0100 (3). Dashed lines represent $k_2 = 0.0075$ for orbital eccentricities 0.25 (2′) and 0.50 (2″).

In order to attain a deeper insight into the physical conditions to be fulfilled by candidate systems, let us now consider the predicted ratio of the relativistic contribution to the classical one. As previously, we will assume a binary system with identical components. In our case, this is statistically a good approximation if the candidate systems should be moderate-mass double-lined eclipsing binaries (Batten 1973). Then we can find the ratio,

$$\alpha \equiv \frac{\dot{\omega}_R}{\dot{\omega}_N} , \quad (10)$$

where $\dot{\omega}_R$ and $\dot{\omega}_N$ are expressed as equations (3) and (6), respectively. This ratio will present a dependence on the orbital period of order of $P^{8/3}$ and will thus increase with the separation between both components. Under the assumption of identical stars, equation (6) can be rewritten using equation (7) and Kepler's third law in the form

$$\dot{\omega}_N = \frac{0.17 f(e, 1)}{(mP^2)^{5/3}} k_2 R^5 , \quad (11)$$

again expressed in degrees per cycle. The quantity $R$ is the radius of each component in solar units, $f(e, 1)$ is a function of the orbital eccentricity alone as defined in equation (9), $m = m_1 = m_2$ and $k_2 = k_{21} = k_{22}$. Finally, the ratio (10) will be given by

$$\alpha = \frac{0.005}{F(e)} \frac{m^{7/3}}{k_2 R^5} P^{8/3} , \quad (12)$$

where $F(e) = (1 - e^2) f(e, 1)$.

To estimate orders of magnitude for this ratio we can now adopt the mass-radius relation for main-sequence stars given by Habets and Heintze (1981) after a thorough statistical study of empirical results in the analysis of double-lined eclipsing binaries, in the form,

$$R = 1.125 m^{0.646} , \quad (13)$$

while the relation $k_2 R^2$, suggested to be approximately constant during main-sequence evolution by Giménez and García-Pelayo (1982), takes the form

$$k_2 R^2 = 0.0047 m^{1.585} , \quad (14)$$

as calculated using the results from theoretical models for different opacities and chemical compositions (Cisneros-Parra 1970; Stothers 1974; Hejlesen 1982).

As a consequence, equation (12) can now be rewritten as

$$\alpha = \frac{0.75}{m^{1.2} F(e)} P^{8/3} . \quad (15)$$

In Figure 4, we have plotted the predicted value of this ratio against the orbital period for an eccentricity of 0.05 and three values of $m$ (1.8, 3.2, and 5.0 solar masses). The relevance of the orbital eccentricity is shown for the 3.2 solar masses model and values of $e = 0.25$ and 0.50.

As predicted, the ratio of both contributions defined by equation (15) increases rapidly with period while the total apsidal motion decreases. We have therefore also plotted in Figure 4 the corresponding values of $\dot{\omega}$, given by equation (1), for equal components, using equations (3) and (11) for the masses and orbital eccentricities adopted before.

## V. THE CANDIDATE SYSTEMS

From the data derived in previous sections, it is clear that a minimum value for $\dot{\omega}$ will fix a maximum orbital period and a

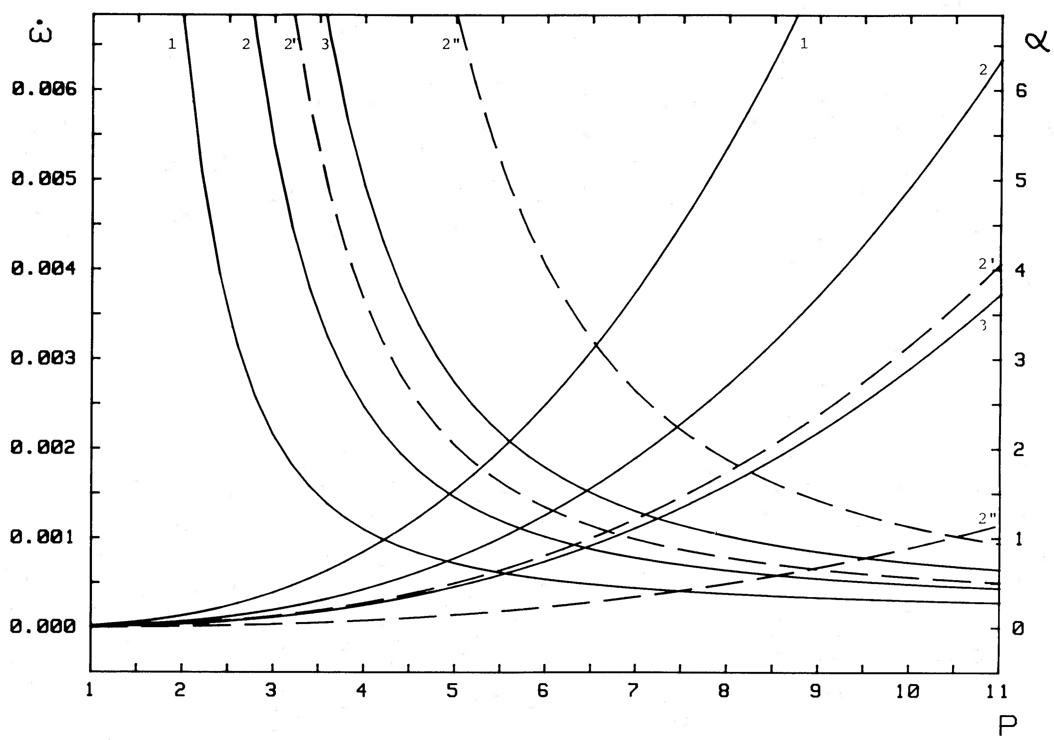


FIG. 4.—Total apsidal motion rate (*descending lines*, *left-hand ordinate*) and relativistic to classical contribution ratio (*ascending lines*, *right-hand ordinate*) vs. orbital period. Continuous lines labeled 1, 2, and 3 correspond to equal components, $e = 0.05$, and values of $m = 1.8$, 3.2, and 5.0, respectively. Dashed lines correspond to equal components and $m = 3.2$ solar masses but orbital eccentricities $e = 0.25$ (2′) and 0.50 (2″).

minimum value of the ratio $\alpha$ will give the lower limit to $P$. Thus, it is convenient to adopt the following observational constraints:

i) The ratio (eq. [12]) should be larger than 0.25 to consider important the relativistic contribution according to the expected errors of the classical term;

ii) The total apsidal motion rate should be well above the detection limit for the expected accuracy of photoelectric measurements, i.e., 0.0003 degrees per cycle, or 3 $\sigma$.

Under these constraints, it is easily shown that

$$P_{\min} \approx 0.66 m^{0.45} F(e)^{3/8} \text{ days }, \quad (16)$$

$$P_{\max} \approx 4.9 m (1 - e^2)^{-3/2} \text{ days }, \quad (17)$$

and, for a given mass and orbital eccentricity, a real binary should obviously verify

$$P_{\min} \lesssim P \lesssim P_{\max} . \quad (18)$$

In Table 1, we give the estimated values of $P_{\min}$ and $P_{\max}$ derived from equations (16) and (17) for a range of masses and orbital eccentricities.

Accordingly, we have made a survey among all known eccentric eclipsing binaries searching for systems which fulfill the mentioned requirements. For this purpose, an updated list of eclipsiing binaries with deep enough eclipses and eccentric orbits has been compiled from different sources (e.g., Batten 1973; Wood 1963; Batten, Fletcher, and Mann 1978; Wood *et al.* 1980). Out of 115 stars, 36 candidates have been selected. These observational targets for the detection of relativistic apsidal motion are listed in Tables 2 and 3. Orbital period, visual magnitude, and spectral types for each system are given in Table 2 which includes those binaries for which the existing information indicates its probable membership. Table 3 contains those systems for which available information is still insufficient but will remain most probably within the list of candidates.

Continuous monitoring of these eclipsing binaries in order to get times of minimum should be most rewarding.

## VI. DETERMINATION OF THE APSIDAL MOTION RATE

A general method for the analysis of apsidal motions in eclipsing binaries has been recently described by Giménez and García-Pelayo (1983; hereafter Paper I). In the study of relativistic periastron advances, there is special interest in the use of Taylor expansions of the general equation (16) of Paper I because no previous knowledge of $\dot{\omega}$ is needed (as required by differential corrections), nor is a large coverage of the apsidal motion cycle (as needed for Fourier analysis).

A particular case of the procedure given by Giménez and García-Pelayo (1983) is found when no curvature is detected in the residuals from equation (29) of Paper I. Then, the first-order Taylor expansion, given by equation (25) of Paper I, perfectly reproduces the observations within their mean errors, and no iteration is needed (i.e., the introduction of $E^2$ does not improve the variance of the linear fit). This simplification will be only true when $P/U$ becomes a very small quantity as expected in the case of relativistic apsidal motions.

The linear least-square fitting will provide "instantaneous" periods, $P_1$ and $P_2$, separately for primary and secondary eclipses directly related to the partial derivatives of the corresponding expansions of equation (16) of Paper I. Thus we can define

$$\bar{P} = (P_1 + P_2)/2 , \quad (19)$$

TABLE 1

MINIMUM AND MAXIMUM ORBITAL PERIODS FOR SELECTION OF CANDIDATES

| | | *e* | | | | | |
|---|---|---|---|---|---|---|---|
| *m* | PERIOD | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| 1.5……… | $P_{\min}$ | 2.3 | 2.4 | 2.6 | 3.0 | 3.5 | 4.4 |
| | $P_{\max}$ | 7.4 | 7.5 | 7.8 | 8.5 | 9.6 | 11.3 |
| 2.0……… | $P_{\min}$ | 2.6 | 2.7 | 3.0 | 3.4 | 4.0 | 5.0 |
| | $P_{\max}$ | 9.8 | 10.0 | 10.4 | 11.3 | 12.7 | 15.1 |
| 3.0……… | $P_{\min}$ | 3.1 | 3.3 | 3.5 | 4.0 | 4.8 | 6.0 |
| | $P_{\max}$ | 14.7 | 14.9 | 15.6 | 16.9 | 19.1 | 22.6 |
| 4.0……… | $P_{\min}$ | 3.6 | 3.7 | 4.0 | 4.6 | 5.5 | 6.9 |
| | $P_{\max}$ | 19.6 | 19.9 | 20.8 | 22.6 | 25.5 | 30.2 |
| 5.0……… | $P_{\min}$ | 3.9 | 4.1 | 4.5 | 5.1 | 6.0 | 7.6 |
| | $P_{\max}$ | 24.5 | 24.9 | 26.1 | 28.2 | 31.8 | 37.7 |

TABLE 2

LIST OF CANDIDATES FOR STUDY OF RELATIVISTIC APSIDAL MOTIONS

| Number | Name | *P* | *V* | Spectra |
|---|---|---|---|---|
| 1……… | BW Aqr | 6.71969 | 10.2 | F7 + |
| 2……… | CD Aqr | 4.83772 | 10.1 | A5 + |
| 3……… | V889 Aql | 11.12088 | 8.7 | B9 + |
| 4……… | V459 Cas | 8.45829 | 10.3 | B9 + |
| 5……… | EK Cep | 4.42780 | 8.2 | A0 + F9 |
| 6……… | TV Cet | 9.10329 | 8.7 | F2 + F5 |
| 7……… | V541 Cyg | 15.33795 | 10.2 | A0 + |
| 8……… | V1143 Cyg | 7.64076 | 5.9 | F5 + F5 |
| 9……… | DI Her | 10.55017 | 8.3 | B4 + B5 |
| 10……… | AI Hya | 8.28968 | 9.7 | F0 + F5 |
| 11……… | KM Hya | 7.75050 | 6.3 | A3m + |
| 12……… | RW Lac | 10.36922 | 10.5 | F2 + |
| 13……… | SS Lac | 14.41629 | 10.1 | B7 + |
| 14……… | ES Lac | 4.45940 | 11.4 | A2 + |
| 15……… | V345 Lac | 7.49186 | 11.1 | B8 + |
| 16……… | RR Lyn | 9.94508 | 5.6 | A7m + F3 |
| 17……… | TZ Men | 8.56900 | 6.2 | A0 + F2 |
| 18……… | UX Men | 4.18110 | 8.2 | G1 + G1 |
| 19……… | EW Ori | 6.93680 | 10.4 | F8 + |
| 20……… | GG Ori | 6.63147 | 10.8 | A2 + |
| 21……… | VV Pyx | 4.59618 | 6.6 | A1 + A1 |
| 22……… | EO Vel | 5.32962 | 11.1 | A0 + |
| 23……… | EQ Vul | 9.29716 | 11.5 | B8 + |

TABLE 3

ADDITIONAL PROBABLE CANDIDATES

| Number | Name | *P* |
|---|---|---|
| 1……… | AA Ara | 8.52070 |
| 2……… | GR Car | 17.13952 |
| 3……… | V383 Cen | 6.78549 |
| 4……… | V384 Cen | 12.63524 |
| 5……… | CO Cep | 4.13759 |
| 6……… | UW Cru | 6.35453 |
| 7……… | UX Cru | 12.29745 |
| 8……… | V501 Mon | 7.02117 |
| 9……… | HH Nor | 8.58313 |
| 10……… | DD Pup | 13.74280 |
| 11……… | V1049 Sgr | 12.04410 |
| 12……… | YZ Vel | 5.48834 |
| 13……… | FQ Vul | 6.26240 |

and

$$\Delta P = (P_1 - P_2)/2 \,, \tag{20}$$

which will become zero only for the cases when $\omega = 0°$ or $180°$.

From the notation and definitions adopted in Paper I, the following expressions can be deduced:

$$\bar{P} = P_s + \left(\frac{\dot{\omega}}{360}\right) P(F_2 \cos 2\omega + F_4 \cos 4\omega) \,, \tag{21}$$

$$\Delta P = \left(\frac{\dot{\omega}}{360}\right) P(F_1 \sin \omega + F_3 \sin 3\omega + F_5 \sin 5\omega) \,, \tag{22}$$

where $P_s = P(1 - \dot{\omega}/360)$ is the sidereal period, and

$$F_1 = e(2 + \cot^2 i) - \frac{3e^3}{4} \cot^2 i + \frac{e^3}{4} (2 + \csc^2 i) \cot^2 i \csc^2 i - \frac{e^5}{8} [\cot^2 i + 2 \cot^2 i \csc^2 i - \cot^2 i \csc^6 i(2 + \csc^2 i)] \,,$$

$$F_2 = e^2(\cot^2 i \csc^2 i + 2 \cot^2 i + \tfrac{3}{2}) + e^4[\tfrac{1}{4} - \cot^2 i + \tfrac{1}{2} \cot^2 i \csc^4 i(2 + \csc^2 i)] \,,$$

$$F_3 = -\frac{3e^3}{4} \left\{(2 + \csc^2 i) \cot^2 i \csc^2 i + 3 \cot^2 i + \frac{4}{3} + \frac{e^2}{4} [2 - 3 \cot^2 i + 3 \cot^2 i \csc^4 i \times (2 \csc^2 i + \csc^4 i + 1)]\right\} ,$$

$$F_4 = -\frac{e^4}{2} \left[\frac{5}{4} + \cot^2 i(4 + \csc^6 i + 2 \csc^4 i + 3 \csc^2 i)\right] ,$$

$$F_5 = \frac{5e^5}{16} \left[\frac{6}{5} + \cot^2 i(5 + \csc^8 i + 2 \csc^6 i + 3 \csc^4 i + 4 \csc^2 i)\right] .$$

All the $F_j$ functions can be directly evaluated with the elements derived from the analysis of the light curve. Moreover, for small values of $\dot{\omega}$, the position of the periastron given by the light curve can be taken as a good mean value to evaluate all the cos $(j\omega)$ and sin $(j\omega)$. Thus, equations (21) and (22) can be solved for the anomalistic period, $P$, and the periastron displacement rate, $\dot{\omega}$. Terms up to $e^5$ have been kept throughout so that the present procedure supersedes previous methods.

A rough simplification of the above given equations was presented in the analysis of V889 Aql (Giménez and Scaltriti 1982) using an expression of $\Delta P$ which neglects terms of the order of $e^3$ and taking $P \approx \bar{P}$. This approximation can still be used to estimate the uncertainties of the derived apsidal motion parameters. Considering only the main contributions, it can be easily seen that

$$\frac{\delta(\dot{\omega})}{\dot{\omega}} \approx \frac{\delta(\Delta P)}{\Delta P} + \frac{\delta(e)}{e} \,. \tag{23}$$

The accuracy of $\dot{\omega}$ will thus depend, as expected, on the actual position of the periastron, i.e., on the possibility of discriminating between the linear periods $P_1$ and $P_2$.

The validity of the applied equations was already discussed in § III and Paper I, while the expansion in terms of the eccentricity has been taken up to $e^5$, which is found to be enough for most practical cases. Nevertheless, for highly eccentric eclipsing binaries, the existing asymmetry between the ascending and descending branches of the eclipses could affect the accuracy of the determination of times of minimum. To avoid this, only the lower part of them should be used together with a parabolic fitting procedure instead of the traditional Kwee and Van Woerden method (Andersen and Giménez 1985). The importance of such asymmetries can be evaluated in any particular case by using the equations given by Kopal (1959), and the value is expected to be a maximum when the periastron is around 0° or 180°.

## VII. CONCLUSIONS

There has been shown in the preceding sections the actual possibility of observing general-relativistic effects outside the solar system in eclipsing binaries. Small relative radii for the component stars have been found to provide the main constraint for the selection of observational targets. In addition, it is clear that relatively low mass systems with later type components are better suited for this kind of study than the early-type binaries previously suggested. A list of observational candidates for the detection of relativistic apsidal motion is given.

The sample of eclipsing binaries showing significant relativistic contributions with the available information, although certainly small up to now, is in perfect agreement with the outlined characteristics of observational targets. In Table 4, the main results are given from the study of four eclipsing binaries for which the relativistic term has been found to be larger than 25% of the total apsidal motion. The observed periastron displacement rates are indicated in column (3). The values predict-

TABLE 4

OBSERVED AND PREDICTED APSIDAL MOTION RATES

| Name (1) | $P$ (2) | $\dot{\omega}^{\text{obs}}$ (3) | $\dot{\omega}^{\text{pre}}$ (4) | $\alpha$ (5) | References (6) |
|---|---|---|---|---|---|
| V889 Aql ......... | 11.120886 | 0.00048 | 0.00048 | 79 | Giménez and Scaltriti 1982 |
| | 7 | 15 | 16 | | |
| EK Cep ........... | 4.427807 | 0.00107 | 0.00096 | 46 | Giménez and Margrave 1985 |
| | 4 | 32 | 36 | | |
| V1143 Cyg ....... | 7.640757 | 0.00071 | 0.00088 | 43 | Giménez and Margrave 1985 |
| | 1 | 4 | 29 | | |
| VV Pyx ........... | 4.596198 | 0.00142 | 0.00158 | 32 | Andersen *et al.* 1984 |
| | 5 | 45 | 22 | | |

ed using equation (1) and the $\alpha$ ratio in percent are given in columns (4) and (5), respectively. References to the detailed analyses are also given, and, in all cases, the adopted procedure for the determination of the apsidal motion period is more or less based on the equations given in § VI. It can also be seen that the agreement between theoretical predictions and observational results is excellent within their mean errors. Nevertheless, the overall significance of these results must be evaluated within a larger context when the number of cases with relativistic apsidal motions has significantly increased.

In any case, it can certainly be concluded that monitoring of the proposed observational targets in order to enlarge the sample of reliable determinations of relativistic periastron advances among eclipsing binaries will provide an important example of the significance of relativistic effects in stellar astronomy.

In a recent paper, Moffat (1984) proposed the observation of periastron advances in eclipsing binaries in order to discriminate between the predictions of general relativity and a nonsymmetric theory of gravitation. In fact, unpublished observations by Guinan, quoted by Moffat (1984), appear to be in much better agreement with the nonsymmetric gravitational theory than with general relativity in the case of DI Her. This second-order kind of analysis certainly deserves further attention, and the proposed candidates in Tables 2 and 3 are very promising for this purpose, although reliable measurements of the rotational velocities of the stars are necessary.

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ALVARO GIMÉNEZ: Departamento de Astrofísica, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid—3, Spain