Mon. Not. R. astr. Soc. (1981) **197**, 633–646

The long-term motion of comet Halley

Donald K. Yeomans *Jet Propulsion Laboratory, 264–664, Pasadena, California 91109, USA*

Tao Kiang *Dunsink Observatory, Castleknock, County Dublin, Republic of Ireland*

Received 1981 March 11

Summary. The orbital motion of comet Halley has been numerically integrated back to 1404 BC. Beginning with an orbit based upon the 1759, 1682 and 1607 observations of comet Halley, our numerical integration was run back in time with full planetary perturbations and non-gravitational forces taken into account at each 0.5 day time-step. The non-gravitational forces were assumed due to the rocket effect of an outgasing water ice nucleus. Small empirical corrections were made to the computed perihelion passage time in 837 and to the osculating orbital eccentricity in AD 800. In nine cases, the perihelion passage times calculated by Kiang from Chinese observations have been redetermined; we have used the unusually accurate observed perihelion passage times in 837, 374 and 141 to constrain the computed motion of the comet. Osculating orbital elements are given at each apparition from 1910 back to 1404 BC.

The dynamic model used to compute the long-term motion of comet Halley successfully represented the ancient Chinese observations over nearly two millennia. This model assumed the comet's non-gravitational forces remained constant from one apparition to the next. Hence it seems likely that comet Halley's spin axis direction and ability to outgas has also remained relatively constant with time over its observed interval.

> 'You see therefore an agreement of all the Elements in these three, which would be next to a miracle if they were three different Comets. . . . Wherefore, if according to what we have already said it should return again about the year 1758, candid posterity will not refuse to acknowledge that this was first discovered by an Englishman.'
>
> E. HALLEY (1752)

1 Introduction

After what he termed a 'prodigious deal of calculation', Halley (1705) published parabolic orbital elements for 24 well-observed comets. He noted the similarities in the orbits for the comets of 1682, 1607 and 1531 and published the first correct prediction for the return of a comet. Although the poor observations did not allow an orbit solution, Halley noted that the comet of 1456 resembled the same periodic comet because it passed retrograde between the Earth and Sun.

Pingré (1783–1784) used additional observations of the comet of 1456 to determine its perihelion passage time. He then assumed the remaining orbital elements were those of comet Halley and noted the similarity between the computed and the observed motion of this comet. Pingré thus confirmed Halley's suspicion that the comet of 1456 was an earlier apparition of the famous comet. Using Chinese observations, Pingré computed crude orbits for the great comet of 837 and the first comet of 1301, but failed to recognize them as comet Halley.

Using a mean period for comet Halley to step backward in time, Biot (1843b) attempted to identify previous apparitions of comet Halley using ancient Chinese observations. Aware of the crudeness of his method, he outlined several sets of possible Chinese observations around the time of each extrapolated time of perihelion passage back to 65 BC. Biot also pointed out that an orbit computed by Burckhardt (1804) for the comet of 989 closely resembled that of comet Halley.

Using Chinese observations from Biot (1843a), Laugier (1843) recognized a comet seen in the Autumn of 1378 as an earlier apparition of comet Halley. Laugier used Chinese observations to compute the comet's perihelion passage time. He then assumed the remaining orbital elements were similar to comet Halley's and computed the resultant apparent path of the comet. This computed apparent path and the observed path were enough alike to allow a correct identification of the Chinese observations with comet Halley. Laugier (1846) used a similar technique to correctly identify apparitions of comet Halley in 760 and 451. Concerning the 1301 apparition of comet Halley, Laugier (1842) based an orbit on a Chinese observation for 1301 September 16 and two observations made in Cambridge, England on September 30 and October 6 of that year. Although four of the five parabolic orbital elements resembled those of comet Halley, he was prevented from making a definitive identification because of a poorly determined longitude of the ascending node.

By stepping backward in time at roughly 76–77 yr intervals and analysing European and Chinese observations, Hind (1850) attempted to identify apparitions of comet Halley from 1301 to 11 BC. Approximate perihelion passage times were often determined directly from the observations and an identification was suggested if Halley-like orbital elements could satisfy existing observations. Although many of Hind's identifications were correct, he was seriously in error for his apparitions of 1223, 912, 837, 608, 373 and 11 BC.

Until the 20th century, all attempts at identifying ancient apparitions of comet Halley were done by either determining orbits directly from the observations or by stepping back in time at approximate 76 yr intervals and testing the observations with the assumed (computed) motion of comet Halley. Using a variation of elements technique, Cowell & Crommelin (1907) began the first effort to actually integrate the comet's equations of motion backward in time. They assumed that the orbital eccentricity and inclination were constant with time and the argument of perihelion and the longitude of the ascending node changed uniformly with time – their rates being deduced from their accepted values over the 1531–1910 interval. By using Hind's (1850) times of perihelion passage or by computing new values from the observations, they deduced preliminary values of the orbital semi-major axis for the perturbation calculations. First-order perturbations for the comet's period were then computed taking into account the effects of Venus, Earth, Jupiter, Saturn, Uranus and Neptune. The motion of the comet was accurately carried back to 1301. Using successively more approximate perturbation methods, Cowell & Crommelin (1907, 1908a–d) carried the motion of the comet back to 239 BC. By 239 BC their integration was in error by nearly 1.5 yr in the predicted time of perihelion passage time. For their earliest apparition, they adopted a time of 240 BC May 15 not from their integration but directly from the observations themselves.

According to Kamienski (1956), the perihelion passage times of comet Halley were computed from 451 back to 622 BC by M. A. Viliew. Using Viliew's perihelion passage times from 622 BC to 451 and those of Cowell & Crommelin from 530 to 1910, Kamienski (1957) fit a Fourier interpolation formula to the orbital periods and while the formula fits the data used to generate it, extrapolation outside the data arc is hopeless. Much as Angström's (1862) similar analysis failed to predict the 1910 perihelion passage time by 2.8 yr, Kamienski's (1962) prediction for the next return (1986.88) will be in error by nine months. In the absence of a dynamical model for the comet's motion, it is unrealistic to investigate the past or future apparitions of comet Halley by using such empirical formulae.

After a complete and careful analysis of the European and Chinese observations, Kiang (1971) used the variation of elements technique to investigate the motion of comet Halley from 1682 back to 240 BC. By considering the effect of perturbations from all the planets on the orbital elements, Kiang was able to determine accurate perihelion passage times and confirm Michielsen's (1968) suggestion that non-gravitational forces are responsible for a deceleration in this comet's mean motion amounting to slightly more than 4 day/(period)^2. Hasegawa (1979) also empirically determined perihelion passage times for comet Halley. For each apparition from 1378 back to 240 BC, Hasegawa computed several ephemerides using Kiang's (1971) orbital elements, except for the perihelion passage times which were chosen to make the best fit with the observations.

Brady & Carpenter (1971) were the first to apply direct numerical integration to the study of comet Halley's ancient apparitions. Using an empirical secular term in the comet's equations of motion to account for the non-gravitational effects, an orbit fitted to observations of the last four apparitions was run back to 87 BC in one continuous integration. Orbital elements for each apparition are given by Marsden (1975). The agreement between their computed times of perihelion passage and the observed times given by Kiang (1971) was quite satisfactory from 1682 back to 218. However, as their backward integration continued, the divergence between observed and computed times of perihelion became quite apparent beginning with the 141 apparition. In 141, the actual comet passed within 0.17 AU of the Earth and experienced perturbations somewhat different from their 'computed' comet. Because their integration was tied to no observations earlier than 1682, the slight differences between the actual and computed motion of the comet were magnified by the Earth–comet close approach in 141.

Using Brady & Carpenter's (1971) orbit for comet Halley, Chang (1979) integrated the comet's motion back to 1057 BC. However, this integration was not based upon any observations prior to 1909 nor were non-gravitational effects taken into account.

A non-gravitational force model, based upon the rocket effect of an outgassing cometary nucleus has been developed by Marsden, Sekanina & Yeomans (1973). Yeomans (1977) used this non-gravitational force model to successfully represent the observations of comet Halley over the 1607–1911 interval. An orbit based upon the 1682, 1759 and 1835–36 observations was numerically integrated backward in time to 837. Due to a close approach of the comet with the Earth in 837 (minimum separation = 0.04 AU) no attempt was made to continue the integration prior to 837.

Although a direct numerical integration technique is the only method available for investigating the motion of comet Halley beyond the well-observed interval, every effort must be made to tie the integration to the ancient observations. When integrating the comet's motion over severe perturbations caused by comet–Earth close approaches, particular care must be taken to rectify the comet's computed motion with observational data. The remainder of this paper will outline the technique we used to include the ancient Chinese observations in our integration of the motion of comet Halley back to 1404 BC.

2 A redetermination of comet Halley's times of perihelion passage using Chinese observations

A Chinese record of comet Halley typically contains a date and a mention of the lunar mansion in which the comet is seen. The date can be converted into a Julian calendar date without ambiguity and the lunar mansion can be identified with some definite range in right ascension. Some records specify the right ascension more precisely by stating the number of degree (*du*), or even fractions thereof, within the lunar mansion. Only occasionally is the position referred to individual stars. Hence the Chinese observations can usually give a good determination of the time of perihelion passage, T_0, and only a very weak determination of the other orbital elements: in this sense they nicely complement the results from dynamical model calculations, which more weakly determine the time of perihelion passage.

Accordingly, in a previous work (Kiang 1971), T_0 was determined as far as possible from the Chinese records while the other elements were calculated according to a purely gravitational model. We have now re-examined the Chinese material and made some new determinations of T_0, complete with error bounds. The new results are given in Table 1.

The re-examination was prompted by three considerations. First, there was the question of the date of a morning observation. In the earlier work, it was assumed that, in general, the date changes at midnight so that any observation made during the second half of the night would be given the new date. However, a recently completed study (Kiang 1980) using over 100 lunar occultation records, has shown that only 15 per cent of such events were dated with the new dates, and these few tend to come from certain particular dynastic volumes and are found almost exclusively amongst the observations made after 3 am local time. It thus appears the Chinese practice was closer to the Korean practice of dating all such observations with the old date (Saito, private communication) than to the Japanese tradition of making a fairly neat divide at the 3 am mark (Saito 1979 and private communication). In the present work, all the observations pertaining to the second half of the night are assumed to be dated with the old date. (The time of dawn, for example, would be the Julian calendar date plus $0^d.9$ approximately.) A second consideration arose out of a search for additional information through the imperial biographies, as distinct from the astronomical chapters, contained in the 25 standard dynastic histories. Although no great hope was entertained,

Table 1. Perihelion passage times determined from Chinese observations.

Return	ΔT_0 Mean Correction To Kiang (1971) Values of T_0 (days)	T_0 (This Paper)
1301	+1.15	1301 Oct. 24.53 ± 0.25
1222	−0.7	1222 Oct. 0.8 ± 1.7
1145	−0.75	1145 Apr. 21.25 ± 0.75
1066	−0.3	1066 Mar. 23.5 ± 0.3
912	0.0	912 Jul. 9.5 ± 1.4
837	+0.77	837 Feb. 28.27 ± 0.05
530	+1.5	530 Sep. 26.7 ± 0.2
374	+1.4	374 Feb. 17.4 ± 0.6
141	+2.35	141 Mar. 22.35 ± 0.25

two very useful records did turn up for the 1066 return. Thirdly, Kanda's (1935) monograph on Japanese astronomical source material has recently been reprinted, making it possible for these records to be examined in the original.

The redetermination of T_0 proceeds as follows. Using the orbital elements given in Table V of Kiang's (1971) paper, and an assumed value of ΔT_0 representing the correction to the tabulated time of perihelion passage, the right ascension, declination, altitude and azimuth of comet Halley are calculated for 'nightfall', midnight and the following 'dawn' of each stated date. 'Nightfall' and 'dawn' are defined to be the times when the Sun is 6° below the horizon. (The value of 6° was found to give more consistent results than the value of 12° used in some previous calculation, when checked with the recorded 'fading into the evening twilight' of the comet on 12 BC October 20 and 1222 October 23.) It is then easily seen whether during the time of night when the comet is up, its position is or is not consistent with the recorded position. More precisely each observation provides an upper and a lower limit in ΔT_0 as follows: let α^+ and α^- be the limiting RA of the observed lunar mansion (or the observed RA plus or minus an assumed tolerance) and let $\alpha_{cal} = \alpha_{cal}(\Delta T_0)$ be the curve of the calculated RA for the stated day and for varying ΔT_0. This particular observation then defines a lower limit in ΔT_0 where α_{cal} crosses α^+ from above (↓) or α^- from below (↑) and it defines an upper limit where α_{cal} crosses α^+ from below (↑) or α^- from above (↓). From the set of observations of a given return, we then take the largest of all the lower limits to be the lower limit and the smallest of the upper limits to be the upper limit, and call the observation(s) involved the crucial observation(s). The midpoint between the lower and upper limits is taken to be the mean correction, and half their distance apart, the error bound. The results are given in Table 1.

It may be noticed that the error bounds for different epochs are very different. The difference in observing accuracy is only one contributing factor here; a more important factor is whether there are any observed positions at times when the apparent motion is large. Such observations are highly sensitive to any assumed value of ΔT_0.

A list of crucial observations that define the limits in ΔT_0 now follows. For the returns of 1301, 837, 530, 374 and 141, the crucial observations were all 'morning' observations and so the change in the dating convention is mainly responsible for their systematically positive ΔT_0. It is understood that all other observations not mentioned in the list are consistent with the values of ΔT_0 delimited by these crucial observations. ΔT_0 is in units of days throughout.

Return of 1301. The comet is said to be at 24 *du* 40 *fen* (1 *du* = 360°/365.25 = 100 *fen*) of *Jing*-22 (Lunar Mansion 22, i.e. the right ascension range defined by μ Gem and θ Cnc) on September 16. We interpret this to mean: α(September 16) = 109°.23 ± 0°.20. We find: at $\Delta T_0 = +0.90$, α(September 16) ⩾ 109°.43 ↓; at $\Delta T_0 = +1.40$, α(September 16) ⩽ 109°.04 ↓. Hence, we adopt $\Delta T_0 = +1.15 \pm 0.25$.

Return of 1222. On September 10, the comet is seen between *Yousheti* (η Boo) and *Zhouding* (β Com). Interpretation: 188°.6 < α(September 10) < 199°.4, $\delta \sim 25°$. We find: at $\Delta T_0 = -2.4$, α = 199°.3 ↓; at $\Delta T_0 = +1.0$, α = 188°.6 ↓. Hence we adopt $T_0 = -0.7 \pm 1.7$.

Return of 1145. On April 14, comet is in *Shen*-21 (δ Ori, μ Gem). Interpretation: 72°.1 < α(April 14) < 82°.8. We find: at $\Delta T_0 = -1.50$, α = 82°.8 ↓; at $\Delta T_0 = 0.00$, α = 72°.1 ↓. Hence adopt $\Delta T_0 = -0.75 \pm 0.75$.

Return of 1066. The *Biography of Emperor Yingzong in the History of Song Dynasty* provides two positions not generally known before: on April 24, comet is in *Mao*-18 (17 Tau, ϵ Tau); on April 25, in *Bi*-19 (ϵ Tau, ϕ^1 Ori). Interpretation: 42°.7 < α(April 24) < 53°.7, 53°.7 < α(April 25) < 71°.0. We find: at $\Delta T_0 = -0.6$, α(April 25) = 71°.0 ↓; at $\Delta T_0 = 0.0$, α(April 24) = 42°.7 ↓. Hence adopt $T_0 = -0.3 \pm 0.3$.

Return of 912. On July 12, comet is in *Zhang*-26 (υ^1 Hya, ν Hya); on July 14, it is W. of *Lingtai* (χ Leo). Interpretation: $134°.8 < \alpha$(July 12) $< 151°.7$, α(July 14) $< 152°.0$. We find: at $\Delta T_0 = -1.4$, α(July 14) $= 152°.0 \downarrow$; at $\Delta T_0 = +1.4$ α(July 12) $= 134°.8 \downarrow$. Hence adopt $\Delta T_0 = 0.0 \pm 1.4$.

Return of 837. On April 8, comet is at *du* 4 of *Nyu*-10 (ϵ Aqr, β Aqr); on April 9, at *du* 10 of *Dou*-8 (ϕ Sgr, β Cap). There are eight other dated positions given nominally to one-half of a *du* ($\sim 0°.5$). Interpretation: α(April 8) $= 300°.0 \pm 1°.5$, α(April 9) $= 273°.2 \pm 1°.5$. The very rapid motion on these dates allow a precise determination: at $\Delta T_0 = +0.72$, α(April 8) $= 298.5 \uparrow$; at $\Delta T_0 = +0.82$, α(April 9) $= 274.7 \downarrow$. In the range $+0.72 < \Delta T_0 < +0.82$, all the 10 recorded positions, except one whose text is suspect anyway, are reproduced to within $1°.5$. Hence adopt $\Delta T_0 = +0.77 \pm 0.05$.

Return of 530. On September 1, comet at 1 'foot' NW of ν UMa. Interpretation: located between $0°.5$ and $2°.0$ NW of ($148°.7$, $40°.7$). We find that this requires $+1.3 < \Delta T_0 < +1.7$. Hence we adopt $T_0 = +1.5 \pm 0.2$. The record on August 29 remains garbled as before.

Return of 374. On April 2, comet is in *Di*-3 (α Lib, π Sco). Interpretation: $200°.9 < \alpha$(April 2) $< 216°.1$. We find: at $\Delta T_0 = +0.8$, α(April 2) $= 200°.9 \uparrow$; at $\Delta T_0 = +2.0$, α(April 2) $= 216°.0 \uparrow$. Hence adopt $\Delta T_0 = +1.4 \pm 0.6$.

Return of 141. On April 16, comet is at *du* 1 of *Kui*-15 (ζ And, β Ari); on April 23, it is in *Jing*-22 (μ Gem, θ Cnc). Interpretation: $348°.6 < \alpha$(April 16) $< 350°.1$, $67°.9 < \alpha$(April 23) $< 100°.7$. We find: at $\Delta T_0 = +2.1$, α(April 16) $= 350°.1 \downarrow$; at $\Delta T_0 = +2.6$, α(April 23) $= +67°.9 \downarrow$. Hence adopt $\Delta T_0 = +2.35 \pm 0.25$.

3 Planetary coordinates

In order to accurately compute the planetary perturbations on the motion of comet Halley, planetary coordinates were required over an interval of three millennia. In a previous work, Yeomans (1971) used planetary coordinates (1800–2000) supplied by Lieske (1968). In a subsequent work on the orbital motion of comet Halley, Yeomans (1977) used an *n*-body integrator by Schubart & Stumpff (1966) and extended the planetary coordinates back to 1600.

For the present study, planetary coordinates were required over a period of three millennia (+1600 to −1600). Although the previous integration of the solar system (1600–2000) was strictly Newtonian, the relativistic advance of Mercury's perihelion over 32 centuries amounts to nearly 23 arcmin. A full relativistic planetary integration computer program, designed at the Jet Propulsion Laboratory by Newhall (1977, private communication), was modified to run with Newtonian equations of motion but using a relativistic potential for the Sun. The Earth–Moon barycentre was integrated as a point mass. H. Fliegel (1977, private communication) determined accurate starting conditions for the Earth–Moon system by an analytic and iterative technique and starting conditions for the remaining eight planets were taken from the JPL planetary Development Ephemeris – DE 97 (E. M. Standish 1977 private communication). The planetary numerical integrator employed a variable order Adams scheme with variable step size control. The absolute, total velocity error at each step was constrained to less than 10^{-13} AU day^{-1}.

A sequence of magnetic tapes was generated from the numerical integration program. Each tape covered an interval of approximately 400 yr with planetary coordinates given at 4-day intervals for all nine planets. These heliocentric, rectangular planetary coordinates were referred to the mean equator and equinox of 1950.0. At the completion of the numerical integrations, consistent planetary coordinates were available for all planets at 4-day increments over the period −1600 to +2000.

1981MNRAS.197..633Y

Table 2. Comparison of numerically integrated planetary positions (column 1) with those analytically calculated (column 2) by Tuckerman (1964) and Stahlman & Gingerich (1963).

Dates (Julian calendar)		Mercury		Venus		Mars		Jupiter		Saturn	
		1	2	1	2	1	2	1	2	1	2
31 March 1211 AD	Longitude	356°.872	356°.88	62°.737	62°.74	233°.145	233°.14	205°.578	205°.59	119°.538	119°.52
(2163 464.5 JD)	Latitude	-2°.474	-2°.47	3°.238	3°.24	0°.993	0°.98	1°.535	1°.53	0°.843	0°.84
16 January 806 AD	Longitude	275°.245	275°.26	287°.348	287°.37	29°.657	29°.64	151°.663	151°.70	212°.147	212°.11
(2015 464.5 JD)	Latitude	1°.368	1°.37	-0°.583	-0°.59	1°.111	1°.11	1°.384	1°.38	2°.471	2°.47
17 October 411 AD	Longitude	203°.814	203°.85	247°.923	247°.95	191°.605	191°.60	64°.114	64°.12	63°.549	63°.48
(1871 464.4 JD)	Latitude	0°.312	0°.31	-2°.368	-2°.37	0°.652	0°.65	-0°.759	-0°.76	-1°.796	-1°.81
17 July 17 AD	Longitude	138°.001	138°.04	70°.711	70°.74	17°.676	17°.56	327°.252	327°.37	267°.852	267°.80
(1727 464.5 JD)	Latitude	-0°.389	-0°.39	-1°.938	-1°.93	-2°.593	-2°.60	-1°.482	-1°.49	0°.300	0°.30
4 May 389 B.C.	Longitude	43°.542	43°.64	72°.072	72°.12	118°.058	118°.011	261°.635	261°.65	356°.328	356°.18
(1579 464.5 JD)	Latitude	1°.070	1°.08	1°.741	1°.74	1°.701	1°.70	0°.074	0°.05	-2°.339	-2°.34
20 February 794 B.C.	Longitude	302°.490	302°	291°.125	291°	352°.995	352°	201°.472	202°	79°.410	79°
(1431 464.5 JD)	Latitude	-2°.063		-0°.358		-0°.343		1°.424		-0°.196	
8 December 1200 B.C.	Longitude	223°.489	224°	290°.686	291°	211°.144	211°	143°.948	144°	172°.957	172°
(1283 464.5 JD)	Latitude	2°.494		-1°.871		0°.361		1°.431		2°.510	

For each planet, column 2 represents the geocentric, ecliptic longitudes and latitudes given by Tuckerman (after 601 B.C.) and the longitudes given by Stahlman and Gingerich (prior to 601 B.C.).

For each 400-yr integration, one or more dates were selected at which to compare these numerically integrated planetary coordinates with analytically computed positions (see Table 2). For the period prior to 601 BC, the analytically computed tables of Stahlman & Gingerich (1963) were used for comparison; after 601 BC, those of Tuckerman (1964) were used. The former work tabulated only planetary longitudes to the nearest degree; the latter work tabulated both longitude and latitude to $0^\circ.01$. In order to effect a comparison between our numerically integrated planetary coordinates and the analytically computed coordinates, our heliocentric, rectangular, coordinates referred to the mean equator and equinox of 1950.0 were transformed to geocentric, celestial latitude and longitude referred to the mean equinox of date. In the transformation from our epoch (0^h ET) to their epoch (16^h UT), we assumed the (ET – UT) correction amounted to 29 s of time per (century)^2. This value was generally accepted at the time Tuckerman (1964) published his tables.

The maximum differences between the analytical, planetary positions and the present numerically integrated, planetary positions were for Saturn's longitude and latitude (see Table 2). The discrepancies are approximately equal to the stated errors in the analytical tables. We believe the discrepancies noted in Table 2 are primarily due to the approximate nature of the analytical calculations. In any case, the generated planetary coordinates are more than accurate enough for our present perturbation calculations.

4 The motion of comet Halley integrated back to 1404 BC

With accurate initial conditions and planetary coordinates, it is theoretically possible to numerically integrate the motion of comet Halley for several centuries. However, the slight errors inherent in the initial conditions result in discrepancies between the computed and actual motion of the comet. The dominant error is the uncertainty in the computed times of perihelion passage. Hence the perihelion passage residual (ΔT) is an excellent monitor for the accuracy of a computed orbit. The task of integrating the motion of comet Halley over a long period of time would be relatively straightforward were it not for occasional severe perturbations due to the Earth. These Earth–comet close approaches often have the effect of magnifying the computed position uncertainty. Unless the computed and actual comet positions are identical, the perturbation will be quite different for the computed and actual comet. Although Earth–comet close approaches can magnify the error of the computed comet's position, a close approach will generally result in excellent position observations of the comet. As is evident from Table 1, the Earth close approaches of 1301, 837 and 374 resulted in well observed apparitions, and hence well determined times of perihelion passage. As will become apparent, the Earth–comet close approaches not only present a dynamical problem, but they also allow the accurate, observations required to overcome the problem.

For the majority of well observed short period comets, obvious non-gravitational accelerations are affecting their motions. By assuming that these non-gravitational accelerations are due to the rocket effect of outgassing volatiles from an icy-conglomerate nucleus (Whipple 1950), these non-gravitational accelerations have been successfully modelled by Marsden *et al.* (1973). The mathematical form of these non-gravitational effects, in the cometary equations of motion, represents an empirical fit to a theoretical plot of water-snow vaporization flux as a function of heliocentric distance. The magnitude of the radial and transverse non-gravitational acceleration, affecting the motion of a particular comet, is indicated by the radial (A_1) and transverse (A_2) non-gravitational parameters. Parameters denoted B_2 and t_0 are used to introduce a time dependence in the transverse term. For comet Halley over the 1607–1911 interval, Yeomans (1977) found that the non-gravitational acceleration was consistent with the outgassing rocket effect of a water-ice cometary nucleus. The transverse

Table 3. Non-gravitational orbits for comet Halley after Yeomans (1977).

Orbit No.	Observation Interval	Solve for*	Mean Residual	Nongravitational Parameters $A_1 \times 10^8$	$A_2 \times 10^8$	B_2	t_o
1	1911–1759	A_1, A_2	11″0	0.2799	0.0159		
2	1836–1682	A_1, A_2	19″1	1.1746	0.0150		
3	1759–1607	A_1, A_2	48″6	0.2767	0.0150		
4	1911–1682	A_2, B_2	13″6	-	0.0159	-0.0115	1911 Oct. 15.
5	1911–1682	A_1, A_2, B_2	13″4	0.1787	0.0159	-0.0112	1911 Oct. 15.

*Differential correction solving for six initial conditions and listed nongravitational parameters.

non-gravitational acceleration, which dominates the non-gravitational changes in the orbital energy, appeared to be time independent.

In an effort to determine whether the transverse non-gravitational acceleration affecting the motion of comet Halley was changing over long periods of time, Yeomans (1977) numerically integrated three different orbits backward in time to 837. These three orbits are numbered 2, 4 and 5 in Table 3. By comparing the computed times of perihelion passage with the observed values given by Kiang (1971), orbit 2 was found to be the most successful integration. However, in 837, the residual between the observed and computed times of perihelion passage amounted to $\Delta T = +5.11$ day when compared to the value in Table 1.

For the present work, orbits 1 and 3 were also integrated back to 837 and the computed times of perihelion passage compared with the observed values given in Table 1 of this paper. In the present work an efficient numerical integration program (Yeomans 1977) was employed and all cometary integrations were run at a constant 0.5 day step size. Although orbit number 1 did not improve upon the previously integrated orbit number 2, orbit number 3 did manage to improve the computed time of perihelion passage in 837. Orbit 3 required a correction of only -0.88 day to bring the perihelion passage time into agreement with the observed value given in Table 1. Hence it was decided to continue the backward integration with orbit 3 instead of orbit 2. Before the integration of orbit 3 was continued backward, the osculating perihelion passage time was given an empirical correction of -0.88 day at epoch 2026840.5 (JD). Even with this empirical correction in 837, the severe perturbation due to the Earth close approach caused a divergence of the computed and observed times of perihelion passage as the integration was continued back prior to 837. In an attempt to reduce this discrepancy, the empirical perihelion passage time correction was altered slightly. However, only a modest improvement in the ΔT residuals was possible with reasonable adjustments to the 837 perihelion passage time.

It was then decided to try an empirical adjustment to the osculating eccentricity in an effort to force the computed motion of the comet through the unusually accurate perihelion passage times in 374 and 141. At epoch = 2013000.5, the osculating eccentricity was iteratively adjusted until the difference between the observed and computed times of perihelion passage reached a minimum for the 374 and 141 apparitions. The optimum adjustment of the eccentricity was only -7.2×10^{-6}. Now with this eccentricity adjustment at epoch = 2013000.5, the integration was continued back to 1404 BC without further rectification. The eccentricity adjustment was made at epoch = 2013000.5 (AD 800) and not at epoch = 2026840.5 (AD 837) because it was more convenient to do it that way. The planetary coordinate tapes were written for 400 yr each and the tape break occurred between epochs

2026840.5 and 2013000.5. Hence to adjust the eccentricity in 837 and integrate back to 400 would have required two tapes and two separate computer runs.

Hence orbit 3 of Table 3 was integrated backward from AD 1607 to 1404 BC with empirical perihelion passage time and eccentricity corrections being made in 837 and 800 respectively. Table 4 presents the final orbital elements from 1910 back to 1404 BC. The orbital elements for 1759–1910 are based upon orbit 1, the remaining elements are based upon orbit 3. Calendar dates prior to 1582 are based upon the Julian calendar, those dates after 1582 are based upon the Gregorian calendar. All dates are in ephemeris time and a given period is an osculating period referenced to the epoch time. The angular elements are in degrees and referred to the ecliptic of 1950.0.

The Earth–comet close approach which took place on 837 April 11 (minimum separation = 0.04 AU), enlarged the differences between the computed and actual motion of the comet so that empirical corrections were required before the computed motion of the comet could be continued back beyond 837. Earth–comet close approaches also took place on 607 April 19 (0.09 AU), 374 April 2 (0.09 AU) and to a lesser extent on 141 April 22 (0.17 AU). Rather than allowing the severe perturbations to increase the discrepancy between the computed and actual motions of the comet, we greatly reduced this discrepancy by using the accurately observed perihelion passage times in 374 and 141 to determine an empirical correction to the eccentricity, as mentioned above. From 374 to 1404 BC, there was no Earth–comet close approaches with a minimum separation less than 0.11 AU. However, on 1404 BC September 7, the computed comet passed within 0.04 AU of the Earth. Without additional observations prior to 1404 BC, the orbit could not be rectified again and we were forced to cease the integration. In the absence of severe perturbations, the inherent accuracy and stability of the numerical integration, as well as the apparent lack of a time dependence in the non-gravitational parameters, allowed the computed motion to closely follow the actual motion of comet Halley over several centuries. Orbit 3 was integrated from 1607 back to 837 with a residual of only -0.88 day between the computed and observed times of perihelion passage in 837. In a sense, the orbit over the interval from 837 to 141 was fit to the observations of 837, 374 and 141. The interval from 141 to 1404 BC was free from severe perturbations so that the computed times of perihelion passage back to 1404 BC are not likely to be in error by more than a month.

Noting a 513-yr periodicity in the perihelion passage time residuals, Brady (1972) erroneously deduced the existence of a massive trans-Plutonian planet. Kiang (1973) showed that this residual periodicity was an inherent property of the idealized Sun–Jupiter–comet model. It is interesting to study this periodicity by comparing our perihelion passage times in Table 2 with those obtained by Chang (1979). Chang initialized a backward numerical integration with an orbit fitted only to 1909–1911 observations. Non-gravitational effects were ignored and no attempt was made to rectify the numerical integration with observational data. Table 5 presents our computed times of perihelion passage (T_c) in the first column. The second column gives the difference between the T_c values and those values (T_0) well determined empirically by Kiang (1971) and updated with our Table 1. The final column gives the differences between Chang's computed times of perihelion (T_{Ch}) and the empirically determined values (T_0). From the last column in Table 5, there is evidence for a rough 600-yr periodicity in the perihelion passage times residuals, $T_0 - T_{Ch}$. Because our computed perihelion passage times (T_c) so closely match the T_0 values, the 600-yr periodicity would also be evident in the $T_c - T_{Ch}$ residuals. However, this residual periodicity degenerates into a secular trend and by the 911 BC return, Chang's perihelion passage time differs by 3 yr from our adopted values. It is interesting to note that a secular trend in the perihelion passage time residuals can also be admissible in the idealized three-body model (Kiang 1979). From

Year	T(E.T.)*		q(AU)	e	P(Yrs)	ω	Ω	i	Epoch (E.T.)	
1910 II	1910 Apr. 20.17771	2418781.67771	0.5871888	0.9672968	76.08	111.71703	57.84670	162.21507	1910 May 9.0	2418800.5
1835 III	1835 Nov. 16.43871	2391598.93871	0.5865423	0.9673860	76.27	110.68555	56.80251	162.25518	1835 Nov. 18.0	2391600.5
1759 I	1759 Mar. 13.06075	2363592.56075	0.5844466	0.9676792	76.89	110.68990	56.52871	162.36893	1759 Mar. 21.0	2363600.5
1682	1682 Sep. 15.28069	2335655.78069	0.5826084	0.9679230	77.41	109.20541	54.85222	162.26569	1682 Aug. 31.0	2335640.5
1607	1607 Oct. 27.54063	2308304.04063	0.5836150	0.9674895	76.06	107.53140	53.05354	162.90204	1607 Oct. 24.0	2308300.5
1531	1531 Aug. 26.23846	2280492.73846	0.5811975	0.9677499	76.50	106.95724	52.34044	162.91385	1531 Aug. 14.0	2280480.5
1456	1456 Jun. 9.63257	2253022.13257	0.5797014	0.9679974	77.10	105.81647	51.15021	162.88607	1456 Jun. 28.0	2253040.5
1378	1378 Nov. 10.68724	2224686.18724	0.5762013	0.9683723	77.76	105.27668	50.30348	163.10897	1378 Nov. 5.0	2224680.5
1301	1301 Oct. 25.58194	2196546.08194	0.5727097	0.9689307	79.14	104.48199	49.43575	163.07179	1301 Nov. 9.0	2196560.5
1222	1222 Sep. 28.82294	2167664.32294	0.5742108	0.9688444	79.12	103.83087	48.58845	163.18782	1222 Oct. 15.0	2167680.5
1145	1145 Apr. 18.56090	2139377.06090	0.5747921	0.9687853	79.02	103.68573	48.33830	163.22004	1145 Apr. 2.0	2139360.5
1066	1066 Mar. 20.93405	2110493.43405	0.5744956	0.9688655	79.26	102.45543	46.90873	163.10814	1066 Mar. 8.0	2110480.5
989	989 Sep. 5.68757	2082538.18757	0.5819144	0.9678887	77.14	101.46581	45.84533	163.39474	989 Aug. 19.0	2082520.5
912	912 Jul. 18.67429	2054365.17429	0.5801559	0.9680692	77.45	100.75913	44.93122	163.30679	912 Jul. 14.0	2054360.5
837	837 Feb. 28.27000	2026830.77000	0.5823182	0.9678055	76.90	100.08403	44.21516	163.44258	837 Mar. 10.0	2026840.5
760	760 May 20.67126	1998788.17126	0.5818368	0.9678541	77.00	99.98016	43.97218	163.43860	760 Jun. 2.0	1998800.5
684	684 Oct. 2.76682	1971164.26682	0.5795841	0.9681495	77.62	99.13197	43.08465	163.41338	684 Sep. 29.0	1971160.5
607	607 Mar. 15.47581	1942837.97581	0.5808315	0.9680396	77.47	98.78209	42.54593	163.47190	607 Mar. 18.0	1942840.5
530	530 Sep. 27.12998	1914909.62998	0.5755915	0.9687113	78.90	97.56504	41.26006	163.38977	530 Oct. 8.0	1914920.5
451	451 Jun. 28.24911	1885963.74911	0.5737438	0.9689123	79.29	97.01122	40.49602	163.47468	451 Jun. 25.0	1885960.5
374	374 Feb. 16.34230	1857707.84230	0.5771940	0.9685857	78.76	96.49409	39.86451	163.53760	374 Mar. 1.0	1857720.5
295	295 Apr. 20.39842	1828915.89842	0.5759148	0.9687528	79.13	95.22565	38.39767	163.36268	295 Apr. 25.0	1828920.5
218	218 May 17.72347	1800819.22347	0.5814660	0.9679755	77.37	94.13158	37.19436	163.56891	218 Apr. 29.0	1800800.5
141	141 Mar. 22.43405	1772638.93405	0.5831377	0.9678439	77.23	93.67835	36.50620	163.43259	141 Mar. 24.0	1772640.5
66	66 Jan. 25.96014	1745189.46014	0.5851046	0.9675458	76.55	92.63672	35.41600	163.57158	66 Feb. 6.0	1745200.5
12 B.C.	-11 Oct. 10.84852	1717323.34852	0.5871999	0.9673664	76.33	92.54399	35.19064	163.58392	-11 Oct. 8.0	1717320.5
87 B.C.	-86 Aug. 6.46171	1689863.96171	0.5856047	0.9676769	77.12	90.76383	33.30553	163.33505	-86 Aug. 23.0	1689880.5
164 B.C.	-163 Nov. 12.56604	1661838.06604	0.5845470	0.9676686	76.88	89.09882	31.35152	163.69946	-163 Nov. 15.0	1661840.5
240 B.C.	-239 May 25.11796	1633907.61796	0.5853647	0.9675871	76.75	88.09919	30.09811	163.46207	-239 Jun. 7.0	1633920.5
315 B.C.	-314 Sep. 8.52367	1606620.02367	0.5874295	0.9673085	76.17	86.86997	28.83174	163.59479	-314 Sep. 29.0	1606640.5
391 B.C.	-390 Sep. 14.36897	1578866.86897	0.5880489	0.9672597	76.12	86.80007	28.61248	163.59935	-390 Sep. 28.0	1578880.5
466 B.C.	-465 Jul. 18.23879	1551414.73879	0.5902897	0.9671425	76.15	85.23459	26.86861	163.25878	-465 Jul. 4.0	1551400.5
540 B.C.	-539 May 10.82702	1524318.32702	0.5917786	0.9669379	75.73	83.57805	25.13235	163.47958	-539 May 13.0	1524320.5
616 B.C.	-615 Jul. 28.50346	1496638.00346	0.5925521	0.9668877	75.70	83.30446	24.68328	163.46465	-615 Jul. 31.0	1496640.5
690 B.C.	-689 Jan. 22.27922	1469421.77922	0.5996958	0.9660847	74.35	82.70312	23.86787	163.37513	-689 Jan. 1.0	1469400.5
763 B.C.	-762 Aug. 5.53008	1442954.03008	0.6016488	0.9659486	74.27	80.54155	21.67254	163.09997	-762 Aug. 12.0	1442960.5
836 B.C.	-835 May 9.30663	1416202.80663	0.5987312	0.9663239	74.97	78.55501	19.55356	163.29189	-835 May 7.0	1416200.5
911 B.C.	-910 May 20.22029	1388819.72029	0.5983184	0.9663753	75.06	78.15720	18.95167	163.24715	-910 Jun. 10.0	1388840.5
986 B.C.	-985 Dec. 2.56395	1361622.06395	0.6020087	0.9660066	74.53	77.71511	18.33702	163.24486	-985 Dec. 21.0	1361640.5
1059 B.C.	-1058 Dec. 3.66376	1334960.16376	0.6118822	0.9648673	72.68	76.65305	17.03099	162.93440	-1058 Dec. 4.0	1334960.5
1129 B.C.	-1128 Apr. 3.84467	1309149.34467	0.6217410	0.9635732	70.52	74.99543	15.24868	162.94390	-1128 Apr. 15.0	1309160.5
1198 B.C.	-1197 May 11.23252	1283983.73252	0.6289955	0.9625705	68.89	74.46894	14.66132	162.95968	-1197 Apr. 18.0	1283960.5
1266 B.C.	-1265 Sep. 5.39589	1259263.89589	0.6341965	0.9619874	68.15	74.03320	14.02581	162.84413	-1265 Aug. 13.0	1259240.5
1334 B.C.	-1333 Aug. 25.50585	1234416.00585	0.6265826	0.9630452	69.82	73.22376	13.08713	162.79898	-1333 Aug. 10.0	1234400.5
1404 B.C.	-1403 Oct. 15.68109	1208900.18109	0.6203761	0.9641090	71.86	71.94040	11.71491	162.49446	-1403 Sep. 26.0	1208880.5

*Julian calendar for dates prior to 1582, Gregorian calendar for post-1582 dates; all calendar dates begin at midnight.

1981MNRAS.197..633Y

Table 5. Comparison of observed and computed times of comet Halley's perihelion passage times.

T_c		T_o-T_c	T_o-T_{Ch}
1531	Aug. 26.24	-0.44	+37.30
1456	June 9.63	-0.53	+41.60
1378	Nov. 10.69	-1.67	+32.52
1301	Oct. 25.58	-1.05	+7.03
1222	Sep. 28.82	+1.98	-37.70
1145	Apr. 18.56	+2.69	-58.25
1066	Mar. 20.93	+2.57	-42.00
989	Sep. 5.69	+3.31	-30.50
912	Jul. 18.67	-9.17	+1.00
837	Feb. 28.27	-	+52.77
760	May 20.67	+1.83	+33.00
684	Oct. 2.77	-4.27	+1.00
607	Mar. 15.48	-2.48	-20.50
530	Sep. 27.13	-0.43	-21.80
451	June 28.25	-3.75	-5.00
374	Feb. 16.34	+1.06	+26.90
295	Apr. 20.40	+0.10	+33.00
218	May 17.72	-0.22	+70.00
141	Mar. 22.43	-0.08	+36.85
66	Jan. 25.96	+0.54	+34.00
12 B.C.	Oct. 10.85	-5.35	+8.00
87 B.C.	Aug. 6.46	-3.96	-13.00

T_c: Computed times of perihelion passage – from Table 4.
T_0: Times of perihelion passage determined by Chinese observations – from Table 1 and Kiang (1971).
T_{Ch}: Computed times of perihelion passage – from Chang (1979).

Table 5, it is clear that the inclusion of non-gravitational effects and the use of ancient observational data is necessary for an accurate representation of comet Halley's long-term motion.

5 Conclusions

The primary reason for integrating the motion of comet Halley back to 1404 BC was to allow possible identifications of ancient cometary observations with this famous comet. The orbital elements in Table 4 were used to generate ephemerides for each apparition and while no comet Halley observations prior to 240 BC have yet been identified in the ancient Chinese records, new observations that come to light can easily be matched against our ephemerides for possible future identifications.

Observations of any comet prior to 240 BC are scarce; Ho Peng Yoke (1964) lists only 16 and several of these reports are vague. Nevertheless, it is worthwhile asking why no reports of comet Halley prior to 240 BC are available. From our ephemeris calculations, it is readily apparent that the apparitions of comet Halley subsequent to 240 BC are generally more favourable than those prior to 240 BC. For the 29 apparitions 240 BC to 1910 there were 14 apparitions during which the Earth–comet distance (Δ) became less than 0.25 AU when the comet was in a dark sky. During the 16 apparitions from 1404 BC to 315 BC, there were only two (1266 BC and 1404 BC). During its past few apparitions, comet Halley has been intrinsically brighter after perihelion. If we only consider post-perihelion close approaches to the Earth, then from 240 BC to 1910 there were eight apparitions for which the comet was observable at $\Delta < 0.25$ AU while for the 1404 BC to 315 BC apparitions there were none. We also note that the 240 BC observations are only probable identifications of comet Halley and it is curious as to why the favourable apparition (minimum $\Delta = 0.1$ AU) in 164 BC went unobserved in September and October of that year.

Concerning the dynamics of comet Halley's long-term motion, we have established that subsequent to a close planetary approach, the comet's motion must be rectified with observational data. Of course, this is true in general for any comet and, where appropriate, non-gravitational effects should be taken into account as well. Yeomans (1977) concluded that the transverse non-gravitational parameter (A_2) for comet Halley was time independent over the 837–1910 interval. This implies that the comet's spin axis is fixed in space without noticeable precessional motion. We have assumed here that A_2 is time independent and the excellent residuals between the observed and computed times of perihelion passage (see Table 5) suggest that the spin axis was stable back to at least 87 BC. Also implied is the relative constancy, over two millennia, of comet Halley's ability to outgas. This result is consistent with the comet's nearly constant intrinsic brightness over roughly the same interval (Broughton 1979).

Acknowledgment

DKY acknowledges that these results represent one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract NAS7-100, sponsored by the Planetary Program Office, Office of Space Science, NASA.

References

Angström, A. J., 1862. *Actes Soc. R. Sci. Upsal, Sér. 3,* **4.**
Biot, E., 1843a. *Connaissance des Temps 1846. Additions* p. 44.
Biot, E., 1843b. *Connaissance des Temps 1846. Additions* p. 69.
Brady, J. L., 1972. *Publs astr. Soc. Pacif.,* **84,** 314.
Brady, J. L. & Carpenter, E., 1971. *Astr. J.,* **76,** 728.
Broughton, R. P., 1979. *J. R. astr. Soc. Canada,* **73,** 24.
Burckhardt, J. C., 1804. *Monatliche Korrespondenz,* **10,** 162.
Chang, Y. C., 1979. *Chinese Astr.,* **3,** 120.
Cowell, P. H. & Crommelin, A. C. D., 1907. *Mon. Not. R. astr. Soc.,* **68,** 111.
Cowell, P. H. & Crommelin, A. C. D., 1908a. *Mon. Not. R. astr. Soc.,* **68,** 173.
Cowell, P. H. & Crommelin, A. C. D., 1908b. *Mon. Not. R. astr. Soc.,* **68,** 375.
Cowell, P. H. & Crommelin, A. C. D., 1908c. *Mon. Not. R. astr. Soc.,* **68,** 510.
Cowell, P. H. & Crommelin, A. C. D., 1908d. *Mon. Not. R. astr. Soc.,* **68,** 665.
Halley, E., 1705. *Synopsis Astronomiae Cometicae,* London.
Halley, E., 1752. *Astronomical Tables,* London.
Hasegawa, I., 1979. *Publs astr. Soc. Japan,* **31,** 257.
Hind, J. R., 1850. *Mon. Not. R. astr. Soc.,* **10,** 51.
Ho Peng Yoke, 1964. *Vistas Astr.,* **5,** 127.
Kamienski, M., 1956. *J. Br. astr. Ass.,* **66,** 127.
Kamienski, M., 1957. *Acta astr.,* **7,** 111.
Kamienski, M., 1962. *Acta astr.,* **12,** 227.
Kanda, S., 1935. *Nihon Tenmon Shiryo* (reprinted Tokyo, 1979).
Kiang, T., 1971. *Mem. R. astr. Soc.,* **76,** 27.
Kiang, T., 1973. *Mon. Not. R. astr. Soc.,* **162,** 271.
Kiang, T., 1979. *IAU Symp. No. 81, Dynamics of the Solar System,* pp. 303–306, ed. Duncombe, R. L., D. Reidel, Dordrecht, Holland.
Kiang, T., 1980. *Acta astr. Sinica,* **21,** 323, English translation in *Chinese astr. Astrophys.,* **5,** 110 (1981).
Laugier, 1842. *Comptes Rendus.,* **15,** 949.
Laugier, 1843. *Comptes Rendus,* **16,** 1003.
Laugier, 1846. *Comptes Rendus,* **23,** 183.
Lieske, J. H., 1968. *PhD Dissertation,* Yale University.
Marsden, B. G., 1975. *Catalogue of Cometary Orbits,* Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.
Marsden, B. G., Sekanina, Z. & Yeomans, D. K., 1973. *Astr. J.,* **78,** 211.

Michielsen, H. F., 1968. *J. Spacecr. Rockets*, **5**, 328.
Pingré, A. G., 1783–84. *Comètographie*, Paris.
Saito, K., 1979. *Sky Telesc.*, **58**, 108.
Schubart, J. & Stumpff, P., 1966. *Veröff. Astr. Rech. Inst. Heidelb., No. 18.*
Stahlman, W. D. & Gingerich, O., 1963. *Solar and Planetary Longitudes for Years −2000 to +2000 by 10-day Intervals*, University of Wisconsin Press, Madison.
Tuckerman, B., 1964. *Planetary, Lunar and Solar Positions, 601 B.C. to 1649*, American Philosophical Society, Philadelphia.
Whipple, F. L., 1950. *Astrophys. J.*, **111**, 375.
Yeomans, D. K., 1971. *Astr. J.*, **76**, 83.
Yeomans, D. K., 1977. *Astr. J.*, **82**, 435.