### Light curves, color curves, and expansion velocity of type I supernovae as functions of the rate of brightness decline

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A photometric classification of supernovae is proposed, using the rate  $\beta$  of decline in photographic brightness after maximum light as a parameter. The principal elements of the light curves, color curves, envelope expansion velocity, and absolute magnitude at maximum light depend on the photometric class of the supernova. The mean absolute magnitude of type I supernovae, corrected for absorption, is determined. The occurrence of the anomalous type I supernovae reported by Bertola is discussed.

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### 1. RATE OF DECLINE AS A PARAMETER FOR PHOTOMETRIC CLASSIFICATION OF SUPERNOVAE

Although our knowledge of the brightness, color, and spectra of supernovae has grown substantially, we still have only fragmentary information about these objects. Investigators have therefore endeavored to reconstruct the light curves from elements that can readily be determined. These elements include the point at which the decline in brightness begins to slow down and the mean rate  $\beta$  of decline in photographic brightness from maximum light to that point (or to the end of the shoulder on the light curve, in the case of type II supernovae). We express  $\beta$ in magnitudes per 100-day interval, as well as the rate  $\gamma$  of fading on the slowly decaying tail of the curve.

Type I supernovae show a range in the parameter  $\beta$  by more than a factor of 2, and type II supernovae may differ in  $\beta$  by as much as three times,  $^{1,2}$  so that the proposal by Barbon et al.3 to subdivide type I supernovae into fast and slow classes is justified. These names, however, conflict with the kinematical significance of the processes involved, because rapidly fading supernovae have envelopes expanding at relatively low velocities, as will be shown presently. Thus in our opinion it would be better to adopt names not connected with the rate of development; for instance, it would be more suitable to call supernovae with small  $\beta$  junior objects, and those with large  $\beta$ , senior ones. But we are convinced that the parameter  $\beta$  can itself serve as a simple, physically sound quantity for establishing a photometric classification of supernovae; indeed,  $\beta$  is in a sense a universal parameter, as it is applicable to supernovae of both types. In this paper the supernova type symbol will be accompanied by the corresponding value of  $\beta$ , which can conveniently be measured from the bright branch of the light curve. For example, supernova 1937c will be assigned to class SN I.12, with the value of  $\beta$  following the decimal point; supernova 1972e, to class SN I.10; supernova 1959d, to class SN II.5, and so on.

A well-defined correlation is observed between  $\beta$  and the other elements of the photographic and visual light curves of supernovae as well as the expansion velocities of their envelopes. Accordingly, a supernova classified with respect to  $\beta$  will possess a whole complex of particular mean photometric and kinematic properties, serving to

facilitate its study.

### 2. REFINEMENT OF POINT OF MAXIMUM LIGHT FOR TYPE I SUPERNOVAE

Without question, maximum light represents one of the chief distinguishing points, although not the only one, on the photographic light curve of a supernova. When a supernova is discovered after maximum light, the establishment of that epoch and of the magnitude there poses a stereotypical practical exercise. We have previously discussed several ways to solve the problem. 1,2 and these are now in use. However, the existence of a relation between  $\beta$  and both the elements of maximum light enables them to be determined more reliably.

Evidently the larger  $\beta$  is, the shorter will be the interval  $\Delta t = t_{K}$  between the epoch  $t_{0} = 0$  of maximum light and the inflection point tk on the light curve for a constant brightness drop  $\Delta m$  during that interval; but the amplitude  $\Delta m$  is itself proportional<sup>3</sup> to  $\beta$ . From the material on type I supernovae collected in Table I, we obtain the following least-squares correlations:

$-2.4 \beta + 58.3 = \Delta t \pm 3  \text{days}$	(27 supernovae)	
$0.084 \beta + 2.12 = \Delta m \pm 0.2$	(24 supernovae)	(1)
$0.07 \text{ B} + 0.65 = \gamma \pm 0.5$	(32 supernovae)	(1)

The rms errors for the computed elements are quoted for the mean photometric class SN I.10, and are typical of the other classes.

The correlations (1) solve the problem of recovering the epoch and magnitude of maximum light for a type I supernova if the inflection point and  $\beta$  are known. In effect,  $\Delta t$  and  $\Delta m$  act as parameters for converting the origin of readings on the light curve from the maximum point to the inflection point, which can be established much more easily than maximum light.

Table I lists the photometric elements for many type I supernovae. The first column gives Zwicky's supernova number. Most of the other columns need to explanation; in the sixth, parenthesized entries give  $\Delta t$  values found from the epoch of maximum determined spectroscopically, while column 13 contains the apparent photographic magnitude of the supernova at maximum light, as estimated from the

Supernova Annual supernov No.	Annual	NGC	Julian day (2,400,000 +) at				_		Julian day (2,400,000 +) at max light				_
	supernova No.	(IC)	max. light	inflec- tion		$\Delta m$	β	γ	по (1)	from spectrum	adopted	m <sub>pg</sub>	References
1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	1895 <i>b</i>	5253	13383	13420	_	_	9	1.1	_	13382	13383	7 <sup>m</sup> .9	[4, 5]
13	1920a	2608	_	22380	-	_	6	0.8	22336	-	22336	11.4	[6]
25	1937c	(4182)	28769	28800	31	3.2	12	1.9	_	-	28769	8.2	[7,8]
26	1937d	1003	28794	28828	34	3.0	12	1.0	-	-	28794	12.9	[7,8]
30	1939a	4636	29290	29319	29	3.1	12	1.7	_	-	29290	12.0	[9, 10]
31	1939 <i>b</i>	4621	29383	29408	25	3.6	14	1.5	-	_	29383	12.2	[10, 11]
50	1954a	4214	_	_	_	_	14	1.6	_	34852	34852	9.0	[12-14]
51	1954 <i>b</i>	5668	_	34895	_	_	12	1.1	34863	_	34863	12.4	[12]
54	1956a	3992	35547	35582	35	3.1	10	1.1	-	_	35547	12.2	[15]
56	1957 <i>b</i>	4374	35968	35998	30	3.0	12	1.7	-	-	35968	12.1	[16]
62	1959c	Anon	_	_	_	_	8	_	-	36751	36751	14.1	[17, 18]
69	1960f	4496	_	37074	(35)	2.9	10	2.0	37040	37039	37040	11.6	[16, 19]
. 86	1960r	4382	_	37323	(37)	2.9	9	1.3	37286	37283	37285	11.7	[16, 20]
87	1961 d	Anon	37304	_	_	-	9		-	-	37304	16.0	[21]
92	1961h	4564	37428	_	_	_	9	_	-	-	37428	11.1	[22]
100	1961p	Anon	37554	37594	40	2.7	8	1.1	-	37551	37552	14.1	[16, 20]
106	1962a	Anon	37689	-	-	-	10	1.0	-	37686	37686	16.0	[23]
117	1962j	6835	-	37940	(41)	-	9	1.5	37900	37899	37900	13.3	[20]
119	19621	1073	38004	38030	26	2.7	11	1.7	_	_	38004	13.4	[24, 25]
145	1962p	1654	37 <b>9</b> 69	38000	31	_	. 9	0.8	-	_	37969	14.5	[26]
126	1963d	4146	-	38084	_	_	10	1.3	38050	-	38050	15.8	[20, 25]
131	1963 <i>i</i>	4178	_	38179	(21)	_	14	-	_	38158	38158	12.3	[20, 27, 28]

### (TABLE I (continued)

Supernova No.	Annual supernova	NGC	Julian day (2,400,000 +) at						Julian day (2,400,000 +) at max light				
	No.	(IC)	max. light	inflec- tion	Δt. days	$\Delta m$	β	Y	по (1)	from spectrum	adopted	m <sub>pg</sub>	Reference
1	2	3	4	5	6	7	8	9	10	11	12	13	14
132	1963 <i>j</i>	3913	38175	38220	45	2.5	6	-	_	_	38175	13.7	[20, 29]
138	1963p	1084	38300	38330	30	3.2	10	2.2	-	38301	38300	13.8	[30]
150	1964e	Anon	38454	38485	31	3.0	11	1.0	_	_	38454	12.5	[31, 32]
159	1964l	3938	_	38771	(32)	_	10	1.1	38737	38739	38738	13.4	[30]
160	1965a	4410	-	38772	-	-	8	1.3	38733	-	38733	14.0	[33]
170	1965i	4753	38928	_	_	_	9	_	_	-	38928	12.5	[34]
175	1965n	3074	39130	-	-	_	l –	2.5	-	-	39130	14.7	[35]
186	1966 <i>j</i>	3198	-	39494	(49)	_	8	1.4	39450	39445	39448	11.3	[36]
198	1966n	Anon	-	39444	_	-	8	1.6	39400	-	39400	14.6	[37]
192	1967c	3389	-	39580	(36)	3.0	11	2.2	39548	39544	39546	12.9	[38-40]
211	1968e	2713	l –	-	-	2.7	6	_	-	39924	39924	13.7	[34, 41, 42]
235	1969c	3811	40254	40294	40	2.7	6	1.1	_	-	40254	14.5	[43]
275	1970j	7619	40865	40900	35	2.9	10	1.0	-	-	40865	14.4	[44]
278	1970 <i>l</i>	2968	_	40900	_	-	-	1.6	-	-	40868:	13	[44]
295	1971g	4165	41055	41080	25	3.5	14	2.2	_	_	41055	13.2	[44-46]
299	1971i	5055	41101	41128	27	3.3	13	1.1	-	41101	41101	11.7	[44, 47, 48]
303	19711	6384	41133	41166	33	2.9	10	1:	-	-	41133	13.5	[44]
307	1971 <i>p</i>	7319	_	41213	(37)	-	7	0.6	41171	41176	41174	_	[49]
336	1972e	- 5253	-	41480	(36)	3.3	10	1.5	41444	41444	41444	7.9	[50, 51]
344	1972 <i>h</i>	3147	-	41558	-	-	9	0.6	41521	-	41521	14.7	[52, 53]
413	1974g	4414	42169	42199	30	3.0	11	_	_	_	42169	i1.9	[54, 55]
419	1975a	2207	42435	- !	_		10		_	_	42435	14.6	[56]

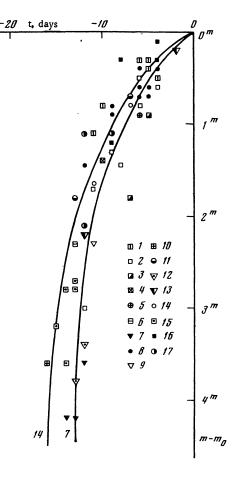


FIG. 1. Rising branches of light curves. Average photographic light curves are shown for supernovae of classes L14-L12 (mean  $\beta$  = 14) and L9-L6 (mean  $\beta$  = 7). 1) SN 1937d; 2) SN 1939a; 3) SN 1939b; 4) SN 1954a; 5) SN 1956a; 6) SN 1957b; 7) SN 1961d; 8) SN 1962a; 9) SN 1962p; 10) SN 1963i; 11) SN 1964e; 12) SN 1965i; 13) SN 1969c; 14) SN 1970j; 15) SN 1971g; 16) SN 1971i; 17) SN 1974g.

second of Eqs. (1) if a dash appears in column 4. Estimates in terms of B magnitudes have been converted to photographic magnitudes according to Arp. 57

The dates of maximum estimated in columns 4, 11, and 12 are in good agreement, except for estimates from one or two spectra separated by a short interval, or in late phases when the changes in the spectra were retarded. On the average the dates of maximum light that we have determined from the spectra are 2-3 days later than those found by Minkowski<sup>8</sup> from series of spectra of supernovae 1937c and 1937d, and values subsequently relying on his series as standards. The disparity arises from our redetermination of the dates of maximum of these supernovae according to average light curves with similar values of  $\beta$ .

TABLE II

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For  $\beta=14-12$  (mean 14)  $t_0=-17$  days,  $p=0.05\pm0.01$  (6 SN), For  $\beta=14-10$  (mean 10)  $t_0=-16$  days, p=0.08 (5 SN), For  $\beta=9-6$  (mean 7)  $t_0=-14$  days, p=0.11 (4 SN).

3. RISING BRANCH OF LIGHT CURVE OF TYPE I SUPERNOVAE

Having revised the estimates of the point of maximum light, we can reexamine the behavior of the light curve prior to maximum. Information on premaximum levels of brightness more than  $1^{\rm m}$  below that of maximum is available for 15 type I supernovae. If we subdivide these into three groups according to the value of  $\beta$ , then the mean light curves for the rising branch will differ slightly (Fig. 1). Each curve of this kind may be represented by an expression of the form

$$m-m_0=-5 \lg (1-t/t_0)+pt.$$
 (2)

By trial and error we obtain the values of  $t_{\boldsymbol{\theta}}$  and p given in Table II.

The author has given elsewhere<sup>58</sup> a simple interpretation of Eq. (2): The first term in the right-hand member corresponds to the enlargement of the radiating surface, and the second, to the drop in the radiation density because of the decline in temperature as the envelope expands. Table II shows that the rise in brightness occurs somewhat more slowly for the senior classes of supernovae (those fading rapidly after maximum) than for the junior classes. This effect is attributable to the lower expansion velocity of the envelope for the senior classes (see Sec. 5 below).

The phase  $t_0$  at which the expansion begins can also be determined from spectroscopic observations of supernovae. In this manner Branch and Patchett<sup>59</sup> have found  $t_0 = -18.2 \pm 3.3$  days, in good accord with our results and with direct observations of supernovae 14-16 days perior to maximum. Searle<sup>60</sup> has obtained the estimate  $t_0 = -10$  days, which is in need of improvement.

# 4. ANALYSIS OF VISUAL LIGHT CURVES OF TYPE I SUPERNOVAE

Only fragmentary data are available for visual light curves, but their analysis materially furthers our understanding of the behavior of supernovae and enables the light curves of galactic supernovae to be reconstructed more accurately. The visual light curves of type I supernovae show a barely perceptible break about 10 days after the inflection point on the photographic light curve. By analogy, then, we may divide the visual light curve as well into segments  $\alpha'$ ,  $\beta'$ , and  $\gamma'$ .

Table III gives the elements of the visual light curves for some type I supernovae. Most of the columns are self-explanatory. The ninth column contains the difference in days between the epochs of the breaks in the visual and photographic light curves; column 10 gives the drop  $\Delta m_V$  in visual brightness between the maximum point and the break (the times of maximum light for the visual and photographic curves are assumed to coincide). Parenthesized values have been computed from the following relation, which permits recovery of a visual light curve of average form from the observed elements:

$$0.38 \beta + 1.9 = \beta' \pm 0.9$$
 (14 supernovae). (3)

The rms error quoted here corresponds to  $\beta$  = 10; it increases from 0.7 to 1.0 as  $\beta$  rises from 6 to 14. The

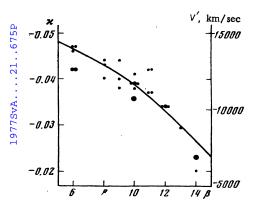


FIG. 2. Dependence of  $\kappa$  and V' on the photometric class  $\beta$  of type I supernovae. The size of the dot increases with the number of measurements of  $\kappa$  (less than 5, more than 5, and more than 20).

parameters  $t_k'-t_k$  and  $\Delta m_v$  are practically independent of the class  $\beta$ ; on the average,  $t_k'-t_k=9$  days and  $\Delta m_v=2.5$ . In practice, visual light curves can be reconstructed more accurately from a photographic curve by means of color curves, because if Eqs. (1) and (3) are used together the error of the result will build up considerably.

### 5. RELATION BETWEEN EXPANSION VELOCITY AND PHOTOMETRIC CLASS OF TYPE I SUPERNOVAE

The rate at which the parameters of a supernova envelope change depends on its expansion velocity V, as defined by the Doppler shift  $\varkappa=(\lambda'-\lambda_0)/\lambda_0$  of absorption minima in the spectrum:  $V=V'/k=c\varkappa/k$ . Here  $\lambda'$  and V' denote the wavelength of the absorption feature and the velocity of the envelope layer in which it is formed,  $\lambda_0$  is the laboratory wavelength of the feature, c is the velocity of light, and the proportionality factor k=3/4 between V' and V has been estimated from measured wavelengths of the blue end of the absorption and  $\lambda'$  for the feature at 6150 Å in spectra of supernova 1937c.

Marked differences in  $\varkappa$  in the spectra of various type I supernovae have been recorded by Mustel'. It is natural to compare the observed  $\varkappa$  values with the super-

nova classes, which determine the rate of photometric evolution. To measure the expansion velocity we have taken the Doppler shift of the most characteristic absorption features:  $\lambda6150$ , identified with the  $\lambda\lambda6347$ , 6371 Si II doublet;  $\lambda4300$  (corresponding to  $\lambda4481$  Mg II) for the first 10 days after maximum light; and  $\lambda4400$  ( $\lambda4584$  Fe II) for later phases. In type I supernovae the absorption features undergo little phase shift, <sup>69</sup> so the  $\varkappa$  determinations are fully consistent.

Table IV gives data on the mean  $\varkappa$  and the Doppler shifts. The first few columns are self-explanatory. Column 5 contains the red shift z of the galaxy; columns 6-8, the wavelengths of the minima at  $\lambda\lambda6150$ , 4300, and 4400 Å (the number of estimates used is parenthesized); column 9, the value of  $\varkappa$  corrected for the red shift of the galaxy; column 10, the velocity V' of the envelope layer; and column 11, references to the sources of the spectra considered. The dependence of  $\varkappa$  on the photometric class  $\beta$  according to Table IV is plotted in Fig. 2. This relation is satisfied by the quadratic law

$$0.000125 \,\beta^2 - 0.051 = \times \pm 0.002$$
 (28 supernovae). (4)

The rms error in  $\kappa$  is quoted for class SN I.10 and is much the same for the other classes. The estimates given in Table IV and in Eq. (4) are provisional, since they rest not on the spectra themselves but on the published measurements and tracings. Direct wavelength measurements of the absorption features should improve the accuracy.

At first glance Eq. (4) is a paradox: The higher the expansion velocity, the more slowly the supernova fades. But this effect is explained in a natural way by a combination of opposing trends that produce the light curve: the drop in temperature and the increase in the radius of the photosphere, 59,86 which continue until the inflection point on the declining branch is reached. For a rapidly expanding envelope the surface area of the photosphere also increases rapidly, and the rise in brightness is swifter, as we have seen; but after maximum the rapidly growing surface of the photosphere checks the fade in the light

TABLE III. Photometric Elements for Visual Light Curves

No. SN No. (IC) $\beta$ v $\beta'$ v inflection point $t^{k-t_k}$ $\Delta^m_v$ Reference in the point $t^{k-t_k}$ $\Delta^m_v$ Reference in $t^{k-t_k$	SN	Annual	NGC		Cl	ass		Julian day at			P. 6	
1 1885a 224 (14) - 7: 2.6 09820 - 3: [61] 25 1937c (4182) 12 1.9 6.0 2.4 28810 10 2.5 [61] 26 1937d 1003 12 1.0 6.5 1.5 28838 10 2.8 [61] 50 1954a 4214 14 1.6 (6.5) 1.6 [62] 51 1954b 5668 12 1.1 7.2 [62] 54 1959c An 8 - 5.2 [62] 62 1959c An 8 - 5.2 [18] 87 1961d An 9 - 5.3 [21] 106 1962a An 10 - (6.0) 2.2 37730 - 2.2 [23] 111 1962e An 10 - 5.0 [63] 192 1967c 3389 11 2.2 5.8 [63] 192 1967c 3389 11 2.2 5.8 [38, 39] 235 1969c 3811 6 1.1 4.2 [43] 275 1970j 7619 10 1.0 5.0 2.0 40910 10 2.0 [44, 64] 295 1971g 4165 14 2.2 6.5 1.8 41088 8 2.2 [44-46, 299 1971g 5055 13 1.7 7.0 2.5 41136 8 2.5 [44, 44, 6]	No.	SN No.	(IC)	. β	Y	β′	ν'	inflec-	tk-tk	$\Delta m_v$	References	
25         1937c         (4182)         12         1.9         6.0         2.4         28810         10         2.5         [61]           26         1937d         1003         12         1.0         6.5         1.5         28838         10         2.8         [61]           50         1954b         5668         12         1.1         7.2         -         -         -         -         [62]           54         1956a         3992         10         1.1         (5.7)         2.1         -         -         -         -         [62]           54         1959c         An         8         -         5.2         -         -         -         -         [15]           62         1959c         An         8         -         5.2         -         -         -         -         [15]           106         1962a         An         10         -         (6.0)         2.2         37730         -         2.2         [23]           111         1962e         An         10         -         5.0         -         -         -         -         [63]           192         1967c	1	2	3	4	5	6	7	8	9	10	11	
336   1972e   5253   10   1.5   5.4   2.5   41495   10   2.3   [50,51,	50 51 54 62 87 106 111 192 235 275 295 299 303 336	1937c 1937d 1954a 1954a 1954b 1956a 1959c 1961d 1962a 1962c 1967c 1969c 1970j 1971g 1971i 1971i	(4182) 1003 4214 5668 3992 An An An An 3389 3811 7619 4165 5055 6384 5253	12 12 14 12 10 8 9 10 10 11 6 10 14 13 10	1.0 1.6 1.1 1.1 - - 2.2 1.1 1.0 2.2 1.7 1:	6.0 6.5 (6.5) 7.2 (5.7) 5.2 5.3 (6.0) 5.0 5.8 4.2 5.0 6.5 7.0 5.5 5.4	2.4 1.5 1.6 - 2.1 - 2.2 - - 2.0 1.8 2.5 - 2.5	28810 28838 	10     10 8 8 8	2.5 2.8 - - - 2.2 - 2.0 2.5 - 2.3 3:	[62] [62] [15] [18] [21]	

Supernova No.	Annual SN No.	NGC (IC)	Class β	z	λ 6150 (Å)	λ 4300 (Å)	λ 4400 (Å)	×	V', .km/sec	References
1	2	3	4	5	6	7	8	9	10	11
3	1895 <i>b</i>	5253	9	0.0013	_	_	4490(1)	-0.044	-13200	[70]
25	1937c	(4182)	12	0.0009	6157 (6)	4332(8)	4435 (9)	-0.034	-10200	[8]
26	1937d	1003	12	0.0020	6160(2)	4420(2)	_``	-0.034	-10200	[8]
50	1954a	4214	14	0.0010	_ ` `	4389 (11)	_	-0.023	-6900	[71–73]
54	1956a	3992	10	0.0035	_		4410(2)	-0.041	-12300	[15, 71]
56	1957b	4374	12	0.0032	6210(1)		4410(2)	-0.034	-10200	[73]
62	1959c	Anon	8	0.0100		4345(1)		-0.040	-12000	[17]
69	1960 <i>f</i>	4496	10	0.0059	6150(8)	4334(3)	4425(3)	-0.039	-11700	[19, 74, 75]
86	1960r	4382	9	0.0026	6125(1)		4405(6)	-0.042	-12600	[20, 76]
92	1961h	4564	9	0.0031	6190(1)	_	_`´	-0.040	-12000	[20]
100	1961p	Anon	8	0.0122	6180(1)	4335(1)	_	-0.043	-12900	[20]
106	1962a	Anon	10	0.0205	_	_	4475(1)	-0.044	-13200	[23]
117	1962j	6835	9	0.0057	6140(3)	4293(3)	_	-0.044	-13200	[20]
131	1963i	4178	14	0.0008	6210(1)	4410(1)	-	-0.020	-6000	[20]
132	1963 <i>j</i>	3913	6	0.0028	6077(7)	_``	4332(5)	-0.042	-12600	[20, 29]
138	1963p	1084	10	0.0049	6120(3)	4357(3)		-0.038	-11400	[30]
150	1964e	Anon	11	0.009:	_	4330(1)	_	-0.042	-12600	[77]
170	1965i	4753	9	0.0045	6143(3)	4330(2)	4440(3)	-0.038	-11400	[34, 78, 79]
186	1966j	3198	8	0.0022	6090(3)	_	4400(8)	-0.042	-12600	[36, 80, 81]
192	1967c	3389	11	0.0043	6170(2)	_	4421 (3)	-0.037	-11100	[40, 73, 81]
211	1968e	2713	6	0.0126	6180(3)	_	4414(4)	-0.046	-13800	[42]
235	1969c	3811	6	0.0140	6150(1)	_	-	-0.047	-14100	[69]
299	1971i	5055	13	0.0017	6178 (5)	4351(3)	_	-0.029	-8800	[44, 69, 82]
303	1971l	6384	10	0.0058	6142(2)		4433(2)	-0.039	-11700	[69]
336	1972e	5253	10	0.0013	6152(10)	4317(6)	4426 (24)	-0.036	-10800	[69, 73, 83]
413	1974g	4414	11	0.0024	6143(6)	_	4427(6)	-0.037	-11100	[55, 84]
419	1975a	2207	10	0.0089	6170(2)	4347 (2)		-0.039	-11700	[85]
420	1975 <i>b</i>	Anon	1 9	0.018:	i –	4300(1)	4400(1)	-0.040	-12000	[85]

more strongly, and the parameter  $\beta$  becomes smaller. For a slowly expanding envelope the situation is reversed: There is a gradual rise in brightness, but a faster decline.

#### 6. COLOR CURVES OF TYPE I SUPERNOVAE

The mean (B-V)(t) and (U-B)(t) color curves for type I supernovae that we have considered previously<sup>1,2</sup> are well confirmed by observations<sup>50,66</sup> of supernova 1972e. A disparity occurs only for (U-B)(t) during the late phases, because of a spurious point on our graph which does not correspond to any observation (compare Fig. 4 and Table 4 of our 1970 paper<sup>2</sup>). However, the mean color curves were merely a preliminary tool for analysis of a small amount of material, and their derivation should now be wholly revised. Even after reddening by the interstellar medium is excluded, the scatter in the mean color curve remains large, because the differences in  $\Delta t$  induce differences in the pace of the color variations and unavoidably cause the color curves to split into a family of curves corresponding to particular photometric classes  $\beta$ .

As indicated above, the break in the photographic light curve is sharp whereas the visual light curve has a smoother bend at a later epoch. Accordingly, as maximum light is passed the (B-V)(t) color curve rises smoothly, reaching maximum reddening at the time of the abrupt change in the rate of decline of the photographic brightness — a point which we define by the first of Eqs. (1). This point occurs at different phases, depending on the class of the supernova, so that strictly speaking no unified mean (B-V)(t) color exists. Color curves must be constructed for the separate classes. We have applied the same method of reconstruction from fragmentary curves as before, 1,2 but we now also have the values given

by Eq. (1) for the phase of maximum reddening.

Unfortunately, the accuracy of the photometric material for supernovae varies, and the scatter in the estimates near the time of maximum reddening and afterward may be as great as 0<sup>m</sup>.4. We have therefore first studied the form of the color curves for classes I.14, I.10, and I.12, which have the most abundant observational data. When the peaks of the color curves for the supernovae of these classes are fitted together, the rise in the reddening to its maximum value is, in general, found to be similar for most classes (Fig. 3). It is interesting to note that the small bends 20 days before maximum reddening correspond closely to a certain change in character of the continuous spectra of type I supernovae at that phase (see the series of spectra published by Minkowski<sup>8</sup> and Kirshner et al.<sup>69</sup>). The reddening diminishes at varying rates, depending on the class of the supernova.

The reddening of supernovae by interstellar matter in our own galaxy and in the galaxy containing the star can dependably be taken into account only if the other galaxy does not have too much internal absorption. We list these cases in Table V. Four normal  $(B-V)_0(t)$  color curves for supernovae of classes I.14, I.12, I.10, and I.6 are shown in Fig. 4. These are decidedly provisional and need to be supplemented by observational material of good quality.

The  $(U-B)_0(t)$  color curves also split into a family of curves (Fig. 5). We have here taken the reddening<sup>2</sup> equal to half of  $E_{B-V}$ . In addition to the maximum reddening at  $t_k$  these color curves also show a relative minimum due to the resumption of the progressive weakening of the ultraviolet part of the supernova spectrum at late phases.

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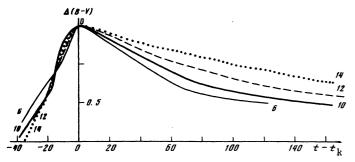


FIG. 3. Arrangement of the family of (B-V)(t) color curves for type I supernovae. The peaks of the curves are matched.

## 7. ABSOLUTE MAGNITUDES OF TYPE I SUPERNOVAE AT MAXIMUM LIGHT

For type I supernovae observed in galaxies with insignificant internal absorption, and for those having measured color indices and belonging to classes with established normal color curves, we can determine the absolute magnitude  $\mathbf{M}_{pg}$  at maximum light somewhat more accurately than hitherto. The chief source of error in this case remains the uncertainty in the distance of the galaxies.

The data required for this purpose are given in Table V. Successive columns contain: 1) the number of the galaxy in the NGC or the IC; 2) its type; 3) the true distance modulus; 4) the supernova number; 5) its number within the year; 6) the photometric class  $\beta$ ; 7) the apparent photographic magnitude  $m_{pg}$  at maximum light; 8) the color excess  $E_{B-V}$ , quoted to two decimal places if estimated by the method of Sharov and the author, 87 the other values being given by the deviation of the B-V color index from the normal color curve of the corresponding class;

9)  $m_0 = m_{pg} - 4E_{B-V}$ ; 10)  $M_{pg}$ ; 11) literature references. The methods of estimating  $m_0 - M$  are designated by letters in column 5 and have been discussed in detail previously<sup>1,2</sup>: S indicates a determination from the brightest star in IC 4182, which has  $m_{pg} = 19.2$ ,  $m_{pg} = -8.0$ , by analogy with the brightest star in IC 1613, for which  $m_{B} = -7.55$  (Ref. 88); H, from the size of the H II regions; Z, from the red shift, taking  $m_{0} = 55 \, \mathrm{km \cdot sec^{-1} \cdot Mpc^{-1}}$ ; L, from the mean absolute photographic magnitude of the galaxy containing the supernova  $m_{0} = 10.0 \, \mathrm{km}$ ; and V, C, U, G, from the membership of a galaxy in the Virgo, Coma, or Ursa Major cluster or the Centaurus group, respectively. With the adoption of a new Hubble constant  $m_{0} = 1.3 \, \mathrm{km}$  all the distance moduli except those obtained by the first two methods have been converted from our previous values  $m_{0} = 1.3 \, \mathrm{km}$ 

For the 16 supernovae in elliptical and lenticular systems,  $\overline{\rm M}_{\rm pg} = -20.0 \pm 0.1$ , while for the 16 type I supernovae in spiral systems,  $\overline{\rm M}_{\rm pg} = -20.3 \pm 0.2$ , practically the same value. This circumstance indicates that we have corrected properly for the absorption. Type I supernovae are entirely suitable as trustworthy distance indicators throughout a region of enormous radius.

A comparison of  $M_{\mbox{\scriptsize pg}}$  with the classes of the supernovae reveals a distinct trend:

$$-21.3+0.11\beta=M_{pg}\pm0.5$$
 (32 supernovae).

Because of this relationship the absolute magnitudes of type I supernovae of junior classes such as I.6 are 0<sup>m</sup>.9 brighter, on the average, than those of class I.14 supernovae.

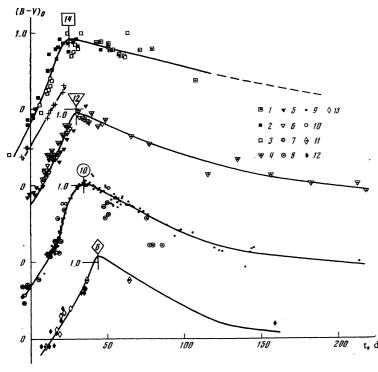


FIG. 4. Normal (B-V)<sub>0</sub>(t) color curves for type I supernovae of classes 6, 10, 12, and 14. The vertical lines mark the peaks of the curves, as determined from  $t_k$  [Eq. (1)]. 1) SN 1954a; 2) SN 1971i; 3) SN 1971g; 4) SN 1937c; 5) SN 1937d; 6) SN 1954a; 7) SN 1975a; 8) SN 1970j; 9) SN 1972e; 10) SN 1956a; 11) SN 1961d; 12) SN 1969c; 13) SN 1968e. Crosses designate color estimates for the anomalous class L12 supernova 1962  $l_*$ , assuming a color excess  $E_{R-V} = 0 \frac{m}{2} 21$ .

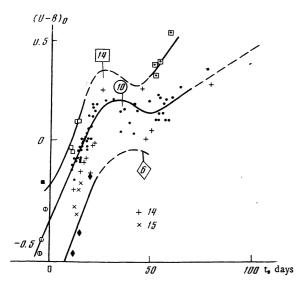


FIG. 5. Normal (U-B)0(t) color curves for type I supernovae of classes 6. 10, and 14. Symbols as in Fig. 4, except: 14) SN 1967c; 15) SN 1954b. The vertical lines mark the

### 8. OCCURRENCE OF ANOMALOUS TYPE I SUPERNOVAE

Bertola<sup>24,30</sup> has reported that the  $\lambda$ 6150 absorption feature is absent from or very weak in the early postmaximum spectra of supernovae 19621 and 19641. A similar anomaly has been found in supernova 1975b. Another property recorded by Bertola as a prototype of this family of anomalous type I supernovae is the enhanced reddening of the radiation, in comparison with the normal color curve. But the galaxy NGC 1073, in which supernova 19621 appeared, is viewed almost in the plane of projection, and strong reddening of a supernova in it would be

unlikely. The reddening in our Galaxy gives  $E_{B-V} = 0^{m}.07$ , and that in NGC 1073, about 0<sup>m</sup>.14. Evidently supernova 19621 had a low-temperature photosphere, so that the Si II lines, which have a high excitation potential and are not resonance lines nor bound to metastable levels, were weakly excited in it. Anomalous type I supernovae have a third distinctive property: If interstellar absorption of moderate amplitude is eliminated, we obtain a low photographic but a perfectly normal visual absolute magnitude for the supernova at maximum light. At the same time the light curves and Doppler shifts of other absorption features are entirely analogous to those found in normal supernovae of the same photometric class.

This category of type I supernovae might possibly include supernova 1957a, which Bertola assigns to type I according to its light curve. It has an analogous "disorder" in its absorption and absolute magnitude at maximum, but the data of Zwicky and Karpowicz suggest a type II spectrum for this supernova. An inspection of the tracings published by these authors shows that in the spectral region photographed, 3500-5000 Å, the hydrogen emission lines exhibit a red shift three times that of the galaxy NGC 2841 in which the supernova was observed. But of the three hydrogen emission lines in this part of the spectrum, only one is prominent; rather than Hy it might be identifiable with the A4358 Hg line of the urban night sky spectrum, and the conflict in the red shift would be removed. Generally speaking, estimates of the type of supernova spectrum based on the blue wavelength range may be in error.95

Among the 30 type I supernovae whose red spectral region has been observed during the phase when the  $\lambda 6150$ absorption was present, three have proved to be anomalous. If the family of anomalous type I supernovae is not associated with a particular type of galaxy, it could include 10%

TABLE V. Reddening and Luminosity of Type I Supernovae

G	alaxy			Deference						
NGC (IC)	type	m <sub>0</sub> —M	SN No.	Annual SN No.	class β	mpg	E <sub>B-V</sub>	m <sub>o</sub>	M pg	References
5253	S0p	28 <sup>m</sup> .2 G	3	1895 <i>b</i>	9	711.9	$0^{m}.07$	7 <sup>m</sup> .6	$ _{-20^m,6}$	[4, 90]
4486	EO	31.6 V	12	1919a	10	11.5	0.00	11.5	-20.1	[91]
4636	E0	31.6 V	30	1939a	12	12.0	0.00	12.0	-19.6	[9, 10]
4621	E5	31.6 V	31	1939b	14	12.2	0.00	12.2	-19.4	[10, 11]
4374	E1	31.6 V	56	1957b	12	12.1	0.00	12.1	-19.6	[16]
4382	S0	31.6 V	86	1960r	9	11.7	0.00	11.7	-19.9	[16]
Anon	E0	35.5 C	87	1961d	9	15.5	0.08	15.2	-20.3	[21]
Anon	S0	36.8 Z	88	1961e	-	17.0	0.00	17.0	-19.8	[92]
4564	E6	31.6 V	92	1961h	9	11.1	0.00	11.1	-20.5	[22]
Anon	Sb	35.5 C	106	1962a	10	16.0	0.08	15.7	-19.8	[23]
Anon	E	35.5 C	125	1963c		15.6	0.08	<b>15</b> .3	-20.2	[93]
4753	S0p	31.6 V	170	1965i	9	12.5	0.3	11.3	-20.3	[34]
7619	E	34.3 Z	275	1970j	10	14.4	0.08	14.1	-20.2	[44]
5253	S0p	28.2 G	336	1972e	10	7.9	0.07	7.6	-20.6	[50, 51]
(4182)	I	27.2 S	25	1937c	12	8.2	0.05	8.0	-19.2	[7]
4214	I	29.1 H	50	1954a	14	9.0	0.14	8.4	-20.7	[12]
1003	Sc	30.4 H	26	1937d	12	12.9	0.7	10.1	-20.3	[7]
5668	Sc	32.5 Z	51	1954b	12	12.4	0.2	11.6	-20.9	[12]
3992	Sb	31.6 Z	54	1956a	10	12.2	0.3	11.0	-20.6	[15]
Anon	SBc	33.6 Z	62	1959c	8	14.1	0.2	13.3	-20.3	[18]
4496	Sc	31.6 V	69	1960 <i>f</i>	10	11.6	0.0	11.6	-20.0	[16]
4178	Sc	31.6 V	131	1963i	14	12.3	0.4	10.7	-20.9	[20, 27, 28]
Anon	SBcp	31.7 U	150	1964e	11	12.5	0.3	11.3	-20.4	[31, 32]
3389	Sc	31.7 Z	192	1967c	11	12.9	0.1	12.5	-19.2	[38, 39]
2713	SBb	34.1 Z	211	1968e	6	13.7	0.05	13.5	-20.6	[34]
3811	Sc	33.7 Z	235	1969c	6	14.5	0.6	12.1	-21.6	[43]
4165	Sc	31.6 V	295	1971g	14	13.2	0.2	12.4	-19.2	[44-46]
5055	Sb	29.9 Z	299	1971i	13	11.7	0.4	10.1	-19.8	[44, 47, 48]
6384	Sb	32.6 Z	303	19711	10	13.5	0.75	10.5	-21.1	[44]
3147	Sb	33.6 Z	344	1972h	9	14.7	0.1	14.3	-19.3	[52]
4414	Sc	30.6 Z	412	1974g	11	11.9	0.4	10.7	-20.3	[54]
2207	Sc I	33.5 Z	419	1975a	10	14.6	0.44	13.0	-20.5	[56]

of all type I supernovae. But if these stars occur only in Sc galaxies, their proportion might reach one-fifth of the type I supernovae in galaxies of this morphological type. In particular, one of the galactic supernovae, the star of 1604, might conceivably be a representative of this family.

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