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Spectrophotometric Investigations of Semiregular and Irregular Carbon Stars

by

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ABSTRACT

The spectrophotometric observations of 15 semiregular and irregular carbon stars carried out at the Toruń Observatory between 1966-71 are discussed. The carbon abundance classes are given for all the investigated carbon stars. The comparison of the changes of the C_2 Swan system and the CN (7, 1) molecular bands as well as of the sodium lines (NaI λλ5890, 5896 Å) with the light phases for two semiregular carbon stars shows that the absorption lines and molecular bands get stronger with a decrease of temperature for hotter carbon stars. There are also population differences in the central depths of sodium lines. The sodium lines are stronger in population I carbon stars than in population II stars. For a low dispersion population criterion, the diagram of central depths of sodium lines against the central depths of the C_2(0, 1) molecular band is proposed.

1. Introduction

Statistical investigations of late-type stars (Ikaunieks 1963) indicate that carbon stars constitute about 1% of the known red giants brighter than $m_{pv} = 12^m$. About 55% of carbon stars are variable stars with a light amplitude higher than 1^m. Approximately 80% of the variable carbon stars have irregular (*Lb* variables) and semiregular (*SR* variables) variations of brightness.

The analysis of the spatial distribution of carbon stars (Ikaunieks 1952, Blanco 1965, Mavridis 1971) has shown that N stars are strongly concentrated toward the galactic plane, while the galactic concentration of R stars is rather weak. Carbon stars are found in globular clusters (Harding 1962, Dickens 1972) as well as in open ones (Hartwick and Hesser 1971, Eggen 1972). Ikaunieks (1952) has discussed the possibility of the existence of carbon stars among the binary stars. Unfortunately, no real physical connection between the components of the analysed systems has been proved up to now. The determinations of the radial velocities for 283 carbon stars by Sanford (1944) have shown that most of the carbon stars possess radial velocities smaller than 70 km/s. The observational data concerning proper motions of carbon stars are very scarce. Dahn's extensive discussion (Dahn 1964) and the kinematic study of carbon stars by Dean (1972) have indicated N stars to be a kinematically flatter system than R stars.

The spatial distribution and kinematical properties of carbon stars were used for the determinations of statistical population indices (Iwanowska and Boenigk 1965, Boenigk and Iwanowska 1965). In Iwanowska's method of statistical population indices (Iwanowska 1966) each morphological group of stars (of equal mass and age) is considered to be composed of two population types according to the birth-place of the stars. The statistical population indices of carbon stars have indicated that more massive and cooler N stars are almost pure population I sample with the negligible admixture of population II individuals in the ratio 1:13; while for less massive R stars this ratio is 1:1.3.

The problem of spectral classification of carbon stars presents difficulties (Richer 1971, Eggen 1972, Scalo 1973), although it could seem that the two parametric Keenan-Morgan's classification had solved this problem definitely (Keenan and Morgan 1941). The common and the best defined effective temperature scale is the scale introduced by Mendoza and Johnson (1965) This temperature scale is based on the color indices, $(R+I)-(J+K)$, formed from measures centered around the wavelength of 1 μm, where approximately there is the maximum of energy distributions of these stars. It seems to be impossible to take the fluxes at longer wavelengths because of the existence of infrared excesses (Woolf and Ney 1969, Gillett et. al. 1971) probably caused by the circumstellar solid particle clouds (Fix 1969, 1970, Gehrz and Woolf 1971). These problems will be considered in detail together with spectrophotometric gradients and carbon abundance classes in Section 4.1.

The abundance determinations as well as the estimations of atmospheric physical parameters are carried out mainly by the curve of growth method (Fujita and Tsuji 1965, Hirai 1969, Utsumi 1970). However, the first simple models of carbon star atmospheres are given by Alexander and Johnson (1972) and Querci *et al.* (1974). The theoretical investigations of the curve of growth (Yamashita and Unno 1963) have shown that this method is not adequate for cool stars because of line blending effects, the difficulties with the determination of the continuum, the non-linear source function and strong changes of the continuous absorption with wavelength. Hence the chemical composition and atmospheric parameters obtained for carbon stars seem to be rather rough estimates. The additional difficulty in all these investigations is the fact that most of the analysed carbon stars are variables. Unfortunately, examinations of the changes of intensities of atomic lines and molecular bands with light phase are missing. Such investigations are urgently needed since the scarcity of constant carbon stars bright enough for the high-dispersion spectroscopic observations has forced the astrophysicists to analyse bright variable carbon stars. The changes of the central depths of C_2 and CN molecular bands as well as the D_1D_2 unresolved sodium dublet with the light phase will be discussed in Section 4.2 of this paper for two semiregular carbon stars.

The present knowledge of absolute magnitudes, masses and physical parameters of carbon stars does not allow one to establish the evolutionary tracks and the origin of these stars conclusively. Therefore, it seems advisable to investigate the relation between the spatial distribution and kinematical properties of different groups of carbon stars, and their physical characteristics and chemical compositions. The correlations between statistical population indices based on stellar distances from the galactic plane and/or the radial velocities and spectroscopic features (central depths of sodium lines) will be shown in Section 4.3 for a sample of semiregular and irregular variable carbon stars.

2. Observations

The spectroscopic observations of 15 semiregular and irregular variable carbon stars have been carried out at the Toruń Observatory in the period 1966-1971. The investigated stars were chosen according to three parameters: brightness, spectral type and population type. The sample is not complete in every respect because of the following reasons:

1. The population II carbon stars are mainly the hotter R stars. Moreover, the cooler carbon stars of population II (N type) are mostly constant stars. There is practically no carbon star of population II and spectral type later than C6 bright enough to be observed with the aid of the 60/90/180 cm Schmidt telescope and objective prism.

2. On the other hand, it was difficult to find bright enough variable carbon stars of population I of early R subtype because the early R stars are mainly constant. Finally, the sample of variable carbon stars of both population types possible for observations at the Toruń Observatory covered the range spectral subtypes from C2 to C6.

Some physical properties taken from the literature for 15 chosen semiregular and irregular carbon stars are given in Table 1. The consecutive columns contain the name of the star, HD number, spectral type in R-N, Keenan-Morgan (K-M) and Yamashita (*Vict*) classification systems respectively, type of variability (*Var*), the photovisual (*pv*) or photographic (*pg*) light amplitudes (Δm), the light period (P), radial velocity (v_r), statistical population index ($\lg(P_I/P_{II})$ derived from the distances to the galactic plane (z) and from these distances and radial velocities (z, v_r).

Table 1.
Physical properties of discussed carbon stars.

Name	HD	Spectral type			Var	Δm	P(days)	v_r(km/s)	$\lg P_I/P_{II}$		Pop
		R-N	K-M	Vict					(z)	(z,v_r)	
ST And	222241	R3e	-	C5:,3:	SRa	8.2-11.8pv	328.0	+ 32	+0.21	- 0.07	II:
UY And	16326	N3	-	-	Lb	10.1-10.9pv	149.0	- 63	-1.34	- 2.02	II!
VY And	-	R8	C3,4	-	SRb	9.6-11.4pv	149	- 7	+0.26	+ 1.49	I
V Ari	13826	R8	C5,5	C4,4CH	SRb	9.8-10.8pg	77:	-176	+0.33	-29.6	II!
RV Aur	46321	Na	-	-	SRb	11.8-13.1pg	229	- 52	-0.22	- 1.42	II
S Cam	36972	R8e	-	C7,3	SRa	8.1-11.0pv	326.4	- 13	+0.67	+ 1.61	I
UV Cam	25408	R8	C5,3	C5,4	SR?	9.3- 9.9pg	294	- 10	+0.41	+ 1.36	I
TT CVn	112869	R6p	Cp,5	C3:,5	Lb	9.2- 9.9pv	-	-135	-0.74	-21.00	II!
UX Cas	-	R2	-	-	SRb	12.0-13.8pg	360	- 11	+0.46	+ 1.57	I
SV Cyg	191738	N3	C5,5	-	Lb?	11.7-13.2pg	-	- 8	+0.37	+ 1.16	I
SY Eri	33404	N0	C5,9	-	SRb	10.4-11.4pg	96:	+ 8	-0.19	+ 0.46	II:
BM Gem	57160	N	-	-	Lb	11.5-12.1pg	-	+ 98	-0.02	- 3.77	II!
W Sex	85319	Nbe	-	-	SR?	11.0-12.1pg	134.0	+ 59	-2.33	- 2.41	II!
TT Tau	30755	N3	C5,2	C4:,3	SRb	8.1- 8.8pv	166.5	+ 20	+0.92	+ 1.57	I!
TU Tau	38218	N3	-	C4:,3	SR?	11.1-12.5pg	190:	- 24	+1.13	+ 1.26	I!

Objective prism spectra of these stars were taken with two telescopes at different dispersions and ranges of wavelengths, as follows:

1. few spectra with the aid of 30/35/75 cm Schmidt camera on Eastman Kodak 103a-F plates. The dispersion of spectra was about 300Å/mm at H_γ.

2. most of the 60/90/180 cm Zeiss Schmidt telescope spectra mainly on Kodak IIa-F plates, occasionally on Kodak IIa-O plates (dispersion about 250 Å/mm at H_γ) and on Kodak I-N plates (dispersion about 1100 Å/mm at telluric band A(O_2) 7615Å).

One, two or three spectra have been made on each plate with the ratio of the exposure times equal to 1:3. Such a large ratio of exposure times results from the steep energy distribution for carbon stars. The occurrence of strong continuous absorption at shorter wavelengths has limited measurements of the energy distributions to the wavelengths longer than 4500 Å for the coolest stars and longer than 4300 Å for the hotter ones. Hence the Kodak IIa-O plates were of little use. There are a few CN molecular bands and telluric H_2O and O_2 bands visible only on the Kodak I-N plates because of the low dispersion and steep instrumental sensivity curve. Thus, the investigations are mainly based on Kodak IIa-F plates covering the wavelength interval from 6600 Å to 4300 Å (or 4500 Å). Two or three plates were made every night for each star. Oke's spectrophotometric standards (Oke 1964) were exposed on the same plates in the best weather conditions. The spectra were widened up to about 0.8-1.0 mm and were calibrated with the aid of a tube sensitometer with two color filters. Two characteristic curves were made for the blue ($\lambda < 5000$ Å) and the red

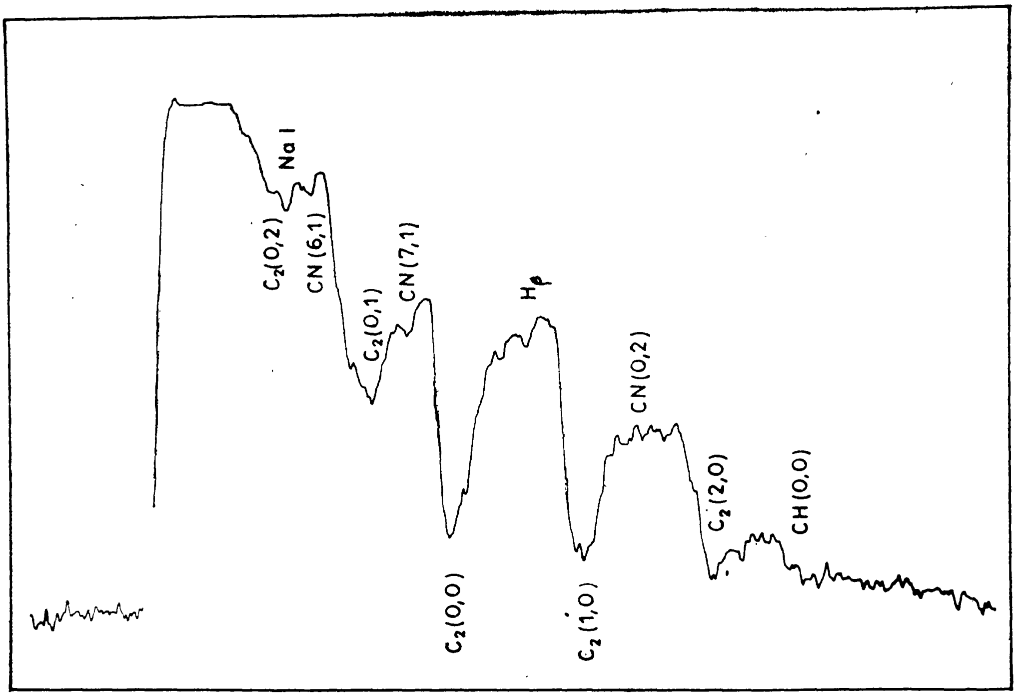

Fig. 1. Microphotometric density tracing of V Ari spectrum taken on a Kodak IIa-F plate with the 60/90/180 cm Schmidt telescope and objective prism.

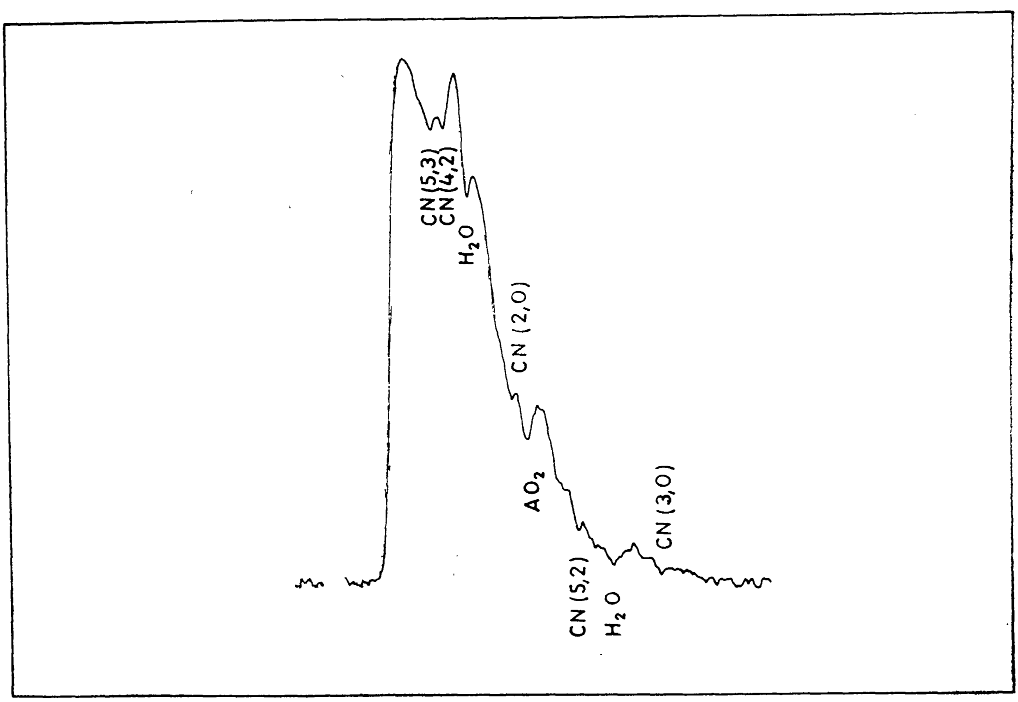

Fig. 2. Microphotometric density tracing of the S Cam spectrum taken on a Kodak I-N plate.

($\lambda > 5000$ Å) regions of spectrum. The spectra were recorded with the Zeiss microphotometer. The microphotometric density tracings of carbon stars are shown in Figs. 1 and 2.

Simultaneously, the photovisual photometry was carried out with the 8-inch Draper astrograph on Eastman Kodak IIa-F plates using the yellow filter Ilford 108 (Krempeć 1973). The light curves were derived from these exposures enabling us to discuss the changes of molecular bands and atomic lines with the light phase.

3. Reductions

3.1. Identification of Molecular Bands and Atomic Lines and the Selection of the Points of Pseudocontinuum.

Strong atomic lines and molecular bands were identified with the aid of identification tables given by Keenan and Morgan (1941) for the range of wavelengths from 3360 Å to 6500 Å or tables given by Nassau and Colacevich (1950) for the range 6880-8700 Å. These tables are suitable for low dispersion spectra. Moreover the "*Atlas des Longueurs d'Onde Caracteristiques des Bandes d'Emission et d'Absorption des Moleceules Diatomiques*" by Barrow *et al.* and "*The Red System* ($A^2\Pi - X^2\Sigma$) *of the CN Molecule*" by Davis and Phillips tables were used. The spectra of carbon stars are very complicated and heavily blended in low dispersion. C_2 bands of the Swan system dominate the visual region. The most conspicuous spectroscopic features are shown in Figs. 1 and 2. It is impossible to measure the true continuum of carbon stars even on high dispersion spectra because of intrinsic blends of lines. The continuous absorption probably caused by C_3 and SiC_2 (Fujita 1966) or by CN^- (Wyckoff 1970) is added to the saturation effect in the photographic and ultraviolet spectral regions. Additional difficulties arose in the course of selection of the pseudocontinuum:

1. It was not possible to find common points of pseudocontinuum for all discussed carbon stars (Fujita *et al.* 1965) and it was necessary to choose the points of pseudocontinuum separately for every star.

2. Due to the variability of the investigated stars, the changes of atomic lines and molecular bands could eliminate some points of pseudocontinuum.

Finally, it was decided to choose the points of pseudocontinuum for each carbon star on the basis of all the "absolute" energy distributions (m_λ) as the highest points in these diagrams. The pseudocontinuum of S Cam is shown as the solid line in Fig. 3. All the possible points of pseudocontinuum in the 8700 – 4200 Å region are given in Table 2. Common points of pseudocontinuum for all the investigated carbon stars have been established in the range of wavelengths from 8700 to 6900 Å (Kodak I-N plates).

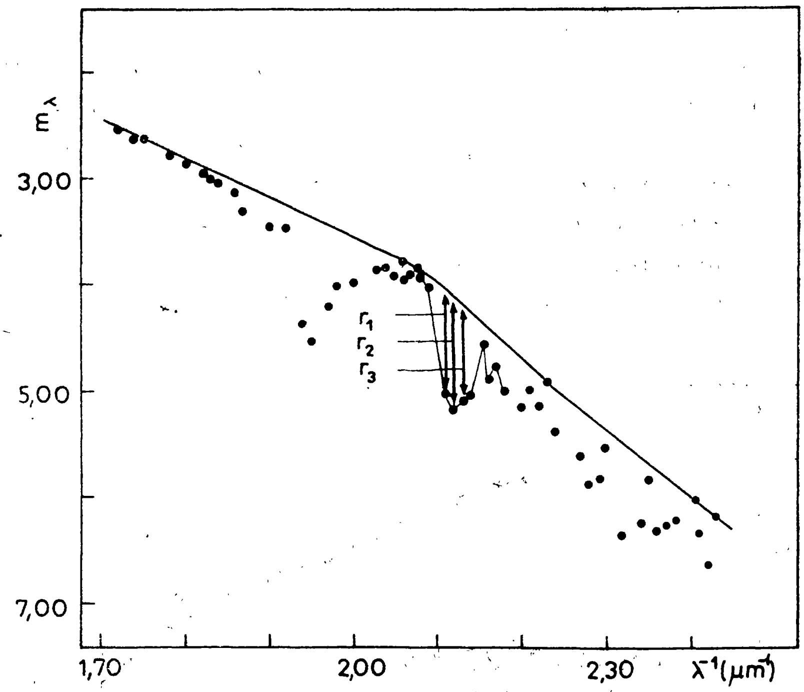

Fig. 3. Energy distribution of S Cam in magnitude scale. Pseudocontinuum is given as the solid line.

In parentheses (Table 2) are given the numbers of carbon stars, for which the same points of pseudocontinuum were chosen for the 6600–4200 Å spectral range (Kodak IIa-F and IIa-O plates).

Table 2.

Wavelengths of the energy peaks adopted as the points of pseudocontinuum for carbon stars.

λ^{-1} (μm^{-1})	λ (Å)	λ^{-1} (μm^{-1})	λ (Å)	λ^{-1} (μm^{-1})	λ (Å)
1.15	8647 ± 10	1.57	6372 ± 6(2)	2.06	4843 ± 3(1)
1.19	8404 8	1.59	6275 4(4)	2.07	4830 6(4)
1.21	8275 7	1.64	6092 10(1)	2.08	4803 10(9)
1.22	8200 8	1.68	5952 10(1)	2.09	4794 8(2)
1.28	7811 5	1.71	5864 9(1)	2.15	4656 10(1)
1.32	7592 7	1.72	5817 8(4)	2.20	4550 9(2)
1.37	7284 6	1.73	5780 8(1)	2.22	4501 5(1)
1.39	7200 9	1.74	5734 7(1)	2.23	4490 4(2)
1.42	7054 8	1.75	5716 6(9)	2.24	4462 5(1)
1.43	6976 10	1.88	5319 5(2)	2.25	4450 5(5)
1.44	6948 10	1.91	5236 6(1)	2.27	4408 6(1)
1.50	6670 5(7)	1.98	5050 6(2)	2.32	4305 6(3)
1.56	6426 ± 2(8)	2.03	4920 ± 4(2)	2.34	4273 ± 5(1)

3.2. The "Absolute" Energy Distributions and Spectrophotometric Gradients.

Firstly, the "absolute" energy distributions m_λ^p (*abs*) over the selected wavelengths were determined for comparison stars from intercomparison with the spectrophotometric standards: αLyr, ξ^2Cet, αLeo, 109 Vir, and 29 Psc, whose energy distributions were taken from Oke (1964). All measurements were corrected for differential atmospheric extinction with the aid of the tables given by Głębocki (1965). No corrections for interstellar reddening of comparison stars were applied. The corrections for the line blocking effect (ε_λ), as estimated by Smoliński (1970) from the parameter "Δ" were applied to the measured monochromatic magnitudes of comparison stars by addition of $\Delta m = 2.5\lg(1-\varepsilon_\lambda)$. As an example the "absolute" continuum of the comparison star HD 222032 (K5III) is drawn in Fig. 4 together with the energy distribution of ε Tau (K1III) taken from Oke

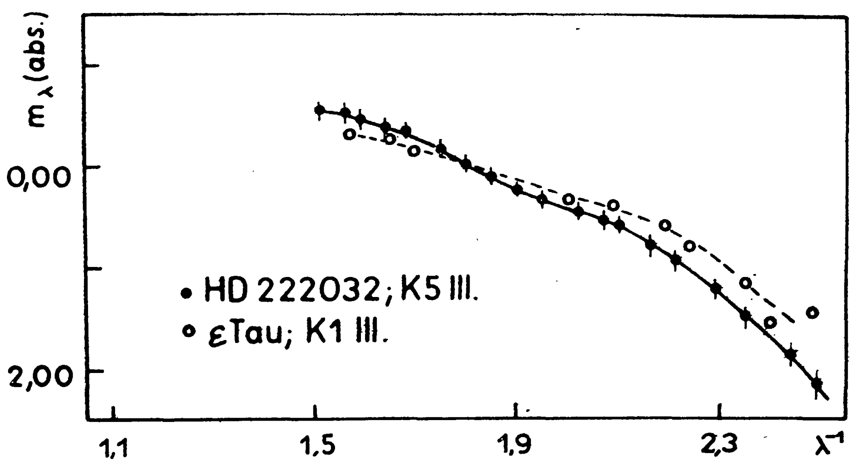

Fig. 4. Comparison of continuous energy distribution of the comparison star: HD 222032 — K5III (the solid line) with that of ε Tau — K1III (dashed line).

and Conti (1966). Usually, the monochromatic magnitudes (m_λ^p) of two comparison stars for each carbon star were used to derive the "absolute" energy distribution of carbon stars. The interstellar extinction in visual range A_v was calculated for carbon stars according to the Parenago formula with the values of parameters taken from Sharov (1963). Then the monochromatic interstellar extinction values A_λ have been calculated using the ratio A_λ/A_v given by Allen (1964). Finally, the following relation was applied to derive the "absolute" energy distributions of carbon stars $m_\lambda^c(abs)$

$$m_\lambda^c(abs) = (m_\lambda^c - m_\lambda^p) + m_\lambda^p(abs) - A_\lambda,$$

where m_λ^c, m_λ^p are observed monochromatic magnitudes of the carbon star and the comparison star respectively; $m_\lambda^p(abs)$ – "absolute" energy distribution of the comparison star. This procedure serves to eliminate the instrumental sensitivity effects as well as the interstellar extinction effects.

The estimated "absolute" energy distribution of S Cam (phase: $\Phi = 0^{\rm P}.74$) is plotted in Fig. 5 for the whole investigated range of wavelengths; the energy distribution of a carbon C6 star BL Ori as determined by Mendoza and Johnson (1965) is also drawn for comparison.

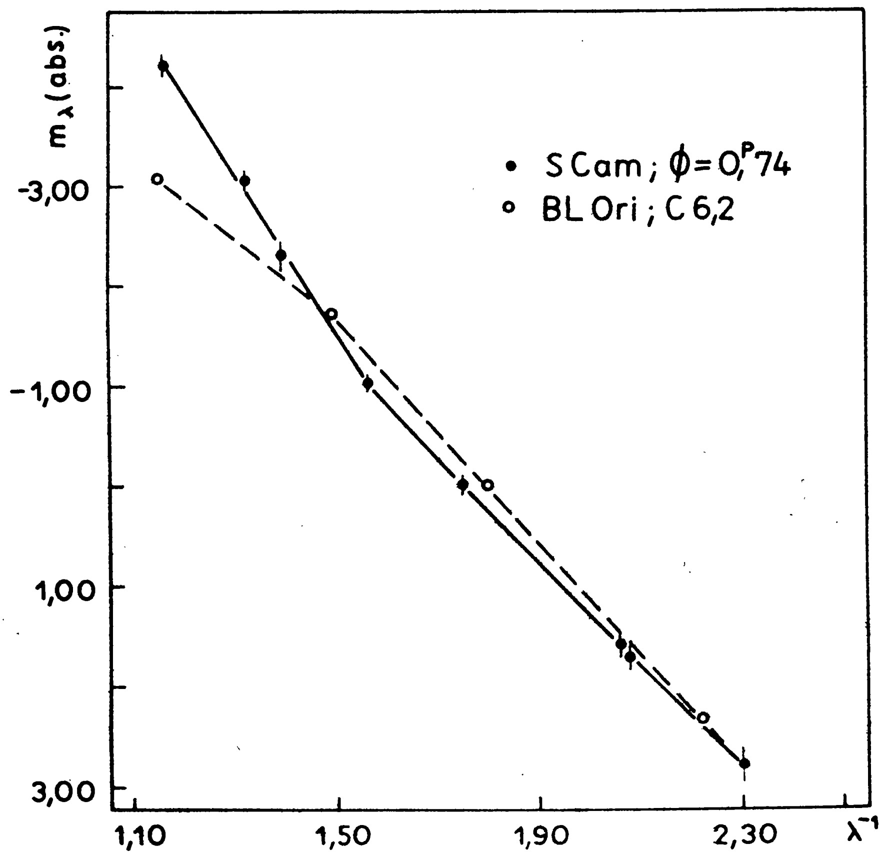

Fig. 5. Comparison of "absolute" pseudocontinuum of the carbon star S Cam for the light phase $\Phi = 0^{\rm P}.74$ (solid line) with energy distribution of the carbon star BL Ori (dashed line).

The relative spectrophotometric gradients were introduced to describe pseudocontinuous spectra of carbon stars for the range of wavelengths from 6400 to 4545 Å. They were obtained according to the formula: $\varphi = 0.921\ \Delta m / \Delta(\lambda^{-1})$ from the slopes of the straight lines drawn on the $(m_\lambda - \lambda^{-1})$ diagrams through the mean points in the range of λ^{-1} from 1.56 to 1.70 and from 2.07 to 2.25 as is shown in Fig. 6.

3.3. The Determination of the Central Depths of Molecular Bands.

The central depths of unresolved C_2 and CN molecular bands as well as the unresolved D_1D_2 sodium dublet blended with the CN (6, 1) band were determined relative to the pseudocontinuum. They have been estimated as the arithmetic means $(r_c = (r_1 + r_2 + r_3)/3)$ of the three strongest depressions (r_1, r_2, r_3) of the band (see Fig. 3). The weighted means of central depths were determined from two or three spectra for every night. Moreover, the weighted means of central depths $\bar{r}_c$ of (0, 1), (0, 0), (1, 0) C_2 bands of the Swan system were estimated. The number of spectra,

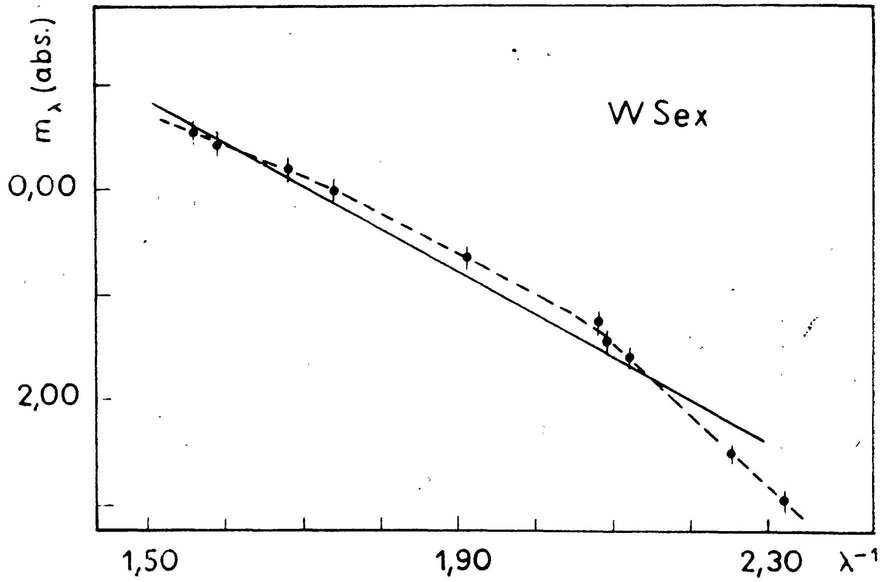

Fig. 6. The method of the determination of relative spectrophotometric gradients (solid line).

usually 2 or 3, were used as the weights. The errors of the weighted means have been calculated according to the formula:

$$\sigma = \left(\frac{\Sigma p_i v_i^2}{(N-1)\Sigma p_i}\right)^{1/2},$$

where p_i are the weights of individual observations, v_i represent the deviations of the individual measurements from the means, N is the number of observations.

3.4. Elimination of Interstellar Sodium Absorption.

Stellar sodium lines are affected by interstellar absorption. To eliminate this effect equivalent widths of interstellar sodium lines estimated for different regions of the sky were used. Early investigations of interstellar sodium lines were carried out among others by Merrill (1937) and Wilson and Merrill (1937). They have shown the existence of a linear relation between the equivalent widths of interstellar sodium lines (D(Å) $= (D_1+D_2)/2$) and the distance r(kpc). Allen (1964) gives the following mean relation: r(kpc) $= 2.0 \cdot D$(Å). Later investigations by Adams (1949) and Hobbs (1969) have indicated the composed profiles of interstellar sodium lines. There are usually a few components in these lines, corresponding to discrete clouds of the interstellar medium. Binnendijk (1952) has reduced all the measured equivalent widths of interstellar H and K CaII lines, D_1D_2 NaI lines as well as 4430 interstellar band to an uniform system. These investi-

Table 3.
Spectrophotometric data.

Date	Phase	$C_2(0,1)$	$C_2(0,0)$	$C_2(1,0)$	C_2	NaI	CN(7,1)	φ	Sp
ST And									
1971.01.30	0^p.08	0^m.57±0.06	0^m.56±0.06	0^m.52±0.08	0^m.55±0.07	0^m.33±0.05	0^m.42±0.07	2^m.72±0.15	C3
1971.02.02	0.09	0.61 0.06	0.72 0.07	-	-	0.31 0.04	0.39 0.06	2.85 0.12	C3
1965.09.23	0.17	0.62 0.06	0.66 0.06	0.65 0.07	0.64 0.06	-	-	3.05 0.15	C3
1965.10.22	0.25	0.75 0.06	0.78 0.07	0.73 0.08	0.75 0.07	-	-	3.20 0.20	C4
1970.08.05	0.55	1.79 0.06	1.87 0.06	1.34 0.07	1.73 0.06	0.82 0.06	0.98 0.07	4.10 0.15	C5
1970.08.12	0.57	1.74 0.06	1.73 0.07	1.42 0.08	1.63 0.07	0.75 0.05	0.94 0.06	4.92 0.15	C6
1969.10.05	0.64	1.38 0.07	1.52 0.07	1.26 0.08	1.42 0.08	0.72 0.08	0.74 0.08	4.95 0.20	C6
1968.12.12	0.73	1.26 0.07	1.26 0.06	1.06 0.06	1.19 0.06	0.52 0.07	0.59 0.06	4.75 0.20	C6
1968.12.14	0.74	-	-	-	-	0.49 0.06	0.49 0.08	3.60 0.22	C5
1971.10.25	0.94	1.08 0.07	0.75 0.06	0.52 0.06	0.84 0.07	0.31 0.08	0.50 0.05	2.20 0.15	C2
UY And									
1970.12.04		1.34 0.08	1.68 0.09	1.57 0.08	1.53 0.08	0.39 0.06	0.90 0.07	3.70 0.14	C5
1971.10.26		1.09 0.07	1.24 0.08	0.72 0.09	1.02 0.08	0.39 0.05	0.82 0.04	2.95 0.21	C3
1971.11.11		1.14 0.07	1.67 0.08	0.97 0.09	1.26 0.08	0.41 0.04	0.81 0.04	3.42 0.16	C5
VY And									
1968.12.15		0.89 0.06	1.30 0.08	0.90 0.09	1.00 0.08	0.55 0.06	0.56 0.07	3.81 0.16	C5
1969.10.12		0.98 0.05	1.33 0.07	1.06 0.07	1.06 0.09	0.58 0.07	0.65 0.08	3.68 0.21	C5
1970.08.05		1.00 0.06	1.35 0.08	1.04 0.07	1.13 0.07	0.60 0.05	0.68 0.04	3.78 0.19	C5
V Ari									C6
1968.12.11	0.41	1.44 0.05	1.70 0.06	1.61 0.07	1.58 0.06	0.62 0.06	0.55 0.05	4.78 0.12	C6
1971.10.25	0.49	1.48 0.07	1.78 0.07	1.67 0.10	1.64 0.08	0.67 0.09	0.52 0.09	4.79 0.10	C6
RV Aur									
1966.02.17		1.09 0.06	1.46 0.09	1.13 0.08	1.23 0.08	0.21 0.07	0.52 0.09	3.32 0.14	C4
1971.03.20		1.30 0.05	1.69 0.06	1.66 0.07	1.55 0.06	0.49 0.06	0.74 0.04	4.06 0.22	C5
S Cam									
1971.04.19	0.02	0.93 0.04	1.35 0.05	0.98 0.05	1.09 0.05	0.50 0.06	0.66 0.04	4.30 0.20	C6
1971.04.28	0.05	0.74 0.07	1.52 0.06	1.01 0.06	1.09 0.06	-	0.69 0.04	4.30 0.17	C6
1971.04.30	0.06	0.80 0.05	1.27 0.06	0.74 0.08	0.94 0.06	0.54 0.07	0.72 0.05	4.44 0.19	C6
1971.05.17	0.11	0.96 0.06	1.30 0.07	0.99 0.08	1.08 0.07	0.55 0.05	0.70 0.07	4.33 0.18	C6
1969.10.12	0.32	0.84 0.05	0.96 0.06	-	-	0.65 0.06	0.62 0.06	5.31 0.20	C7
1968.12.11	0.33	0.88 0.07	1.73 0.10	0.84 0.06	1.15 0.08	0.80 0.07	0.59 0.06	5.31 0.20	C7
1968.02.20	0.35	0.87 0.06	1.43 0.04	-	-	0.60 0.08	0.59 0.06	5.37 0.20	C7
1970.09.22	0.39	0.78 0.07	-	-	-	0.90 0.06	0.87 0.06	5.63 0.15	C8
1967.04.11	0.53	0.91 0.05	1.03 0.08	-	-	0.89 0.05	0.90 0.08	6.67 0.16	C9
1971.10.25	0.60	0.70 0.05	1.38 0.08	0.67 0.05	0.92 0.06	0.88 0.05	0.68 0.07	5.85 0.15	C8
1971.11.10	0.65	-	-	-	-	0.78 0.07	0.58 0.06	5.88 0.17	C8
1971.12.08	0.74	0.75 0.03	1.38 0.06	1.16 0.06	1.10 0.05	0.56 0.06	0.56 0.08	4.92 0.19	C7
1972.01.20	0.87	0.96 0.03	1.20 0.06	0.92 0.05	1.03 0.05	0.52 0.07	0.66 0.04	4.53 0.18	C6
1971.03.20	0.93	0.80 0.07	1.65 0.05	1.11 0.09	1.19 0.07	0.50 0.04	0.80 0.03	4.31 0.16	C6
1971.03.23	0.94	0.84 0.05	1.53 0.08	1.19 0.08	1.19 0.07	-	0.73 0.05	4.39 0.21	C6
1971.03.31	0^p.96	0.81 0.03	1.51 0.06	1.30 0.08	1.21 0.06	0.46 0.06	0.58 0.10	4.33 0.18	C6
UV Cam									
1965.02.02		0.88 0.06	1.44 0.06	1.07 0.07	1.13 0.06	0.27 0.05	0.35 0.07	3.04 0.12	C5
1966.02.17		0.74 0.05	1.05 0.07	1.27 0.09	1.02 0.07	0.27 0.06	0.26 0.04	1.09 0.20	C1
1968.12.11		0.90 0.06	1.44 0.07	1.01 0.08	1.12 0.07	0.22 0.08	0.46 0.05	1.64 0.18	C2
1970.09.22		0.89 0.04	1.34 0.05	1.01 0.06	1.08 0.05	0.31 0.05	0.84 0.04	3.41 0.21	C6
TT CVn									
1966.02.16		0.84 0.05	0.88 0.06	0.75 0.07	0.82 0.06	-	-	1.77 0.22	C1
1966.04.16		0.59 0.04	0.56 0.05	0.54 0.05	0.56 0.05	-	-	1.32 0.19	C0
1966.04.22		0.79 0.03	0.72 0.05	0.78 0.04	0.76 0.04	-	-	1.69 0.17	C1
1968.12.12		0.60 0.06	0.56 0.07	0.45 0.08	0.54 0.07	0.25 0.06	0.32 0.07	1.48 0.25	C0
1970.06.09		0.80 0.04	0.86 0.06	0.80 0.05	0.82 0.05	0.42 0.05	0.64 0.06	2.30 0.18	C2
1970.08.05		1.02 0.05	1.07 0.07	1.04 0.08	1.04 0.07	0.55 0.06	0.67 0.06	3.24 0.15	C4
UX Cas									
1964.09.04		1.10 0.07	1.59 0.10	1.04 0.09	1.24 0.09	0.67 0.06	0.82 0.06	3.89 0.18	C5
1970.12.04		1.06 0.07	-	-	-	0.65 0.05	-	3.89 0.14	C5
SV Cyg									
1965.09.02		0.62 0.04	0.82 0.06	0.86 0.07	0.77 0.06	0.61 0.05	0.36 0.07	2.45 0.19	C2
1965.09.20		0.64 0.04	0.82 0.06	0.92 0.08	0.76 0.06	0.61 0.07	0.43 0.08	2.35 0.19	C2
1965.09.30		1.10 0.04	1.06 0.08	1.03 0.09	1.07 0.07	0.62 0.06	0.63 0.07	2.66 0.19	C3
1965.10.22		0.88 0.05	1.06 0.09	1.01 0.10	0.96 0.08	0.76 0.07	0.66 0.08	2.31 0.20	C2
1966.04.22		1.01 0.05	1.23 0.07	1.07 0.09	1.08 0.07	0.62 0.05	0.75 0.07	2.85 0.22	C3
1968.08.22		1.59 0.07	1.53 0.08	1.25 0.09	1.46 0.08	0.95 0.08	0.72 0.08	4.17 0.16	C5
1970.08.11		1^m.39±0.06	1^m.53±0.08	1^m.15±0.09	1^m.36±0.08	0^m.72±0.06	0^m.64±0.06	4^m.30±0.14	C6

Date	Phase	$C_2(0,1)$	$C_2(0,0)$	$C_2(1,0)$	C_2	NaI	CN(7,1)	φ	Sp
SY Eri									
1971.12.08	$0^p.50$	$0^m.84 \pm 0.06$	$1^m.27 \pm 0.07$	$1^m.35 \pm 0.07$	$1^m.00 \pm 0.07$	$0^m.19 \pm 0.06$	$0^m.64 \pm 0.05$	$3^m.42 \pm 0.14$	C5
1971.02.03	0.80	0.80 0.08	-	-	-	0.23 0.06	0.50 0.07	3.37 0.12	C5
1971.11.10		0.94 0.07	1.23 0.09	1.04 0.07	1.00 0.07	0.16 0.05	0.49 0.06	3.53 0.18	C5
BM Gem									
1966.02.18		0.96 0.06	1.73 0.09	1.30 0.08	1.33 0.08	0.25 0.06	0.48 0.08	3.23 0.25	C5
1966.03.11		1.44 0.07	1.67 0.08	1.06 0.08	1.39 0.08	0.28 0.05	0.50 0.04	3.27 0.22	C5
1968.12.12		0.98 0.05	1.63 0.07	1.35 0.10	1.32 0.08	0.46 0.05	0.78 0.07	3.34 0.14	C5
1 {1971.03.23, 1971.04.09		1.36 0.03	1.26 0.05	1.38 0.05	1.34 0.04	0.31 0.04	0.72 0.05	3.65 0.10	C5
1971.10.15		0.94 0.06	1.66 0.07	1.63 0.08	1.41 0.07	0.32 0.07	0.64 0.07	4.33 0.21	C6
1971.12.08		0.93 0.05	1.47 0.08	1.69 0.09	1.37 0.08	0.20 0.08	0.50 0.06	3.94 0.16	C5
W Sex									
1966.03.25		0.88 0.06	1.08 0.08	1.06 0.09	1.01 0.08	0.21 0.06	0.85 0.07	3.54 0.26	C5
1967.04.10		0.89 0.08	1.44 0.10	1.07 0.08	1.13 0.09	0.21 0.05	0.96 0.06	3.51 0.19	C5
1971.03.23		1.21 0.06	1.72 0.10	1.20 0.09	1.38 0.09	0.32 0.08	0.86 0.08	3.51 0.21	C5
TT Tau									
1969.10.07	0.36	0.88 0.10	0.97 0.10	1.16 0.05	1.04 0.09	0.45 0.07	0.47 0.06	3.48 0.18	C4
1971.01.15	0.50	0.96 0.07	1.37 0.09	1.42 0.08	1.25 0.08	0.50 0.05	0.60 0.04	3.68 0.15	C5
TU Tau									
1964.03.07		0.87 0.06	1.42 0.08	0.90 0.08	1.06 0.07	0.76 0.08	0.87 0.09	5.56 0.22	C8
2 {1966.02.17		0.92 0.05	1.36 0.05	0.98 0.08	1.09 0.06	0.73 0.06	0.68 0.07	6.70 0.25	C9
2 {1968.12.11		$1^m.34 \pm 0.07$	$1^m.41 \pm 0.08$	$1^m.14 \pm 0.10$	$1^m.30 \pm 0.09$	$0^m.84 \pm 0.06$	$1^m.10 \pm 0.08$	$6^m.50 \pm 0.24$	C9

1 - no changes of brightness

2 - the spectral type obtained from C_2 molecular bands is C5

gations were made for different regions of the sky and it was possible to obtain the changes of equivalent widths of interstellar sodium lines with the distance for the considered regions. Generally, the equivalent widths of interstellar sodium lines increase with the distance for all the investigated regions of the sky but there are some differences in the slopes of these relations. We have obtained the equivalent widths of interstellar sodium lines for the distances of carbon stars given by Ikaunieks (1952). Then we have transferred the estimated equivalent widths into the central depths using the empirical relation for the Toruń observational system (Zaleski 1968). Certainly it is a rough method since the relation between central depths and equivalent widths depends on the portion of the curve of growth on which measured equivalent widths of sodium lines have fallen. The estimated central depths of interstellar sodium lines are contained in the interval from $0^m.05$ to $0^m.11$. These corrections are taken off the obtained central depths of sodium lines in spectra of carbon stars.

All the measured spectrophotometric data are given for the analysed carbon stars in Table 3 and they will be discussed in the next Section.

4. Results and Discussion

4.1. Changes of the Spectral Features with the Spectrophotometric Gradients and the Determinations of the Carbon Abundance Classes.

The spectral classification of carbon stars is an open question up to now. A two parametric C classification introduced by Keenan and Morgan

(1941) and reexamined by Yamashita (1967, 1972) was criticized by Richer (1971), Eggen (1972) and Scalo (1973). Yet, the new Richer's classification system using the ionized triplet calcium lines (CaII $\lambda\lambda$ 8498,8543,8662 Å) fails when these lines appear in emission near the light maxima of many variables. On the other hand, the good agreement of changes of molecular band intensities along spectral subtypes with the theoretical consideration of molecular dissociative equilibrium (Tsuji 1964, 1972) suggests that Keenan-Morgan's C classification system is the best one at present. There is also a relation between the effective temperatures of carbon stars obtained from the differences of color indices $[(R+I)-(J+K)]$ (Mendoza and Johnson 1965) and C-subtypes.

In order to analyse the behaviour of spectral features with the increase of spectrophotometric gradients as well as to look for the population differences, the central depths of D_1D_2 sodium lines (NaI $\lambda\lambda$ 5890,5896 Å) and C_2 and CN molecular bands are plotted against the spectrophotometric grandients φ in Figs. 7-12. The spectral subtypes corresponding to the given

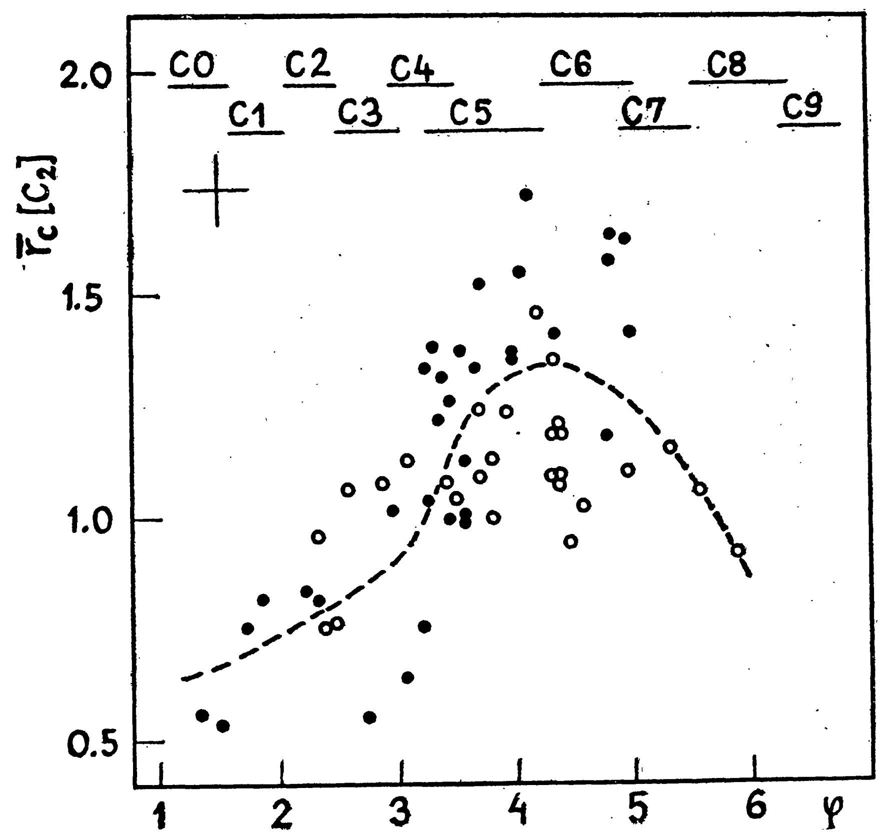

Fig. 7. Changes of the mean central depths ($\bar{r}_C$) of C_2 (0,1; 0, 0; 1, 0) molecular bands with the spectrophotometric gradients φ (except two last points for TU Tau). Open circles-population I carbon stars, dots-population II carbon stars. The ranges of spectrophotometric gradients for the particular spectral subtypes are drawn as the horizontal lines on the top of the figure. In the left upper corner the maximum errors of φ and r_C are shown as the big cross.

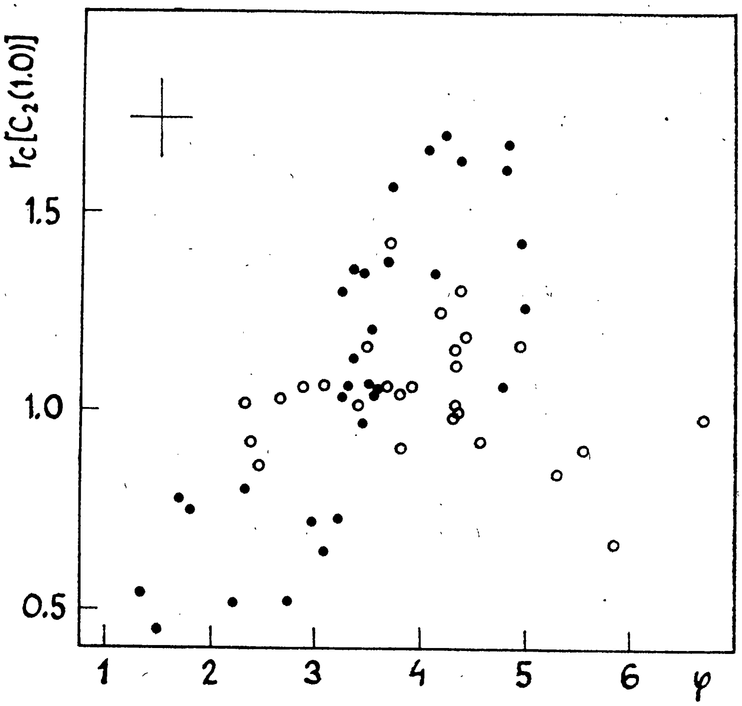

Fig. 8. Same as Fig. 7 for C_2 (1, 0) λ4737 Å.

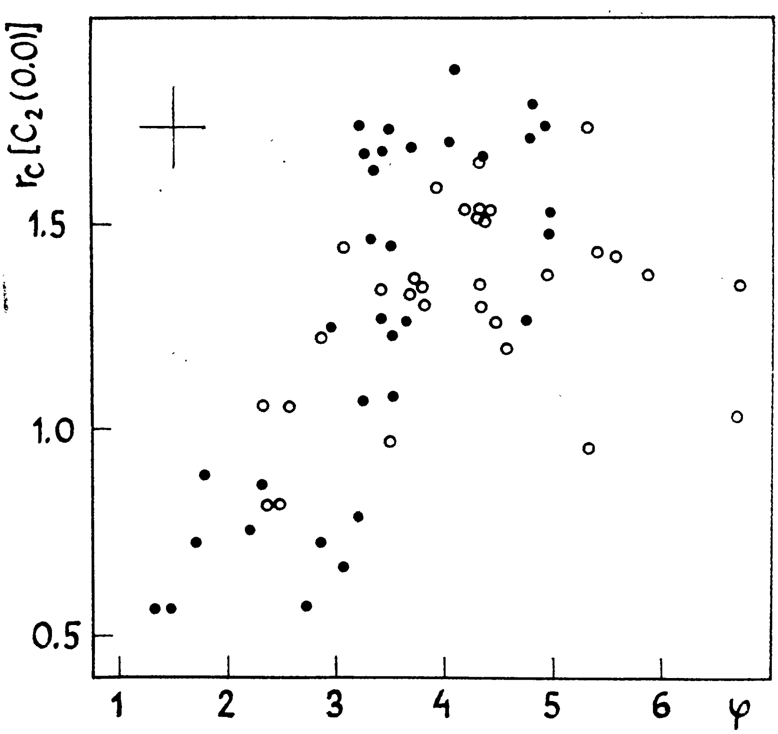

Fig. 9. Same as Fig. 7 for C_2 (0, 0) λ5165 Å.

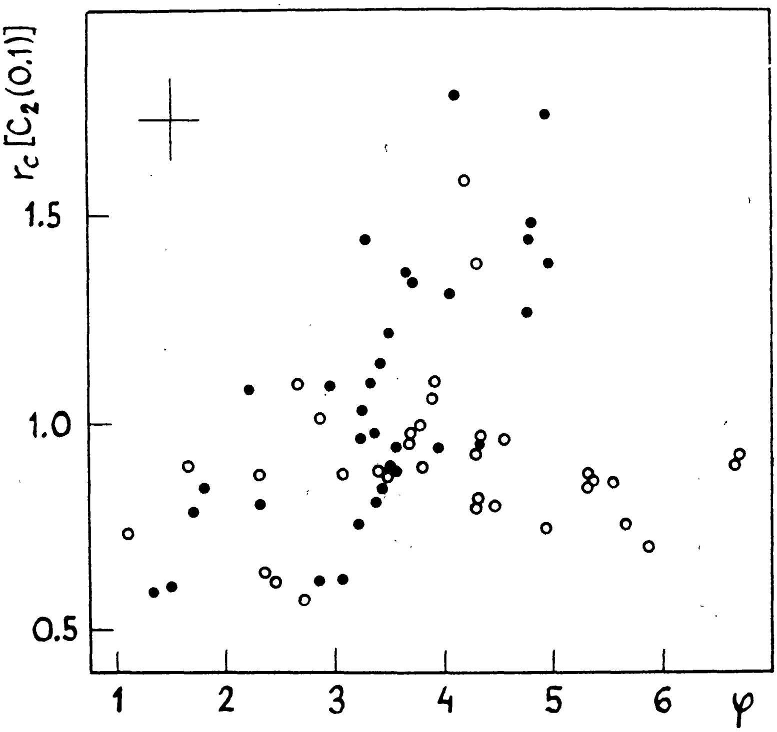

Fig. 10. Same as Fig. 7 for C_2 (0, 1) λ5635 Å.

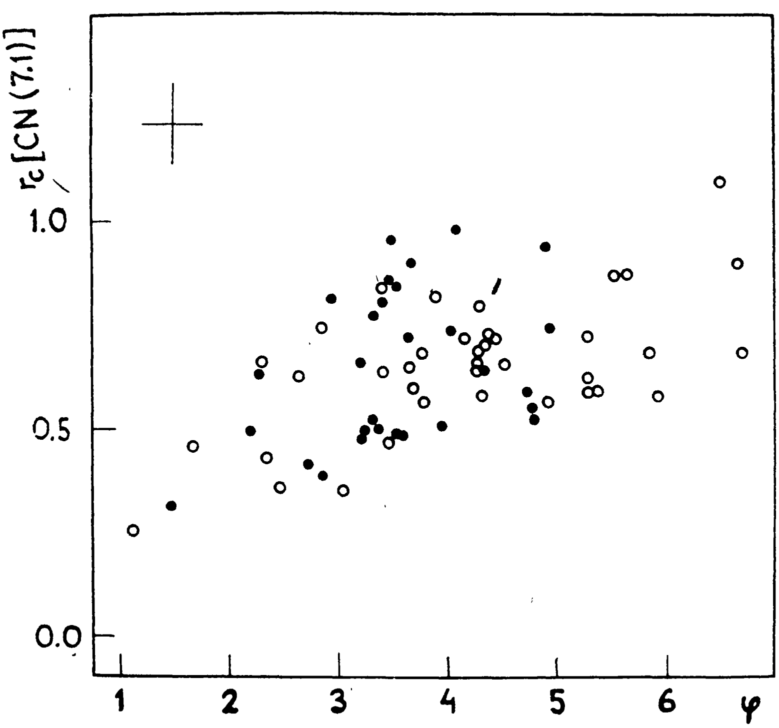

Fig. 11. Same as Fig. 7 for CN (7, 1) λ5239 Å.

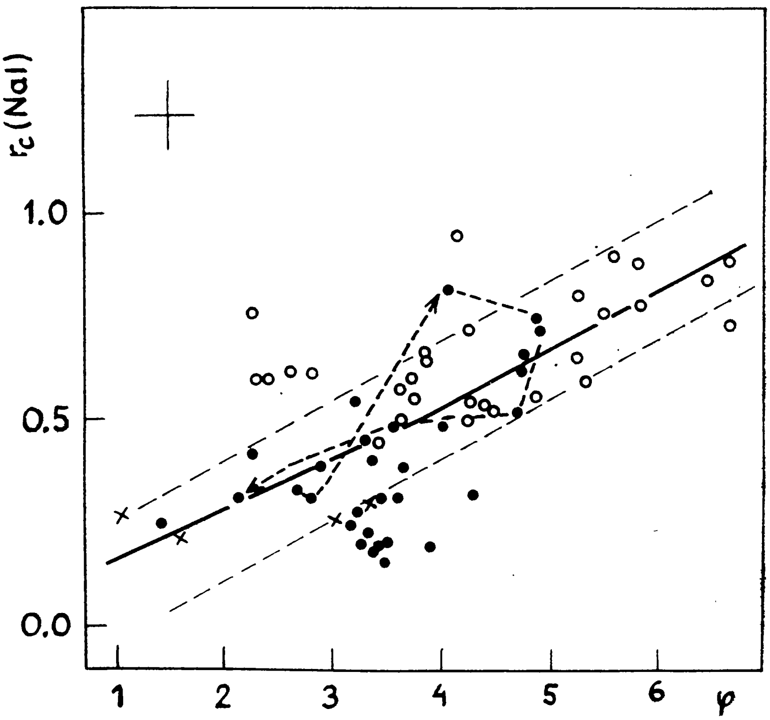

Fig. 12. Same as Fig. 7 for D_1D_2 sodium lines (NaI λλ5890, 5896 Å). Crosses — the data for UV Cam — a peculiar carbon star of population I. The changes of the central depths with light phase of ST And are shown by a dashed line with arrows.

ranges of spectrophotometric gradients are drawn schematically as the horizontal lines on the top of the figures. They are obtained from the Mendoza-Johnson relation between $(B-V)$ color and spectral C-subtypes, using in our material as $(B-V)$ color the differences between $m_\lambda^c(abs)$ at λ^{-1} equal to 1.80 and 2.22 respectively. This method can give systematically earlier spectral subtypes because of the partial elimination of molecular and line blanketing in our measurements relatively to broad-band photometry given by Mendoza and Johnson (1965). However, it is believed that even in those parts of the paper, where one has used the spectral subtypes these effects will not alter the basic conclusions. The inspection of Figs. 7-12 leads to the following remarks:

1. The central depths of all C_2 molecular bands rise up for spectrophotometric gradients between 3.5-5.0 (C4 — C5 spectral subtypes) and then decrease for the larger spectrophotometric gradients (or later C-subtypes, Figs. 7-10), while the central depths of the CN (7, 1) molecular band increase up to $\varphi \approx 4.0$ (C5 — C6 subtypes) and then remain nearly constant (Fig. 11). The behaviour of C_2 molecular bands agrees quite well with the theoretical considerations of molecular dissociative equilibrium made by Tsuji (1964, 1972). The decrease of the central depths of these bands for spectral subtypes later than C5 (for temperatures lower than

2800 K) is probably caused by the formation of multiatomic molecules such as HCN, C_2H, C_2H_2 etc. On the other hand, it is difficult to understand the behaviour of the CN (7, 1) molecular band in terms of dissociative equilibrium considerations. Wyckoff (1970) has obtained a similar result for this band. Moreover, Wing (1969) and Baumert (1970) have distinguished four groups of carbon stars having different CN intensities in the diagram of CN infrared index against temperature. This points to various nitrogen abundances.

2. There are no marked population differences for C_2 and CN molecular bands. However, there is a large scatter of central depths of C_2 and CN molecular bands within the same temperature class. This scatter has appeared for low dispersion measurements (Keenan and Morgan 1941) as well as for the higher dispersion ones (Yamashita 1967, 1972). It has forced Keenan and Morgan to introduce the second classification parameter measuring the carbon abundance. Furthermore, it is well known from Tsuji's theoretical considerations, that this large range of carbon and cyanogen band intensities for the same temperatures has a physical meaning and is explained by following factors.

(a) The variations of C/O ratios among stars of a given temperature range, which can strongly change the intensities of carbon molecular bands; a small deviation of C/O ratio from (1) can drastically change a star from an oxygen star into a carbon one.

(b) The differences in N/C ratios, even for the fixed relative abundances H:C:O. Unfortunately, two methods of the determinations of N/C ratios, namely one from the equations of dissociative equilibrium for the ratios of partial pressure $p(CN)/p(C_2)$ (Maron 1971) and another from fitting synthetic and observed spectra (Marenin 1970) give different N/C ratios for the same stars.

(c) The luminosity differences. The rise of luminosity leads to the decrease of the intensities of C_2 molecular bands for early C subtypes and to their increase for the later subtypes.

The influence of the differences in the C/O ratio on C_2 central depths is most important and we have tried to obtain carbon abundance classes therefrom. This has been done for the investigated variable carbon stars by taking the deviations of carbon central depths $\bar{r}_c(C_2)$ from the mean relation between these spectral features and spectrophotometric gradients (Fig. 7) derived from all discussed carbon stars. It is believed that the influence of the temperature on C_2 central depths is eliminated in such a way. We have tried to remove the effect of the differences of absolute magnitudes too. It was done by the separate consideration of irregular and semiregular, carbon stars, since different authors (Wilson 1939, Ishida 1960, Alksne

1969, Baumert 1974) have indicated that stars of the same type of variability have small dispersions in absolute magnitudes. We have introduced five carbon abundance classes for the following intervals of the deviations of C_2 central depths from the mean values (Fig. 7) for the given spectrophotometric gradients (given temperatures):

$$\text{class } 1: \ -0.5 \leqslant \bar{r}_c < -0.3,$$
$$2: \ -0.3 \leqslant \bar{r}_c < -0.1,$$
$$3: \ -0.1 \leqslant \bar{r}_c < +0.1,$$
$$4: \ +0.1 \leqslant \bar{r}_c < +0.3,$$
$$5: \ +0.3 \leqslant \bar{r}_c < +0.5.$$

The estimated carbon abundance classes are given in Table 4.

We have found that the influence of the differences of absolute magnitudes is rather small and the dispersion of C_2 central depths is mainly caused

Table 4.
Carbon abundance classes.

Name of star	Abundance class
Irregular variables	
UY And	3.0
TT CVn	2.8
SV Cyg	3.4
BM Gem	3.6
Semiregular variables	
ST And	2.8
VY And	1.7
V Ari	5.0
RV Aur	3.5
S Cam	2.5
UV Cam	3.0
UX Cas	3.0
SY Eri	2.0
W Sex	2.7
TT Tau	2.5
TU Tau	4.0

by the differences of C/O ratios for carbon stars. In the future, when C/O ratios will be estimated from high dispersion spectra for a greater number of carbon stars it will be possible to calibrate the relations between C/O ratios and carbon abundance classes. Then, it will be possible to obtain O/C ratios for carbon stars with known C_2 central depths.

3. Generally, the central depths of the unresolved D_1D_2 sodium dublet

corrected for the interstellar sodium absorption (Fig. 12) increase with the decrease of temperature (the region contained between broken parallel lines). However, most of population II stars have weak sodium lines compared to population I stars, except the peculiar carbon star of population I, UV Cam (crosses in Fig. 12) which has a weak sodium line. UV Cam has peculiar infrared color indices and is deficient in metals. Thus, one can suspect systematic population differences in the central depths of D_1D_2 sodium lines. But these lines can be strongly affected by the light changes and variations of luminosities of carbon stars. The changes of central depths of sodium lines with the light variations are shown schematically for ST And by the broken line with arrows in Fig. 12. These changes will be discussed in the next Section , while the population differences will be analysed in Section 4.3.

4.2. Changes of Spectral Features with the Light Phases for two Semiregular Carbon Stars: ST And and S Cam.

Two of the observed semiregular carbon stars, ST And and S Cam, behaved rather regularly in the examined interval of time and offer the possibility to discuss the variations of their spectral features with the light phases. Since ST And belongs to early spectral subtypes, while S Cam is the later subtype star, these stars will be examined separately. Irregular behaviour of other stars did not allow us to carry out such a discussion and in Table 3 we have given only the moments of observations for these stars.

4.2.1. ST And. The spectroscopic observations of ST And were carried out from 1965 to 1971. The simultaneous photovisual photometry (Krempeć 1973) was used to obtain the photovisual light curve (Fig. 13 — lower curve). We have assumed $0^{\mathrm{P}}.0$ and $1^{\mathrm{P}}.0$ phase for the light maximum. The changes of spectrophotometric gradients with lihgt phases are also shown in Fig. 13 as the upper curve with the marked corresponding spectral subtypes. The various signs refer to different light cycles. The small number of observations on the declining branch of the light curve is caused by the joint action of two factors, namely, the seasonal inaccessibility of this region of sky in the March-May period of each year and the 328 days' light period. The light curve of ST And is asymmetric and has a sharp and deep light minimum. The light changes for the individual cycles agree quite well except for two points near the light maximum of 1971 which show a brightening by $0^{\mathrm{m}}.5$. The photovisual amplitude (Δm_{pv}) is about $1^{\mathrm{m}}.70$. The curve of the variations of spectrophotometric gradients (φ) with light phases has a similar shape as the light curve and it is generally nearly identical for the particular light cycles. The spectrophotometric

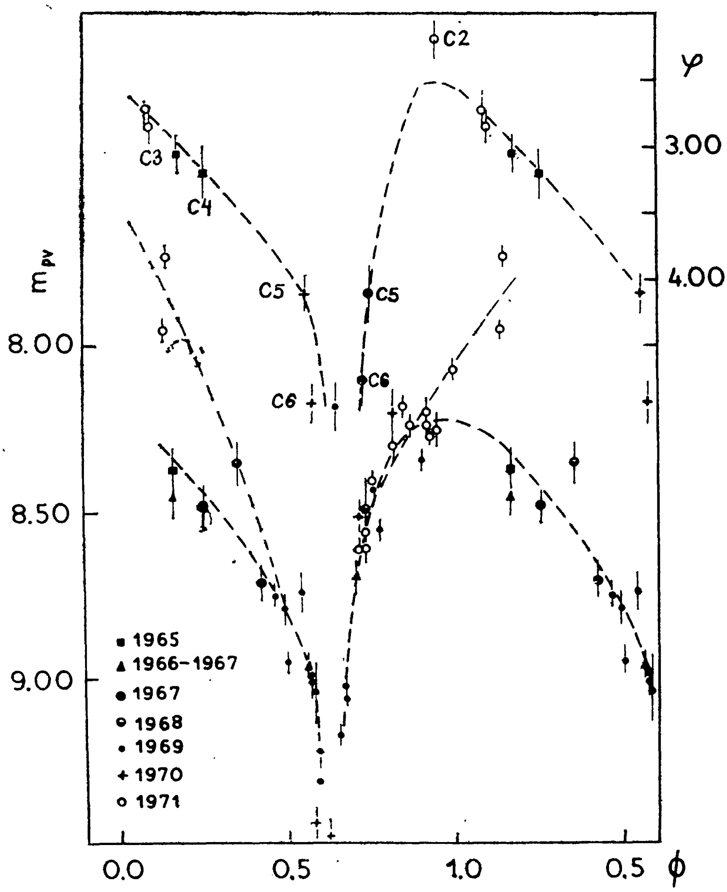

Fig. 13. Changes of spectrophotometric gradients with the light phase (top) and the photovisual light curve (bottom) for ST And.

gradient increases from 2.70 to 4.95 with the decrease of light and then decreases to the value of 2.20 with the rise of brightness. This change of spectrophotometric gradient of the order of 2.75 is associated with the change of spectral type from C2 (light maximum) to C6 (light minimum) and with the temperature decrease of about 700°K. The temperature difference has been estimated from the relation between the effective temperatures and C subtypes. The variations of the spectrophotometric gradients (stellar temperature) are connected with the changes of spectral lines and molecular bands (Fig. 14). In spite of using different light cycles, the changes of $C_2(0, 1)$, $(0, 0)$, $(1, 0)$ molecular bands are well determined by the two successive light cycles in 1970 and 1971. The mean central depth of all the analysed C_2 molecular bands has changed by about $1^m.30$, while the central depths of the CN (7, 1) band and the unresolved D_1D_2 sodium dublet have increased from the light maximum to the light minimum by about $0^m.60$ and $0^m.50$ respectively. The changes of the central depths of C_2 bands and sodium lines with the spectrophotometric gradients are shown

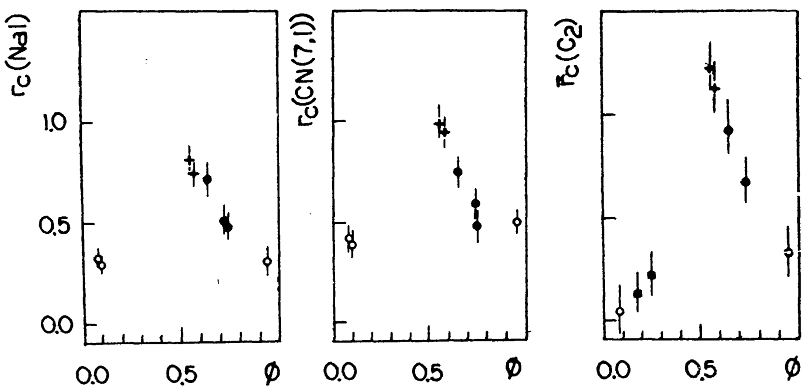

Fig. 14. Changes of the central depths of C_2(0, 1; 0, 0; 1, 0), D_1D_2 sodium lines and CN (7,1) with the light phase for ST And.

in Fig. 15. The values referring to successive light cycles in 1970 and 1971 are joined by a broken line. The arrows show the direction of phase changes. One can notice the different behaviour of C_2 bands as well as of the sodium line for the rising and declining branches of the light curve. The large scatter in these figures can be explained, at least partially by the various forms of the spectral changes with spectrophotometric gradient for different light cycles. However, there is a clear rise of central depths of the C_2 band

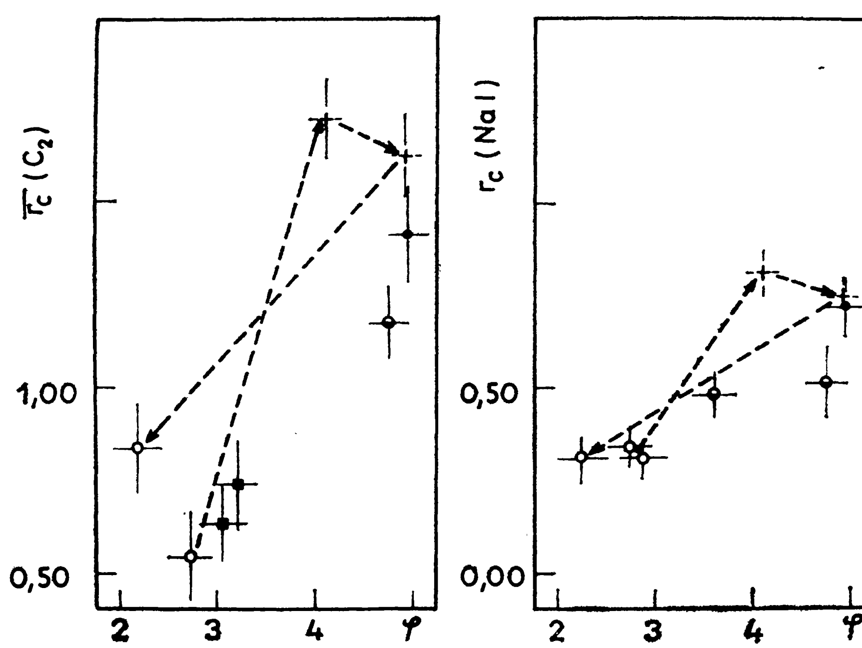

Fig. 15. Changes of the mean central depths of carbon molecular bands and sodium lines with the spectrophotometric gradients for ST And. The dashed line joins the points referring to light cycles in 1970 and 1971. The arrows show the direction of light changes.

as well as of sodium lines with the increase of the spectrophotometric gradient (with the decrease of temperature).

4.2.2. S Cam. The spectroscopic observations of S Cam were carried out from 1967 to 1972, but most of observations were made in 1970 and 1971. It is known from photovisual photometry of S Cam (Krempeć 1973), that the brightness of this star has changed in two different ways in the (1965-1971) time interval (Fig. 16 — lower curve). The photovisual amplitude in the (1965 — 1966) cycle and perhaps in the 1967 and 1969 cycles was above $1^m_.20$, while in the 1971 cycle and probably in the (1970-1971) cycle, it was smaller than $0^m_.90$. The change of spectrophotometric gradients (Fig. 16 — upper curve) from 4.50 at light maximum to 6.70 at light

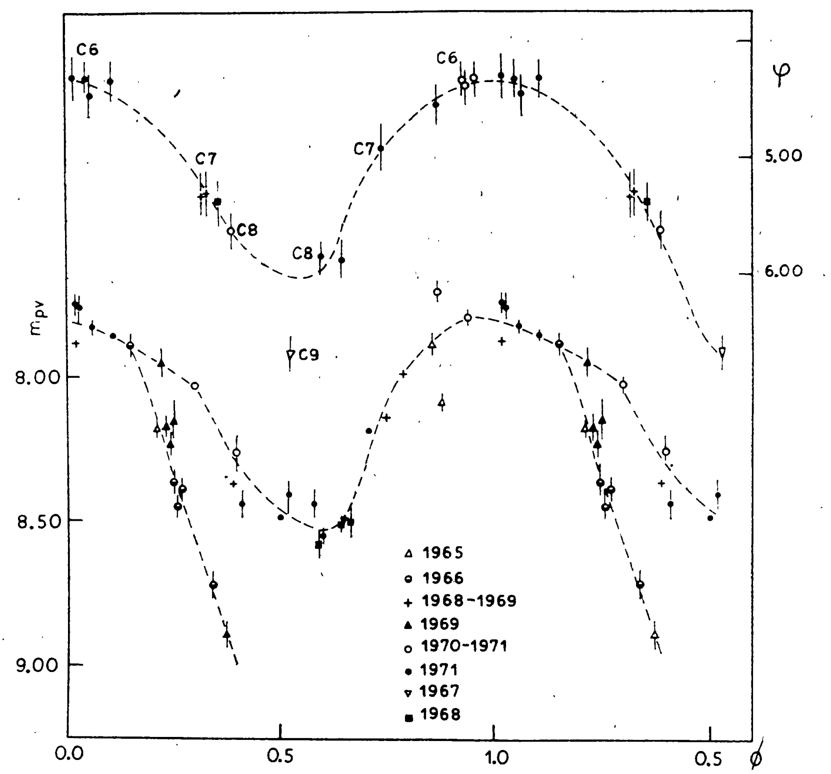

Fig. 16. Photovisual light curve (bottom) and the changes of spectrophotometric gradients (top) with the light phase of S Cam.

minimum corresponded to the variation of spectral type from C6 to C9 and to the decrease of the temperature by about 300°K. It was interesting to examine the behaviour of spectral characteristics for such late spectral subtypes (Fig. 17), where the continuous absorption may be very important. The change of the central depths of the sodium line is equal to $0^m_.45$. The direction of this change is consistent with the general increase of intensities

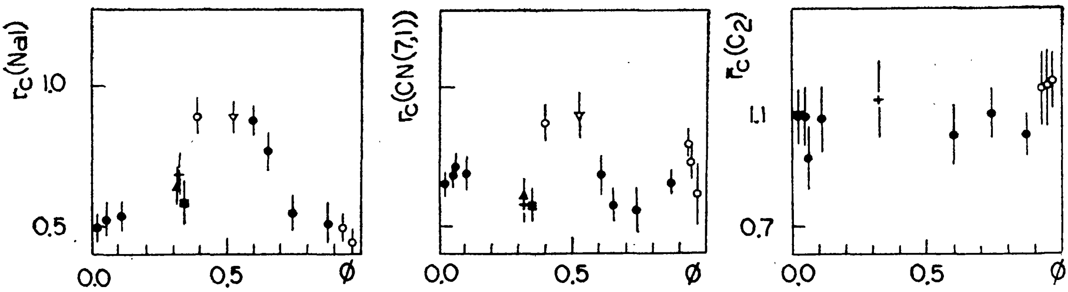

Fig. 17. Same as Fig. 14 for S Cam.

of sodium lines with the temperature decrease. C_2 as well as CN molecular bands do not behave typically. No regular changes of these spectral features with the light phases of S Cam can be traced and the scatter is very large. It seems that the scatter might be partly interpreted as the differences of central depths at the same light phases for different cycles and as the different effect of continuous absorption. The central depths of the sodium line increase also with the rise of the spectrophotometric gradient, while there is no marked relation between the central depths of C_2 bands and φ (Fig. 18).

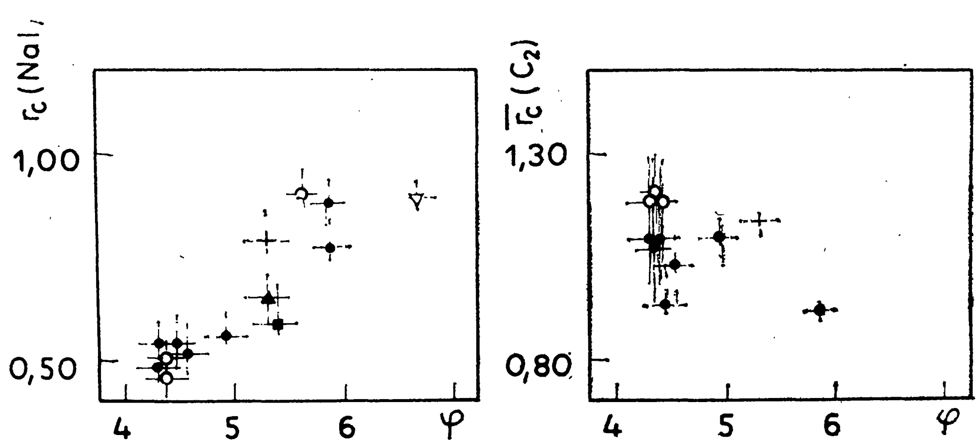

Fig. 18. Changes of central depths of sodium lines and C_2 molecular bands with the spectrophotometric gradients for S Cam.

The hydrogen emission lines of the Balmer series (H_β, H_γ, H_δ) have appeared during the (1970-1971) light maximum. It is difficult to discuss their central heights because of the low dispersion and blended spectra. The light maximum with hydrogen emission lines was followed by a flat minimum (Fig. 16). It is possible that the additional continuous emission caused the decrease of light amplitude.

4.2.3. Discussion. The measured photovisual magnitudes of late type stars are strongly affected by continuous absorption, and by molecular and atomic line blanketing, and they correspond to the outer atmospheric layers of late type stars. The Toruń photovisual system is the superposition of the sensitivity curve of Kodak IIa-F plates and the pass-band of the Ilford gelatine filter 108. Our photovisual magnitudes are mainly affected by $C_2(0,2)$ and $CN(4,0)$ molecular absorption and partially by $C_2(0,1)$ and $C_2(1,0)$ molecular bands. The influence of the molecular and line absorptions is smaller, of course, for the approximately monochromatic magnitudes measured from the spectra of carbon stars and therefore for the estimated energy distributions and spectrophotometric gradients. The determinations of these influences on the color indices and magnitudes of carbon stars are still lacking. This is closely connected with the poor understanding of the appearance and origin of continuous absorption in the U and B bands (partially also in V), the large range in C_2 molecular band intensities for a given temperature, the existence of many unidentified line absorptions in the spectra of these stars and the impossibility of obtaining the true continuum for these stars in high dispersion. As the result of these difficulties, it was decided not to introduce the "fictitious" corrections for molecular and line blanketing. Then the obtained changes of spectrophotometric gradients and of the photovisual magnitudes measure the changes of the color temperature of carbon stars but the spectrophotometric gradients, less affected by molecular and line blanketing, should measure the color temperature of a little deeper layers. The largest spectrophotometric gradients or the lowest color temperatures (latest spectral subtypes) appear at light minimum. The changes of spectral subtypes between the light minimum and light maximum are four and three subtypes for ST And S Cam respectively. It corresponds to the variations of color temperature by about 700°K for ST And and 300°K for S Cam, if we have neglected the influence of variations of line and molecular blanketing. The large changes of the temperature for the hotter carbon star, ST And, may be the result of the addition of the increasing C_2 absorption effects to the real changes of the color temperature. On the other hand, for the carbon star of later subtype, S Cam, C_2 band intensities diminish with the decrease of the temperature and the influence of C_2 molecular blanketing on spectrophotometric gradients is smaller.

The variations of spectral features with light phases are probably mainly caused by the temperature changes, although the continuous opacity as well as line and molecular blanketing can affect all the spectral features especially for late subtype carbon stars.

It seems necessary from the previous discussion to look at the light phases in all the investigations of variable carbon stars at least in photographic and photovisual spectral regions.

4.3. The Population Investigation of Carbon Stars.

The division of carbon stars into different population types according to their spatial distribution (distance to the galactic plane) and/or kinematical properties and the search for the relations between these properties and the chemical composition of the stars are very important for the investigations of the stellar state of the evolution of the Galaxy. In this Section we shall discuss in detail the spectral differences between carbon stars of populations I and II according to their statistical population indices (Iwanowska and Boenigk 1965, Boenigk and Iwanowska 1965). Such differences have already been analysed (Krempeć 1970) for the visually measured intensities of the molecular bands and atomic lines in slit spectra published by Yamashita (1967) for 73 constant and variable carbon stars. It was obtained that calcium (CaI $\lambda 4227$ Å), sodium (D_1D_2 NaI $\lambda\lambda 5890$, 5896 Å), and lithium (LiI $\lambda\lambda 6707.8$, 6707.9 Å) lines and the C^{13}/C^{12} ratio are stronger in population I carbon stars, while barium (BaII $\lambda\lambda$ 4554, 4934 Å) and hydrogen absorption (H_β, H_γ, H_δ) are stronger in carbon stars of population II. The CH/CN ratio was given as a population criterion.

The present material of objective prism spectra restricted the number of analysed spectral features to a few characteristics only, namely, C_2, CN molecular bands and D_1D_2 sodium lines. The "G" band, which was previously used as the parameter separating the population I and II carbon stars, was underexposed on the prismatic spectra of carbon stars except some few cases. It was possible to estimate the ratios of CH(O, O)$\lambda 4300$/CN (0, 1)$\lambda 4216$ central depths for two population II carbon stars and one population I carbon star only, as follows:

Star	TT CVn(C4, II)	V Ari(C6, II)	UV Cam(C6, I)
CH/CN	1.79	1.31	0.84

The investigated population I carbon star, UV Cam, has a lower CH/CN ratio than two extreme population II carbon stars in agreement with expectation.

The preliminary and rough analysis of the population differences in the central depths of C_2, CN bands and sodium lines was done in Section 4.1. One will possibly expect from this analysis the population differences for sodium lines only. However, all the spectral properties change mainly with the temperature, but they are also affected by the differences of the absolute magnitudes and the light phases. The intensities or central depths of stellar sodium lines can also be affected by interstellar sodium absorption. It is necessary to eliminate the effects of temperature, luminosity, light phases and interstellar sodium before the discussion of population differences. Interstellar sodium was taken into account by the method described in Section 3.3.

It was possible to eliminate all three factors, namely, temperature, absolute magnitudes (Richer 1971) and light phases for two semiregular carbon stars of different population types only. The measured central depths of C_2, CN bands and stellar sodium lines at the light minimum as well as population types for these two stars are given in Table 5. No population differences in central depths of C_2 and CN bands are visible for these stars

Table 5.
Spectral properties of carbon stars of different population types.

Name	Phase	Sp	M_V	Pop	NaI	CN(7,1)	C_2(0,1)	C_2(0,0)	C_2(1,0)	C_2
TT Tau	$0^{p}.5$	C5	$-2^{m}.7$	I!	$0^{m}.50 \pm 0.05$	$0^{m}.60 \pm 0.04$	$0^{m}.96 \pm 0.07$	$1^{m}.37 \pm 0.09$	$1^{m}.42 \pm 0.08$	$1^{m}.25 \pm 0.08$
SY Eri	0.5	C5	-2.7	II:	0.19 0.06	0.64 0.05	0.84 0.06	1.27 0.07	1.35 0.07	1.00 0.07
					Irregular carbon stars					
SV Cyg		C5		I	0.92 0.07	0.72 0.08	1.59 0.07	1.53 0.08	1.25 0.09	1.46 0.08
BM Gem		C5		II!	0.30 0.06	0.60 0.06	1.10 0.06	1.57 0.08	1.40 0.08	1.36 0.08
UY And		C5		II!	0.40 0.05	0.86 0.06	1.24 0.08	1.68 0.08	1.27 0.08	1.40 0.08
					Semiregular carbon stars					
SY Eri		C5		II:	0.19 0.06	0.54 0.06	0.86 0.07	1.25 0.08	1.20 0.07	1.10 0.08
W Sex		C5		II!	0.24 0.07	0.89 0.07	0.99 0.07	1.41 0.10	1.11 0.09	1.17 0.09
RV Aur		C5		II	0.49 0.06	0.74 0.04	1.30 0.05	1.69 0.06	1.66 0.07	1.55 0.06
VY And		C5		I	0.58 0.06	0.63 0.07	0.96 0.06	1.33 0.08	1.00 0.08	1.10 0.08
TT CVn		C5		II!	0.50 0.05	0.60 0.04	0.96 0.07	1.37 0.09	1.42 0.08	1.25 0.08
UX Cas		C5		I	0.66 0.06	0.82 0.06	1.08 0.07	1.59 0.10	1.04 0.09	1.21 0.09
TW Tau		C5		I!	$0^{m}.84 \pm 0.06$	$1^{m}.10 \pm 0.08$	$1^{m}.34 \pm 0.07$	$1^{m}.41 \pm 0.08$	$1^{m}.14 \pm 0.10$	$1^{m}.30 \pm 0.09$

but there is a difference between the central depths of sodium lines. TT Tau is an extreme population I carbon star and the central depth of its sodium lines is about $\Delta m = 0^{m}.31 \pm 0^{m}.08$ stonger than the central depth of sodium lines of the population II carbon star, SY Eri, which have an extremely large space velocity (357 ± 45 km s^{-1}).

Next we have considered separately semiregular and irregular carbon stars of the same spectral subtypes (the same temperatures) in order to eliminate the effects of the temperature and luminosity. Therefore we compared the spectral features of the irregular and semiregular (Table 5) carbon stars of different population types at randomly chosen light phases. It seems from the data of Table 5, that there are no significant population differences in the central depths of CN (7, 1) and the mean value of the C_2 Swan system bands, although there can exist small differences in the central depths of individual C_2 bands of the Swan system. However, there is again a marked population difference in the central depths of sodium lines. This difference is equal to $\Delta m = 0^{m}.57 \pm 0^{m}.08$ and $0^{m}.34 \pm 0^{m}.08$ for irregular and semiregular stars respectively in the sense that sodium lines are stronger in population I carbon stars.

Finally, we have made the correlational analysis and statistical discussion of the differences of the central depths of the sodium lines for semiregular stars (with the hope that the temperature and absolute magnitudes are eliminated) as well as for all the discussed stars. For the latter case we have eliminated the temperature only by taking the deviations from the mean relation between the central depths of sodium lines and the spectrophotometric gradients. The differences ($\mathscr{R}$) between the mean central depths (or deviations) of sodium lines of population I and II carbon stars have been calculated and are given in Table 6. We have also derived the correlation coefficients (r) between the central depths (or deviations) of sodium lines and statistical population indices obtained from

Table 6.

Population differences ($\mathscr{R}$) and correlation coefficients (r) for sodium lines in semiregular and all the discussed carbon stars.

Case	$\mathscr{R} \pm \sigma_{\mathscr{R}}$	$r \pm \varepsilon_r$	
		(z)	(z, v_r)
Semiregular stars	$0^m.35 \pm 0.11$	$+0.63 \pm 0.35$	$+0.51 \pm 0.39$
All stars	0.18 ± 0.06	$+0.38 \pm 0.27$	$+0.12 \pm 0.29$

the distances of the stars from the galactic plane ($r(z)$) and from the distances and stellar radial velocities ($r(z, v_r)$). The estimated correlation coefficients (Table 6) are positive in both cases but they are smaller in a case of statistical population indices based on the distribution and radial velocities of the stars. They are also lower for the case when all the stars are considered namely, when the temperature is eliminated only. Generally, the sodium lines are probably stronger in I population carbon stars more concentrated toward the galactic plane and probably more massive than in less concentrated and less massive population II carbon stars.

We have also looked for a low dispersion empirical criterion separating carbon stars of population I and population II. The investigations of the relations between the central depths of sodium lines and the nearest spectral features in the wavelength (in order to eliminate even partially the differences in the continuum level) give the best separation for the $C_2(0, 1)$ band (Fig. 19 — broken line). Each point in Fig. 19 is an individual measurement from Table 3. These measurements are corrected for the influence of the interstellar sodium lines only. The diagram allows one to assign the carbon stars into the appropriate population type on the basis of low dispersion spectra.

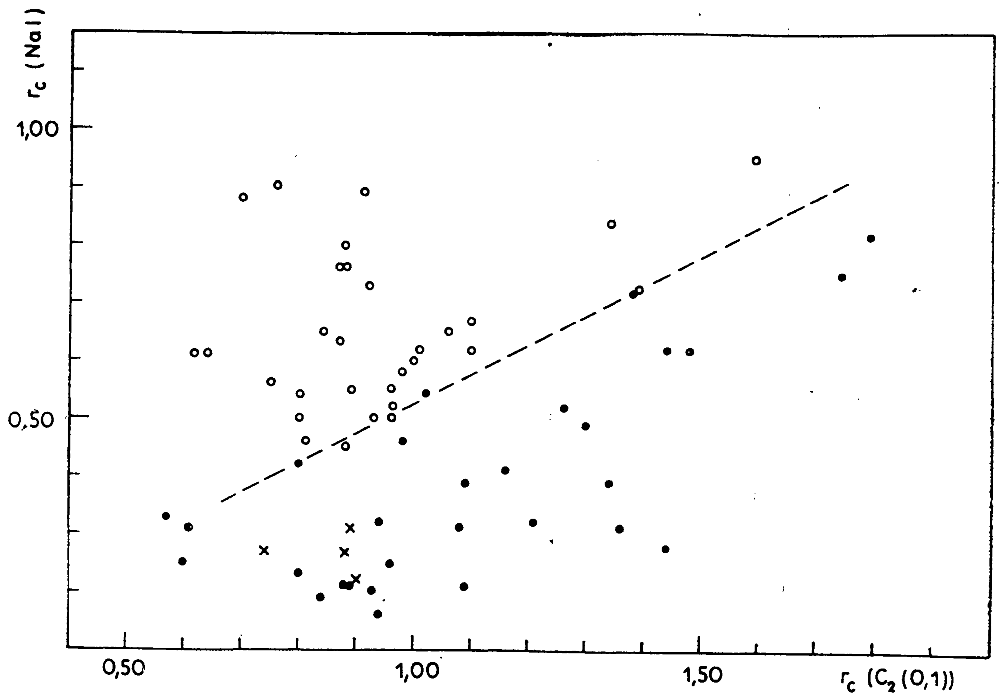

Fig. 19. Separation of carbon stars of different population types (broken line). Open circles — population I carbon stars, dots — population II carbon stars, crosses — UV Cam.

5. Conclusions

The principal results of the spectrophotometric investigations of 15 semiregular and irregular carbon stars are:

1. The carbon abundance classes based on the deviations of C_2 central depths from the mean relation $\bar{r}_c(C_2)-\varphi$ are introduced for all the investigated carbon stars.

2. There are considerable changes of $C_2(0, 1;\ 0, 0;\ 1, 0)$ and CN(7, 1) molecular bands as well as D_1D_2 sodium absorption lines with the light phases for variable carbon stars. As the variable star gets fainter during its light cycle, the absorption lines and molecular bands get stronger for the hotter carbon star. These changes can be mostly interpreted as a consequence of the temperature changes, although especially for cooler stars some other factors as formation of solid particles or luminosity may be important.

3. D_1D_2 sodium lines are stronger in population I carbon stars which are more concentrated to galactic plane than in less concentrated stars of population II. The diagram of the central depths of the $C_2(0, 1)$ mole-

cular band against the central depths of sodium lines (Fig. 19) is given as the population criterion for low dispersion investigations.

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