THE ASTROPHYSICAL JOURNAL, 176:1–19, 1972 August 15

ON THE INFALL OF MATTER INTO CLUSTERS OF GALAXIES AND SOME EFFECTS ON THEIR EVOLUTION*

JAMES E. GUNN
Hale Observatories, California Institute of Technology, Carnegie Institution of Washington

AND

J. RICHARD GOTT III
Princeton University Observatory
Received 1972 January 24

ABSTRACT

A theory of infall of material into clusters of galaxies is developed and applied to the Coma cluster. It is suggested that the infall phenomenon is responsible for the growth of cluster galaxies. The generation of a hot intracluster medium is discussed and its relation to the observed absence of normal spirals in rich clusters investigated. The inference made earlier by Gott and Gunn that the observed X-ray luminosity of Coma puts severe constraints on the deceleration parameter q_0 is further elucidated. We discuss the relation of these phenomena to the morphology of clusters, and find that some observed regularities in their observed properties can be explained.

I. INTRODUCTION

In this paper we develop the point of view investigated by many workers—most recently Peebles and Yu (1970), Silk (1968), and Field (1972), among others—that galaxies and clusters of galaxies develop from small density perturbations in the primeval medium which have survived until the time that the plasma recombines. Peebles (1970) has recently performed numerical computations for a model Coma cluster, and has obtained reasonable fits to the observations. We here make use of some of his results and consider some interesting aspects of cluster evolution *after* the initial collapse and "fast" relaxation.

In particular we treat the behavior of matter outside the main body of the perturbation that makes the initial cluster, which is bound to the cluster and eventually collapses into the cluster proper.

After treating the dynamics of the infalling matter, we turn to some astrophysical implications for clusters like Coma. We have already shown (Gott and Gunn 1971) that severe constraints can be placed on the gas density in intergalactic space with these results; we are here concerned mostly with effects in the cluster itself. Possible connections with the origin of supergiant D galaxies and the absence of ordinary spirals in regular clusters are discussed.

Recombination occurs reasonably quickly at an epoch corresponding to redshift $1 + z_i) \sim 1000$, or electron and radiation temperatures $\sim 3000°$ K (Peebles 1968). Let us suppose that there exists at this epoch a spherical region of radius R_i, which has uniform density slightly higher than the (uniform) density of the surrounding region. (One can almost as easily treat arbitrary spherical perturbations, but the results are qualitatively the same and this example is particularly transparent.)

The expansion is assumed still uniform at this epoch (we comment on this assumption later), so that the velocity $\boldsymbol{v}$ is given by

$$\boldsymbol{v} = H_i \boldsymbol{r} , \tag{1}$$

* Supported in part by the National Science Foundation [GP-28027, GP-27304].

where H_i is the Hubble parameter appropriate to the epoch z_i. Let ρ_{ci} be the *critical density* at this epoch:

$$\rho_{ci} = 3H_i^2/8\pi G . \tag{2}$$

For densities less than the critical density, the expansion is unbounded; the total energy at the critical density is just zero, and for densities in excess of ρ_{ci}, the material will expand to a maximum radius and collapse again. Let ρ_{ei} be the external density, which we assume to be representative of the universe, and let $\rho_{ci} + \rho_+$ be the density in the perturbation. (Note that the first assumption requires that such clusters as we are considering be a negligible contribution to the total mass.) Then in terms of the present Hubble parameter H_0, the present deceleration parameter q_0, and the epoch z_i, one can easily show that

$$\rho_{ei} = \rho_{ci} \frac{2(1 + z_i)q_0}{(1 - 2q_0) + 2q_0(1 + z_i)} \tag{3}$$

and

$$H_i^2 = H_0^2[2q_0(1 + z_i)^3 + (1 - 2q_0)(1 + z_i)^2]$$

for Friedmann models with vanishing pressure and cosmological constant. (See, for example, Sandage 1961.) Now q_0 is proportional to the density:

$$3H_0^2 q_0 = 4\pi G \rho_0 , \tag{4}$$

where the subscript zero always refers to the present. Oort (1958) has shown that the density in galaxies accounts for enough mass to make $q_0 \gtrsim 0.02$. Since $z_i \sim 10^3$, we may approximate the relations (3) with an accuracy of at least about a percent by

$$\rho_{ci} - \rho_{ei} = \rho_{ci} \frac{1 - 2q_0}{2q_0(1 + z_i)} , \quad H_i^2 = 2q_0 H_0^2 (1 + z_i)^3 . \tag{5}$$

In the following we shall continue to neglect terms of order $(1 + z_i)^{-1}$ in comparison to unity or $2q_0$; this is done primarily to simplify the presentation, and incurs no error larger than about a percent in any calculation. Note that $\rho_{ci} - \rho_{ei}$ is itself much smaller than ρ_{ci} in absolute value.

In the following we will restrict ourselves to discussion of perturbations of the chosen form; it is perhaps well at this point to discuss the effect of this choice. As mentioned above, our simple rectangular form gives qualitatively the same results as any *positive* spherical perturbation. The outcome can be changed radically only if outside the protocluster proper there is a shell with a large *negative* value of $\rho - \rho_{ci}$; and unless this quantity is large in absolute value compared to ρ^+, even here there is no substantial change. It is only in the case that each cluster is surrounded by a shell of very low density that the treatment we shall give is invalid; and since the picture we are using assumes random fluctuations, the situation seems *a priori* very unlikely.

II. DYNAMICS

If we let the radius of any shell of matter whose radius at the fiducial time t_i is r_i be

$$r(r_i, t) = r_i a(r_i, t) , \tag{6}$$

we easily find that $a(r_i, t)$ satisfies

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3a} \bar{\rho}_i(r_i) + \frac{8\pi G}{3} (\rho_{ci} - \bar{\rho}_i) , \tag{7}$$

where we assume only that matter is conserved and the pressure vanishes. It should be noted that equation (7) is correct relativistically, but that its relativistic correctness is of no concern here, and we can think entirely in terms of Newtonian mechanics. The quantity $\bar{\rho}_i(r_i)$ is the *average density* inside r_i at t_i; analytically,

$$\begin{aligned}\bar{\rho}_i &= \rho_{ei} + (\rho_{ci} + \rho_+ - \rho_{ei})R_i^3/r_i^3\,, & r_i > R_i\,;\\ &= \rho_{ci} + \rho_+\,, & r_i \leq R_i\,. \quad (8)\end{aligned}$$

These relations hold so long as the radii are small compared to the radius of curvature of the constant cosmic-time hypersurface; this condition is well satisfied for clusters of galaxies. Equation (7) can be written

$$\left(\frac{da}{dt}\right)^2 = H_i^2\left(\frac{\beta}{a} + \gamma\right), \quad (9)$$

where

$$\beta = \bar{\rho}_i/\rho_{ci}\,, \qquad \gamma = \frac{\rho_{ci} - \bar{\rho}_i}{\rho_{ci}}\,, \quad (10)$$

and has as solution the standard Friedmann relations

$$a = \frac{\beta}{2\gamma}[C_k(|\gamma|^{1/2}\theta) - 1]\,, \quad H_i t = \frac{\beta}{2\gamma|\gamma|^{1/2}}[S_k(|\gamma|^{1/2}\theta) - |\gamma|^{1/2}\theta] + H_i t_+\,, \quad (11)$$

where

$$d\theta = \frac{H_i dt}{a}\,, \quad k = -|\gamma|/\gamma\,, \quad C_k(\theta) = \begin{cases}\cos\theta,\ k = +1\\ \cosh\theta,\ k = -1\,,\end{cases}$$

$$S_k(\theta) = \begin{cases}\sin\theta,\ k = +1\\ \sinh\theta,\ k = -1\,,\end{cases} \quad (12)$$

and appropriate limits can be taken for the singular case $\gamma = 0$. It is clear that the correction t_+ in equation (11) is small; much smaller, in fact, than t_i, since it represents a departure in the backward-extrapolated behavior of the perturbation relative to the background. Since even t_i is negligible compared to the overall timescale, we will henceforth neglect t_+. (See, however, § V.) If one sets $a = 1$ in equation (11) ($t = t_i$), and makes use of the fact that γ is small to expand the trigonometric functions, it is found that

$$t_+ = \frac{2\gamma}{15H_i}\,, \qquad \theta_i = 2 - \tfrac{2}{3}\gamma\,, \quad (13)$$

to within terms of order γ^2. We return to a discussion of these matters in § V in connection with the correctness of the initial conditions.

We now notice an interesting point, which forms the basis of the first part of this paper. If the perturbation is bound (i.e., $\gamma < 0$, density inside *greater* than the critical density), it will, of course, eventually collapse to form a bound system. But the *mean* density *outside* the perturbation is greater than critical for some finite distance also (cf. eq. [8]), and the material here is bound to the cluster and will eventually fall in unless prevented from doing so by subsequent physical processes. (If $q_0 > \frac{1}{2}$, the *universe* is "bound," of course, and in this case the material in question will fall into the cluster before the final catastrophe. We will find here, as is invariably so, that no marked difference in behavior through the present epoch occurs as one increases q_0 past $\frac{1}{2}$.)

Let us thus confine our attention to those shells for which $\gamma < 0$, $k = +1$. For this material, the time of ultimate collapse clearly corresponds to $|\gamma|^{1/2}\theta = 2\pi$, or

$$t_c = \frac{\pi\beta}{H_i|\gamma|^{3/2}} = \frac{\pi\bar{\rho}_i\rho_{ci}^{1/2}}{H_i(\bar{\rho}_i - \rho_{ci})^{3/2}} \approx \frac{\pi\rho_{ci}^{3/2}}{H_i(\bar{\rho}_i - \rho_{ci})^{3/2}} \quad (14)$$

to lowest order in the perturbation. The time t_c is clearly a function of r_i, and we can find the derivative dt_c/dr_i straightforwardly, combining equations (8) and (14). But the mass in a shell at r_i is

$$dM = 4\pi\rho(r_i)r_i^2dr_i \approx 4\pi\rho_{ci}r_i^2dr_i\ ,$$

so the *infall rate* at any epoch later than the collapse time T_c for the body of the perturbation is

$$\frac{dM}{dt} \approx 4\pi\rho_{ci}r_i^2\left(\frac{dt_c}{dr_i}\right)^{-1}_{t_c=t}, \tag{15}$$

and we can find the cluster mass as a function of time by integrating equation (15). Let us perform this calculation for the model case. If we set $\bar{\rho}_i - \rho_{ci} = \rho_+$ for the region $r_i < R_i$, we obtain

$$T_c = \frac{\pi}{H_i}\frac{(\rho_{ci} + \rho_+)\rho_{ci}^{1/2}}{\rho_+^{3/2}} \approx \frac{\pi}{H_i}\left(\frac{\rho_{ci}}{\rho_+}\right)^{3/2}; \tag{16}$$

and, combining equations (16), (14), and (8), we find, for $r_i > R_i$,

$$\left(\frac{t_c}{T_c}\right)^{2/3} \approx \frac{\rho_+}{(\rho_{ei} - \rho_{ci}) + (R_i/r_i)^3(\rho_{ci} + \rho_+ - \rho_{ei})}\ . \tag{17}$$

Equation (17) is correct so long as ρ_+ and $\rho_{ei} - \rho_{ci}$ are small compared to ρ_{ci}. We will see subsequently that this approximation is excellent for all clusters of current interest. Now the mass interior to $\zeta = r_i/R_i$ is given by

$$M \approx \tfrac{4}{3}\pi R_i^3\rho_{ci}\zeta^3\ , \tag{18}$$

or, solving expression (17) for ζ^3 and setting $t_c = t$,

$$M(t) \approx \tfrac{4}{3}\pi R_i^3\frac{\rho_{ci}(\rho_{ci} - \rho_{ei} + \rho_+)(t/T_c)^{2/3}}{\rho_+ + (\rho_{ci} - \rho_{ei})(t/T_c)^{2/3}}\ . \tag{19}$$

Thus we obtain

$$\begin{aligned} M(t) &= M(T_c) + \tfrac{4}{3}\pi R_i^3\rho_+\rho_{ci}\frac{(t/T_c)^{2/3} - 1}{\rho_+ + (\rho_{ci} - \rho_{ei})(t/T_c)^{2/3}} \\ &= M(T_c)\left[1 + \rho_+\frac{(t/T_c)^{2/3} - 1}{\rho_+ + (\rho_{ci} - \rho_{ei})(t/T_c)^{2/3}}\right], \end{aligned} \tag{20}$$

where $M(T_c)$ is the mass at T_c, i.e., the mass of the density perturbation proper, the mass contained within R_i. Let $M(T_c) \equiv M_i$, the *initial cluster mass*.

Our next task is evidently to find the parameter ρ_+ in terms of observable characteristics of the cluster. This can be done only if we assume that the original energy of the cluster is conserved; we will do this, and return to its justification later.

If the cluster *now* is more than about 1.5 collapse times old, numerical calculations indicate that it will closely satisfy the virial theorem (Peebles 1970; Henon 1964). Thus the total energy

$$E \approx -\tfrac{1}{2}M_i\langle v^2\rangle \approx -GM_i^2/2R_g\ , \tag{21}$$

where $\langle v^2\rangle$ is derivable from radial-velocity data. It should be noted that in the times available, two-body relaxation has hardly begun, so $\langle v^2\rangle$ should *not* be a function of the masses of the galaxies. R_g is the gravitational radius, the mass-weighted harmonic mean of the distance between any two mass points in the cluster, and may be obtained from counts (Schwarzschild 1954) if the distance is known.

Computing the total energy at t_i is a straightforward exercise (cf. § III) and yields

$$E/M_i = -\tfrac{3}{10}H_i^2R_i^2\frac{\rho_+}{\rho_{ci}} = -GM_i/2R_g \tag{22}$$

for the initial perturbation itself. If we make use of relations (2) and (18), we find

$$M_i = \frac{H_i^2R_i^3}{2G}, \tag{23}$$

and, using equation (5) to calculate H_i,

$$\frac{\rho_+}{\rho_{ci}} = \frac{5}{6}\left(\frac{GM_i}{q_0H_0^2R_g^3}\right)^{1/3}\frac{1}{1+z_i}. \tag{24}$$

Note that this ratio is, to within factors of order unity, the cube root of the ratio of the present mean density of the cluster to the density at t_i. If we take parameters appropriate to the Coma cluster (Peebles 1970) of $M = 2 \times 10^{15}\, M_\odot$, $R_g = 2.8$ Mpc, and use $H = 75$ km s^{-1} Mpc^{-1}, or $H^{-1} = 1.3 \times 10^{10}$ years, we obtain (it can be easily shown that if the mass is the virial mass, then the result is independent of H_0)

$$\rho_+/\rho_{ci} = 4.0 \times 10^{-3}(2q_0)^{-1/3}\mathfrak{M}^{1/3}\mathfrak{R}_g^{-1}, \tag{25}$$

where $1 + z_i$ has been taken to be 1000 (it will become apparent that the results, except for extrapolated initial conditions like ρ_+ and R_i, are independent of z_i, so long as it is large), and $\mathfrak{M}$ and $\mathfrak{R}_g$ are the mass and radius of the cluster in units of the adopted values for Coma, 4×10^{48} g and 9×10^{24} cm.

The collapse time can be likewise calculated (cf. eq. [16]), and yields

$$T_c = \pi\left(\frac{6}{5}\right)^{3/2}\left(\frac{R_g^3}{2GM}\right)^{1/2} = 5.0 \times 10^9\ \mathrm{yr}\left(\frac{\mathfrak{R}^3}{\mathfrak{M}}\right)^{1/2}, \tag{26}$$

and, as expected, depends not at all on initial conditions and cosmology.

All of this is correct, of course, only so long as the mass which has fallen in *since* the initial collapse is small; we shall find that in general this is *not* the case. We shall find it most convenient to take as independent parameter the collapse time T_c and fit the observed properties of the cluster. It is clear that the effect of the infall, again in the absence of significant dissipation, is to make the cluster more *loosely* bound, thus giving an erroneously *long* characteristic collapse time if ignored.

We have, from equations (15), (16), and (20), the ratio of total to initial mass, given T_c:

$$\frac{M(t)}{M_i} = 1 + \frac{(t/T_c)^{2/3} - 1}{1 + (1 - 2q_0)(H_0t/2\pi q_0)^{2/3}}. \tag{27}$$

At this point we introduce some quantities which will greatly simplify the appearance of subsequent analysis. Let

$$\mu_c = \frac{\rho^+}{\rho_{ci} - \rho_{ei}} = \left(\frac{2\pi q_0}{H_0T_c}\right)^{2/3}(1 - 2q_0)^{-1},$$

$$\mu_0 = \mu_c\left(\frac{T_c}{t_0}\right)^{2/3} = \left(\frac{2\pi q_0}{H_0t_0}\right)^{2/3}(1 - 2q_0)^{-1}, \qquad m(t) = \frac{M(t)}{M_i} - 1. \tag{28}$$

Thus equation (27) becomes

$$m(t) = \mu_c\frac{(t/T_c)^{2/3} - 1}{\mu_c + (t/T_c)^{2/3}}; \tag{29}$$

and for the case of $q_0 < \frac{1}{2}$, in which the universe has an infinite nonsingular future, it is seen that μ_c is the limiting value for the ratio of added to original mass as $t \to \infty$. It is likewise not difficult to show that μ_0 is the corresponding ratio of added mass to *present* mass as $t \to \infty$.

Some useful points should be noted. The *fractional infall rate* at present is

$$\frac{1}{M}\frac{dM}{dt}\bigg|_0 = \frac{1}{1+m}\frac{dm}{dt}\bigg|_0 = \frac{2}{3t_0}\frac{\mu_0}{1+\mu_0}\,. \tag{30}$$

Since μ_0 is *independent of* T_c, we obtain the result that the infall rate at present (and hence at any time) depends *only* upon the accumulated mass up until that time. The result is a simple consequence of the fact that the behavior of a shell with given initial conditions depends only upon the contained mass.

The infall rate for the Coma cluster with the assumed parameters for the present mass and gravitational radius as a function of q_0 is given in figure 1. For q_0 in the neighborhood of $\frac{1}{2}$, the rate is of order $10^5\ M_\odot$ per year, or about a cluster mass per Hubble time. The fate of this material will be discussed extensively in § VI. We return now to the question of energetics and comparisons with observations.

III. THE ENERGETICS OF THE TOTAL ACCUMULATED MASS

Let us now calculate the total energy of the matter in the cluster as a function of time, including the material which has fallen in since the initial collapse. Consider a shell of initial radius $r_i > R_i$. The initial kinetic energy of this shell is

$$d\mathfrak{T} = 2\pi r_i^4 \rho_{ei} H_i^2 dr_i\,,$$

and the gravitational potential energy referred to the center is

$$d\mathfrak{W} = -\tfrac{16}{3}\pi^2 G r_i^4 \bar{\rho}_i(r_i)\rho_{ei} dr_i\,; \tag{31}$$

so, using equation (2), we obtain

$$\begin{aligned} d\mathfrak{E} = d\mathfrak{T} + d\mathfrak{W} &= 2\pi r_i^4 H_i^2\left[1 - \frac{\bar{\rho}_i(r_i)}{\rho_{ci}}\right]\rho_{ei} dr_i \\ &= -2\pi H_i^2 r_i^4 \rho_{ei} dr_i\left(\frac{\rho_{ei} - \rho_{ci}}{\rho_{ci}} + \frac{\rho_{ci} + \rho_+ - \rho_{ei}}{\rho_{ci}}\frac{R_i^3}{r_i^3}\right). \end{aligned} \tag{32}$$

The total energy is obtained by integrating equation (32) from R_i to the desired initial shell radius and adding the contribution from equation (22) for the initial cluster mass.

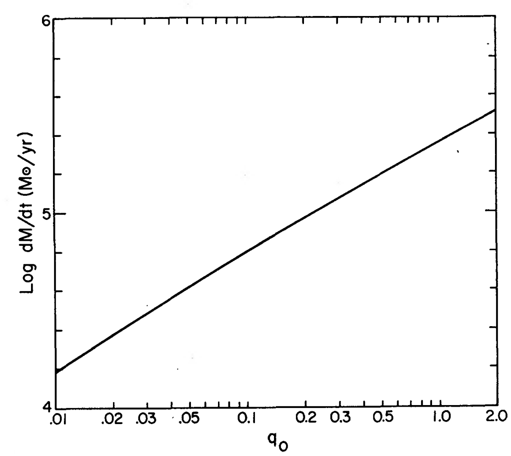

FIG. 1.—The present mass infall rate as a function of q_0 for the adopted Coma cluster parameters

Recalling that the density is everywhere very nearly equal to ρ_{ci} initially, so that $(4/3)\pi r_i^3 \rho_{ci} \approx M = M_i(1 + m)$, we can write the result as

$$E = -3H_i^2 R_i^2 M_i \left\{ \frac{\rho_+ + \rho_{ci} - \rho_{ei}}{\rho_{ci}} \left[\frac{(1+m)^{2/3}}{4} - \frac{3}{20} \right] - \frac{\rho_{ci} - \rho_{ei}}{10\rho_{ci}} (1+m)^{5/3} \right\}. \quad (33)$$

We can use the relations for ρ_+, $\rho_{ci} - \rho_{ei}$, and the definition of μ_c introduced earlier to reduce this to the form

$$\frac{E}{M} = -3\left(\frac{2\pi GM}{T_c}\right)^{2/3} (1+m)^{-5/3} \left\{ \left(1 + \frac{1}{\mu_c}\right) \left[\tfrac{1}{4}(1+m)^{2/3} - \tfrac{3}{20}\right] - \frac{1}{\mu_c} \frac{(1+m)^{5/3}}{10} \right\}. \quad (34)$$

This result needs some discussion. As calculated, it is the energy of all material which has fallen into the cluster (including the initial perturbation itself) since $t = 0$. If the infalling material is mostly *gas*, it will certainly interact significantly with the gas in the cluster (and itself) and, as we shall see in § VI, will distribute itself roughly like the other mass in the cluster. If the mass falling in is mostly in the form of galaxies (or stars, or rocks, or black holes), it will *not* interact, and will travel through the cluster unimpeded to emerge again. It is clear, however, that it never gets farther from the center than its initial maximum radius of expansion. We shall see in the following section that, for Coma, this radius is about 6 Mpc for material currently falling in, and slightly less (about 4 Mpc) for the material most distant which *has* fallen in in the past. Thus, while it remains close to the cluster, it is not concentrated to the center, and may be missed in calculating the energy from observations. Whether the material is galaxies or gas, the dissipation processes are, as we shall see, very slow compared to the Hubble time, so the assumption of energy conservation is a good one.

IV. THE SPACE DISTRIBUTION OF THE INFALLING MATERIAL

In the foregoing we have been primarily concerned with simple time development and have not considered the motion in detail. There are several interesting points worthy of attention. (1) Where is the "zero-velocity" surface—i.e., where is the shell which is just now turning around? (2) Where is the "critical surface" beyond which material is not bound to the cluster at all? (3) What is the radial density distribution of the infalling stuff?

The location of the zero-velocity surface is the maximum expansion radius of that shell whose collapse time is *twice* the present age of the universe. One obtains from equation (7) by setting da/dt to zero that the maximum expansion radius $R_{\max}$ for any shell is

$$R_{\max} = r_i a_m = r_i \frac{\bar{\rho}_i}{\bar{\rho}_i - \rho_{ci}}. \quad (35)$$

From equation (14), however,

$$\frac{\bar{\rho}_i - \rho_{ci}}{\rho_{ci}} = \left(\frac{\pi}{H_i t_c}\right)^{2/3}.$$

Setting $t_c = 2t_0$, we can find $\bar{\rho}_i - \rho_{ci}$, and find r_i from equation (8). The result is

$$R_{v=0} = \left(\frac{8GM_i t_0^2}{\pi^2}\right)^{1/3} \left[\frac{1+\mu_c}{1 + 2^{-2/3}\mu_0}\right]^{1/3}; \quad (36)$$

or, in terms of the present cluster mass,

$$R_{v=0} = \left(\frac{8GMt_0^2}{\pi^2}\right)^{1/3} \left[\frac{1+\mu_0}{1 + 2^{-2/3}\mu_0}\right]^{1/3}. \quad (37)$$

We can write, in general, that the zero-velocity radius at any epoch t is

$$R_{v=0}(t) = \left(\frac{8GM(2t)t^2}{\pi^2}\right)^{1/3} . \tag{38}$$

The radius of the critical shell (which exists, of course, only for $q_0 < \frac{1}{2}$) is found by calculating the initial radius for which $\bar{\rho} = \rho_{ci}$, and noting that this shell expands always with $r \propto t^{2/3}$. The result is

$$R_{\rm crit} = \left(\frac{9GMt_0^2}{2}\right)^{1/3}(1 + \mu_0)^{1/3} = \left(\frac{9GM_\infty t_0^2}{2}\right)^{1/3} . \tag{39}$$

To find the run of density in the infalling material, we need only use the fact that, since the average density decreases outward, infalling shells do not cross (i.e., the collapse time is an increasing function of radius), so we need only find the volume a given shell occupies as a function of time. From equation (6), we find

$$dr = a(r_i, t)dr_i + r_i \frac{\partial a(r_i, t)}{\partial r_i} dr_i = adr_i\left(1 + \frac{\partial \ln a}{\partial \ln r_i}\right) ; \tag{40}$$

and, since mass is conserved,

$$\rho = \rho_i a^{-3}\left(1 + \frac{\partial \ln a}{\partial \ln r_i}\right)^{-1} . \tag{41}$$

This result is not very useful computationally, however. It is clear from equation (11) that we can write

$$a(r_i, t) = a_{\rm max}(r_i)A[t/t_c(r_i)] , \tag{42}$$

where $A(x)$ is a universal function which increases from 0 to 1 and decreases back to zero again as x goes from 0 to 1. But (cf. eqs. [16], [35])

$$a_{\rm max} = a_{mc}(t_c/T_c)^{2/3} , \tag{43}$$

where a_{mc} is the value of $a_{\rm max}$ for the perturbation proper, and we can write equation (17) as

$$t_c = T_c\left[\frac{\mu_c}{(R_i/r_i)^3(1 + \mu_c) - 1}\right]^{3/2} = t_0\left[\frac{\mu_0}{\eta^{-3}(1 + \mu_0) - 1}\right]^{3/2} , \tag{44}$$

where $\eta = r_i/R_f$, and R_f is the initial radius of the shell just now falling in, i.e., for which $t_c = t_0$; η is a convenient radial variable to label the shells. Hence

$$\frac{\partial \ln a}{\partial \ln r_i} = \frac{d \ln t_c}{d \ln \eta}[\tfrac{2}{3} - B(t/t_c)] , \tag{45}$$

where $B(x) = d \ln A(x)/d \ln x$. Making use of relations derived before (cf. eqs. [35], [16], [4], and [5]) connecting the expansion of the shells and the universe with the density, we obtain

$$\rho(t_0, \eta) = \rho_0 \frac{[\eta^{-3}(1 + \mu_0) - 1]^3}{[2q_0/(1 - 2q_0)]^3\{1 + (d \ln t_c/d \ln \eta)[\frac{2}{3} - B(x)]\}[A(x)]^3} \tag{46}$$

with $x = t_0/t_c$. It is easy to show that $\rho(t_0, \eta) \to \rho_0$ as $\eta \to \infty$.

The radius of this shell is just

$$r(t_0, \eta) = r_{\rm max}(\eta)A(t_0/t_c) ; \tag{47}$$

and from equation (38) and the definition of η, we have

$$r(t_0, \eta) = \left(\frac{2GM_0 t_c^2}{\pi^2}\right)^{1/3} \eta A(t_0/t_c) . \tag{48}$$

The radial velocity of the shell is then given simply by

$$v(t_0, \eta) = \frac{r(t_0, \eta)}{t_0} B(t_0/t_c) . \tag{49}$$

Thus, given the function $A(x)$, we can describe the whole flow pattern parametrically in η for the range $1 < \eta < \infty$. For $q_0 < \frac{1}{2}$, of course, there is a last bound shell, but an almost identical analysis works for the unbound shells (one can regard the collapse time as imaginary in this case, but caution as to signs must be exercised). The results of some of these calculations are presented in figures 2 and 3, and a summary in table 1. The quantity v_f there is the infall velocity at R_g; $E_{,t}$ is the energy input. This completes our discussion of the dynamics, but we digress briefly before considering the implications.

V. ON THE QUESTION OF INITIAL CONDITIONS

The choice of a perturbation in density alone, with no perturbation in velocity, is arguable—in fact, we shall argue below that it is not strictly correct, but that it really doesn't matter.

It is clear that the criterion we must apply to the perturbation in order that it be "reasonable" is that it remain a small perturbation in some sense, as we approach the singularity from above. Let us think first in Newtonian terms, and then argue that everything still makes sense in the proper relativistic setting. It is clear here that a nec-

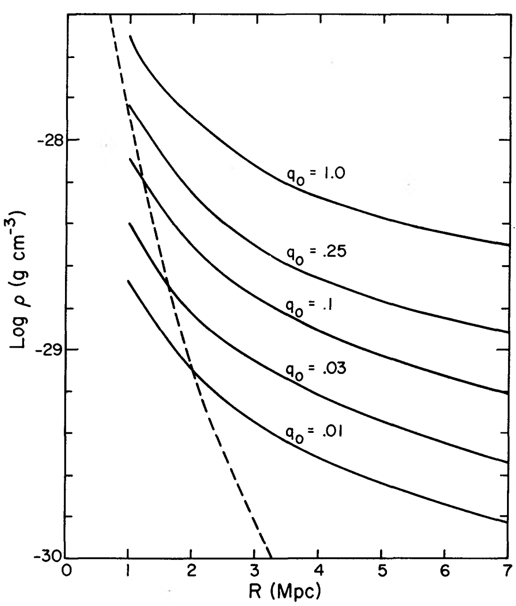

FIG. 2

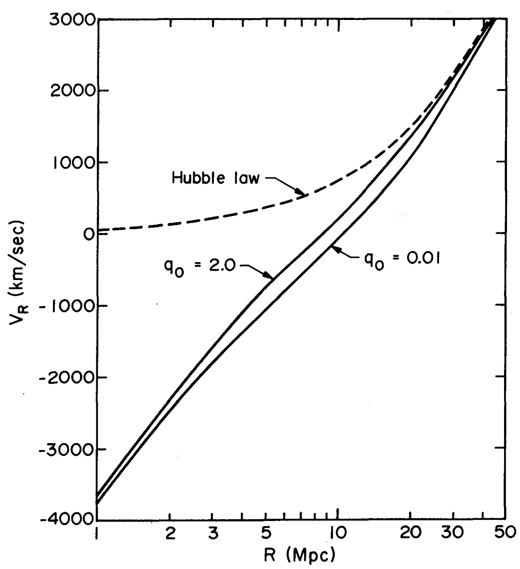

FIG. 3

FIG. 2.—The run of density of the infalling material with radius for several values of q_0 for the adopted Coma cluster parameters. The dashed line is the adopted intracluster gas density.

FIG. 3.—The radial flow velocity as a function of radius for the extreme values of q_0 for the adopted Coma cluster parameters. The dashed line is the Hubble expansion velocity; it is seen that the velocity perturbation due to the presence of the cluster is essentially independent of q_0 except at large radii, where the perturbation is small compared to the expansion velocity.

TABLE 1

INFALL PARAMETERS

q_0	T_0 (10^9 yr)	T_c (10^9 yr)	M_i ($10^{15}\,M_\odot$)	$M_{,t}$ ($10^5\,M_\odot$ yr^{-1})	V_f (10^3 km s^{-3})	$E_{,t}$ (10^{46} ergs s^{-1})
0.01....	12.4	4.35	1.75	0.15	1.84	1.6
0.03....	11.9	4.05	1.56	0.30	1.82	3.1
0.1.....	10.9	3.64	1.29	0.62	1.77	6.1
0.25....	9.7	3.44	1.13	1.05	1.70	9.6
0.5.....	8.6	3.54	1.11	1.55	1.62	12.9
1.0.....	7.4	3.90	1.20	2.27	1.51	16.3
2.0.....	6.1	4.42	1.47	3.30	1.33	18.5

essary criterion is that infinite density occur at the same time for all shells of matter; i.e., that $r = 0$ at some time $t = 0$ for all shells. It is also clear that this is incompatible with equation (7) and the condition that $H_i = (r_{,t}/r)_i$ be the same for all shells at the initial epoch; shells whose average internal density is highest will have been decelerated more than the others, and hence their radii will vanish "soonest" as one approaches $t = 0$ (i.e., at larger t). The discrepancy, in fact, has been noted and its magnitude evaluated (see eq. [13]).

Making use of equations (11) and (14), one can easily show that the behavior at small t for a given shell for which $r = 0$ at $t = 0$ is

$$r = (r_m/4)(\kappa t)^{2/3}[1 - \tfrac{1}{20}k(\kappa t)^{2/3}]\,, \tag{50}$$

where k is the energy parameter defined in equation (12) and $\kappa = 12\pi t_c^{-1}$ for bound ($k = +1$), $6V_\infty{}^3(GM)^{-1}$ for unbound ($k = -1$) shells. Here V_∞ is the terminal expansion velocity. From this relation one finds

$$H = r_{,t}/r = \frac{2}{3t}\left[1 - \frac{k}{20}(\kappa t)^{2/3}\right], \qquad q = -r_{,tt}r/r_{,t}{}^2 = \tfrac{1}{2}[1 + \tfrac{1}{4}k(\kappa t)^{2/3}]\,. \tag{51}$$

The average internal density is most easily found by making use of the relation (4) at this epoch:

$$\bar{\rho} = \frac{3H^2q}{4\pi G} = \frac{1}{6\pi t^2 G}\left[1 + \frac{3k}{20}(\kappa t)^{2/3}\right]. \tag{52}$$

The question now arises: To what may we compare this density? In the preceding we compared it to a "critical" density which we shall here call ρ_{cH}:

$$\rho_{cH} = \frac{3H^2}{8\pi G} = \frac{.1}{6\pi t^2 G}\left[1 - \frac{k}{10}(\kappa t)^{2/3}\right]. \tag{53}$$

This density, however, is a function of position as well as epoch, since κ is different for different shells. It is also clear that this "critical" density is *not* the density of a critical shell which has been expanding since $t = 0$; for such a shell,

$$\rho = \rho_{ct} = \frac{1}{6\pi t^2 G}\,. \tag{54}$$

Thus ρ_{ct} is a suitable constant "background" against which to measure the perturbation. Then

$$\bar{\rho} - \rho_{ct} = \frac{k(\kappa t)^{2/3}}{40\pi t^2 G}\,, \tag{55}$$

while

$$\bar{\rho} - \rho_{cH} = \frac{k(\kappa t)^{2/3}}{24\pi t^2 G} . \tag{56}$$

Thus constant H and constant t are *not* equivalent, but the *only* difference in the description of the initial conditions is a constant factor of 5/3 in the density contrast, in the sense that a density contrast evaluated at a single epoch of 60 percent of that which we have considered produces the same subsequent evolution—*but the specification of initial conditions on an initial hypersurface of constant H is in every way equivalent to the specification at constant epoch.* The amplitude of the perturbation is nowhere of interest to us except insofar as it is reflected in the later behavior, which we have calculated correctly.

The relativistic equations of motion are the same for a proper choice of radial coordinate, and the same remarks apply to the initial hypersurface. The only real difficulty comes because the very early universe was almost certainly radiation-dominated, so our equations of motion do not hold for very early epochs. It is possible that a large perturbation in the primeval radiation field could change the character of the subsequent motion in the nearly-empty models, for which the epoch of radiation dominance ends a relatively short time before recombination. The other factor we have neglected is radiation viscosity *before* recombination, but this is a significant effect for the mass range we are interested in again only for the nearly-empty models (Peebles and Yu 1970).

VI. SOME ASTROPHYSICAL IMPLICATIONS

a) The Growth of Cluster Galaxies prior to $t = T_c$

Let us first consider the times previous to T_c, the collapse time for the cluster. It is widely believed on the basis of the kind of energy arguments we have made for clusters that galaxies formed very early in the expansion of the universe, their collapse time corresponding roughly to their present dynamical times, of the order of 10^8 years. This corresponds to perturbations of much larger amplitude than those required to make Coma-style clusters (cf. eq. [16]). It should be noted, however, that the environment of cluster galaxies during the first few billion years of their existence is very different from galaxies of the field; in essence, the cluster galaxies find themselves in a universe with a much larger deceleration parameter than those in the field. We cannot expect the infall theory developed here to apply quantitatively to galaxy-sized perturbations because of radiation-pressure effects on the velocity field in early epochs, but it is instructive to consider some qualitative aspects. If t_c is the galaxy collapse time and T_c the cluster collapse time, the parameter μ_c (eq. [28]) for the galaxy in the background of the cluster becomes

$$\mu_{gc} = -(T_c/t_c)^{2/3} , \tag{57}$$

and the galaxy mass $\mathfrak{M}_g$ grows, for $t > t_c$, like

$$\mathfrak{M}_g(t) = \mathfrak{M}_0\left[\frac{(t/t_c)^{2/3} - (t/T_c)^{2/3}}{1 - (t/T_c)^{2/3}}\right] \tag{58}$$

as long as the galaxies are still a minor perturbation on the total mass of the cluster. $\mathfrak{M}_g(t)$ goes to infinity, of course, at $t = T_c$, and this result is clearly unphysical; exhaustion of material, if nothing else, will limit the growth. This result suggests that cluster galaxies grow by spherical accretion until the material in the cluster is essentially exhausted. It is important to note that the bremsstrahlung cooling time for the gas falling into young galaxies is quite short; that time is approximately (Spitzer 1962)

$$\tau_{\text{brems}} = 9 \times 10^7 n_e^{-1} T_8^{1/2}\ \text{yr} , \tag{59}$$

where T_8 is the temperature in units of 10^8 ° K. The gas will be about 10^6 °–10^7 ° K if its infall velocities are randomized, and n_e is unity or greater if the gas at any time makes up even a few percent of the mass of the galaxy; thus the cooling time is short compared to the dynamical time of the galaxy, and the accreted material can be easily incorporated into the interstellar material and thence into star formation in the galaxy. It is important to note that energy is *not* conserved in this process, and there is no reason to suppose that the binding energy per unit mass of these accreting galaxies will be appreciably smaller than that of their cousins in the field. (The field galaxies are also accreting, of course, though much more slowly, and there is evidence from the high-velocity clouds that our own galaxy may still be doing so; see also Larson [1972], who has commented on this point.) It thus seems likely that clusters are much more efficient than the field in galaxy-building, even though the cluster did not exist as an entity at the epoch of galaxy formation; and it is possible that essentially all the material in the protocluster is incorporated into galaxies prior to T_c.

It is interesting to speculate also that in a more realistic perturbation than the rectangular one considered here, the central portions might well have higher densities and shorter T_c's, and that the galaxies there might accrete more efficiently than those in the outskirts. This might provide a mechanism for formation of the supergiant galaxies sometimes found in the centers of clusters. Another possible mechanism for their growth is discussed in the next section.

b) Events at and after the Collapse

One seemingly inescapable consequence of the collapse is the shock randomization of the infall energy of any remaining gaseous debris. We shall show below that any such debris cannot have survived as relatively cool, dense clouds. Thus, following the collapse, the gaseous intracluster medium will have a temperature corresponding roughly to the three-dimensional rms velocities of the galaxies, about 1700 km s^{-1} for Coma, or about 7×10^7 ° K for a mixture that is 90 percent H and 10 percent He by number. To discuss the fate of this material, we need to have a model for the cluster's density and gravitational field. We adopt the following distribution for the total density:

$$\rho = \frac{\rho_0 e^{-r/L} a^2}{r^2 + a^2}\,, \tag{60}$$

with $a = 170$ kpc, $\rho_0 = 2.6 \times 10^{-25}$ g cm^{-3}, $L = 1.7$ Mpc. This fits the derived density distribution of Peebles (1970) to within the errors of determination of that distribution, and fits his theoretical wings fairly well where the count data are poor. This distribution mocks up an isothermal distribution with $\langle v^2 \rangle^{1/2} = 1700$ km s^{-1} with fair accuracy to radii of about 1 Mpc, but gives lower densities for larger radii in agreement with the count data. The total mass is 4.0×10^{48} g, our adopted value. *We assume that the gas is similarly distributed*, for want of a better hypothesis. We shall see that it is likely that the gas is never an appreciable contributor to the total density (after T_c). (One can, of course, solve the hydrostatic equation for gas in a gravitational field generated by the distribution [60], but the dynamical times in the outer parts are comparable to the total times of interest, and there are ram-pressure and flow effects from the infall, so a more careful treatment, short of a detailed numerical model, seems hardly justified.) Numerical models of the collapse (Henon 1964; Peebles 1970) indicate that a density distribution differing only slightly from the present one is established in the period $T_c < t < 3T_c/2$, after which the cluster is essentially quiescent, but during which the dynamical evolution is quite violent.

Once the gas has been heated, the pressureless accretion onto galaxies that we have discussed clearly ceases, and since the kinetic velocities in the gas are very large compared to the kinetic velocities in galaxies, there should be no accretion at all. There are, however, some interesting effects *if there is any gas left*. We here discuss briefly some of these.

Consider first a cluster with a hot, smooth (the distribution *must* clearly be smooth if the temperature is high) intracluster medium, and a galaxy moving through that medium. The interstellar material in the galaxy feels the ram pressure of the intracluster medium as it flows past. This ram pressure is

$$P_r \approx \rho_e v^2 \,, \tag{61}$$

where ρ_e is the external (to the galaxy, i.e., the intracluster) density, and v the velocity of the galaxy. If the galaxy is a typical spiral, this material will be held in the plane by a force per unit area which cannot exceed

$$\mathfrak{F} = 2\pi G \sigma_s \sigma_g \,, \tag{62}$$

where σ_s is the star surface density and σ_g the gas surface density on the disk of the galaxy. For a typical large spiral with a mass of $10^{11}\, M_\odot$ and a radius of 10 kpc, $\sigma_s \sim 0.06$ g cm^{-2}; a gas layer 200 pc thick with a density of one atom per cm^3 has a surface density of 10^{-3} g cm^{-2}, corresponding to a restoring force of about 2.5×10^{-11} dyn cm^{-2}. The ram pressure, from equation (61), is, for a galaxy moving at the rms velocity of 1700 km s^{-1}, $5 \times 10^{-8} n$ dyn cm^{-2}; where n is the intracluster number density. *Thus if the intracluster density exceeds 5×10^{-4} atoms cm^{-3}, then a typical galaxy moving in it will be stripped of its interstellar material.* The central density corresponding to the distribution (60) has $n = 0.16$, so if as little as 3×10^{-3} of the mass of the cluster is in gas, a galaxy moving through the central regions will be stripped. We will see below that the X-ray data indicate that the *present* gas density comprises roughly 3 percent of the cluster mass, so we expect *no normal spirals* in the central regions of clusters like Coma. The lack of such systems is, of course, observed, and it was originally suggested (Baade and Spitzer 1951) that galaxy collisions were responsible, though modern cluster parameters make that somewhat unlikely.

Consider next a very massive galaxy near the center of the cluster, moving slowly. This situation will be met, for instance, in the case of the central supergiants in many clusters, and occurs not quite so "purely" in the case of NGC 4874 and 4889, the giant central double in Coma. The interaction of the interstellar material and intracluster material in this case is very different. The exact behavior probably cannot be predicted without recourse to detailed numerical calculation, but a few features can be investigated qualitatively. Two processes compete in this case: on the one hand, heat is conducted rather efficiently from the hot, tenuous gas into the cooler gas in the galaxy, and on the other hand, energy is radiated efficiently in bremsstrahlung by the cooler, denser galactic gas. The relative importance of these processes can be estimated under some rather restrictive assumptions. The conclusions are reasonably clear; if conduction "wins," the interstellar medium will be heated to 10^8 ° K and dissipated; if cooling is dominant, the ensuing behavior is likely to be much more complicated but one effect is that the galaxy acts as a trap for the hot gas; thus the cD galaxies might grow once again at the expense of the intracluster medium. One can estimate the effect easily, once parameters for the galaxies are derived. In Coma, one can get an estimate of the masses involved by the simple observation that NGC 4889 and 4874 completely dominate the central regions of the cluster. If one makes the reasonable assumption that these objects are not centrally located by chance, that they "belong" in the center, then they move there essentially under their mutual gravitation alone. A lower limit to the mass is then obtained by assuming that the plane of their orbit contains the line of sight, and that they are at their maximum separation. Their projected separation is 170 kpc for $H = 75$ km s^{-1}, and their velocity difference is 700 km s^{-1} (Humason, Mayall, and Sandage 1956). The computed systemic mass is then $2 \times 10^{13}\, M_\odot$. The projection effects and the other galaxies near the center contribute oppositely, and we take $10^{13}\, M_\odot$ each as a likely mass.

This seems very large, but well-exposed photographs show that the envelopes of these objects are very extensive and probably overlap; a detailed photometric and dynamical study of the central regions is under way (Sargent and Gunn, in preparation).

We take 30 kpc for the gravitational radius of these galaxies, a value which comes from rather uncertain direct measurement on deep photographs; fortunately, neither this quantity nor the mass need be very accurately known for the argument below. These values and the virial theorem give rms velocities of 1000 km s^{-1}, and equivalent temperatures of 3×10^7 ° K, not terribly much smaller than for the cluster itself.

The conductive input is of order

$$J = 4\pi r\kappa(T_e - T_i) , \tag{63}$$

where r is some radial scale for the system and κ is the conductivity; T_e is the exterior temperature, and T_i the interior temperature. This relation is satisfied only if the mean free path λ is small compared to r; $\lambda \approx 50$ pc n^{-1} (Spitzer 1962) for electrons or protons at 3×10^7 ° K. The conductivity κ is, in the absence of a magnetic field, about $10^{14} T_8^{5/2}$ (Spitzer 1962) in cgs units, so J is about 10^{44} ergs s^{-1}. The bremsstrahlung luminosity is of order (Spitzer 1962)

$$L_b \approx 6 \times 10^{-23} T_{8i}^{1/2} n_i^2 r^3 \text{ ergs s}^{-1} \approx 2 \times 10^{46} n_i^2 \text{ ergs s}^{-1} . \tag{64}$$

Thus if, at the time of collapse, the central galaxies have interstellar densities of about 0.1 cm^{-3} or more, they can trap and refrigerate the intracluster gas. Under the crude assumption that every proton impinging upon them is trapped, each collects in this way about $2 \times 10^6 n_e M_\odot$/yr; we shall see that the present central density n_e is likely about 5×10^{-3} cm^{-3}, so if the primordial density were also of this order, the central galaxies could double their mass in 10^9 years.

The conductive input into smaller systems competes with the ram-pressure effects in removing interstellar material, but is not so important. If the intracluster density becomes high, the heating of gas in cluster galaxies by bremsstrahlung X-rays can also be important, though here again the ram pressure is so effective that it matters little that it has help.

Note that, at least in Coma, the total amount of gas that could have been incorporated into the giant central members is small compared to the cluster mass, since their total mass presently is a small fraction of the cluster mass. Since the total mass of gas at present presumably contains some contribution from infall and is still a small fraction of the cluster mass, we conclude that the gas has never made a significant contribution to the mass since the collapse.

c) The Intracluster Medium at Present: Infall

The cluster, on our model, has been quiescent (dynamically) for, typically, 4–6 $\times$ 10^9 years. Observationally, the cluster seems to consist entirely of elliptical and S0 galaxies with old stellar populations (Turnrose and Rood 1970). The minimum age of these systems is a little difficult to estimate without a detailed stellar content analysis, for which there are as yet insufficient data, but it is difficult to see how systems so red could be less than 10^9 years old—i.e., at least 10^9 years have elapsed since epochs of significant star formation. Indeed, if the picture prescribed here has any validity, the material out of which stars form should have been absent in nearly all the systems in the cluster since the collapse, with only the central giants being possible sites for star formation. From a sample of 100 spectra of cluster galaxies obtained for a dynamical analysis (Sargent and Gunn, in preparation), the only two with emission lines ([O II] λ3727) are the central giants, but the absorption spectra of these objects show no apparent differences from any of the others except possibly for effects which are traceable to their very large velocity dispersions.

The X-ray results of Meekins *et al.* (1971) and Gursky *et al.* (1971) indicate that the cluster is an X-ray source with a spectrum like that of optically thin bremsstrahlung with a temperature of about 7×10^7 ° K, and a flux of about 0.03 photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ in the band 0.7–4 keV. With the gas distribution given in equation (60), with $H = 75$, and assuming that the gas is 10 percent He by number and the rest H, one obtains for the bremsstrahlung flux

$$J_E = 4.2\alpha^2 \frac{kT}{E} e^{-E/kT} \text{ counts cm}^{-2}\ \text{s}^{-1}\ \text{keV}^{-1}\,, \tag{65}$$

where α is the fraction of the cluster mass which is in gas. From this, using $T = 7 \times 10^7$ ° K, one obtains $\alpha = 0.03$. This value is somewhat dependent on the Hubble constant; it can easily be shown to be proportional to $H_0^{-3/2}$ (remembering that the virial mass and the size of the cluster also depend on H_0).

The naïve interpretation of the result is, in the present context, somewhat shattering; we will discuss it briefly and then attempt to defend it as the only reasonable one. A brief discussion of some of these points has appeared previously (Gott and Gunn 1971).

Consider the possible origin of the gas. Some may have been left over from the beginning; certainly any remaining gaseous material at collapse must contribute to it. The interstellar material which was swept from the spirals and irregulars (if any) must contribute to it. Infall must contribute, and here one meets a difficulty. From figure 1 it is seen that for a "reasonable" value of q_0 (near $\frac{1}{2}$), the present mass of gas, 6×10^{13} $M_\odot$, will accumulate in 4×10^8 years at the present infall rate, and that the rate (eq. [31]) was *higher* in the past. Furthermore, one must go to small values indeed for q_0 in order that the accumulated mass since the collapse be as small as 3 percent of the virial mass; values smaller than 0.01 are indicated, for which our simple theory is inadequate, since for such small values clusters cannot be assumed to contribute negligibly to the total mass density. Such small values are absurd, of course, since *galaxies* already make up a density sufficient to make $q_0 \approx 0.02$ (Oort 1958; Shapiro 1971). For very small q_0, the infall rate and the total infallen mass since collapse goes approximately as $q_0^{2/3}$ (cf. eq. [31]). Thus, if we assume that *all* the gas is infallen material, that no gas has been lost, and that galaxies make up a density corresponding to $q_0 = 0.02$, one obtains a value for q_0 by demanding that

$$(M_0 - M_i)\left(\frac{q_0 - 0.02}{q_0}\right) = M_{\text{gas}}\,, \tag{66}$$

or, approximately, since $M_0 - M_i \approx 1.6\, q_0^{2/3} M_0$, and $M_{\text{gas}} \approx 0.03\, M_0$,

$$q_0 - 0.02 = 0.02 q_0^{1/3}\,, \tag{67}$$

or $q_0 \approx 0.026$. The value 0.02 for the galaxy density is very uncertain, but the conclusion is that the intergalactic gas density is *smaller than or, at most, comparable to the density in galaxies*, and q_0 itself must be very small, *unless* there is much unseen mass in other forms, either in zero-rest-mass fields or in collapsed, low-luminosity objects such as lower-main-sequence stars, small black holes, rocks, or whatever.

This conclusion can be escaped in a number of possible ways, none of which we believe are viable, though the severity of our limit may be eased somewhat. Clearly any gas remaining in the cluster after collapse, and the remnant interstellar gas of the member galaxies, makes the problem worse (i.e., it decreases the allowable gas infall).

Some gas can have been incorporated into the central galaxies, but not much, as we have seen. The gas cannot be making galaxies presently, since there are no young systems in the cluster. The age limit of 10^9 years for the youngest galaxies is a bit uncertain, but if we say that all the infalling mass were incorporated into galaxies until 10^9 years ago, and none thereafter, we obtain with reasoning essentially the same as before that $q_0 \lesssim$

0.1; the hypothesis, however, seems a bit artificial. Indeed, the gas upon infall suffers an adiabatic compression, and if the adiabatic exponent is greater than 4/3, it can easily be shown that such compression is a stabilizing influence against gravitational fragmentation. The relevant value for these conditions is clearly 5/3, so that infall should not induce the formation of galaxies. The infall velocities are quite high (cf. fig. 3), and at some point must be randomized in an essentially stationary shock. The location of this shock depends on the internal and external gas density, and is sensitive to the form of the density distribution assumed, but typically will occur in the vicinity of the gravitational radius, about 3 Mpc. The resulting temperatures are of the order of the kinetic temperature of the cluster (and of the X-ray spectrum as fitted to bremsstrahlung), about 7×10^7 ° K. Once so heated, galaxies can clearly not form.

But does the intracluster medium have to be smooth? Perhaps there are clouds in which the gas is trapped and cooled; perhaps the intergalactic medium is sufficiently clumpy that there is no shock, the clouds avoiding collisions altogether.

That this cannot be the case can be demonstrated very simply. In order that there be clouds in the cluster, it is necessary that they be stable against tidal disruption and gravitational contraction (since there are no young stellar systems, and therefore their cooling time must be long), and they must avoid collisions with each other. There is, in addition, the powerful constraint imposed by the X-ray flux.

We approximate the cluster with a uniform sphere of radius $R = 2$ Mpc, mass $M = 2 \times 10^{15}\ M_{\odot}$, and kinetic temperature $T_{\rm kin} = 7 \times 10^7$ ° K (for our standard composition). A fictitious cloud of gas with the same mass, radius, and temperature would have a bremsstrahlung cooling time $t_{\rm cool}$ of about 1.3×10^{10} years. Now let the cluster contain N clouds of radius $r = \beta R$, density $\rho = \gamma \bar{\rho}_{\rm cluster}$, and temperature $T = \delta T_{\rm kin}$. Let the ratio of external pressure (from the intracluster medium) to the cloud internal pressure be f. Then the virial theorem demands, if the clouds are in equilibrium,

$$\delta(1 - f) = \beta^2 \gamma \, . \tag{68}$$

Tidal stability requires that

$$\gamma \gtrsim 1 \, . \tag{69}$$

Let the cloud cooling time be $t_{\rm cloud} = \tau t_{\rm cool}$; then

$$\delta^{1/2}/\gamma = \tau \, . \tag{70}$$

Let p be the formal average number of collisions with another cloud in the time since collapse. Then taking the collapse time to be 4×10^9 years and the velocity to be the cluster rms velocity, we have

$$p = (t_0 - T_c)\pi r^2 v \frac{3N}{4\pi R^3} = (t_0 - T_c)\frac{v}{R}\frac{3N\beta^2}{4} \approx 4N\beta^2 \, . \tag{71}$$

These relations can be solved to yield

$$N = \frac{p}{4\tau^2\gamma(1 - f)} \, , \qquad \delta = \gamma^2\tau^2 \, , \qquad \beta = \tau\gamma^{1/2}(1 - f)^{1/2} \, . \tag{72}$$

The fraction α of the cluster mass that is in clouds is

$$\alpha = N\gamma\beta^3 = \tfrac{1}{4}p\gamma^{3/2}\tau(1 - f)^{1/2} \, , \tag{73}$$

and the X-ray data require that if $\delta \gtrsim 0.1$ (so that the X-rays are observed),

$$\frac{\alpha\gamma}{\delta^{1/2}} = \tfrac{1}{4}p\gamma^{3/2}(1 - f)^{1/2} < 0.03 \, , \tag{74}$$

since the observed X-ray luminosity is 0.03 of that appropriate to our gaseous cluster.

Now τ must be larger than about $\frac{1}{2}$ if the clouds are to survive to the present epoch, and p must, of course, be of order unity or smaller, since collisions at the rms velocity of the cluster must certainly be disruptive. First, assume that $f = 0$. Then inserting $\gamma \gtrsim 1$, $p \lesssim 1$, $\tau \gtrsim \frac{1}{2}$ in equations (72) yields $N \lesssim 1$, $\delta \gtrsim \frac{1}{4}$, $\beta \gtrsim \frac{1}{2}$, and we have the marginal possibility of *one* hot cloud in the cluster center—but the X-ray luminosity of such a cloud would be larger than the gaseous cluster and is ruled out by the observations. Actually, such a single cloud need not satisfy the tidal requirement, but its existence is precluded by the X-ray data.

Clearly, objects one would like to call "clouds"—things small compared to the cluster radius—are possible only if $1 - f \ll 1$, i.e., clouds which are essentially in pressure equilibrium with negligible self-gravitation.[1] Such objects must still be hot, however, to avoid cooling and collapse, and the X-ray data preclude their contribution of even as much mass as the supposed hot intracluster medium (which must still be present to provide pressure equilibrium).

A more realistic picture is even more discouraging. The cluster is strongly centrally condensed, so γ must exceed not just unity, but the highest average interior density it encounters—a number which might be more like 5 or 10—which would rule out clouds altogether, since it would require that any clouds be fewer than one in number, hotter and bigger than the cluster (the "external" pressure must clearly be small in that case!).

We conclude that the intracluster medium is smooth and hot, and that the gas which has fallen has not been incorporated into condensed objects in the cluster.

One possibility remains, that the gas has somehow been held at bay or even ejected. Two possibilities present themselves: normal stellar winds from the galaxies, and large-scale violent events. If q_0 is even as small as 0.1, the mass infall to the present epoch is 35 percent of the cluster mass; the energy input to the cluster of this material (taking as velocity the infall velocity at R_g) is 6×10^{46} ergs s^{-1}. To keep the material at bay requires an average input of at least this much, in mass outflow; and if the mass outflow is not to contribute *itself* to the intracluster medium, it must occur at velocities large compared to the kinetic velocities. With present ideas of quasar luminosities, it would require an equivalent of 10 "average" QSOs at all times in the cluster to disperse the material; stellar winds are woefully inadequate energetically, and their velocities too low anyhow.

Thus, our simple calculation earlier stands, and with it the inference about the smallness of q_0.

The other possible weakness of our arguments lies at the very beginning, in the assumption that clusters grow from small density perturbations; a highly inhomogeneous, turbulent early universe might produce "isolated" clusters which do not interact significantly with their surroundings, but there seems no natural way to do this, or to avoid the situation, on some sort of average, that we discuss here. The Coma cluster, of course, may be an atypical example in the smallness of its X-ray luminosity and inferred low gas density, but the situation will become much more difficult if other rich, collapsed clusters also display those characteristics.

d) Other Clusters

The collapse timescales for Coma are representative of relatively rich, tightly bound clusters; the times are enormously longer for small aggregates like the Local Group. *The present is very much the epoch of cluster formation*, and, from the results we have obtained, it is perhaps possible to draw a few conclusions about clusters in general.

The violent heating at collapse and subsequent energy input via infall should be a phenomenon common to all clusters that have collapsed. Thus it becomes possible to

[1] Silk and Goldsmith have shown recently (1972) that collapsed low-temperature clouds supported by *rotation* can exist within very narrow parameter limits. It is not clear to the authors why these objects do not cool and become galaxies.

understand the difference between the "open" clusters and the "regular" clusters and their populations (Abell 1965). The open aggregates are irregular because they have not undergone violent quasi-relaxation; they have not yet collapsed. For the same reason, they have generated no hot intracluster medium, and they can (and do) contain normal spiral galaxies because these have not been swept clean. The "regular" objects *have* collapsed, and contain no spirals (except for an occasional one just falling in from outside). Thus irregular, open clusters should *not* be X-ray sources—at least not "hot" X-ray sources.

e) The Local Group and Infall on the Galaxy

It is perhaps instructive to consider the infall problem for the Local Group—or, more precisely, *in* the Local Group, i.e., the accretion by the Galaxy. Oort (1970) has recently estimated that the Galaxy accretes about 0.9 percent of its mass per 10^9 years, as evidenced by the high-velocity clouds. (This estimate includes not only the observed clouds; inferences are made about a general flow, and the *total* infall is calculated.) It is difficult to attach an error to this figure—Oort estimates that it is correct to, perhaps, a factor of 2.

If we assume that the Local Group is bound with a collapse time $T_c > t_0$, then we can show, by using equations (58) and (28), that the present infall rate is

$$\frac{1}{M}\frac{dM}{dt}\bigg|_0 = \frac{2}{3t_0}\frac{\alpha_0}{1-(t_0/T_c)^{2/3}}, \tag{75}$$

where α_0 is the present mass fraction in gas. The assumption that the Local Group is just marginally bound ($T_c \rightarrow \infty$) is consistent with consideration of the dynamics (Herbst 1969) and also with a detailed consideration of the M31–Galaxy system (Oort 1970). The characteristic total mean density in the Local Group is 3×10^{-29} g cm^{-3} (the mass of the Galaxy and M31 spread over a sphere 600 kpc in radius). If q_0 is small, $H_0t_0 \approx 1$, and we expect a cluster of galaxies that is marginally bound ($T_c \rightarrow \infty$) to have a mean density of $\sim 4.7 \times 10^{-30}$ g cm^{-3} at the present epoch. A cluster with $T_c = 2t_0$ should have a mean density of $\sim 2.6 \times 10^{-29}$ g cm^{-3} (cf. eqs. [39] and [38]). The expected density with $T_c = 2t_0$ agrees well with the observed value for the Local Group. Thus, let us assume that for the Local Group $2t_0 < T_c < \infty$. Then using Oort's rate we find $0.07 < \alpha_0 < 0.18$. The mass fraction in gas, while somewhat uncertain, seems not unreasonable in comparison with our formerly derived figure of some 30 percent in the field and 3 percent in Coma. Our values of α_0 correspond to gas densities of 1 to 3×10^{-6} atoms cm^{-3} within the Local Group.

VII. SUMMARY AND CONCLUSIONS

In the first half of the paper we have developed a simple theory of spherically symmetric, pressureless infall from an originally expanding medium, and in the second half we have applied those results to some phenomena in clusters of galaxies, specifically the Coma cluster.

The salient points in the development are the following.

1. It seems likely that infall onto cluster galaxies during the early phases prior to dynamical collapse of the cluster can result in extremely high "efficiency" of galaxy-building. It is possible that little or no intergalactic gas is left in this process.

2. The remaining medium, if any, is shock-heated at the time of collapse to approximately the kinetic temperature of the cluster; normal spirals are swept clean of interstellar material by this gas, and under certain circumstances central giant galaxies can grow further at its expense.

3. Infalling gas from outside is heated in a standing shock to the temperature of the intracluster gas and distributes itself smoothly within the cluster. The X-ray data strong-

ly limit the amount of gas that may have fallen in, and one infers that the gas density in intergalactic space is very low; in a pressureless Friedmann cosmology, the deceleration parameter is smaller than 0.1, and is likely much smaller. The universe must be open (hyperbolic) unless most of its mass is hidden in zero-rest-mass fields or in collapsed low-luminosity objects.

4. The distinction between open, irregular clusters and compact, regular ones is suggested to be simply that the latter are older, and the former younger, than their collapse times. In view of point (2) above, this explains why spirals are found in open clusters and are not found in compact ones. X-ray observations provide an easy method of checking this assertion.

The authors would like to thank Drs. James Bardeen, W. W. Morgan, Jim Peebles, Martin Rees, W. L. W. Sargent, Joseph Silk, and Lyman Spitzer for helpful discussions during the long course of this research. It was begun while both authors were at Princeton; one of us (J. E. G.) enjoyed a visiting position at the University of Washington in Seattle during part of its final phases. J. R. G. acknowledges the support of an NSF predoctoral fellowship.

REFERENCES

Abell, G. 1965, *Ann. Rev. Astr. and Ap.*, **3**, 1.
Baade, W., and Spitzer, L. 1951, *Ap. J.*, **113**, 413.
Field, G. 1972, chapter in *Galaxies and the Universe*, of *Stars and Stellar Systems*, Vol. **9**, ed. A. and M. Sandage (Chicago: University of Chicago Press) (in press).
Gott, J., and Gunn, J. 1971, *Ap. J.* (*Letters*), **169**, L13.
Gursky, H., Kellogg, E., Murray, S., Leong, C., Tananbaum, H., and Giacconi, R. 1971, *Ap. J.* (*Letters*), **167**, L81.
Henon, M. 1964, *Ann. d'ap.*, **27**, 83.
Herbst, W., 1969, *Pub. A.S.P.*, **81**, 619.
Humason, M., Mayall, N., and Sandage, A. 1956, *A.J.*, **61**, 97.
Larson, R. 1972, *Nature*, **236**, 21.
Meekins, J., Fritz, G., Chubb, T., and Friedman, H. 1971, *Nature*, **231**, 107.
Oort, J. 1958, *Inst. de Phys. Solvay, Brussels*, **2**, 163.
———. 1970, *Astr. and Ap.*, **7**, 381.
Peebles, J. 1968, *Ap. J.*, **153**, 1.
———. 1970, *A.J.*, **75**, 13.
Peebles, J., and Yu, J. 1970, *Ap. J.*, **162**, 815.
Sandage, A. 1961, *Ap. J.*, **133**, 355.
Schwarzschild, M. 1954, *A.J.*, **59**, 273.
Shapiro, S. 1971, *A.J.*, **76**, 291.
Silk, J. 1968, *Ap. J.*, **151**, 459.
Silk, J., and Goldsmith, D. 1972, *Ap. J.* (*Letters*) (in press).
Spitzer, L. 1962, *Physics of Fully Ionized Gases* (New York: John Wiley & Sons).
Turnrose, B., and Rood, H. 1970, *Ap. J.*, **159**, 133.