On Inertia and Inertial Frames of Reference

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1. INTRODUCTION

In recent years papers have appeared concerned with what is called 'the origin of inertia' (1), (2), and others (3), (4) with the associated idea of 'inertial frames of reference'. The phrase, *origin of inertia*, is at least paradoxical. The ordinary meaning of inertia is the property (if it can be so called) which a body possesses of complete inability to change its state, so that, if its state does change, that is conclusive evidence of the action on it of some external influence. It is therefore a natural problem to inquire, in that case, what the external influence is. But if it does not change, what sense is there in asking for the 'origin' of its immutability? What can we imagine would happen to it if that originating something were absent? One would have thought that the absence of an occurrence was the one thing that needed no explanation: what, then, are we looking for when we try to discover the origin of inertia?

It is to be presumed that those who discuss this subject have some definite problem in mind, but it does not appear that they perceive the desirability of stating it explicitly; one must infer from what is written—necessarily with some uncertainty—precisely what they are trying to do. It is not the purpose of this paper to make such inferences, though it is possible that what is to be written may have some indirect bearing on that matter. The main object, however, is to portray the background of existing knowledge and thought in this field of study, that has given meaning to the terms used in it, and against which any further investigations must inevitably be seen and appraised by those who are aware of it. The value of the paper, I hope, will be not only in the assistance it may give towards the elucidation and assessment of what has already been written, but also in the inducement and ability which it may provide for future writers to express their ideas in terms most readily understood by those familiar with the history of the subject.

2. GENERAL CONSIDERATIONS

The basic problem of mechanics or gravitation theory or cosmology or whatever this 'mechanical philosophy' may be called reduces to this:

given any system whatsoever of discrete bodies, with their positions relative to an observer at one or more particular instants of time, to determine their relative positions at all other instants of time. Strictly speaking, our problem is not concerned with any arbitrary system but with the actual universe, but since we cannot survey the universe in its entirety, and cannot even say whether it is finite or infinite in extent, we must perforce aim at a general description, applicable to all conceivable distributions of bodies, and therefore to the actual universe however it may be constituted.

This statement presupposes that we have means of measuring times and positions of bodies. In fact we have, so we need not enter into problems concerning scales of time and distances, or the most suitable instruments for measuring times and distances: it is enough for our purpose that we can speak of the position of a body at a given time with the knowledge that it can, in principle, be determined unambiguously, whatever we may consider to be the most suitable way of interpreting the readings of our instruments. We can, of course, derive secondary concepts—velocity, acceleration, . . .—from the primary spaces and times, and these may be found more convenient as parameters in which to express our results. Furthermore, the statement may require some analysis of the concept of 'body' if the problem which it defines is to be soluble, for it may not be the case that all bodies, whatever their nature, will behave similarly in similar circumstances. We may therefore restate the problem as that of determining the positions at all times of all the bodies in an arbitrary specified system, having given their natures, positions and motions at certain instants—leaving it to be discovered what qualities are necessary to define the 'nature' of a body, and whether 'motion' is to be represented by velocity, acceleration, or some more complicated function of its successive positions. Our problem is solved when we have obtained a formula in which the quantities we desire to know are related uniquely to those which we can here and now determine.

The traditional approach to this problem has been that adopted in a much wider field of study—namely, the assumption that each body is in itself 'inert', i.e. that unless something external interferes with it, it remains in the same state, so that, if it is observed to change its state, then it must be acted upon by an external 'force'. From this assumption it follows automatically that the inert state is that which would characterize a single body alone in otherwise completely empty space, since there would be nothing external to such a body to exert force on it. The problem treated in this way then becomes three problems: we must define a 'body', then define its 'inert state', and then describe, in terms of measurable quantities, the 'forces' that must be acting upon it to change it from the inert to the observed state.

None of these problems is as simple as it might appear. Take first the definition of the *body*. That this in general involves some difficulty may be seen best by considering a non-physical—but, in terms of the general philosophical approach to experience of which physical science is a part, a quite analogous—problem, that of human free-will. The perennial and inconclusive discussion of this problem continues because of the difficulty in reaching agreement on the definition of the 'individual'. Certain of a man's actions are regarded as the effect of his will. If his will is *sui generis*, a part of his intrinsic nature, then, with respect to those actions, he is a free agent, for it is he himself who determines them. On the other hand, if his will is the resultant of all the external forces that influence him, then his actions are externally determined and his freedom does not exist. There is still no generally agreed choice between these alternatives—the nature of *the self* is a traditional philosophical problem—and this indicates how necessary it is to have a clear definition of the body or system or whatever it may be that is under consideration before we can approach the second problem—that of specifying its inert state—and so defining what we mean by *inertia*.

Fortunately, in the astronomical problem this difficulty is reduced to a minimum. We are concerned only with discrete, easily identifiable, inanimate material bodies—the Earth, the Moon, an apple, . . .—and our definition is complete if it includes only those observable and measurable quantities that are necessary to enable us to define the 'state' sufficiently to make a solution of our complete problem possible. (It is true that, in the development of mechanics, energy has been added to matter as an equally 'real' constituent of the physical universe. That, however, pertains to a late stage in the investigation of the problem along particular lines. Initially the problem concerns only matter: if we can predict the positions and motions of all observable pieces of matter, with or without using the concept of energy, we shall have solved it.) Furthermore, it is always assumed (whether justifiably or not we are not here considering) that the mechanical motions of a body are susceptible of treatment independently of electric, magnetic, vital or other motions (electromagnetic inertia is an analogue, not a particular case, of mechanical inertia), so that, up to the present at least, it has been sufficient to state the *mass* of a body in order to define it completely enough for our purpose.

But although the difficulty of defining *body* is here at a minimum, it is not entirely absent. In the more recent literature of the subject a distinction is often made between the *inertial* mass and the *gravitational* mass of a body, which, though they are said always to be equal to one another, are yet regarded as distinct quantities which might, in fact, have been unequal. I shall refer again to this in connection with

Newtonian mechanics, and it is sufficient here to express the view that this distinction is quite unnecessary and merely introduces a complexity that serves no useful purpose. But, however that may be, we can regard the problem of defining *body* as soluble, and each investigator will doubtless regard it as solved: we may therefore pass on to the definition of the inert state.

This is a very much more difficult problem. The particular state that we are concerned with is the state of motion (including rest), and we confine our attention to apparently 'free' motions, i.e. those in which no independently observable force acts on the body considered, such as would occur if it collided with another body. (Of course, general physics must take account of such cases, but it must then amplify its definition of *body* by the inclusion of additional concepts, such as elasticity, thermal capacity, etc.) Take the Moon as an example. It has a motion: is that motion changing or remaining the same throughout a month, say? It is universally agreed to regard the Moon as an inert body: therefore, whether, during the month, it is changing its state of motion or not depends on what we regard as an inert state. If the motion which the Moon actually has is an inert state, then it is free from external forces and that motion is the natural motion of an inert body. If, however, the natural motion of an inert body is defined otherwise, then the Moon is acted upon by a force.

How, then, shall we define the natural state of motion of an inert body? Nature does not help us at all, for she provides no means of temporarily annihilating all the bodies in the universe but one: we are therefore compelled to make our own choice. Various choices have been made in the course of history. The Greeks, and Galileo at times, regarded circular motion as natural, or inertial, motion—at least for heavenly bodies; Boulliau (5) rejected this in favour of elliptical motion; Newton (6) chose rectilinear motion; Swedenborg (7) preferred motion in a spiral; Einstein (8) attempted no definition, but denied the objective existence of any non-observable disturbing forces, so that every apparently freely moving body was in fact freely moving and therefore remaining in a constant state of motion. It is futile to ask which of these is 'right', for there is no criterion by which the answer to such a question can be assessed. The one to be chosen is that which, taken together with the consequent forces necessary to change the inertial motion to that which is observed, gives the most satisfactory description of the motions actually occurring in the universe.

Certain comments may be made on this procedure. First, whatever we choose as the natural state of motion of a body, the force (if any) that changes its state must be defined wholly by its power of doing so.

We have no other justification for asserting its existence, and therefore no justification for assigning to it any properties other than those that enable it to change the inertial to the observed state of motion. That means that it may be represented simply as itself a motion which, by the principle of *composition* of motions, can interact with the inertial to produce the observed motion as a resultant.

This, however, at once introduces an ambiguity, or even a more complex indefiniteness. If the principle of composition of motions be granted, we cannot even be sure whether a body that appears to move differently from our choice for a free body is, in fact, acted upon by a force or is, nevertheless, free. For example, suppose we decide, with the ancients, that the inertial motion is circular, and we observe a body moving rectilinearly. The observed motion can then be ascribed either to the influence of a force or to the combination of two inertial circular motions in opposite directions which combine to make the body appear to move in a straight line. This combination was well known to the Arabs in the Middle Ages. If the motion is described in this way, then no forces can be postulated. It is possible to assert, of course, as Simplicius did in Galileo's dialogue (9), that a body cannot possess more than a single motion at any time, but to do so would be to reject the argument of Sagredus and return from Galileo to Aristotle; it would no longer be possible to say that the Earth had motions of rotation on its axis and revolution about the Sun. We must accept the fact that the analysis of motion into an ideal inertial component and a component caused by a disturbing force leaves a large amount of arbitrariness in our description.*

Let us turn now from the geometrical form to the magnitude of a body's motion. We have here an indefinite number of possibilities to choose from. We may define the state of motion of a body by reference to its position, its velocity, its acceleration, its rate of change of acceleration, and so on without limit. Accordingly, we may say that the amount of motion is unchanged if the position, the velocity, the acceleration, . . . of the body remains the same, and we are free to choose as we will between these possibilities. As before, it is futile to ask which is 'right': we can only recognize our freedom of choice and use it to the best advantage.

The ancients chose position: every body had its own natural place, and it was inert only when it had reached that place. If it was not there, then its urge to get there (what in later times would have been

*It should be noted, however, that Newton's inertial motion escapes this ambiguity, since all combinations of uniform rectilinear motions yield uniform rectilinear motions.

called a force) produced a motion towards the place. Positions were specifiable by reference to the centre of the universe, and since this coincided with the centre of the Earth, it was locatable by observation.

Galileo and Newton faced the problem at a time when the centre of the universe was no longer locatable, and the position of a body had become meaningless except in relation to another body. They chose the next simplest state of motion as the inertial state: a body was unacted upon by a force if its velocity remained constant at any value at all, including zero. This, of course, involved the tacit assumption that the velocity of a body was uniquely specifiable, but for practical purposes all that was necessary was that *constancy* of velocity was uniquely determinable, for, if that were granted, whatever the magnitude of the velocity might be, the forces in the universe could be uniquely determined from the observable changes of velocity which they produced.

This choice of inertial motion is expressed by Newton's first law, and its success, when amplified by the other laws and the definition of 'gravitational force', has been so great that it has been called (by Mach and others) *the* law of inertia. Submission to the temptation of regarding a successful theory as established truth is the last infirmity of noble mind in this subject, and we must accept the fact that those who—at any rate, up to the time of Einstein's gravitational theory—identify inertial motion with uniform motion in a straight line have custom and the authority of the Oxford English Dictionary as their justification. That standard work defines inertia as 'that property of matter by virtue of which it continues in its existing state, whether of rest or of uniform motion in a straight line, unless that state is altered by external forces'. In view of later work it is clear that the phrase, 'whether of rest or of uniform motion in a straight line', should have been omitted, and the literal meaning of impotence, without further specification, alone retained. Certainly, if those who now write on the subject wish to be clearly understood, they should not use the term without stating exactly what they mean by it.

Einstein, as has been said, made a fundamental change by discarding the analysis of observed motions into natural and disturbed components. A body's inertial motion (in the original sense) was its *actual* motion, so that no forces needed to be postulated. Rather paradoxically he expressed this by appearing to deny the existence of inertial motion instead of that of the disturbing component, but it is clear that what he is denying is the division of an observed, apparently free, motion into natural and disturbed parts, and from that point of view neither of the 'parts' has meaning and therefore may be eliminated without prejudice to the proper designation of the undivided whole.

When everyone is somebody,
Then no one's anybody.

Accordingly, from Einstein's point of view it means the same thing to say that there is no inertial motion as that all motion is inertial: he returns to the Aristotelian contention of Simplicius that 'To one single movable there can naturally apply but one motion and no more' (9).

In all this it has been assumed that the quantities concerned are, in principle, unambiguously determinable: we must now consider the conditions necessary to secure this. The quantities in question are position, velocity and its derivatives, and mass, and these are meaningless except with reference to standards. A position must be so far in a certain direction from a standard position; a velocity must be so fast in a certain direction from a resting standard; acceleration and higher derivatives are then made definite automatically; a mass must be so many times a standard mass. The question then arises: are these standards absolute—i.e. provided by nature, so that, although others are of course thinkable, they cannot provide measures that will fit satisfactorily into an acceptable cosmology?

Mediaeval cosmologists did not concern themselves much with measurements, but their ideas were such that we can see what they implied with regard to this question. Mass played no part in the scheme (except indirectly in such beliefs as that the motion of a falling body varies with its weight, but this was only a minor feature of the whole conception). There was an absolute position—the centre of the universe—which was made manifest by the fact that it coincided with the centre of the Earth. The Earth was absolutely at rest and, being of finite size, could therefore serve as an absolute standard of rest for both translation and rotation. There was no absolute direction: the radii of the stellar sphere were indistinguishable, and the circle described by the Sun had no more or less fundamental significance than that described by a star or any other body. There was an absolute time scale, determined by the fact that the stellar sphere revolved uniformly, and after the synthesis of Aristotelian philosophy with Christianity there was an absolute zero point of time—that of the Creation. The various modes of motion—uniform, difform, uniform-difform, . . . that corresponded to our velocity, acceleration, . . .—were thus absolutely specifiable in magnitude, and, when the Copernican cosmology made its appearance, a fundamental distinction was drawn by many between the 'reality' of the absolute structure and motions, and the merely convenient representations of them which that cosmology afforded.

With the downfall of the mediaeval system of thought, which roughly coincided with the invention of coordinate geometry by Descartes, the problems associated with coordinate systems, and the distinction between absolute and relative measurements in its modern form, had their origin. Of the various cosmological schemes that arose, two only have so far succeeded in commanding universal serious attention, namely, those of Newton and Einstein. I shall now examine these, not with the aim of giving a complete account of them, but in order to set out their basic conceptions and the meanings of the terms which they employ, for unless those terms are clearly understood and used only in their original senses (or in such modifications of them as later developments of language in general make desirable, care being taken not to introduce changes or amalgamations of distinct ideas with changes of nomenclature) nothing but confusion can result. It is, of course, open to anyone to propose other schemes, introducing other conceptions, but in that case those conceptions should be represented by other words, and those employed by the earlier writers reserved for the meanings which those writers gave them.

3. NEWTONIAN COSMOLOGY

As I have said, when Newton began his work it was no longer possible to locate in the universe any absolute position or any absolute scale of time, for the celestial spheres, and with them their centres, had been abolished, and the rotation of the Earth could not claim any absolute character as a universal clock. He had therefore to choose between postulating absolute standards which were inaccessible to direct observation and denying the existence of absolute standards. As is well known, he chose the former, distinguishing absolute time and space from the relative times and spaces that we must necessarily use when we choose particular phenomena to which to refer our measurements. The only phenomena which he felt the time was ripe to consider in this connection were mechanical motions, and his cosmology is concerned with them alone; his comments on the fundamentals of other phenomena—light, magnetism, etc.—were mainly confined to queries and speculations clearly described as such. His main problem, then, was to deduce from observations of the relative motions of bodies as much as possible about their real, or absolute, motions.

Newton's conception of space was the ordinary commonsense one that the average man still holds—a vast homogeneous emptiness in which bodies are distributed. As an objective reality it was immovable, and therefore constituted an absolute standard of rest, but it was inaccessible to direct observation through the phenomena of motions because the real motion of a body produced no observable mechanical effects. We could observe the motion of one body relatively to another, but that revealed nothing about the real motion of either. We could, however, choose some particular body and, quite arbitrarily, decide to regard it as resting. By that act we chose our *apparent* space, in which all observable motions are specifiable. It is identical with real space in everything but the fact that its motion is unknown. It is not a different entity from real space, but a partial conception of real space, just as 'the present Prime Minister of Great Britain' is not a different entity from 'Mr Harold Wilson', but a partial conception of that person. Newton is less specific about time and, although asserting a similar distinction between real and apparent time, thereafter treats them as indistinguishable.

Since all observable motions are merely differences of real motions and are the same as differences of apparent motions, it would appear to be impossible to infer real motions from observation, 'yet', says Newton, 'the thing is not altogether desperate'. What is not possible to pure reason alone may become possible if we make certain postulates which, it is true, are not *a priori* necessities, but which, if their observational requirements are found always to be verified, may be regarded as highly probable. He accordingly postulated that the real motion of a body alone in space was unchangeable. He tacitly assumes that there are identifiable absolute directions in both real and apparent spaces, and such a body, in the absence of any interference, would continue moving with constant speed (which might be zero) in the same direction for ever. It then follows that if the relative motion of two bodies is not constant in both direction and magnitude, a force must act on at least one of them, for it is impossible that both maintain a constant motion. This is something absolute. Hence, although we cannot distinguish one state of constant real motion from another, we can detect changes of real motion, or accelerations. It then follows from the postulate that we can detect real forces, a force being defined as the cause of *change* of motion.

The way is now open to the discovery of the forces acting in the universe, and Newton then proceeds to his law of gravitation. For this he has to amplify his definition of motion, to include some property of the moving body, for it is a fact of observation that the same force produces different effects on different bodies. He therefore defines 'body or mass' as the quantity of matter in the body, and as identical, except in our manner of conceiving it, with the *vis insita* or (better) *vis inertiæ*—'force of inactivity'. This seems a contradiction in terms, but receives its justification from another postulate which asserts that a body opposes an equal and opposite reaction to every force that acts on it. Thus, from one point of view the quantity of matter in a body is inert—incapable of changing the body's state of motion; from another point of view that very incapability changes the state of anything that interferes with it and therefore acts as a force on that thing.

In view of subsequent progress we can, without altering the essential conceptions, simplify this terminology by defining *mass* (a measurable quantity) as equal to the force necessary to produce unit acceleration in a body; *inertia* (a non-measurable property) as the absence of any inherent power in a body to change its state of motion; and *force* as anything that changes the state of motion of a body—necessarily external to the body, of course. Newton's scheme can then be described unequivocally and faithfully without introducing such paradoxical terms as 'inertial force'. There are other modifications, however, which are frequently made, which distort Newton's ideas and do him injustice. Two of these are the introduction of coordinate systems and the supposed distinction between inertial and gravitational mass.

Although he might have done so, Newton did not introduce coordinate systems into his scheme. All his geometry is of the pre-Cartesian type, and he never entertained the idea of imaginary reference frames; his discussion was confined to the relation between actual observations and the real but unobservable constituents of the natural world. A coordinate system is an imaginary structure of axes assumed to be at rest, and it may be situated anywhere, have any orientation and any motion with respect to an observable body. If such conceptions are introduced, then it is permissible to regard as objective reality only that which holds good for all possible choices of coordinate system, for it is a contradiction to call that objective which varies with subjective choice.

The self-imposed task thus laid on us to see that this condition is fulfilled may indeed be worth while, but it did not exist for Newton, for he did not introduce coordinate systems whose effects had then to be eliminated. He secured all the advantages of such systems that he considered he needed by his conception of the unknown motion of each single body, so that the Earth, for instance, could have any real motion at all without affecting any of his measurements. The statement frequently made that his scheme implied that only so-called 'inertial systems' were permissible is erroneous (except, of course, in a context in which it is understood that Newton's scheme is being compared with some other in coordinate language). You could choose to regard any body you wished as at rest in apparent space, the apparent spaces being as numerous as the bodies in the universe; and, when it was chosen, the motion of another body relative to it, if uniform or non-existent, showed that the force (if any) was the same on each body and could therefore be ignored in considering their relations with one another, and, if variable, showed that inevitably the forces acting on the bodies were different. In the latter case, if you chose an apparent space in which one of the bodies was at rest, then, in that space, a perfectly definite force acted on the other. But essentially, without regard to your choice, there was a precisely specifiable differential force acting between them, which was independent of the particular real force acting on each. The feature of Newton's cosmology that is mistakenly regarded as the postulation of objectively existing inertial systems is the postulation of real space, with an unknown state of motion relatively to an observable body, in which bodies have no intrinsic power to change their motion, and all his calculations are automatically free from any assumption that such motion, in the case of any observable body at all, is uniform. The other outstanding difference between this and the idea of objective inertial systems is that there is only one 'real space' but an infinite number of inertial systems if there is any at all.

As a particular example of the misinterpretation of Newton's theory, consider the following passage from a fairly recent paper (1): 'The inertial properties of matter imply that at each point of space there exists a set of reference frames in which Newton's laws of motion hold good—the so-called "inertial frames". . . . The question then arises: what determines the inertial frames? Newton asserted that they were determined by absolute space.' In fact, 'the inertial properties of matter', in any intelligible sense of the phrase at all, imply nothing whatever about the 'existence' of reference frames, inertial or otherwise; reference frames are, by their very nature, purely imaginary. Newton never even imagined them, and Einstein called inertial frames 'ghosts', which should be expelled from physics (10). The statement that Newton asserted that they were 'determined by absolute space' is thus a plain libel. The nearest he came to such a statement, so far as I have discovered, is the remark that 'it may be that there is no body really at rest, to which the places and motions of others may be referred'—which, if the body alluded to is claimed as a 'reference frame' (it appears to be the only possible candidate for this title in Newton's work), asserts its possible non-existence. We may, if we wish, express Newton's theory in new terms of our own devising, but to suppose that it is called upon to deal with problems arising only when it is so transformed is altogether illegitimate.

A subsidiary advantage of dispensing with coordinate systems is that one is then not liable to the errors arising from the identification of such systems with observers. All the observations Newton had to deal with were those obtained by Earthbound instruments—i.e. effectively those of a single observer—and it probably never occurred to him that he was under any obligation to speculate about what an astronomer on Mars might see. He had all the freedom he was entitled to, and all he needed, in the freedom to give the Earth any apparent motion he desired.

A second illegitimate criticism of Newton's cosmology is that it contains an unexplained equality of inertial and gravitational mass—an equality that ought to appear as a necessity in a satisfactory theory, but in his theory is an accidental coincidence. I have dealt with this at length elsewhere (11), and here need only point out that the distinction between gravitational and inertial mass appears only when we transform Newton's particle theory into a field theory. As with the employment of coordinate systems, it may be that the complications thus introduced are justified by the greater scope of application so given to the theory (and, of course, it may not), but it is grossly unfair to Newton to charge him with an apparent defect arising only from our distortion of his work. Newton defined only one property of a body—mass—which happens to be equivalent to what in the distorted form is called inertial mass, and he says when defining it: 'It is this quantity that I mean hereafter everywhere under the name of body or mass'. Later, in his 'Rules of Reasoning in Philosophy, he says: 'Not that I affirm gravity to be essential to bodies: by their *vis insita* I mean nothing but their inertia'. He then discovers by experiment or observation that their mutual gravitation depends on their masses as so defined—a discovery of exactly the same character as the discovery that the velocity of sound in a gas depends on the ratio of the principal specific heats of the gas. No one on that account thinks it necessary to speak of a thermal ratio and an acoustical ratio, the mysterious equality of which points to a defect in physical theory.

The mutual gravitation of two bodies, as a primary observation, is a single phenomenon. In terms of the field concept we analyse it into a field produced by one of the bodies to which the other reacts, and we then regard each body as possessing two properties, a field-creating and a field-responding property. This device was first used by Faraday in his investigations of electricity and magnetism, and there it has a justification not apparent in the problem of gravitation, for the two properties are actually separable. A magnet possesses both the field-creating and the field-responding property; a piece of unmagnetized iron possesses the field-responding but not the field-creating property; and a piece of wood possesses neither. Similarly, a circuit may or may not possess the property of responding to a current in another circuit, without possessing the power of creating a field. But gravitation exhibits no such separation. There is in every case but one force, which depends on the masses of both of the bodies concerned, and the procedure of representing this essentially relative phenomenon as the mysterious interaction of two supposititious absolute phenomena is something that itself requires justification rather than constituting a ground for criticizing the direct expression of the facts.

To sum up, then, Newton's cosmology consists of the postulation of mass and inaccessible real time and space for which, for the purpose of describing the relations between observable motions, apparent times and spaces are adequate substitutes. The motion of each body consists in general of a uniform real motion on which is superposed a variable motion arising from the forces acting on it. The real difference between the forces on any two bodies is determinable from their apparent relative accelerations and is wholly expressible in terms of the relative motion of the bodies (a mass, according to Newton's definition and all legitimate variations of it, is expressible only in terms of its relation to an arbitrarily chosen unit; the mass of a single body alone in space would have no empirical meaning) and their distance apart. Epistemologically it would seem that this scheme is not open to criticism except possibly on aesthetic grounds or on its possible failure to explain observed phenomena. It defines the meanings of inertia, mass, force, space and time—terms which should not be used with other meanings by those who wish to be generally understood.

Before leaving Newton's scheme it should be noted that, in its relation to motion, his absolute space is only another name for ether as that conception later took shape. To Newton it certainly existed; it was, indeed, the 'sensorium' of God. It would therefore be quite consistent with his philosophy if an etherial medium were found to be necessary to convey optical waves or to constitute the seat of electromagnetic phenomena: it would be equally consistent therewith if absolute space were totally inaccessible, directly or indirectly, by any kind of observation.

4. EINSTEIN'S RELATIVISTIC COSMOLOGY

Einstein departs from Newton at the very beginning. Faced with the choice which the collapse of mediaeval cosmology exposed, he denied the existence of absolute standards of space, time and motion, granting physical significance only to *relative* positions and motions. But he made a further departure from Newton's procedure which is much less generally realized. Having made a different choice of concepts out of which to construct a theory, he dealt with those concepts in a quite different way. Newton took his 'realities', accessible or inaccessible, as primary data between which he sought relations. Einstein ignored observations in the first instance, reserving them as the ultimate arbiters of the correctness of a theory which he proceeded to build up from *a priori* postulates unrestricted by any considerations of observability. Having constructed such a theory, he then correlated its concepts with observable quantities and staked the tenability of the theory on the possibility that such a correlation could be made fittingly, without defect or superfluity. He was like a modern Leonardo who, instead of producing independent works of art and of science, aimed at making the same construction serve both ends. Whether or not his theory will have greater survival value in science than Leonardo's flying machines remains to be seen, but, however that may be, it is likely to command equal respect to that paid to Mona Lisa, in the scientific Louvres of the future. (Of course, we cannot doubt that, psychologically, the goal at which he was aiming, which he could not have visualized without the knowledge that Newton's theory had brought to light, was an essential factor in determining the kind of construction he made, but logically that construction stands in its own right as a purely ideal creation.)

Since this aspect of Einstein's work is so relevant to the present subject and, as Einstein himself realized, is so little appreciated, it is necessary to call upon his own precise declarations to substantiate it. These are to be found in various writings, but nowhere, I think, so clearly as in a letter to the late Viscount Samuel, written for the direct purpose of contrasting his approach with that of Newton (12). Here are a few extracts: 'There is only one way from the data of consciousness to "reality", to wit, the way of conscious or unconscious intellectual construction, which proceeds completely free and arbitrarily. . . . These facts could be expressed in a paradox, namely that reality, as we know it, is exclusively composed of "fancies". Our trust or our confidence in our thoughts referring to reality is solely based on the fact, that these concepts and relations stand in a relation of "correspondence" with our sensations. . . . We are free to choose which elements we wish to apply to the construction of physical reality. The justification of our choice lies exclusively in our success. . . . Newton recognized with complete clarity that in his system space and time were just as real things as material points. For if one does not accept, besides material objects, space and time as real things, the law of inertia and the concept of acceleration lose all meaning.'

Einstein's construction was that of a four-dimensional field—called space-time or ether or what you will when you come to correlate it with experience, but intrinsically best called *field* or *continuum*. It was defined by a *metric*, expressed in terms of *coordinates*, of which four were necessary and sufficient to specify each point of the field. All possible coordinate systems were permissible and, one having been chosen, it was possible to express in terms of it certain special paths called *geodesics*. An infinite number of such fields are conceivable, each characterized by its own metric.

That was the ideal part of the theory: it remained to relate it to observation. For this purpose each observable distribution of bodies in space and time corresponded to a particular field, i.e. to a particular metric. But whereas, for Newton, the bodies, space and time were independent concepts, for Einstein the space and time were merely the physical correlates of components of a largely arbitrary analysis of the coordinates, and the bodies or masses were then a function of the coordinates of the points at which the bodies were held to be situated. It was not necessary, when one contemplated only the basic four-dimensional world, to choose a coordinate system and so to divide the field into parts corresponding to space and time. When that were done, for the subsequent correlation with experience, what corresponded to physical nature was simply a particular field with particular geodesics threading through it, and the actual world that confronts us was distinguished from all other possible worlds by the particular pattern of geodesics represented by it—a 'static' picture, so to speak, if one makes the analogy (it can be nothing else) of picturing the field as familiar space. If, however, as for scientific and practical purposes we must do, we adopt a coordinate system and so distinguish space from time, each geodesic becomes a particle moving from point to point of space as time goes on.

The essential feature of all this for our purpose is that its concepts are *wholly* different from those of Newtonian theory. Einstein's theory needs no other concepts than the four-dimensional field represented by a metric, coordinate systems with the corresponding coordinates, and geodesics. Newton's theory needs only inertia, mass, space and time (real and apparent), and force. Newton's theory does not need coordinates though it can use them as a calculating device in particular problems if desired; in Einstein's theory they are essential to connect the characteristics of the field with observation. Einstein's theory has no meaning for inertia, which is a primary concept for Newton since his theory begins with the analysis of motion into the inertial and the disturbed parts. The theories are fundamentally totally dissimilar, and although, since they are both intended to represent the same world of experience, some kind of correspondence must exist between their constituents, nothing but confusion can result from speaking of particular concepts as though they could belong to either. A statement such as 'the mass of the Sun is $1\frac{1}{2}$ km' is literally nonsense, for by the very definitions of mass and kilometre it is impossible that one shall be expressible in terms of the other. What it means is that the characteristic of the observable object known as the Sun which is represented in Newton's theory by a mass of 2×10^{33} g is represented in Einstein's theory by a length of $1\frac{1}{2}$ km. 'Since the days of Faraday and Maxwell', wrote Einstein (**12**), 'the conviction has established itself that "mass" has to be replaced by "field" as a basic element or brick for constructing "reality". . . . The programme of the field-theory has the great advantage that it makes a separate concept of space (as distinguished from space-content) superfluous. The space is then merely the four-dimensionality of the field, and no longer something existing in isolation. This is an achievement of the General Theory of Relativity which, so far, seems to have escaped the attention of the physicists.' The papers which have made this one necessary show that it has escaped the attention of the astronomers also.

It is pertinent here to add a note on the so-called 'Mach's principle', since this is a focal point of much of the confusion of Newtonian with Einsteinian ideas. Mach, in principle, was wholly Newtonian. He accepted Newton's cosmology completely, with the one exception that the standard of reference implied in Newton's first law, which according to Newton was absolute space, was in Mach's view the whole system of bodies in the universe (13). This was relativistic in the sense that it abolished unobservable standards of reference, and it was compatible with all the observations that had led Newton to believe that he had detected acceleration with respect to absolute space, but it fell short of Einstein's postulate of relativity, according to which there was *nothing* in nature, observable or unobservable, corresponding to the idea of absolute rest. There is an element of contingency in Mach's postulate because it leads one to suppose that if the matter in the universe had happened to be differently distributed, a free body in space would no longer move uniformly, i.e. Newton's first law of motion would no longer hold. It is possible that this predisposed Einstein to abandon his youthful adherence to Mach's view, but his ultimate dissent came from his fundamental disagreement with the whole Newtonian–Machian scheme of scientific thought which I have indicated. 'In my younger years', he wrote towards the end of his life, 'Mach's epistemological position also influenced me very greatly, a position which today appears to me to be essentially untenable. For he did not place in the correct light the essentially constructive and speculative nature of thought and more especially of scientific thought.' (14). It is clear, therefore, that any attempt to associate 'Mach's principle' with Einstein's general relativity is a sure sign of basic misunderstanding.

5. INERTIAL REFERENCE FRAMES

Implicit in the foregoing considerations is the meaning properly to be attached to the term, *inertial frames of reference*, but in view of the frequency with which this term is used, and the confusion arising from the unjustified meanings which have been given to it, it is necessary to give it explicit attention. I have dealt elsewhere (15) with the almost limitless variety of uses of this term by physicists; here I illustrate the same phenomenon by reference to two recent discussions by astronomers—those of G.M.Clemence entitled 'Inertial Frames of Reference' (3) and P.A.Wayman entitled 'Determination of the Inertial Frame of Reference' (4). (The contrast between singular and plural in these titles, though regrettable, is of minor significance; I think both authors would agree that any frame of reference moving uniformly and rectilinearly with respect to an inertial frame of reference, if such exists, would itself be an inertial frame, though I suspect that both, for a reason that would well repay reflection, would regard one particular member of the infinite company as having special significance.) I propose briefly to consider certain expressions used in these discussions in the light of the following facts:

(1) A frame of reference is an arbitrary standard with respect to which the positions and motions of observed bodies can be assigned definite and unique values. It is itself, by definition, at rest, for if it be supposed to have a motion, of either translation or rotation, that supposition necessarily implies another standard, which is at rest, with respect to which it has that motion, and this second standard then becomes the one with respect to which the motions assigned to the observed bodies are referred. For example, if a body is observed to move eastward at 5 metres a second with respect to the laboratory, and the laboratory is chosen as a standard and postulated to move at 20 metres a second westward, then the velocity of the body must be stated as 15 metres a second westward, and that is a velocity with respect to a resting standard.

(2) The motions of all astronomical bodies are, at present, necessarily observed with respect to the Earth, but we can choose a frame of reference with respect to which the Earth has any motion we please, and then, from the observations, we can assign a motion to each body with respect to that frame.

(3) If the principle on which Newtonian mechanics is based—i.e. the principle that the motion of a body is analysable into a *natural*, or *inertial*, motion which it would possess if undisturbed, and a motion resulting from the disturbance of the body by an external force or forces—is valid, then there may be supposed to exist in nature a reference frame (an *inertial* frame) with respect to which an undisturbed body has that inertial motion. In Newtonian mechanics such motion is uniform and rectilinear and of indefinite magnitude. An inertial frame in Newtonian mechanics would thus, by definition, be a frame with respect to which a free body rests or moves with constant velocity in a straight line, and any departure from such a state indicates the existence of a force.

(4) In relativity mechanics motion is *not* uniquely analysable into inertial and disturbed parts (though one may make any such analysis

for purposes of convenience) and the inertial frame becomes, in Einstein and Infeld's words (10), a 'ghost' which should be expelled from physics.

In the light of these facts, consider the following definitions, expressed or implied, at the beginning of Clemence's and Wayman's investigations: 'An inertial frame of reference is defined as a frame which is free from all accelerations, linear and rotational' (Clemence (3)). 'Basically the only way to determine the inertial frame is to analyse the motions of a mechanical system, artificial or occurring naturally, sufficiently thoroughly to identify, to the required degree of accuracy, the coordinate system which is to be regarded, in Newtonian mechanics, as non-rotating'—from which it may be assumed that an inertial frame is defined as a frame which, in Newtonian mechanics, is non-rotating (Wayman (4)). It is at once clear that these are definitions, if of anything, of any reference frame at all, for, as is stated above, a reference frame which was not free from accelerations (in Clemence's terms) or rotations (in Wayman's terms) would merely imply the existence of one which was, if its acceleration or rotation were specifiable, and, if it were not, the frame would be completely unidentifiable.

Next, if one supposes that an inertial frame exists, one is necessarily committed to the acceptance of Newtonian mechanics (or at least to the fundamental principle on which Newtonian mechanics is based; but it is reasonable to suppose that all other choices of inertial motion than that of Newton can now be ignored) and a rejection of relativity mechanics. Discrepancies between Newtonian mechanics and observation—e.g. that associated with the perihelion of the orbit of Mercury—can then have only two possible explanations. Either the inertial motion has not been properly identified, or the forces acting on bodies have not been correctly or completely specified. What is not permissible is to introduce some *deus ex machina*, however respectably disguised, in order to maintain the legitimacy of Newtonian mechanics notwithstanding its disagreement with observation. But that is just what Wayman does. Having adopted Newtonian mechanics in order to give a meaning to an inertial frame, he says that in seeking it 'we ought to consider the significance of relativity', and goes on to speak of 'relativistic corrections to Newtonian dynamics'. But that is quite illegitimate. Relativity mechanics is not Newtonian mechanics plus a correcting factor; it is a completely independent system. If you choose one you necessarily discard the other wholly, and no combination of them, of any kind at all, has any meaning.* Of course, it does not

*It might be remarked that, although relativity mechanics seems at present to have the advantage with respect to agreement with observation, the differences between it and Newtonian mechanics are so slight and uncertain (16) that it would be very unwise to assume that this advantage is final.

follow from this that, for practical purposes, there is any objection, if you adopt relativity mechanics, to making your first calculations by Newton's theory and then adjusting the results by applying what may be called a 'relativity correction'—so long as you do not imagine that this has anything to do with 'the significance of relativity'; nor does it follow that the investigations of Clemence and Wayman are meaningless. Whichever mechanics you adopt you have to make a choice—of the motion you assign to the Earth, the necessary observing place, in Newtonian mechanics, and of the coordinate system in relativity mechanics—and the choice will be determined by considerations of simplicity. It is important to make the simplest choice, not only from an acceptance (if you do accept it) of Newton's belief that 'Nature is pleased with simplicity', but also because the true system of mechanics, whatever it may be, is most likely to be discovered when the observations are described in the simplest terms. On the potential value of Clemence's and Wayman's work I am therefore making no comment at all; I am merely saying that whatever value it possesses is merely obscured by the terms in which it is expressed. This is not a trifling matter; on the contrary, it is of the most basic importance, for we shall never reach the truth in anything unless we keep our conceptions and language clear and definite. As a single example of the fundamental errors that can arise when we fail to do this, the work of Sherwin (17) may be cited, in which it is claimed that the existence of inertial frames has been experimentally established, and, at the same time, that this verifies a prediction of Einstein, to whom inertial frames were ghosts. To such an approach to the subject all great illusions are possible.

6. CONCLUSION

Neither Newton nor Einstein can be assumed to have said the last word on this matter. It is open to anyone to approach the problem afresh, and to introduce new conceptions which he may find useful. But if he wishes to be understood by those familiar with the history and present state of the subject he will respect the meanings that Newton and Einstein gave to the concepts which they employed, and neither confuse them nor tacitly change them to something which the reader is left to divine. If Newton had been asked what he considered to be the origin of inertia he would doubtless have replied that the only answer to such a strange question would be that everything in nature—its material and its laws—originated in the work of God, and if you wanted to know about that you should read the Bible and not his Principia, which only tried to discover 'the true motions from their causes, effects, and apparent differences, and the converse. . . . For to this end it was that I composed it'. Einstein would have denied that there was anything called inertia that could have an origin, and if asked what he considered to be the origin of space-time, he would have replied, with a modest blush, that 'it was just an idea of mine'—'for which, however,' he would have hastened to add, 'I was much indebted to Minkowski'. The point of view from which inertia and space-time appear as objective entities whose origin it is an intelligible task to discover, is so inaccessible to those who have understood their only legitimate meanings as to make discussions of that task almost impossibly difficult to follow. If such discussions were translated into justifiable language it might well be that they would require fewer words and equations (perhaps even none at all) and so become more generally intelligible and more conducive to edification. I recommend this as a desirable objective to future writers on these subjects.

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