RADIAL SYSTEMS IN THE HEAD OF THE COMET 1908 III (MOOREHOUSE)

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A. Eddington [1] studied the evolution of envelopes in the head of the comet 1908 III and concluded that the envelopes were formed exclusively by massive repulsive accelerations due to the sun (from 700 to 16,000 in the units of solar gravitational acceleration) with initial emission velocities of molecules from the nucleus of from 10 to 100 km/sec. Since accelerations in the tails of comets (found from the motion of cloud formations) are only 200, with initial velocities of about 1 km/sec, accelerations in the heads of the comets should be about unity, and the velocity less than 1 km/sec.

The author [2] has shown that the solution of the problem can be reduced to two equations with three unknowns and is therefore indeterminate.*

Eddington used only one of the possible solutions ($M_2-M_1=\tau_2-\tau_1$). If one assumes that the nuclei of comets rotate and that the emission of molecules from the nucleus is due to thermal energy then the values $1 + \mu < 5$ are physically admissible, the initial velocities being $q < 1$ km/sec. The spectra of the ends of envelopes, as well as, in general, of the heads of comets, should consist of CN (or C_2) bands.

A. Eddington [1] succeeded in tracing the appearance and evolution of envelopes in the head of the comet 1908 III. His investigations were helped by the large scale photographs (reflector D = 30" and 1 mm = 1') and the very short exposures (about 10^m). The top of the envelope appeared first, then the envelope itself, extending left and right, entered the nucleus and enveloped it with its rapidly lengthening ends. These ends, like leaves of a fan, quickly crossed and finally merged with the tail, and the envelope disappeared. At the same time, at the point where the top of the first envelope was formed, the top of the second new envelope appeared and this went through an identical evolutionary cycle.

In order to explain these shifts, Eddington [1] assumed that the emission of molecules from the nucleus took place in bursts. At any moment in such a burst the initial velocities of all the molecules emitted in all the possible directions will be close to each other because otherwise the sharp outlines of envelopes would not appear. The velocities of emission of molecules towards the end of an outburst systematically decrease. Two or more envelopes could be observed at the same time.

Near the head of the comet, the force field due to the sun is practically uniform and this means that the molecules emitted from the nucleus with velocities g move under the action of a repulsive acceleration $1 + \mu$ due to the sun and along parabolas relative to the comet. For this reason the outline of the head of the comet is also a parabola, with the nucleus as the focus, and the head itself is a paraboloid of revolution [2].

The equation of the envelope in terms of cometocentric coordinates ξ, η (the origin of the coordinate system being at the nucleus while the ξ-axis is along the radius vector and the η-axis is perpendicular to both the ξ-axis and the line of sight) has the form

$$\eta^2 = 2\xi\eta_0 + \eta_0^2,$$

*This indeterminacy is quite understandable on purely mechanical grounds: it is impossible to determine acceleration from two measurements of position (in Eddington's case, two positions of the envelope and the interval of time between them).

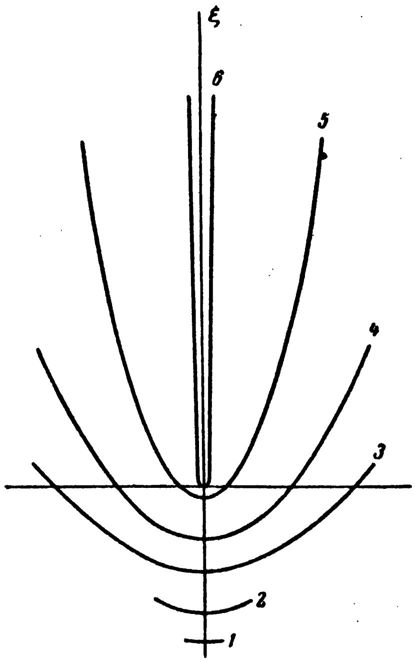

Fig. 1.

where $\eta_0 = \frac{g^2}{R}$, and $R = \frac{k^2(1+\mu)}{r^2}$.

The time τ necessary for a particle to reach the envelope after emission from the nucleus is given by

$$\tau = \sqrt{\frac{2}{R}}\sqrt[4]{\xi^2+\eta^2}.$$

The envelopes are undoubtedly gaseous and the repulsive forces due to the sun are none other than the radiation pressure and therefore the quantities $1 + \mu$ should be discrete, and we are justified in assuming that any separate envelope is formed only by one definite repulsive force due to the sun.

If one takes two photographs of the head, one after another, at the moments M_1 and M_2, and on each of them one determines the coordinates ξ and η for a number of points and hence $\eta_0 = -\xi + \sqrt{\xi^2 + \eta^2}$, then it is always possible to find points having the same η_0 on both the first and the second photographs.

Since $\eta_0 = g^2/R$ and R is the same for all points in an envelope, it follows that g (the initial velocity) increases with increasing ξ (cf. Table 2), for example, points 5 and 11 for which $\eta_0 = 0.20 \cdot 10^{-3}$. Molecules at these points are emitted from the nucleus with the same velocity ($g^2 = R\eta_0$) and therefore, according to Eddington, simultaneously (Table 2).

It is therefore possible to write

$$\tau_1 = \sqrt{\frac{2}{R}}\sqrt[4]{\xi_1^2+\eta_1^2} \quad \text{and} \quad \tau_2 = \sqrt{\frac{2}{R}}\sqrt[4]{\xi_2^2+\eta_2^2}.$$

The quantities τ_1, τ_2 and R are unknown to us but since, according to Eddington, the molecules are emitted from the nucleus simultaneously $\tau_2 - \tau_1 = M_2 - M_1$ and from the formula

$$M_2 - M_1 = \sqrt{\frac{2}{R}}\left(\sqrt[4]{\xi_2^2+\eta_2^2} - \sqrt[4]{\xi_1^2+\eta_1^2}\right)$$

it is possible to calculate R and consequently $1 + \mu$ and g. Eddington [1] obtained the values given in Table 1 for $1 + \mu$.

TABLE 1

Data	$1+\mu$
September 17	4 000
October 2	730
2	930
14	8 000
15	10 500
27	12 000
27	11 000
27	16 000
27	12 000
November 3	700
10	3 500

These enormous values of accelerations $1 + \mu$ (from 700 to 16,000) and initial velocities g (from 10 to 100 km/sec) differ considerably from the well known values of $1 + \mu$ and g in the tails of comets (obtained from motion of cloud formations; $1 + \mu$ from 25 to 200 and $g = 1$ km/sec approx.) and in the head ($1 + \mu$ from 0.5 to 4.5 and g about 1 km/sec).

Eddington himself thought that the values he obtained were improbable, but he could not find any other suitable mechanism which could explain the observed shifts of envelopes.

The mechanism of the bursts remained incomprehensible (enormous velocities of up to 100 km/sec, their systematic decrease, and the strange property of giving the same velocity to the emitted molecules to begin with [the tops of the envelopes usually appear at the same distances from the nucleus]).

Apparently, there is only one admissible mechanism which leads to the emission of molecules from the surface layers of nuclei, i.e., their thermal energy. It should be noted that at a given temperature an appreciable number of molecules have thermal velocities which do not differ very much from the most probable velocities,

TABLE 2

	$\zeta \cdot 10^{-8}$	$\eta \cdot 10^{-8}$	$\sqrt{\zeta^2+\eta^2} \cdot 10^{-8}$	$\eta_0 \cdot 10^{-8}$
		17.3417 September		
1	−0.062	0.20	0.209	0.26
2	+0.42	0.55	0.687	0.28
3	1.59	1.06	1.910	0.32
		17.3792 September		
4	0.240	0.162	0.163	0.14
5	0.648	0.539	0.843	0.20
6	1.47	0.848	1.697	0.23
7	3.73	1.617	4.069	0.34
8	5.21	2.004	5.578	0.37
		17.4196 September		
9	0.258	0.248	0.359	0.10
10	1.250	0.649	1.408	0.16
11	2.595	1.039	2.795	0.20
12	5.857	2.013	6.193	0.34

which at a distance r from the sun are given by

$$g = \frac{2.02}{\sqrt{M}\sqrt[4]{r}}\ \text{km/sec} \qquad \text{or} \qquad g = \frac{2.51}{\sqrt{M}\sqrt[4]{r}}\ \text{km/sec},$$

the first of these expressions being applicable to surface layers which transmit thermal energy into the nucleus, and the second to completely nonthermally conducting surfaces. For C_2N_2 ($M = 52$) at a distance $r = 1.3$ AU from the sun (September 17th) we have $g = 0.3$-0.5 km/sec while Eddington obtained 10-100 km/sec.

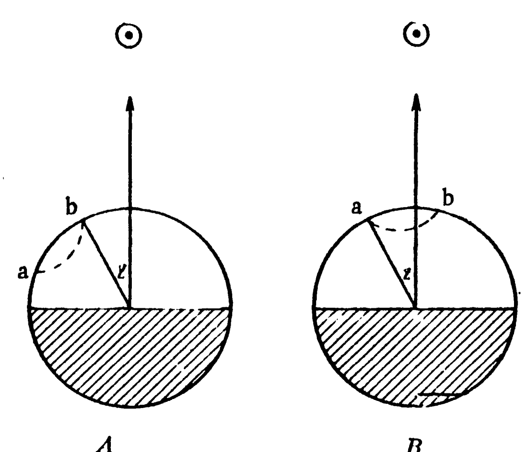

Fig. 2.

TABLE 3

$1+\mu$	$\eta_0 \cdot 10^{-8}$	$\Delta\tau$	g, km/sec
3 000	0.20	0^h04	20
1 000	0.20	0.07	12
500	0.20	0.10	7.4
5	0.20	1.0	0.83
1	0.20	2.22	0.37

In the case of the rotation of the nucleus and the thermal velocities a repetition of the same values of g after finite time intervals is inevitable. Then $M_2 - M_1 < \tau_2 - \tau_1$ and from all these possible values one must choose the admissible one. Here it is better to replace $\tau_2 - \tau_1$ by $\Delta\tau$.

Suppose that on the surface of the nucleus there is a region ab where a preferred emission of molecules occurs. The highest temperature in this region will occur when its center is as close as possible to the radius vector, and this will, of course, also correspond to maximum speed of molecules (Figure 2). We may assume that the degree of heating at various points of the region will be proportional to cos z (Figure 2). In the position A, the point b, heated by solar rays, will have a definite temperature and therefore also definite emission velocities, and the point a will for a time have the same temperature. In this case $\Delta\tau$ will not be equal to $M_2 - M_1$. Herein lies the solution to the problem of the enormous accelerations obtained by Eddington. In fact, we have two equations with three unknowns R, g and $\Delta\tau$:

$$\eta^2 = 2\xi \frac{g^2}{R} + \frac{g^4}{R^2} \quad \text{and} \quad \Delta\tau = \sqrt{\frac{2}{R}}\left(\sqrt[4]{\xi_2^2 + \eta_2^2} - \sqrt[4]{\xi_1^2 + \eta_1^2}\right).$$

From all of the possible solutions (cf. Table 3) we should choose only those which are physically admissible assuming that the reason for the emission of molecules is thermal energy of the surface layers of the nucleus. Hence g cannot be greater than 1 km/sec and $1 + \mu$ cannot be greater than 5 (cf. Footnote on page 231).

A few solutions of the equation

$$\Delta\tau = \sqrt{\frac{2}{R}}\left(\sqrt[4]{\xi_2^2+\eta_2^2} - \sqrt[4]{\xi_1^2 + \eta_1^2}\right)$$

are given in Table 3 with $\eta_0 = 0.20 \cdot 10^{-3}$ and arbitrary values of $1 + \mu$ (3000, 1000, etc.).

I would maintain that enormous accelerations and large initial velocities are not in fact present. Radial systems are nothing but the ends of envelopes with the associated small repulsive accelerations ~1 and initial velocities ~1 km/sec. In the spectra of these ends of envelopes one should see CN or C_2 bands just as, in general, in all heads of comets.

LITERATURE CITED

[1] A. Eddington, Monthly Notices, Roy. Astron. Soc. 70, 442 (1910).

[2] S.V. Orlov, The Head of a Comet and the Classification of Cometary Forms (State Technical Press, 1945), p. 31.

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