ON NUCLEAR REACTIONS OCCURRING IN VERY HOT STARS. I. THE SYNTHESIS OF ELEMENTS FROM CARBON TO NICKEL

F. HOYLE*

MOUNT WILSON AND PALOMAR OBSERVATORIES
CARNEGIE INSTITUTION OF WASHINGTON
CALIFORNIA INSTITUTE OF TECHNOLOGY

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ABSTRACT

The present paper aims to show that the abundances of the chemical elements over the portion of the periodic table from carbon to nickel are consistent with the view that the elements originate at the high temperatures that probably occur in the interiors of certain types of star. The argument takes its most definite form in the discussion of the synthesis of elements below sodium, where it seems that the abundances are in fairly close accord with the known properties of the various nuclei. For sodium and heavier elements sufficiently precise nuclear data are not available, and hence the argument becomes more qualitative than quantitative. Nevertheless, the general nature of the agreement between calculation and observation for these latter nuclei makes the discussion of them worth presenting.

The early parts of the paper are concerned with the relation to cosmogony of the theory of the origin of the elements.

I. INTRODUCTION

The purpose of the investigation of which this paper forms a part is to examine the changes of chemical composition that occur when material inside a star is heated to higher and higher temperatures. The heating of the material is due to the gravitational energy released by the shrinkage of the star. In this connection it may be noted that, so long as the shrinkage is not extremely rapid (i.e., not in a few hours, minutes, or seconds), the condition of hydrostatic equilibrium must be satisfied in a high degree of approximation. A generalized form of the virial theorem can then be established, requiring that the total gravitational potential—measured positively—must be equal to twice the total heat content of the star. Thus, since the gravitational potential of the star increases during shrinkage, the heat content (and hence the temperature) must increase correspondingly. The temperatures under consideration will lie considerably above the temperatures occurring in ordinary main-sequence stars and are to be thought of as temperatures attained in a collapsing sequence following the exhaustion of hydrogen in luminous stars.

Perhaps the best observational indication of the existence of collapsing stars comes from the faint blue stars present in the globular clusters and also in regions of the sky near the galactic pole. These stars are of type II.

Now supernovae of type I occur among the type II stars. Although it is not known with certainty where in a color-magnitude diagram the presupernova state lies, the balance of evidence would favor the view that it should be taken as among these same blue collapsing stars. That is to say, the same stars in which exceptional nuclear reactions occur at very high temperatures may well be the stars that by explosion come to scatter their materials into space. It is suggested that it is by this process that elements other than hydrogen and helium are built up and distributed in the universe. The present paper is, however, restricted to a discussion of elements in the periodic table ranging from carbon to nickel.

This suggestion derives some plausibility from the circumstance that the total quantity of material passing through the process can be seen to be roughly of the required

* Normally at St. John's College, Cambridge.

amount. Let us begin the argument with the globular-cluster stars. Recent observational evidence[1] indicates that the number of stars in active evolution in a typical globular cluster is of the order of 30,000. Moreover, the evidence suggests that the number of star deaths that have occurred in a globular cluster is probably also of the same order—the argument for this being that the observations require the number of star deaths in the *next* three billion years to be of the order of 30,000, and presumably the number that occurred during the *last* three billion years was of a similar order. Next, we may say that if the difference in luminosity between the type II population in M31, for example, and the luminosity of a typical globular cluster is due simply to a difference in the number of evolving stars in the two cases and not to some systematic difference between the stellar populations, the comparison requires the number of evolving stars in M31 to be of the order of 3×10^8, the number in our own Galaxy being by implication of a similar order. On this basis the total number of type II star deaths in M31, and in the Galaxy, must have been about 3×10^8. A last step would be to say that *if* each star death produced an explosion that distributed an average of 1 solar mass in space, then the total mass involved in the process would be of the order of $3 \times 10^8 \odot$. This mass, in the form of elements heavier than helium, is close to the amount that is required to explain their observed total abundance in the Galaxy.

Objection can be taken to the last step, however, on the grounds than 3×10^8 supernovae of type I is too great a number to have occurred in the history of even a large galaxy such as our own. Thus, even if we take the age of the Galaxy to be as high as 10^{10} years and even if the type I supernova rate is taken as an average of 1 per 300 years, the total number of supernovae of type I is still only about 3×10^7.

The explanation of this discrepancy is probably that the masses of the majority of dying stars are of the order of $1.2\odot$, whereas only a minority of stars with masses exceeding Chandrasekhar's limit (about $1.44\odot$ for stars without hydrogen) become supernovae. Thus most dying stars probably evolve without explosion, the evolution proceeding smoothly until the white-dwarf state is reached. This view is supported by the circumstance that, although 3×10^8 is too large a number for the supernovae that have occurred in the Galaxy, it is not too large for the number of stars that have evolved into white dwarfs.

These considerations suggest that the material provided by supernovae of type I should probably be placed at about $3 \times 10^7 \odot$, not at $3 \times 10^8 \odot$, and this reduced estimate is now too low by a factor of about 10. This may be taken as showing that supernovae of type I alone are not adequate to explain the total abundance of all elements heavier than helium. The supernovae of type I are indeed adequate to explain the total abundance of elements heavier than neon, but not that of the carbon, oxygen, and neon group, which exceeds the total abundance of heavier elements by a factor of about 10, this being just the order of the discrepancy revealed by the present discussion.

An attractive resolution of the difficulty would be to argue that the bulk of the lighter elements—carbon, oxygen, and neon—are produced in ejecting stars other than the supernovae, in particular, in the ejecting stars of population I.

It should be stressed at this stage, however, that the theory developed below is in no way dependent on the acceptance of such cosmogonic speculations. The theory deals with the nuclear processes that arise in stellar material at high temperatures (about 10^8 ° K), irrespective of the processes of evolution that cause such temperatures to arise. Nevertheless, discussions of the origin of the elements raise so many questions, both cosmogonic and cosmological, that some consideration of them, by way of introduction, does not seem out of place.

With this in mind, it is perhaps permissible to go on to a brief discussion of the particular cosmological framework into which the writer would seek to fit the present theory.

[1] I am indebted to A. R. Sandage and H. C. Arp for information concerning the results of their counts of stars in globular clusters.

This is outlined in Figure 1. The galaxies, which are taken as forming from an extragalactic cloud, are shown as consisting of interstellar gas (this is a nearly negligible component in the case of elliptical galaxies), ordinary noncollapsing stars, and collapsing stars. Heavy elements are generated and scattered into space by the latter, or by a proportion of them. The scattering process may lead either to the heavy elements being trapped inside interstellar gas or to their escaping entirely from the parent-galaxy into the extragalactic medium. In this connection it may be noted that the velocities of ejection from many exploding stars—the novae and supernovae in particular—considerably exceed the velocities of escape from the galaxies. Consequently, unless stopped by collisions with interstellar gas, the material ejected from such stars must move out into extragalactic space. Trapping of ejected material by interstellar gas may be taken as considerable in the spiral galaxies but not in the ellipticals.

The enrichment in this way of the heavy-element component of an extragalactic gas cloud means that even if the cloud were initially pure hydrogen, only the first galaxies formed could be of pure hydrogen. Subsequent galactic condensations must contain

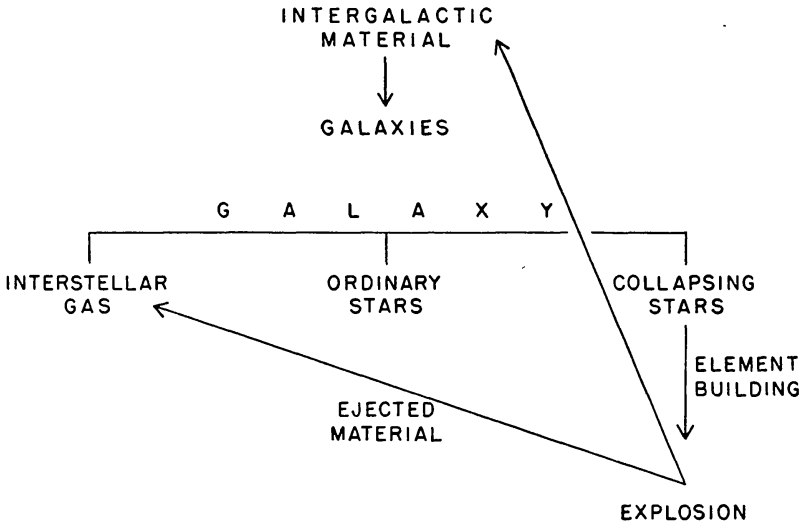

FIG. 1.—The general cosmological framework assumed for this discussion

heavy elements ejected from the "first" galaxies. Strictly speaking, the concept of "first" galaxies can be applied only in cosmologies that assign a finite age to the universe. In the steady-state theory of the universe, on the other hand, there are no "first" galaxies in the present sense, since, in this theory, the universe has an infinite past. Consequently, at all times the extragalactic material must contain heavy elements that were ejected from previously existing galaxies. Accordingly, all galaxies should possess heavy elements at the time of their birth.

An interesting estimate can be made on the basis of the steady-state cosmology. Let M be the total mass of the galaxies within a volume V, which is taken large enough to contain a reasonable average sample of the universe (say of dimensions 10^7 psc). As an order-of-magnitude estimate, we take M/V to be 10^{-29} gm/cm^3. Then, since in the steady-state cosmology[2] the mean space density of matter is 10^{-28} gm/cm^3, the mass of the uncondensed material in the volume V must be about $10M$. Now the extragalactic material is renewed (according to the theory) by continuous creation of new hydrogen in a time equal to one-third of Hubble's constant T, now estimated as 3.6×10^9 years, the

[2] A correction has to be applied to the numerical values given by Hoyle (1948), so as to take account of Baade's new distance scale.

older material being dispersed by expansion. Accordingly, effective enrichment with heavy elements must be regarded as confined to the time scale of order T.

Next, let the material condensed into galaxies produce in time T an amount $M\kappa$ ($\kappa \ll 1$) of heavy elements. Suppose, further, that, *on the average*, half these heavy elements are retained within their present galaxies and half are distributed within the extragalactic medium. Then the average concentration of heavy elements becomes $\kappa/2$ within the galaxies and $\kappa/20$ in the extragalactic medium. The latter concentration determines the heavy-element content of a galaxy at the time of its origin.

Consider now a galaxy, such as our own, that contains considerable quantities of interstellar material and hence must manage to trap an appreciable fraction—say a half—of the heavy elements that are produced within it. The initial heavy-element concentration at the time of formation of the galaxy would be about $\kappa/20$, as just indicated. This represents the heavy-element concentration of the first stars to form. After a time T, however, the heavy-element concentration rises to about $\kappa/2$, owing to the trapping process. Hence stars that form at a time of the order of T after the birth of the galaxy must contain about 10 times the heavy-element concentration of the oldest stars. This result would appear to be in good agreement with the difference of composition between stars of type I in the Galaxy and the oldest stars (probably represented among near-by stars by the subdwarfs). The present result is independent of κ and M but depends on the fraction of the material of the universe that is condensed into galaxies and on the assumption that half the heavy elements are, on the average, retained within their present galaxies.

II. PRELIMINARY DATA

Later calculations are dependent on certain theoretical data that will now be itemized.

1. The chemical composition of the material ejected from a hot star depends on the maximum temperature, τ, to which the material was heated during the collapsing phase of the star. To obtain theoretical estimates of the relative abundances of the elements, it is therefore necessary to specify a heating function whose nature will now be explained. Take an average over all the material ejected from a large number of stars, and write $H(\tau)d\tau$ for the fraction of the material heated prior to ejection to temperatures between τ and $\tau + d\tau$. Then the dependence of $H(\tau)$ on τ is a necessary datum of the theory. Unfortunately, this dependence cannot be specified from observation, although it might be said that for $\tau > 10^{8}\,^{\circ}$ K, $H(\tau)$ probably decreases with increasing τ. In the sequel it will be assumed that

$$H(\tau) \propto \frac{1}{\tau^2}. \tag{1}$$

The calculated abundances are not particularly sensitive to the assumed form of $H(\tau)$. It should, however, be pointed out that form (1) is a somewhat favorable choice for the theory. That is to say, other choices for $H(\tau)$ might be made that would not turn out to give such good agreement between the theoretical and the observed abundances of the elements.

2. It will be assumed that there is no upper limit to τ, or, more precisely, that there is no upper limit less than $5 \times 10^{9}\,^{\circ}$ K. The physical basis for this assumption rests on Chandrasekhar's limiting mass for degenerate configurations. When hydrogen is absent, a nonrotating star cannot be supported in hydrostatic equilibrium by degeneracy pressure alone when its mass is greater than about $1.44\odot$. This means that an unlimited degree of collapse occurs in such cases—unlimited, that is to say, when angular momentum and general relativity effects are ignored.

To understand this conclusion clearly, it is necessary to take account of several further points. The statement that hydrostatic equilibrium cannot be satisfied in hydrogen-poor, nonrotating stars with masses greater than $1.44\odot$ applies only when the internal temperature of the star is zero. At a finite internal temperature, equilibrium can be maintained. Now in actual stars there is always a finite internal temperature, and, except in

very unusual circumstances (which will form the background of the second paper of this series), this temperature is such as will maintain equilibrium in a high degree of approximation. Equilibrium can only be exact, however, if the energy escaping from the surface of the star is exactly balanced by a process of nuclear-energy generation in the interior. During certain periods in the history of the star this condition is satisfied, but it cannot be satisfied indefinitely, for the reason that there are no permanent sources of energy. Thus a finite internal temperature can, and usually does, prevent catastrophic shrinkage (in a few hours, minutes, or seconds), but it cannot prevent a slow steady collapse, except during finite periods of time when suitable nuclear sources of energy are available. The only way in which permanent equilibrium can be achieved is through degeneracy pressure at zero temperature, there being then no escape of radiation from the surface of the star; but even this is impossible in hydrogen-poor, nonrotating stars with masses greater than about $1.44\odot$. Hence the indefinite collapse of such stars.

These remarks refer only to nonrotating stars. Indefinite collapse cannot occur in rotating stars without highly flattened configurations arising, and such configurations are believed[3] to be rotationally unstable. Rotational instability causes material to be ejected from the star, as is required by the present theory. Now the moment of onset of the instability must depend on the initial angular momentum of the star. This may be taken as being variable from one star to another, thereby leading to a corresponding variability in the moment of onset of the instability. It is just this variability that introduces temperature differences between one exploding star and another, these temperature changes being contained in the heating function $H(\tau)$. A high value of τ, it may be noted, corresponds to a comparatively high degree of shrinkage and hence to an initially small angular momentum.

3. We shall also require the relation between the density ρ and the temperature T maintained during the collapse of a star. A star of mass $1.5\odot$, with uniform molecular weight $\frac{4}{3}$, built on the Cowling model, would satisfy the relations

$$\rho \sim 10^6 T^3, \qquad T \propto \frac{1}{r}, \tag{2}$$

at its center, where ρ is in grams per cubic centimeter, T is in units of $10^9\,^\circ$ K, and r is the radius. Relation (2) will be used throughout most of the subsequent work, on the basis that, first, the stars under consideration will be regarded as not differing much in mass from $1.5\odot$; second, the molecular weight in the absence of hydrogen can vary only from $\frac{4}{3}$ to about 2, in which range there is not an extensive change in the relationship of ρ to T; and, third, the Cowling model is probably fairly representative of the density distribution within collapsing stars, at any rate up to values of T of order $5 \times 10^9\,^\circ$ K. Moreover, in the Cowling model ρ/T^3 does not vary at all sensitively with distance from the center of the star, so that relation (2) can be used as a general approximation throughout the main mass of the star. In addition, most of the later calculations will be rather insensitive to the exact correspondence between ρ and T. In an exceptional case, however, where a sensitive dependence does indeed arise, we shall write $\rho = \alpha 10^6 T^3$ and shall consider different values of the parameter α separately.

III. NUCLEAR-PHYSICS REQUIREMENTS

Nuclear reaction rates when levels are widely spread.—In an assembly at kinetic temperature T the number of collisions per unit volume per unit time in which nuclei of

[3] Highly flattened rotating configurations are certainly rotationally unstable when the material is incompressible. Some doubt has been expressed as to whether the same conclusion also applies when, as in the stars, the material is compressible. In a recent lecture at the California Institute of Technology, I expressed the opinion that the unstable property is particularly likely to hold if a magnetic field happens to be present. This opinion is supported by the calculations of S. Chandrasekhar, who has kindly informed me that a rotating mass of gas tends to behave rather as an incompressible fluid when a strong magnetic field is present.

species 0 and 1 have a collision with kinetic energy, relative to their mass center, that lies between E and $E + dE$ is given by

$$\frac{4n_0n_1E\exp(-E/kT)}{(2\pi m)^{1/2}(kT)^{3/2}}\,dE, \tag{3}$$

where n_0, n_1 are the number densities of the two nuclear species under consideration, and $m = m_0m_1/(m_0 + m_1)$; m_0 and m_1 are the masses of the two species. The number of these collisions that result in the formation of a compound nucleus and in the subsequent emission of a particle i, which may be a material particle or a γ-ray, can be estimated by multiplying expression (3) by

$$\frac{\lambda^2}{4\pi}\,\frac{\omega\Gamma_s\Gamma_i}{(E_R-E)^2+\Gamma^2/4}, \tag{4}$$

where ω is a statistical-weight factor of the order of unity that depends on angular-momentum considerations; Γ_s is the width for elastic scattering; Γ_i is the width for the emission of the particle i; $\Gamma = \Sigma\Gamma_i$, the summation being over all particles that the compound nucleus is energetically capable of emitting; $\lambda = h/\sqrt{2Em}$, h being Planck's constant; and E_R is the energy level of the compound nucleus that is "nearest" to E. It is particularly to be noted that E_R is measured relative to the sum of the rest masses of nuclei 0 and 1 as the zero of energy, since E is the kinetic energy in the center-of-mass reference system.

Expression (4) is the Breit-Wigner one-level formula. It gives a good approximation when Γ is small compared to the separation of the energy levels of the compound nucleus and when E is sufficiently close to the one particular level E_R. When E falls fairly evenly between two levels, the use of expression (4), applied to one or the other of the two levels, may lead to an appreciable error (Teichmann 1950)—possibly by as much as a factor of 10.

The statistical-weight factor ω will be taken as equal to unity throughout the following work. This represents a further approximation in many cases, but the approximation is necessary, since ω is not known for many of the reactions of interest in the present paper.

Resonance contributions.—The yield, given by multiplying expressions (3) and (4), varies with E. In particular, the yield possesses a sharp maximum at $E = E_R$. The integral of the yield over this peak represents the resonance contribution (Gamow and Teller 1938) from the level E_R. A simple reduction leads to the following value of this contribution:

$$2.53\times10^{-13}n_0n_1\left(\frac{A_0+A_1}{A_0A_1}\right)^{3/2}\frac{\Gamma_s\Gamma_i}{T^{3/2}\Gamma}\qquad\exp\left(-\frac{E_R}{kT}\right), \tag{5}$$

where A_0 and A_1 are the atomic weights of the colliding species; T is now expressed in units of 10^9° K; Γ_s, Γ, Γ_i are in mev, and n_0 and n_1 are the numbers of nuclei per cubic centimeter.

There is a resonance contribution of the form (5) from each level, E_R, of the compound nucleus, such that E_R is appreciably greater than Γ, this being a necessary condition, since E—a kinetic energy—cannot be negative. For the sake of completeness it may be noted that in an unusual case where E_R differs from zero by an amount of order Γ, there is a partial resonance contribution, given by integrating over a portion of the resonance peak. It will be recalled, in this connection, that E_R is measured not relative to the ground state of the compound nucleus but relative to the sum of the rest masses of nuclei 0 and 1.

The width Γ_s may be represented as a product of a width "without barrier," G_s, and a Coulomb factor, P. Writing as a general function of E, we put

$$\Gamma_s = G_s P(E) , \tag{6}$$

where

$$P(E) = E^{-1/2} \exp\left[\frac{8\pi e}{h}(2mZ_0Z_1R)^{1/2} - \frac{4\pi e^2 Z_0 Z_1}{h(2E/m)^{1/2}}\right], \tag{7}$$

Z_0e and Z_1e being the nuclear charges and R, the collision radius, taken in the following as $1.6 \times 10^{-13}(A_0 + A_1)^{1/3}$ cm.

In applications of expression (5), the energy E in equations (6) and (7) must be put equal to E_R. Also, in many applications of expression (5), $\Gamma_i = \Gamma_\gamma$, corresponding to the emission of a γ-ray, no other behavior of the compound nucleus being energetically allowed, except, of course, scattering back into the nuclei 0 and 1. Then Γ reduces to $\Gamma_s + \Gamma_\gamma$. In addition, in such applications Γ_s is considerably less than Γ_γ when P is small, in which case expression (5) takes the simplified form

$$2.53 \times 10^{-13} \frac{n_0 n_1}{T^{3/2}} \left(\frac{A_0 + A_1}{A_0 A_1}\right)^{3/2} G_s P(E_R) \exp\left(-\frac{E_R}{kT}\right), \tag{8}$$

where E_R and G_s are in mev, T is in units of 10^9 ° K, and kT is in mev.

The nonresonance contribution.—Although the yield possesses a sharp maximum at each resonance, the resonance contributions may still not be dominant. This arises in cases where all levels have E_R values that make the factor $P(E_R) \exp(-E_R/kT)$ in expression (5) extremely small. A nonresonance contribution must then be calculated.

To illustrate the method of calculation, consider the important case in which the emission of the particle i corresponds to the emission of a γ-ray. Then, for values of E between two resonance peaks, expression (4) can be replaced by

$$\frac{\lambda^2}{4\pi} \frac{\omega \Gamma_s \Gamma_\gamma}{(E_R - E)^2} . \tag{9}$$

The nonresonance contribution for the range E to $E + dE$ in the collisional energy spectrum is now given by multiplying expressions (3) and (9). Again, writing equation (6) for Γ_s, we see that the nonresonance contribution depends on E, according to the factor

$$\frac{P(E) \exp(-E/kT)}{(E_R - E)^2} . \tag{10}$$

In all nonresonance applications of present interest the factor $(E_R - E)^2$ varies slowly with E, in contrast with both $P(E)$ and $\exp(-E/kT)$.

Now the product of $\exp(-E/kT)$ with the exponential term in $P(E)$ possesses a strong maximum with respect to E at a value $E = E_0$, given by the equation

$$E_0^{3/2} = (2m)^{1/2} \pi^2 e^2 h^{-1} Z_0 Z_1 kT . \tag{11}$$

Relation (11) is often more usefully expressed in the form

$$E_0 = 0.123 \left(\frac{A_0 A_1 Z_0^2 Z_1^2 T^2}{A_0 + A_1}\right)^{1/3} , \tag{12}$$

where E_0 is now in mev and T is in units of 10^9 ° K.

In the neighborhood of E_0 the value of expression (10) is given to a good approximation by

$$P(E_0)\,(E_R - E_0)^{-2} \exp\left[-\frac{E_0}{kT} - \frac{3\,(E_0 - E)^2}{4E_0 kT}\right], \tag{13}$$

a quantity of the order of $(E_0 - E)^3$ being neglected in the exponent in the exponential term. The decline as E changes from E_0 is shown by the Gaussian exponential in expression (13). This may be expressed by saying that the nonresonance yield comes largely from the range of E near E_0. This is the well-known Gamow peak. Integration over the peak leads to

$$P(E_0)\,(E_R - E_0)^{-2} \left(\frac{4\pi E_0 kT}{3}\right)^{1/2} \exp\left(-\frac{E_0}{kT}\right). \tag{14}$$

This result, when multiplied by the factors not depending on E in the product of expressions (3) and (9), gives the nonresonance yield, which, after some reduction, gives a formula of the type first derived by Atkinson and Houtermanns (1929),

$$8.46 \times 10^{-15} n_0 n_1 \left(\frac{A_0 + A_1}{A_0 A_1}\right)^{4/3} \frac{(Z_0 Z_1)^{1/3} \Gamma_\gamma G_s}{T^{2/3}\,(E_R - E_0)^2} P(E_0) \exp\left(-\frac{E_0}{kT}\right), \tag{15}$$

the statistical-weight factor being again taken as unity.

In examples in which one of the nuclear species is an α-particle and the other is an α-particle nucleus (Be^8, C^{12}, O^{16}, Ne^{20}, Mg^{24}, . . .), we can write $A_0 = 2Z_0$, $A_1 = 2Z_1$, in which case, using equations (7) and (11), $E_0^{1/2} P(E_0) \exp(-E_0/kT)$ becomes simply the antilogarithm to the base 10 of the expression

$$\frac{Z_0 Z_1}{(Z_0 + Z_1)^{1/3}} \left(0.92 - \frac{2.34}{T^{1/3}}\right), \tag{16}$$

where T is again in units of 10^9 ° K.

Further remarks on the resonance contribution.—The discussion just given of the dependence of $P(E) \exp(-E/kT)$ on E shows that resonance effects must be particularly strong when an energy level, E_R, of the compound nucleus falls near the center, E_0, of the Gamow peak. In such cases, the resonance contribution may be written as

$$2.53 \times 10^{-13} \frac{n_0 n_1}{T^{3/2}} \left(\frac{A_0 + A_1}{A_0 A_1}\right)^{3/2} G_s P(E_0) \exp\left(-\frac{E_0}{kT}\right). \tag{17}$$

Nuclear reaction rates when levels are narrowly spaced.—In certain cases in which a compound nucleus is excited to very high energies (of the order of 16 mev above ground level), the total width Γ is of the order of 0.1 mev. This arises because the Coulomb-barrier factors for protons and α-particles emitted by the nucleus are now small, so that the proton width and the α-width become effectively the widths without barrier. Now a width of 0.1 mev is much greater than the separation of the levels at high energies of excitation. The Breit-Wigner one-level formula must then be abandoned, even as an approximation. In its place we use

$$\frac{\lambda^2}{4\pi} \frac{\Gamma_s \Gamma}{D^2}, \tag{18}$$

where D is the average level spacing (Blatt and Weisskopf). The width Γ_s still contains an important Coulomb-barrier term, however. Thus $\Gamma_s = G_s P(E)$, and, on multiplying expression (3) by expression (18), we again obtain an integral of $P(E) \exp(-E/$

kT) with respect to E. This is again just the integral over the Gamow peak. Accordingly, we obtain the following modified formula for the yield

$$8.46\times10^{-15}\left(\frac{A_0+A_1}{A_0A_1}\right)^{4/3}\frac{(Z_0Z_1)^{1/3}G_s\Gamma}{T^{2/3}D^2}P(E_0)\exp\left(-\frac{E_0}{kT}\right), \tag{19}$$

where E_0 is again given by equation (12).

Photodisintegration reaction rates.—Photodisintegration reactions, in which a nucleus, C, is disintegrated by γ-ray absorption into two nuclei, 0 and 1, are inverse to reactions already considered. Their rates can therefore be obtained by using the equation of thermodynamic balance for

$$C+\gamma\rightleftarrows 0+1\ , \tag{20}$$

together with results already obtained. We quote the well-known thermodynamic equation (possibly neglecting a weight factor of the order of unity) in the form

$$\log_{10}\left(\frac{n_C}{n_0n_1}\right)=-34.08-\tfrac{3}{2}\log_{10}T+\tfrac{3}{2}\log_{10}\left(\frac{A_0+A_1}{A_0A_1}\right)+\frac{5.04\chi}{T}, \tag{21}$$

where n_C is the number density of the nuclei C; χ is the smallest γ-ray energy, expressed in mev, that is required to disrupt the ground state of C; and T is once again in units of 10^9 ° K.

In addition, under conditions of detailed balancing we have the yield of nuclei C per unit volume per unit time equal to

$$n_C\times(\text{Probability of photodisintegration of nuclei } C \text{ per unit time}). \tag{22}$$

Combining formulae (21) and (22) gives

log (Probability of photodisintegration)

$$=34.08+(\tfrac{3}{2})\log T+(\tfrac{3}{2})\log\left(\frac{A_0A_1}{A_0+A_1}\right)-\frac{5.04\chi}{T}+\log\left(\frac{\text{Yield}}{n_0n_1}\right), \tag{23}$$

from which it is seen that a calculation of the yield rate implies a knowledge of the disintegration rate.

It is to be emphasized that equation (23) remains valid even when thermodynamic conditions do not in fact hold among the nuclei, provided always that the Planck distribution of radiation is maintained. It can readily be shown that under the conditions occurring inside hot stars electron–positive ion collisions maintain this latter requirement (Hoyle 1946).

IV. THE BUILDING OF CARBON, OXYGEN, AND NEON

It was pointed out some years ago by Bethe (1939) that effective element-building inside stars must proceed, in the absence of hydrogen, by triple α-particle collisions as a starting point:

$$3\alpha\rightarrow C^{12}+\gamma. \tag{24}$$

It is particularly to be noted that a serious difficulty, encountered by Gamow, Alpher, and Herman in their work on the origin of the elements, does not occur here. This difficulty arises only at the comparatively low density used by these authors, for reaction (24) cannot then be used to bridge the well-known gap caused by the instability of nuclei of atomic weight 5.

Recently Salpeter (1952) has developed these considerations and has suggested that the energy released in reaction (24) may have an important place in the theory of stellar

evolution and that the C^{12} so produced may provide a basis for explaining the excess carbon that is detected in the atmospheres of the R and N stars.

Now the release of energy by reaction (24) and by the further reactions

$$C^{12} + \alpha \rightarrow O^{16} + \gamma, \tag{25}$$

$$O^{16} + \alpha \rightarrow Ne^{20} + \gamma, \tag{26}$$

amounts to a total of about $Mc^2/1000$, where M is the mass of the star and c is the velocity of light. Thus the time scale for a collapse through the temperatures at which reactions (24), (25), and (26) occur must be of the order of $10^{-3}\ Mc^2/L$, where L is the luminosity of the star. For the stars under consideration, M/L may be taken as about 10^{-3} (for stars of $1.5\odot$ this would imply a bolometric magnitude of about -3, this being a reasonable value after effectively all the hydrogen has been exhausted). The time scale for collapse through the relevant temperatures is, accordingly, of the order of 10^{15} seconds. This result will be used in the following work. The conclusions reached are not at all sensitive to this particular value of the time scale.

It is convenient to replace reaction (24) by

$$\alpha + \alpha \rightleftarrows Be^8, \qquad Be^8 + \alpha \rightarrow C^{12} + \gamma. \tag{27}$$

This is a permissible step, since the lifetime of the unstable Be^8 is appreciably longer than the time required for a "nuclear" collision of two α-particles; that is, longer than the α-particle radius divided by the relative velocity. The merit of reaction (27) is that the number density n_4^8 of Be^8 nuclei is given in terms of the number density n_2^4 of α-particles by the equation of statistical equilibrium (Hoyle 1946),

$$\log n_4^8 = 2 \log n_2^4 - 34.53 - \tfrac{3}{2} \log T - \frac{0.464}{T}, \tag{28}$$

where the disintegration energy of Be^8 is taken as 0.092 mev, and T is in units of $10^{9}\,^{\circ}$ K. We shall henceforth always use this unit for measuring T, unless otherwise stated.

The yield of C^{12} per cubic centimeter per second is given immediately by inserting $A_0 = 4$, $Z_0 = 2$, and $A_1 = 8$, $Z_1 = 4$, in the formulae of the previous section. The important energy level of the C^{12} nucleus in the present problem is one very recently identified by Dunbar, Pixley, Wenzel, and Whaling (1953). This level occurs at about 7.68 mev above ground level, which corresponds to a value of E_R of about 0.31 mev. (It will be recalled that E_R is measured relative to the sum of the masses of Be^8 and α-particle, this being about 7.37 mev above the ground level of C^{12}.) Assuming, as we shall do in this paper, that the $Be^8 + \alpha$ reaction through this level is not forbidden by strict selection rules, the resonance contribution from it quite overwhelms not only the nonresonance yield but also the resonance contributions from other levels.

The application of expression (8) gives, on eliminating n_4^8 by using equation (28),

$$1.71 \times 10^{-48} (n_2^4)^3 T^{-3} 10^{-0.464/T} G_s P(E_R) \exp\left(-\frac{E_R}{kT}\right), \tag{29}$$

where, for the moment, we retain the general form in terms of E_R.

Now it would be possible to put $E_R = 0.31$ mev in expression (29), to work out the yield of C^{12} over a time scale of 10^{15} seconds, and to show that for sufficiently high T the yield becomes comparable with $n_2^4/3$, thereby indicating that an appreciable fraction of the original helium has become built into carbon. It is not necessarily correct, however, to make the further step of arguing that a high concentration of C^{12} must then have been produced, for this assumes that the C^{12} so built up is not destroyed by other reactions.

It is plain that carbon will accumulate only if it is not destroyed by other reactions as fast as it is produced.

It can be shown that reaction (25) is the most effective in destroying C^{12}. Hence, to decide how far C^{12} accumulates, it is necessary to compare the rates of reactions (24) and (25). For the latter reaction the value of E_R of main interest is -0.05 mev, corresponding to a level at 7.10 mev above the ground state of O^{16}. This gives a nonresonance contribution

$$4.47 \times 10^{-15} n_6^{12} n_2^4 \frac{\Gamma_\gamma G_s}{T^{2/3} (E_R - E_0)^2} P(E_0) \exp\left(-\frac{E_0}{kT}\right), \tag{30}$$

where E_0 is given by equation (12), and $n_6^{12}$is the number density of C^{12} nuclei. There are resonance contributions to the yield of O^{16}, but these occur at such large positive values of E_R that, on account of the factor $\exp(-E_R/kT)$ in the resonance formula, these contributions are small compared with expression (30).

We now use expressions (29) and (30) in the construction of two differential equations. First, we write

$$\frac{dn_2^4}{dt} = -3A (n_2^4)^3 - B n_2^4 n_6^{12}, \tag{31}$$

where A is the coefficient of $(n_2^4)^3$ in expression (29), and B is the coefficient of $n_2^4 n_6^{12}$ in expression (30). Equation (31) is approximate in that it neglects the removal of α-particles in reactions other than (24) and (25). We also have

$$\frac{dn_6^{12}}{dt} = A (n_2^4)^3 - B n_2^4 n_6^{12}. \tag{32}$$

If f represents the fraction by mass of the material in the form of helium and g is the fraction in the form of C^{12}, then

$$\log n_2^4 = 23.18 + \log f + \log \rho, \qquad \log n_6^{12} = 22.70 + \log g + \log \rho.$$

After an elementary reduction it can be shown that, for changes at constant density, equations (31) and (32) lead to

$$\frac{dg}{df} = 3 \frac{\kappa g / f^2 - 1}{\kappa g / f^2 + 3}, \tag{33}$$

where

$$\kappa = \frac{2.2 \times 10^{-24} B}{A \rho}. \tag{34}$$

Instead of expressing ρ in terms of T by means of the order-of-magnitude relation (2), we now obtain increased accuracy by writing

$$\rho = \alpha 10^6 T^3,$$

where α is a parameter of the order of unity that may vary from one sample of material to another. With this value of ρ, expression (34) for κ becomes (writing in the values of A and B)

$$\kappa = 5.7 \times 10^{3 + 0.464/T} T^{-2/3} \alpha^{-1} \frac{S}{J}, \tag{35}$$

where

$$S = \Gamma_\gamma G_s (E_R - E_0)^{-2} P(E_0) \exp\left(-\frac{E_0}{kT}\right),$$

all quantities in S being calculated for the $C^{12} + \alpha$ reaction, and

$$J = G_s P(E_R) \exp\left(-\frac{E_R}{kT}\right), \tag{36}$$

all quantities in equation (36) being calculated for the $Be^8 + \alpha$ reaction.

Next we consider how g varies with f, subject to the following two approximations:

i) The temperature T and the total density ρ of the material remain constant as f decreases from 1 to 0.

ii) The removal of α-particles by reactions other than (24) and (25) is neglected.

With these approximations, κ remains constant as f changes. Hence the nature of the solutions of equation (33) can be seen by specifying various values of κ. In Figure 2 the variation of g with f is shown for four prescribed values of κ.

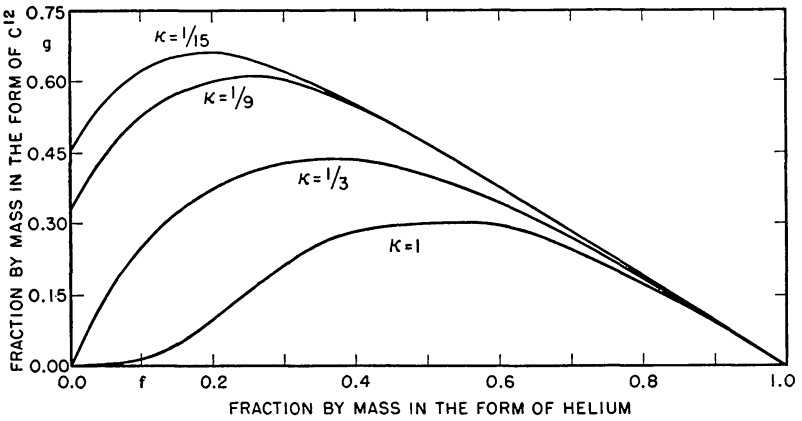

FIG. 2.—The formation of C^{12} from helium as a function of κ. The quantity f represents the fraction of the mass which is helium; g is the fraction which is C^{12}; the balance, $1-f-g$, is O^{16}.

The ratio of the mass of O^{16} produced to the mass of C^{12} produced may be estimated as

$$\left(\frac{\overline{1-g}}{g}\right)_{f=0}. \tag{37}$$

To agree with observed cosmic abundances, this ratio should take a value of about $\frac{2}{1}$, which would imply a value of κ not much greater than $\frac{1}{9}$. It is, however, to be emphasized that different samples of material may be expected to correspond to somewhat different values of κ. This arises because κ is proportional to α^{-1}, and α must vary from one sample to another. Accordingly, it is not a question of determining one unique value of κ, chosen so as to give the required ratio of oxygen to carbon. Rather, we should proceed in the following manner:

Suppose that $\alpha = 1$ for an average sample of material. If for $\alpha = 1$ we put $\kappa = \frac{1}{9}$, a typical sample of material would be built into a mixture with a C^{12} to O^{16} ratio of about $\frac{1}{2}$. On the other hand, in some exceptional samples α may well be as small as 0.3, in which case we have $\kappa = \frac{1}{3}$, and the material is then without C^{12}. Thus a variability of composition arises, C^{12} being produced in high abundance in some samples but not in others (a circumstance that may turn out to be of importance in relation to the carbon stars).

In accordance with these remarks, we see that astrophysical considerations suggest that we put $\kappa = \frac{1}{9}$ when $\alpha = 1$ in equation (35), thereby giving

$$J = 5.1 \times 10^{4+0.464/T} T^{-2/3} S. \tag{38}$$

The reaction widths being supposed given, equation (38) yields a relation between E_R and T, it being remembered that E_R represents the appropriate level in the C^{12} nucleus (E_R close to 0.31 mev). Hence equation (38) can be used to determine the temperature T at which these processes must take place if we are to obtain the required proportions of O^{16} and C^{12}.

But a second relation between E_R and T can be determined from the requirement that the time scale in which f is reduced to zero must correspond with the time of collapse of the star, which we have seen must be of the order of 10^{15} seconds. This enables us to make a strict test of the theory, since this second relation must yield the same value of T as that given by equation (38).

To carry out this test, we note that equation (31), after some reduction, can be written in the form

$$\frac{d(\log f)}{dt} = -0.040\,(\kappa g + 3f^2)\,\rho^2 T^{-3} 10^{-0.464/T} J\,, \tag{39}$$

again making the assumption that ρ and T remain constant.

Consider the application of equation (39) to the case $\kappa = \frac{1}{9}$. The main contribution to the time required for f to decrease effectively to zero comes from the late stages, when f is small compared to unity. The helium concentration then declines exponentially to zero, the characteristic time for the decline being close to

$$25\kappa^{-1} g_0^{-1} \rho^{-2} T^{-3} J^{-1} 10^{0.464/T}, \tag{40}$$

where g_0 is the value of g at $f = 0$. This characteristic time must be of the order of 10^{15} seconds. Putting $\kappa = \frac{1}{9}$ and $g_0 = 0.322$, we require

$$J = 7.0 \times 10^{-13} T^3 \rho^{-2} 10^{0.464/T}.$$

Remembering that the case $\kappa = \frac{1}{9}$ corresponds to $\alpha = 1$ in $\rho = \alpha 10^6 T^3$, this gives, on eliminating ρ,

$$J = 7.0 \times 10^{-25} T^{-3} 10^{0.464/T}. \tag{41}$$

Equation (41) gives the second determination of T (E_R supposed known).

It remains to test whether equations (38) and (41) do in fact give the same value of T (if they do not, we infer that the identification $\kappa = \frac{1}{9}$ with the case $\alpha = 1$ is in error and hence that the appropriate proportions of C^{12} and O^{16} are not produced). The test can be carried out in two ways. We could insert $E_R = 0.31$ mev in equations (38) and (41) and, making suitable estimates of the reaction widths, solve both equations for T. Or we can treat E_R as unknown and proceed to solve equations (38) and (41) for *both* E_R and T, the value of E_R so obtained being then compared with the value $E_R = 0.31$ mev, given by nuclear physics. Of these two procedures, the latter is preferable and will now be carried out.

When we put G_s for the $Be^8 + \alpha$ reaction equal to 0.1 mev, equation (41) gives

$$P(E_R) \exp\left(-\frac{E_R}{kT}\right) = 7.0 \times 10^{-24} T^{-3} 10^{0.464/T}, \tag{42}$$

and when we put $G_s/\sqrt{E_0} = 0.1$ mev, $\Gamma_\gamma = 10^{-6}$ mev, and $(E_R - E_0)^{-2} = 25$ mev^{-2} for the $C^{12} + \alpha$ reaction, equation (38) gives

$$P(E_R) \exp\left(-\frac{E_R}{kT}\right) = 1.3 T^{-2/3} 10^{0.464/T} \left(\text{antilogarithm of } 5.52 - \frac{14.04}{T^{1/3}}\right), \tag{43}$$

where expression (16) has been used to determine the value of $E_0^{1/2}P(E_0)\exp(-E_0/kT)$ for the $C^{12}+\alpha$ reaction. Solving equations (42) and (43) gives

$$T = 0.14, \qquad E_R = 0.33 \text{ mev.} \tag{44}$$

The agreement of this deduced value of E_R with the experimental value is highly satisfactory, thereby confirming our identification of $\kappa = \frac{1}{9}$ with the case $a = 1$ and hence giving the required relative concentrations of C^{12} and O^{16}.

An important step yet remains. Going back to expression (40), which determines the time scale required for the helium to become entirely used up, we must now consider cases where κ is greater than $1/9$ ($a < 1$). It transpires when we consider the cases $\kappa = \frac{1}{3}$ and $\kappa = 1$, for example, that the time scale diverges for small f. The time scale for the range of f from 1 to 0.15 is substantially the same as that calculated above, but as f approaches zero the time scale tends to infinity, since g tends to zero in these cases. This divergence is an outcome of the assumptions that were introduced into the calculations, particularly the assumption that reactions (24) and (25) are the only ones that contribute to the removal of α-particles. So long as either f is not too small or g does not become too small, this assumption is a good approximation, as it is in the case $\kappa = \frac{1}{9}$, considered above. But when both f and g become small, reactions (24) and (25) become unimportant compared with reaction (26), which converts O^{16} into Ne^{20} by α-particle addition. Thus, in the case $\kappa = \frac{1}{3}$, the last 10 per cent or so of the helium is consumed, not in the production of C^{12} and O^{16}, but in converting O^{16} into Ne^{20}. It should be noted that this does not mean that in such cases the amount of Ne^{20} produced is only 10 per cent of the amount of O^{16}, for the amount of Ne^{20} produced must have 5 times the mass of the helium that is used up in its production. Thus, if the helium so used up comprises 10 per cent of the total mass, the mass of the Ne^{20} produced must be of the order of 50 per cent of the total. That is to say, in the case $\kappa = \frac{1}{3}$ the Ne^{20} produced must have a total mass comparable with the mass of O^{16}. Accordingly, since we are regarding $\kappa = \frac{1}{3}$ as a not unusual case, it follows that Ne^{20} production must be roughly comparable with the production of O^{16}.

It should be added that the $O^{16}+\alpha$ reaction does not proceed with sufficient rapidity at $T = 0.14$. The temperature $T = 0.14$ still applies to the stages where f is reduced from unity down to about 0.1, but (in the case $\kappa = \frac{1}{3}$) not when f becomes so small that the production of C^{12} and O^{16} effectively ceases. Since energy is then no longer being released in any important degree by the nuclear reactions, the star must contract, and, as it does so, the temperature must rise until eventually the $O^{16}+\alpha$ reaction is speeded up sufficiently for the energy it provides to slow up the contraction of the star once again. The star then halts its collapse for a while until the helium concentration falls effectively to zero.

It is a straightforward calculation, of the type already considered above, to obtain the temperature at which the last of the helium becomes used up in the formation of Ne^{20}. The known levels in the Ne^{20} nucleus provide an important resonance corresponding to $E_R = 0.654$ mev, it being assumed that this resonance is not rendered ineffective by selection rules. Using this value for E_R, it turns out that the $O^{16}+\alpha$ reaction is adequately speeded up when T has risen to about 0.16. The criterion for deciding what constitutes an adequate speed is simply that the value of f must fall effectively to zero in a time of the order of 10^{15} seconds. It appears, therefore, that no large rise of temperature is required for neon production to set in.

To conclude the present section, we may sum up the main points considered above. Provided that α-particle resonance in a level in the C^{12} nucleus at about 7.7 mev above the ground state is not forbidden by selection rules, the theory is capable of giving roughly the astrophysical abundance ratios $\frac{1}{3} : 1 : 1$ for $C^{12} : O^{16} : Ne^{20}$. These abundances refer to samples of material where the relation between density and temperature is not much different from relation (2). In material of abnormally high density the produc-

tion of C^{12} would exceed oxygen and neon. In material of abnormally low density, on the other hand, the production of C^{12} would be negligible. Carbon and oxygen production occur at about $T = 1.4 \times 10^8$° K, and neon production at about $T = 1.6 \times 10^8$° K.

An implicit assumption is contained in the above remarks concerning neon production. It is assumed that the Ne^{20} is not destroyed by the reaction

$$Ne^{20} + \alpha \rightarrow Mg^{24} + \gamma \tag{45}$$

as soon as it is produced. In the absence of full information concerning the levels in the Mg^{24} nucleus, it is not possible to decide whether this assumption can be justified or not. It can, however, be said that at $T = 0.16$ the Ne^{20} is *not* destroyed by reaction (45) unless the Mg^{24} nucleus has a level giving an E_R value that is very close to the value making $P(E_R) \exp(-E_R/kT)$ a maximum, that is, close to the value of E_0 defined by equation (12). It will be assumed in the sequel that this is not the case. Later considerations contain an uncertainty on this account.

V. MINOR EFFECTS AMONG THE LIGHT ELEMENT GROUP

So far we have considered the nuclear reactions occurring in the temperature range around 0.14 as if the material were initially entirely pure helium. It is, of course, the case that the material must initially contain traces of other elements. From the present point of view the important trace elements are C^{13} and N^{14}.

The stars in question have masses of the order of, or greater than, $1.5\odot$. For such stars the energy production that occurred during the stage *before* the initial hydrogen content became exhausted would be through the carbon-nitrogen cycle, not through the proton-proton chain. Recent estimates[4] by Fowler would place the ratios $C^{12} : C^{13} : N^{14} : N^{15}$ occurring during this phase at about 1 : 0.25 : 10 : 0.004, the temperature for the operation of the carbon-nitrogen cycle being taken as about 2×10^7° K. That is to say, the main trace element operating in the carbon-nitrogen cycle is N^{14}.

Now, with the heating of the material to temperatures in excess of $T = 0.1$, the presence of N^{14} leads to the reactions

$$N^{14} + \alpha \rightarrow F^{18} + \gamma\,, \tag{46}$$

$$F^{18} \rightarrow O^{18} + \beta^+\,, \tag{47}$$

$$O^{18} + \alpha \rightarrow Ne^{22} + \gamma\,, \tag{48}$$

it being noted that reactions such as $N^{14} + \alpha \rightarrow O^{17} + p$ do not occur in any appreciable intensity on account of energy requirements.

The next step is to estimate the concentrations of O^{18} and Ne^{22} that are built up by these reactions. In *normal* stellar material the ratio of hydrogen to N^{14} is about $\frac{1000}{1}$. It follows that when the hydrogen is converted into helium, the ratio He^4 to N^{14} is about $\frac{250}{1}$. Thus, if the N^{14} were to become converted mainly into O^{18} and the He^4 were to become mainly converted into equal quantities of O^{16} and Ne^{20}, the ratio of O^{18} to O^{16} would be about $\frac{1}{30}$, which may be compared with the known terrestrial ratio of about $\frac{1}{500}$. Thus the present process is capable of providing considerably more O^{18} than is required. This result should not be regarded as a discrepancy, however, since it probably implies that most of the O^{18} produced by reactions (46) and (47) is converted to Ne^{22} by reaction (48). Thus the Ne^{22} to Ne^{20} ratio would then become about $\frac{1}{30}$, as compared with the observed terrestrial ratio of about $\frac{1}{10}$. In view of the order-of-magnitude character of the present argument, this may be taken as satisfactory agreement. We see that the

[4] Private communication. I am much indebted to Dr. Fowler for this information.

trace element N^{14} is capable of yielding approximately the required amounts of O^{18} and Ne^{22}.

The presence of the trace element C^{13} likewise leads to the reactions

$$C^{13} + \alpha \rightarrow O^{17} + \gamma\,, \tag{49}$$

$$O^{17} + \alpha \rightarrow Ne^{21} + \gamma\,. \tag{50}$$

Remembering that the C^{13} concentration is less than the N^{14} concentration by a factor of about 40, it follows that, since the N^{14} concentration led to ratios of the order of $\frac{1}{30}$, the O^{17} to O^{16} ratio given by reaction (49) is of the order of $\frac{1}{1200}$, as compared with a terrestrial value of $\frac{1}{4000}$. Again, it would seem that an appreciable fraction of the O^{17} produced by reaction (49) is destroyed by reaction (50), although not such a large fraction as in the case of O^{18}. The ratio of Ne^{21} to Ne^{20} given by reaction (50)is then about $\frac{1}{1200}$, as compared with the terrestrial value of $\frac{1}{300}$. This moderate discrepancy is, again, not very serious in an order-of-magnitude calculation. It is, in any case, mitigated by the considerations of a later section that provide an additional process for the formation of Ne^{21}.

It remains to be added that F^{19} is produced by the reaction

$$N^{15} + \alpha \rightarrow F^{19} + \gamma\,. \tag{51}$$

For a N^{15} to N^{14} ratio of $\frac{1}{4000}$, as indicated by present estimates for the carbon-nitrogen cycle, the F^{19} to O^{16} ratio would be about $1/10^5$. According to Urey (1952), the meteoritic abundance of fluorine yields a ratio of just this order. On the other hand, astrophysical estimates suggest a greater fluorine abundance. The discrepancy between the present value and the astrophysical requirements may possibly be due to the N^{15} to N^{14} ratio, as given by calculations on the carbon-nitrogen cycle, being too low. Thus, if we were to put the N^{15} to N^{14} ratio equal to the terrestrial value of about $\frac{1}{260}$, the F^{19} to O^{16} ratio would then become about $\frac{1}{8000}$.

VI. NOTABLE OMISSIONS AMONG THE LIGHT ELEMENTS

Among elements lighter than Ne^{22}, the nuclei C^{13}, N^{14}, and N^{15} are not synthesized by the processes described above. The view is here taken that these three nuclei owe their origin to the carbon-nitrogen cycle operating in normal stars, not in the very hot stars at present under consideration. The C^{12} that forms the basis of this cycle is indeed produced by the reactions already discussed. We have the scheme shown in the accompanying tabulation. The present suggestion is supported by the circumstance that it now seems

	C^{12}	C^{13}	N^{14}	N^{15}
Produced in very hot stars (per cent).....	100	0	0	0
Produced after C^{12} has been put through the carbon-nitrogen cycle in ordinary stars with temperatures of order 2×10^{7}° K (approximate per cent).....	9	2	89	0.022

possible to understand the wide variability of the C^{13} to C^{12} ratio that is found in different stars. The first sample in the accompanying table has a zero C^{13} content, such as seems to occur in hydrogen-poor stars.[5] The second sample has a ratio of 1/4.5, as in some giant

[5] I am indebted to W. P. Bidelman for a private communication on this point. See also Bidelman (1953).

stars, while a mixture of the two samples in equal proportions gives a ratio of about $\frac{1}{55}$—of the same order as the terrestrial value.

There remains, however, a serious discrepancy in the N^{15} to N^{14} ratio, which, at $\frac{1}{4000}$, is too low by a factor of more than 10 to explain the terrestrial ratio of $\frac{1}{260}$. It is possible that the above estimate of the N^{15} concentration stands in need of revision. This possibility may be related to the remarks of the previous section concerning the abundance of F^{19}.

VII. THE SYNTHESIS OF Na^{23}

The discussion has reached the point where an increase of temperature up to about 1.6×10^{8}° K causes the initial helium to be converted into elements lighter than Na^{23}. It must be stressed that considerations from this point become much more tentative. Not enough is known concerning the detailed properties of the nuclei to enable really reliable calculations of reaction rates to be made. Nevertheless, some progress toward a qualitative understanding of the further effects likely to arise at still higher temperatures can be made. With this limitation in mind, the present and following sections attempt in very brief outline to indicate what the main processes of element-building may be from sodium to nickel.

Any new reactions that arise as the temperature rises still further must come from either (i) collisions between the nuclei themselves or (ii) photodisintegration of the nuclei—protons, neutrons, or α-particles being released and then proceeding to contribute to further reactions. An analysis of the situation has shown that, with increasing T, the first important reaction to occur is probably of type i. It is

$$C^{12} + C^{12} \rightarrow Mg^{24}, \tag{52}$$

the Mg^{24} being excited to about 16 mev above ground level and decaying mainly by emitting an energetic proton or α-particle: thus

$$Mg^{24} \begin{cases} \rightarrow Ne^{20} + \alpha\,, \\ \rightarrow Na^{23} + p\,. \end{cases} \tag{53}$$

The temperature at which reactions (52) and (53) become important can be shown to be close to $T = 0.6$.

At first sight it might seem that Na^{23} is readily given by reaction (53), but this is not so, for the following reason: the protons released in the formation of Na^{23} quickly become attached to the heavy nuclei, and much the largest attachment probability is for

$$Na^{23} + p \rightarrow Ne^{20} + \alpha\,. \tag{54}$$

This is because attachment to other nuclei takes plase only through (p, γ) reactions, which are less probable than the exothermic reaction (54) by a factor of the order of 10^5. Thus the Na^{23} produced by reaction (53) is effectively wholly destroyed by reaction (54). The formation of Na^{23} would therefore appear to be confined to a less probable mode of decay of the highly excited Mg^{24} nuclei, namely, to the reaction

$$Mg^{24} \rightarrow Mg^{23} + n, \qquad Mg^{23} \rightarrow Na^{23} + \beta^{+}. \tag{55}$$

Reaction (55) is less probable than reaction (53) because the excited Mg^{24} nucleus formed in reaction (52) is energetically incapable of emitting a neutron unless the relative kinetic energy E of the colliding C^{12} nuclei exceeds the energy E_0 at the center of the Gamow peak by about 0.65 mev. For such an energy of collision, $P(E) \exp(-E/kT)$ is less than $P(E_0) \exp(-E_0/kT)$ by a factor close to 10. It appears, therefore, that reaction (55) is

less probable than reaction (53) by a factor of the order of 10—the neutron width for reaction (55) being taken as comparable with the proton and α-particle widths in reaction (53).

The Na^{23} produced by reaction (55) is not appreciably destroyed by neutron addition, since neutrons add with roughly equal probability to all nuclei present. Thus, since Na^{23} is present in low abundance compared with Ne^{20}, O^{16}, and C^{12}, most of the neutrons become attached to these latter nuclei. Attachment to O^{16} tends, however, to be impermanent because the photodisintegration

$$O^{17} + \gamma \rightarrow O^{16} + n$$

is already effective at the temperature at which reactions (52), (53), and (55) become important (the neutron binding in O^{17} is as low as 4.15 mev). Thus the neutrons produced in reaction (55) are mainly used up in the formation of Ne^{21}, either directly through the reaction

$$Ne^{20} + n \rightarrow Ne^{21} + \gamma$$

or indirectly through the reaction

$$C^{12} + n \rightarrow C^{13} + \gamma\,,$$

$$C^{12} + C^{13} \rightarrow Mg^{25} \rightarrow Ne^{21} + \alpha\,.$$

This mode of formation of Ne^{21} is of interest because the process discussed in a previous section was inadequate to explain the observed abundance of Ne^{21} by a factor of about 4. On the present basis the cosmic abundance of Ne^{21} should be closely comparable with the abundance of Na^{23}. Accepting the terrestrial value for the ratio Ne^{21}/Ne^{20}, this requirement would seem to be well satisfied. Thus

$$(Ne^{21}/Ne^{20})_{\text{terrestrial}} = 0.0030, \qquad (Na^{23}/Ne^{20})_{\text{astrophysical}} = 0.0045,$$

the astrophysical value being determined from the spectroscopic study of stellar atmospheres.[6]

The α-particles produced in reactions (53) and (54) are mainly absorbed by C^{12} in $C^{12} + \alpha \rightarrow O^{16} + \gamma$. This means that only about two-thirds of the C^{12} is effective in reaction (52), the remaining third being converted to O^{16} by α-particle addition. Accordingly, one Na^{23} nucleus is produced in the destruction of about 30 C^{12} nuclei (this assumes reaction [53] to be 10 times more probable than reaction [55]). Accepting a ratio of $\frac{1}{3}$ for C^{12}/O^{16}, this would imply a Na^{23} to O^{16} ratio of about $\frac{1}{90}$.

At first sight we might be inclined to accept this as an estimate of the average Na^{23} abundance in material ejected from stars, but this would be incorrect, since it would ignore the heating function, $H(\tau)$, defined in section II, 1, above. It will be recalled that, taking an average over the material ejected from a large number of stars, the fraction of material heated to temperatures in the range τ to $\tau + d\tau$, before ejection into space takes place, was written as $H(\tau)d\tau$. Thus, in obtaining the abundance ratio of two nuclear species, it is necessary to take into account the ratio of

$$\int_{\tau_f}^{\tau_d} H(\tau)\, d\tau \tag{56}$$

for the two nuclei. The lower limit, τ_f, in this integral represents the temperature of formation of the nuclear species, and τ_d is the temperature at which the species is effec-

[6] Cf., for example, Aller (1953).

tively destroyed (all heavy nuclei are destroyed by photodisintegration if τ is sufficiently high). In many cases τ_d is so much higher than τ_f that integral (56) may be approximated by

$$\int_{\tau_f}^{\infty} H(\tau)\, d\tau\,. \tag{57}$$

In the work of previous sections the nuclei whose abundances were compared were formed at such nearly equal temperatures, and τ_d was so large compared with τ_f, that integral (56) was effectively the same for all of them. Consequently, the present considerations could be ignored. They cannot be ignored, however, in comparing Na^{23} and O^{16}, for O^{16} is formed at about $\tau_f = 0.13$, whereas $\tau_f = 0.6$ is required for Na^{23}. When we put $H(\tau) \propto 1/\tau^2$, as indicated in section II, 1, the modification arising from integral (57) is a factor of 4.6, in the sense of a reduction of Na^{23} relative to O^{16}. The calculated abundance ratio of Na^{23} to O^{16} therefore becomes about $\frac{1}{400}$, as compared with a ratio of $\frac{1}{200}$ from astrophysical data (Aller 1953). In view of the approximations made above, this may be considered good agreement.

It may be added that in the calculation of the rate of occurrence of reactions (52) and (53), on which the temperature $T = 0.6$ (quoted above) was based, formula (19) was used. This is because the average spacing of the levels within the excited Mg^{24} nucleus is small compared with the total width Γ for decay. The width Γ_s in expression (19) corresponds to the width for the carbon-carbon collision (52). This contains an important Coulomb-barrier factor. The following values were used in making the estimate $T = 0.6$:

$$G_s = 0.1 \text{ mev}, \qquad \Gamma = 0.1 \text{ mev}, \qquad D = 0.01 \text{ mev},$$

while the time-scale criterion employed was that reactions (52) and (53) should be capable of destroying the C^{12} in 10^{15} seconds.

It may be wondered why the same time scale was used in the present case as in the synthesis of C^{12} from helium. The reason for this is that the reactions that lead to C^{12}, O^{16}, and Ne^{20} being built into heavier elements are exothermic and yield an amount of energy per unit mass of material that is of the same order as the energy released in the building of C^{12}. Hence we expect that the lifetime for the building of C^{12}, O^{16}, and Ne^{20} into such elements as Mg^{24}, Si^{28}, and S^{32} must also be of the order of 10^{15} seconds.

VIII. THE SYNTHESIS OF Mg^{24}

The next most important reaction is the photodisintegration of Ne^{20},

$$Ne^{20} + \gamma \rightarrow O^{16} + \alpha\,, \tag{58}$$

requiring a quantum with energy greater than 4.74 mev. The rate of this reaction may be calculated by using the method described in section 3. The quantity χ in equation (23) is now 4.74, and the yield from the inverse of reaction (58) is calculated for resonance in the neon level corresponding to $E_R = 0.654$ mev—it being assumed that this is not an entirely forbidden transition. Subject to this assumption, investigation of the photodisintegration process then shows that disruption becomes important when T rises to 0.8, the criterion of "importance" in the present sense being that an appreciable fraction the Ne^{20} is disintegrated in the time scale for the collapse of the star—namely, in about 10^{15} seconds. The α-particles produced in reaction (58) are rapidly absorbed at $T = 0.8$. The main source of absorption comes ultimately from

$$Ne^{20} + \alpha \rightarrow Mg^{24} + \gamma\,, \tag{59}$$

the Mg^{24} being highly stable against photodisintegration at $T = 0.8$. Thus the net effect of the photodisintegration of Ne^{20} is to produce Mg^{24} and O^{16}:

$$2Ne^{20} \rightarrow O^{16} + Mg^{24}.$$

The abundance of Mg^{24} can now be calculated relative to O^{16}. With an original Ne^{20} abundance comparable to that of O^{16}, the ratio of Mg^{24} to O^{16}, in the particular sample of material under consideration, is about $\frac{1}{3}$; but once again this does not give the required abundance ratio for the average of all material ejected from stars. The ratio of integral (56) for the two nuclei must once again be taken. Writing $\tau_f = 0.13$ for O^{16} and $\tau_f = 0.8$ for Mg^{24}, and quoting a result to be obtained in the next section—that τ_d is probably about 1.35 for both O^{16} and Mg^{24}—the required ratio becomes about 13.5 in the sense of a decrease of Mg^{24} relative to O^{16}. This leads to an Mg^{24} to O^{16} ratio of about $\frac{1}{40}$, as compared with an astrophysical value of about $\frac{1}{9}$. In view of the uncertainty concerning the form of $H(\tau)$, this may be considered a satisfactory agreement.

It may be noted in addition that, although Ne^{22} is stable against photodisintegration at these temperatures, a considerable proportion of Ne^{22} is nevertheless destroyed by the reaction

$$Ne^{22} + \alpha \rightarrow Mg^{25} + n, \qquad Mg^{24} + n \rightarrow Mg^{25} + \gamma\,.$$

On this basis the ratio Mg^{25}/Mg^{24} should be comparable with the ratio Ne^{22}/Ne^{20}. According to the composition of terrestrial material, this is indeed the case.

IX. ELEMENTS FROM ALUMINUM TO PHOSPHORUS

Of the nuclei so far built up, the most abundant are O^{16} and Mg^{24}. Of these, O^{16} is the less refractory. Calculation indicates that O^{16} undergoes photodisintegration,

$$O^{16} + \gamma \rightarrow C^{12} + \alpha\,, \tag{60}$$

at about $T = 1.35$, this result being based on the resonance effect of a level in O^{16} at 8.6 mev above ground level. The C^{12} so produced is mainly destroyed through the reaction

$$C^{12} + O^{16} \rightarrow Si^{28} \begin{cases} \rightarrow Mg^{24} + \alpha\,, \\ \rightarrow Si^{27} + n\,, \\ \rightarrow Mg^{26} + D\,. \end{cases} \tag{61}$$

Aluminum is then produced by the β-decay of Si^{27}. The disintegration of the Si^{28} in reaction (61) into $Al^{27} + p$ has been omitted for the reason that the protons so produced are removed by the reaction

$$Al^{27} + p \rightarrow Mg^{24} + \alpha\,,$$

this reaction, being exothermic, having a far higher probability than reactions of the (p, γ) type.

The situation is further complicated because at about the same temperature, $T = 1.35$, the following reaction becomes about equally effective in destroying the O^{16}:

$$O^{16} + O^{16} \rightarrow S^{32} \begin{cases} \rightarrow Si^{28} + \alpha\,, \\ \rightarrow P^{30} + D\,, \\ \rightarrow S^{31} + n\,. \end{cases} \tag{62}$$

The decay of S^{32} into $P^{31} + p$ has been omitted in reaction (62) for the reason that the reaction $P^{31} + p \rightarrow Si^{28} + \alpha$ reduces this case to a decay into $Si^{28} + \alpha$, exactly as in the case $Al^{27} + p$, discussed above.

If reactions (60) and (61) are more important than reaction (62), then approximately one α-particle is produced for every *two* O^{16} nuclei that are destroyed. Then, since the magnesium abundance is about one Mg^{24} nucleus for every *three* O^{16} nuclei, it follows that an excess of α-particles is made available, over and above the α-particles necessary to convert the magnesium entirely to silicon through the reaction

$$Mg^{24} + \alpha \rightarrow Si^{28} + \gamma\,. \tag{63}$$

It is to be noted that, so long as Mg^{24} remains available, reaction (63) is favored as compared with other (α, γ) reactions, such as

$$Si^{28} + \alpha \rightarrow S^{32} + \gamma\,,$$

by possessing a lower Coulomb barrier.

In contrast, if reaction (62) is more important than reactions (60) and (61), then the α-particles made available are fewer than in the case just discussed and may indeed be fewer than an average of one α-particle for every *three* O^{16} nuclei destroyed. Should this be so, there are insufficient α-particles to convert entirely the Mg^{24} into Si^{28} by reaction (63).

The discussion of the following section shows that the synthesis of the elements in the sulphur-calcium group is closely dependent on which of these two cases (an α-particle excess over Mg^{24} or vice versa) holds. To decide the matter, more precise nuclear data are required than are at present available.

From the point of view of the present section, however, the situation is substantially the same whichever alternative we adopt, for in both cases the amount of Si^{28} produced by reaction (63) gives an Si^{28} abundance comparable with that of Mg^{24}, as observation demands. In addition, reactions (61) and (62) provide for the origin of the nuclei Mg^{26}, Al^{27}, Si^{29}, Si^{30}, and P^{31}.

X. ELEMENTS FROM SULPHUR TO CALCIUM

The number of reactions that affect nuclei heavier than silicon increases so rapidly as T rises above 1.5 that it is difficult to obtain quantitative estimates for the abundances of such elements. For this reason, the present section will be concerned mainly with qualitative issues.

The procedure so far followed has an underlying feature that will now be explained. Thus far we have been following a set of nuclei (C^{12} to Si^{28}) that have, apart from local deviations, become increasingly stable as the atomic weight increased. The general effect of this increased stability toward magnesium and silicon has been to make both additive and disintegrative processes favor a progression along the sequence of atomic weights: if at a particular temperature a nucleus became unstable then we simply moved on to nuclei of higher atomic weight that were stable at the temperature in question. Consider the effect of the photodisintegration of Ne^{20}, for example.

Now the situation is entirely altered as soon as we come to consider nuclei from sulphur to calcium, for all these nuclei are considerably *less* stable than silicon. Consequently, it is pointless to attempt to explain the origin of these nuclei by increasing the temperature until silicon becomes unstable—at such temperatures all the nuclei in question would be extremely unstable against photodisintegration processes. What would then result would be an approximation to statistical equilibrium, and we shall see in the next section that the nuclei would then belong almost entirely to a group centered around Fe^{56}, not to the sulphur-calcium group.

It may be useful in this connection to give a table of the binding energies of the proton,

neutron, and α-particle for several nuclear species (see the accompanying tabulation). These binding energies give the χ-values for use in equation (23), which determines the effects of photodisintegration. It should be emphasized that the stability of a nucleus is

	Binding Energies in Mev															
	Mg^{26}	Al^{27}	Si^{28}	Si^{29}	Si^{30}	P^{31}	S^{32}	S^{33}	Cl^{35}	A^{36}	Cl^{37}	A^{38}	K^{39}	Ca^{40}	Ca^{42}	Ti^{46}
Proton	13.9	8.3	11.6	12.3	12.6	7.2	8.8	9.4	6.3	8.5	10.4	10.1	6.7	8.3	10.2	10.4
Neutron	11.1	13.8	17.0	8.5	10.6	12.4	14.5	8.7	12.7	14.7	10.3	11.7	13.1	15.7	11.3	13.3
α-Particle	10.4	10.1	10.0	11.1	10.7	9.6	6.9	7.0	6.8	6.4	8.2	7.0	7.5	7.3	6.6	7.7

decided by the most sensitive reaction: a nucleus that is unstable against proton removal, for example, cannot be classed as stable because it happens to be stable against α-particle removal or against neutron removal. Thus the stability of Cl^{35} against neutron removal does not make it a stable nucleus; on the contrary, it would seem rather markedly unstable to proton removal.

The above remarks pose a serious question as to the mode of origin of the nuclei in the sulphur-calcium group. The answer to the apparent dilemma would seem to be in the possibility of the existence of a moderate α-particle excess produced by the reactions that destroy O^{16}, an excess, that is to say, over and above the α-particles that are required for the conversion of Mg^{24} into Si^{28}. We saw in the previous section that the number of α-particles must be comparable with, and may indeed be somewhat greater than, the number of Mg^{24} nuclei. If this is so, then there is an excess of α-particles not used in the reaction $Mg^{24} + \alpha \rightarrow Si^{28} + \gamma$ and available for the reaction

$$Si^{28} + \alpha \rightarrow S^{32} + \gamma\,. \tag{64}$$

Thus an α-particle excess forces Si^{28} nuclei through into the sulphur-calcium group. Indeed, the extent of the α-particle excess would seem to determine the abundance of the latter group. A 33 per cent α-particle excess would give an abundance ratio, (sulphur-calcium group)/Si^{28}, of about $\frac{1}{2}$, which is in accordance with the astrophysical data.

When we come to consider individual nuclei within the sulphur-calcium group itself, once again the tendency is for increasing stability to go with increasing atomic weight. Thus Ca^{40} is more stable than either A^{36} or S^{32}, and A^{36} is probably slightly more stable than S^{32}, in spite of the fact that the α-binding in A^{36} is somewhat less than that in S^{32}. The reason for this is that the Coulomb barrier for the reaction

$$A^{36} + \gamma \rightarrow S^{32} + \alpha \tag{65}$$

is higher than that for the reaction

$$S^{32} + \gamma \rightarrow Si^{28} + \alpha\,. \tag{66}$$

This means that, although S^{32} is first formed by reaction (64), a temperature is eventually reached at which reaction (66) supplies α-particles that promote the inverse of reaction (65), namely,

$$S^{32} + \alpha \rightarrow A^{36} + \gamma\,. \tag{67}$$

At a slightly higher temperature, reaction (65) likewise provides α-particles that promote the reaction

$$A^{36} + \alpha \rightarrow Ca^{40} + \gamma, \tag{68}$$

the Ca^{40} so produced being stable at the temperature in question; it seems likely that Ca^{40} is stable up to about $T = 1.75$.

In addition to reactions (67) and (68), the reactions

$$P^{31} + \alpha \rightarrow Cl^{35} + \gamma\,, \tag{69}$$

$$Cl^{35} + \alpha \rightarrow K^{39} + \gamma\,, \tag{70}$$

occur with appreciable probability. It is accordingly possible to understand qualitatively how chlorine and potassium may originate. There is, however, a serious quantitative difficulty: it seems that both Cl^{35} and K^{39} must be highly unstable against proton removal by photodisintegration. Indeed, it is surprising that these nuclei can persist, even at $T = 1.35$, for the usual time scale of 10^{15} seconds. Yet, if they do not so persist, we should expect to find only extremely low abundances of these nuclei. While it is true that meteoritic data do indicate unusually low abundances for these nuclei, the abundances are not nearly so low as quantitative estimates might suggest. The resolution of the difficulty may turn out to lie in the fact that the energy-level diagrams of these two nuclei are somewhat unusual. It may be that the energy levels are peculiarly distributed, in the sense that there is no level at all close to the disintegration energies—that there is no level close to 6.3 mev above the ground level of Cl^{35} and no level close to 6.7 mev above ground level in K^{39}.

XI. STATISTICAL EQUILIBRIUM AND THE IRON PEAK

The present section encroaches on the topics to be discussed in the second paper of this series. Thus far we have regarded the collapse of the star as being brought about by the escape of radiation from its surface into space. However, as the temperature in the deep interior rises above $T = 2$, other effects come to control the collapse of the star. Thus, at about $T = 3$ the loss of energy through neutrinos escaping from the star—the Gamow urca process—becomes more effective than the escape of radiation in promoting the collapse of the star. At about $T = 5$, neutrino emission greatly dominates the collapse, producing an important shrinkage, not in the former time scale of 10^{15} seconds, but in a time of the order of a year. At temperatures above $T = 5$, nuclear reactions other than β-processes take over control of the shrinkage, at which stage the collapse occupies no more than a few seconds of time.

Thus the state of the star changes, as T rises above 2, from one of quasi-stability to one of severe instability. The division between the present paper and a future paper is made by the fact that we are here concerned with the discussion of the quasi-stable case, whereas future work will be concerned with the high-temperature states of dynamical instability. The two cases differ not only in the time scales to be applied to them but also in the relation between the density of material and its temperature. The relation used frequently in the above work, namely,

$$\rho = 10^6 T^3,$$

belongs to the quasi-stable case and cannot necessarily be applied under conditions of serious instability.

Now, although the behavior of stellar material at temperatures near $T = 5$ comes mainly under the terms of reference of the investigation of a later paper, there is one consideration relating to temperatures of this order that it is useful to discuss here, for the reason that it throws light on all the work of the present paper.

At temperatures near $T = 5$, in spite of the shortening of the time scale due to the rapid dynamical evolution of the star that then ensues, nuclear reactions occur so rapidly and in such profusion that an approximation to statistical equilibrium among them must of necessity be set up. Both associative and photodisintegrative processes happen very

quickly: the photodisintegration of the refractory nucleus Si^{23}, for example, takes less than a second at $T = 5$, as compared with more than 10^{15} seconds at $T = 2$.

The equations of statistical equilibrium are well known. The form in which they will now be used was arrived at in a former paper. Thus, we quote[7] the following equation,

$$\log n_A^Z - 34.08 - \tfrac{3}{2}\log T - \tfrac{3}{2}\log A = \frac{A}{A'}(\log n_{A'}^{Z'} - 34.08 - \tfrac{3}{2}\log T - \tfrac{3}{2}\log A') + \frac{5.04}{T} A \left(\frac{Q_A^Z}{A} - \frac{Q_{A'}^{Z'}}{A'}\right) + A\theta\left(\frac{Z}{A} - \frac{Z'}{A'}\right), \quad (71)$$

where logarithms are to the base 10; θ is the logarithm of the ratio of the density of free protons to the density of free neutrons; T is again in units of 10^9 ° K; Q_A^Z and $Q_{A'}^{Z'}$ represent the binding energies in mev of nuclei (A, Z) and (A', Z'), respectively; and n_A^Z and $n_{A'}^{Z'}$ are the number densities of these nuclei. For nuclei with experimentally determined binding energies, equation (71) serves to determine statistical abundance ratios when T, θ, and the total density are specified. In the accompanying tabulation the logarithms of abundance ratios, relative to Fe^{56}, are given for $T = 5.04$, $\theta = 2$, and $\log n_{56}^{26} = 28$—these values being chosen so as to be also consistent with statistical equilibrium between

ABUNDANCE RATIOS RELATIVE TO Fe^{56} (LOGARITHMS TO BASE 10)

	Nucleus											
	Ti^{48}	Cr^{52}	Fe^{54}	Mn^{55}	Fe^{57}	Fe^{58}	Ni^{58}	Co^{59}	Ni^{60}	Ni^{62}	Cu^{63}	Zn^{64}
Theoretical abundances	−3.3	−0.38	−0.47	−2.3	−2.3	−2.0	−1.5	−2.6	−0.67	−3.1	−4.8	−5.0
Meteoritic abundances	−2.6	−1.9	−1.2	−2.0	−1.6	−2.5	−1.8	−2.4	−1.4	−2.7	−3.4	−3.9
Adopted binding energy per nucleon (mev) (only differences affect the calculated abundances)	+8.702	+8.773	+8.737	+8.764	+8.769	+8.793	+8.733	+8.768	+8.785	+8.782	+8.741	+8.725

neutrons and protons (Hoyle 1948). A value of 492 mev was adopted for the binding energy of Fe^{56}.

These values provide a satisfactory explanation of the well-known sharp peak of abundances centered at Fe^{56}. Statistical balancing fails, however, to explain the abundances of elements in the far wings of the peak; the abundances are too low for elements lighter than titanium and for elements heavier than nickel. Taking different values for T and ρ (θ being determined by statistical considerations when T and ρ are given) does not alter this situation: a lowering of the temperature produces a *narrowing* of the peak (the abundances only being slightly altered by moderate variations of ρ). We may quote the result of a future paper that a significant increase of T above 5 is inapplicable to the building of these nuclei. The origin of elements from calcium to titanium and of elements heavier than nickel accordingly remains to be explained. Elements heavier than nickel will be considered in detail in a subsequent paper, but the elements from calcium to titanium will not receive consideration either in this or in the subsequent paper. The reason for this omission is that the members of the calcium-titanium group of nuclei probably originate at values of T only slightly above 2, before any useful approximation to statistical equilibrium is attained but at temperatures such that the reactions occurring are very numerous. Thus this group of nuclei probably belongs to an intermediate range of tem-

[7] Hoyle (1946, p. 375). The present eq. (71) is equivalent to eq. (50) of this former paper. It may be noted that a numerical difference arises in the coefficient that multiplies $(Q_A^Z/A - Q_{A'}^{Z'}/A')$, because Q_A^Z and $Q_{A'}^{Z'}$ are measured in mev in the present paper, as compared with the m.m.U. used in the former paper.

peratures in which it is extremely difficult to make quantitative estimates—where the reactions that occur are too numerous to be traced individually but where statistical equilibrium cannot be applied.

The present results put the work of earlier sections in a clearer light. We have seen that the general tendency with rising temperature has been for nuclei of increasing atomic weights to be synthesized. In this work we have been examining the details of a slow drift toward statistical equilibrium. If all additive and disintegrative reactions had taken place sufficiently rapidly at lower temperatures, the statistical distribution of nuclei centered around Fe^{56} would have been set up. The fact that this did not happen was due to the slowness of the reactions, but those reactions that did occur had the general effect of favoring the drift toward the iron peak.

Two further points must now be made concerning the abundances shown in the table on page 144. At temperature $T = 5$ an assumption used implicitly in previous sections cannot be safely made. Thus it has been assumed so far that the composition of material is not altered during the process that causes its ejection from the star. At temperatures at which nuclear reactions take place slowly, in a time of the order of 10^{15} seconds, for example, this is certainly valid, since the ejection presumably takes place in only a few seconds. At values of T near 5, however, many reactions also take place on a time scale of a few seconds. Accordingly, it is no longer valid to suppose that the composition remains unchanged during the expansion of the material. The freezing of the composition must then take place in a complex manner, and it does not follow that material with, say, the composition shown in the table on page 144 will necessarily reach a dispersed state without substantial changes taking place. Second, it is to be emphasized that the statistical equation (71) neglects the internal partition functions of the nuclei. Inclusion of the partition functions might well alter the calculated abundances by a factor of as much as 10. Furthermore, it may be added that the nucleus binding energies adopted above may be in error by a margin sufficient to alter the calculated abundances also by a factor 10.

Finally, we may note that, since Fe^{56} is the most abundant nucleus at $T = 5$, the absolute abundance of Fe^{56} must be roughly comparable with the abundance of Si^{28}, which, it will be recalled, was the most abundant nucleus at $T = 2$. This is in accordance with observation.

XII. CONCLUSIONS

It remains to collect in a table (see the accompanying tabulation) the main results deduced above (with the exception of the abundances of elements in the iron group, which have already been given in tabular form on p. 144). It is particularly stressed that

Results Deduced	Astrophysical Estimates	Terrestrial and/or Meteoritic Abundances
$C^{12}/O^{16} \doteq \frac{1}{3}$	$C^{12}/O^{16} = \frac{1}{3}$	
$O^{16}/Ne^{20} \doteq 1$	$O^{16}/Ne^{20} = 1$	
$O^{17}/O^{16} \leq \frac{1}{1200}$		$O^{17}/O^{16} = \frac{1}{4000}$
$O^{18}/O^{16} \leq \frac{1}{30}$		$O^{18}/O^{16} = \frac{1}{500}$
$Ne^{22}/Ne^{20} \doteq \frac{1}{30}$		$Ne^{22}/Ne^{20} = \frac{1}{10}$
$F^{19}/O^{16} \doteq \frac{1}{8000}$	?	?
$Ne^{21}/Na^{23} \doteq 1$	$Na^{23}/Ne^{20} = 0.0045$	$Ne^{21}/Ne^{20} = 0.0030$
	$Ne^{21}/Na^{23} = \frac{2}{3}$	
$Na^{23}/O^{16} \doteq \frac{1}{400}$	$Na^{23}/O^{16} = \frac{1}{200}$	
$Mg^{24}/O^{16} \doteq \frac{1}{40}$	$Mg^{24}/O^{16} = \frac{1}{9}$	
$Ne^{22}/Ne^{20} \doteq Mg^{25}/Mg^{24}$		$Ne^{22}/Ne^{20} = Mg^{25}/Mg^{24}$
$Si^{28}/Mg^{24} \doteq \frac{2}{3}$	$Si^{28}/Mg^{24} \doteq \frac{1}{2}$	
$(S^{32}, A^{36}\ Ca^{40}) \doteq 0.5 Si^{28}$	$(S^{32}, A^{36}, Ca^{40}) \doteq 0.5 Si^{28}$	
$Si^{28} \doteq Fe^{56}$	$Si^{28} \doteq Fe^{56}$	$Si^{28} \doteq Fe^{56}$

the results given in the first column were obtained subject to certain assumptions and accordingly have not been demonstrated in a manner free from all doubt. Nevertheless, the number of assumptions made was much less than the number of results obtained.

The comparisons shown above can, in the view of the writer, be summed up in the statement that they provide considerable support for the belief that the elements from carbon to nickel have been synthesized by nuclear reactions occurring inside stars—albeit inside the special collapsed type of star that has been under consideration throughout the paper.

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