Summary

The present paper gives the results of a photographic photometry of 39 nearby galaxies. The results are mainly based on observations made by the author as guest investigator at the Mount Wilson and Palomar Observatories. The material comprises nebulae belonging to the local group and the two groups associated with Messier 81 and 101. The investigation has principally aimed at a determination of the luminosity function of nebulae and a study of the relation between total magnitude and magnitude of brightest resolved stars.

Chapter I gives an account of the selection of the material. The known members of the local group are listed in Table 1, whereas Tables 2 and 3 contain the members of the M 81 and M 101 groups. The latter members have in most cases been selected on the basis of the radial velocity. A careful search for faint group members has been made on survey plates taken with the Mount Wilson 10-inch refractor, the plates covering circular areas with a diameter of 20 degrees. The final material, 29 nebulae, presumably represents a given volume of space.

Chapter II presents the results of the photometric investigation. Total photographic and photovisual magnitudes, on the international scale, have been determined by numerical integrations of the luminosity distributions of the nebulae as obtained from microphotometer tracings. The plate material has been secured by the writer in 1947—1948, mainly by means of the Mount Wilson 60-inch telescope. A number of additional plates have later on been obtained from the 60-cm and 100-cm reflectors of the Hamburg Observatory. Each plate has been calibrated by means of extrafocal exposures of stars of the North Polar Sequence. The final results are contained in Tables 5 and 6. The 168 individual magnitudes are based on analyses of about 2800 photometer tracings. The internal agreement indicates a mean error of one plate of $0^{m}.056$. The average color indices for different types of nebulae range from +0.33 to +0.84 (Table 7). The total luminosities are about one magnitude brighter than the values of the Shapley-Ames catalogue (Fig. 6).

Chapter III gives the frequency distributions of absolute luminosities and dimensions, as derived from the data collected for the present material of nebulae. Both distributions possess a considerable skewness, the maximum frequencies corresponding to faint, and small systems (Fig. 7). The luminosity curve has a mean of —13.5 and a dispersion of not less than 1.85 magnitudes. Both values deviate considerably from those previously adopted.

In *Chapter IV* an investigation is made of the relation between total magnitudes (M_t) of nebulae and magnitudes (M_s) of brightest resolved stars. The correlation between these quantities, which seems to be rather pronounced, is described by the equation, $M_s = 0.60\ M_t + 2.4$. The constant term is not the same for different types of nebulae. The variation is interpreted as a systematic effect in the total luminosities, caused by differences in stellar content. The correlation between M_s and M_t will, unfortunately, reduce the value of the apparent magnitude of resolved stars as a distance indicator. A similar relation between total magnitude and magnitude of brightest stars has been derived for stellar populations in spheres of different radii around the sun and for open galactic clusters (Fig. 9). A theoretical analysis shows that the relation may be explained entirely as a result of statistical selection, the most luminous stars favoring the largest samples.

I wish to express my sincere gratitude to the Director of Mount Wilson and Palomar Observatories, Dr. I. S. Bowen, who has made possible this investigation by putting the telescopes and other facilities of the observatories at my disposal. I extend my thanks to Dr. W. Baade for his aid and advice in the course of the work and for his permission to make use of unpublished results. My thanks are also due to Dr. E. Hubble, Dr. M. L. Humason, and Dr. N. U. Mayall, who have supplied me with unpublished data. By the kind courtesy of Dr. O. Heckmann, Director of the Hamburg Observatory, it has been possible to enlarge the material considerably. I am indebted to Dr. P. Wellmann and Mr. J. Stock, who have given valuable assistance in the observational work at Hamburg. The measuring and reduction work has been done at Lund at I wish to acknowledge my gratitude to Dr. K. Lundmark, Director of the Lund Observatory, who has furthered the investigation in several ways. Finally, I am indebted to the *Naturvetenskapliga Forskningsrådet*, to the *Kungl. Fysiografiska Sällskapet*, and to the trustees of the *Lennanderska Fonden*, who have given the economical support necessary to carry through this investigation.

CHAPTER I

Scope of the Investigation

1. *Introduction.* The present paper gives the results of a photographic photometry of a number of nearby extragalactic nebulae. Total photographic and photovisual magnitudes have been determined by numerical integrations of luminosity distributions as obtained from microphotometer tracings. The plate material has been collected mainly by means of the Mount Wilson 60-inch reflector. A detailed account of the material and the procedure followed in the magnitude determinations is given in the following chapter.

The principal aim of the photometric investigation has been the accumulation of data for a study of the *absolute* magnitudes of extragalactic nebulae and their distribution function. The recent discoveries of several very faint nearby galaxies seem to indicate that the number of systems of low luminosities has been underestimated and that the dispersion in the absolute magnitudes may be considerably larger than the usually adopted value of about one magnitude. The magnitude distribution may prove to be unsymmetrical, the maximum frequency being shifted towards fainter luminosities. The absolute magnitudes of brightest resolved stars and their relation to the total magnitudes of nebulae represent another important problem to be investigated in this connection.

The study of absolute luminosities has been confined to the members of three metagalactic groups, and the results thus refer to a *given volume of space*. The confinement to a definite and limited volume reduces the number of nebulae to be included in the final analysis but the material will, on the other hand, be free from disturbing systematic errors due to statistical selection.

The investigation is, in the first place, based on the local group of nebulae. More or less reliable distance determinations are available for practically all of these nearby systems. Since the number of objects is too small for a statistical analysis, the material has been increased by the inclusion of nebulae belonging to the groups associated with the bright spirals M 81 and M 101. In these cases we have to replace the distances of the individual objects, which are generally not known, by the mean distances of the groups. As is shown below, the addition of the members of the latter two groups doubles the material, and the total number of nebulae will amount to 29.

TABLE 1.

The local group.

Object	Type	m_{pg}	V	Remarks
Galactic syst.	Sb (Sc?)	—	—	member
NGC 147	Ep	10.46	—	»
NGC 185	Ep	10.17	−270 (M)	»
NGC 205	Ep	8.89	−239 (H)	»
NGC 221	E2	9.06	−220 (H)	»
NGC 224	Sb	4.33	−267 (H)	»
NGC 598	Sc	6.19	−190 (H)	»
NGC 6822	Ir	9.21	− 34 (H)	»
IC 1613	Ir	10.00	−235 (H)	»
LMC	Ir	1.2	+276 (W)	»
SMC	Ir	2.8	+168 (W)	»
Fornax syst.	Ep	9.1	+149 : (H)	»
Sculptor syst.	Ep	8.8	—	»
Wolf-Lundm. syst.	Ir	11.13	—	prob. memb.
NGC 6946	Sc	11.1	+ 34 (H)	poss. memb.
IC 10	Sc	—	−343 (H)	» »
IC 342	Sc	—	− 20 (H)	» »
Leo syst.	Ir	—	—	» »
Sextans syst.	Ir	—	—	» »

The final material is rather limited, but it may not be possible for the present to get any additional contributions of importance. An investigation of absolute luminosities and their distribution function can, naturally, be based only on those groups (clusters) of nebulae with known distances where a reliable separation between members and field nebulae is feasible. In the Virgo cluster, for instance, we have to expect a considerable number of galaxies as faint as the sixteenth apparent magnitude, and a complete separation of these cluster members from background nebulae seems to be difficult with the observational data available today.

2. *The local group.* The comparatively high space density of nebulae in our metagalactic neighborhood indicates that the galactic system forms part of a small group, or cluster of galaxies. The group mainly consists of the triple system formed by the Galaxy and the two Magellanic Clouds, and of the quintet containing the Andromeda nebula and its companions (NGC 147, 185, 205, and 221). The conspicuous spiral system M 33 may, perhaps, also be referred to the latter sub-group.

Table 1 gives a list of nebulae which may be accepted as members of the local group. The nebulae, at least those contained in the upper part of the table, are all situated at distances smaller than 300 kiloparsec, which may be assumed to represent the outer limit of the group. Most of the objects are the same as those included in a list recently published by W. BAADE[1].

The second column of the table gives the types of the nebulae, and the third column the total photographic magnitudes. The values given with two decimals represent

[1] Mt Wilson Contr. 697 = ApJ 100, p. 147 (1944).

measures by the writer. The magnitude for NGC 6946 refers to the SHAPLEY-AMES catalogue[1]. The remaining magnitudes, which are based on investigations at Harvard and at Lembang, are the same as those adopted by BAADE in his above-mentioned list. The fourth column gives the observed radial velocities[2], expressed in km/sec, according to M. L. HUMASON (H), N. U. MAYALL (M), and R. E. WILSON[3] (W). The velocities are, with four exceptions, negative.

According to the remarks given in the last column, thirteen of the nebulae listed in the table may be regarded as rather definitive members of the local group. The Wolf-Lundmark nebula[4] has been designated »probable member», the distance modulus in this case being based only on the magnitudes of the brightest resolved stars. A comparison of the bright end of the stellar luminosity curve, as determined by W. BAADE (Cp. Ch. III), with the luminosity curve of IC 1613 gives a distance modulus (corr. for gal. abs.) of about 22.0. For the five nebulae in the lower part of the table, which are listed as »possible members», no reliable distance determinations are available. Three of these, NGC 6946, IC 10, and IC 342, which have been more closely studied by E. HUBBLE[5], are situated in low latitudes and are heavily obscured by galactic absorption. The large angular dimensions and the low velocities indicate that the distance moduli of these nebulae are comparatively small. The two last systems, which are of the irregular type, have been discovered by F. ZWICKY[6] on photographs taken with the 18-inch Schmidt telescope at Mount Palomar.

It seems certain that the above list includes all high-luminosity nebulae, which belong to the local group. As regards faint systems, especially those of the elliptical and irregular types, we probably have to expect future additions. The discovery by H. SHAPLEY of the extremely faint stellar agglomerations in Fornax and Sculptor seemed, at first, to indicate that the number of dwarf systems had been seriously underestimated. Subsequent examinations at Harvard[7] of small-scale plates covering more than one third of the sky have, however, failed to reveal any more objects of the same type. A number of nebulae belonging to the local group may be hidden behind absorbing material in low galactic latitudes. An omission of possible members

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[1] Harvard Ann. 88, No. 2 (1932).

[2] The radial velocities contained in Tables 1—3 represent, for the most part, new observations and revised values from the Mount Wilson and Lick observatories which have not yet been published. The writer wishes to acknowledge his gratitude to Dr. HUMASON and to Dr. MAYALL for the permission to use their measures in the present discussion.

[3] Lick Publ. 13, p. 185 (1918).

[4] This irregular system ($\alpha = 0^h\ 0^m$, $\delta = -15^\circ\ 45'$, 1950) was first noted by M. WOLF at Heidelberg and was later rediscovered by K. LUNDMARK and P. J. MELOTTE (MN 86, p. 636, 1926).

[5] *The Realm of the Nebulae*, Yale University Press (1936).

[6] Phys. Rev. II, 61, p. 489 (1942). The coordinates (1940) are: $\alpha = 9^h\ 55^m.9$, $\delta = +31^\circ\ 2'$ (Leo syst.), and $\alpha = 10^h\ 8^m.1$, $\delta = -4^\circ\ 24'$ (Sextans syst.).

[7] Wash. Nat. Ac. Proc. 25, p. 565 = Harvard Repr. 183 (1939). In a private communication to the writer Dr. W. Baade has reported that two new systems of the Sculptor type have recently (April 1950) been found on survey plates taken with the 48-inch Schmidt telescope on Mount Palomar. Both systems seem to be members of the local group.

in these regions will be of no importance in the following statistical discussion, since it is equivalent to a reduction of the volume of space investigated.

3. *The Messier 81 group.* The conspicuous spiral system M 81 (NGC 3031) in Ursa Major is surrounded by a number of bright nebulae, which apparently form a cluster more or less similar to the local group. The most well-known of these objects are the irreglar nebulae NGC 3034 and 3077, and the spiral NGC 2976.

Since the M 81 group is going to be included in the following statistical investigation, it has been necessary to make a detailed examination of the region in order to discover possible faint members of the group. The survey has been restricted to a circular area, centered on M 81, with a diameter of twenty degrees. If we assume a mean distance modulus of the group of about 24 (Cp. Ch. III), this diameter will correspond to a tangential distance of approximately 200 kiloparsec. Although the diameter is small compared to the extension of the local group, the survey has for practical reasons been restricted to a region of this size.

The search for faint members has been based on a number of plates taken by the writer with the Mount Wilson 10-inch refractor, in this case equipped with the 5-inch Ross lens. Each of the plates (Eastman 103 *a—O*, 8″ x 10″), which are partly overlapping, covers an area of about 13 x 16 square degrees. Exposures of 80 minutes clearly bring forth all nebulae down to the fourteenth apparent magnitude, which, with the distance modulus suggested above, presumably represents the limiting magnitude for group members. Careful examinations of the plates reveal a number of nebulae which have not before been catalogued. These objects have been included in a working list, together with all the NGC- and IC-objects to be found within the survey area.

The separation of possible group members from background nebulae has, in the first place, been based on radial velocities, which are available for a comparatively large number of the objects. If radial velocities are missing, it has been necessary to use apparent dimensions and degree of resolution as criteria of membership. Most of the doubtful objects have been photographed earlier with the 60-inch and 100-inch telescopes, and are included in the large plate collection of the Mount Wilson observatory. The remaining objects have, together with the new nebulae discovered in the survey, been photographed by the writer with the 60-inch telescope. A study of the reflector plates shows that most of the nebulae undoubtedly belong to the background. Only a few of the objects may, with a reasonable degree of certainty, be assigned to the group.[1]

The results of the survey of the M 81 region are reproduced in Table 2. The list of nebulae given here is however, with a few exceptions, confined to objects contained in the Shapley-Ames catalogue. The different columns are explained in the same way as those of Table 1. The magnitudes (third column) given with two decimals have been measured by the writer, whereas the remaining ones have been

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[1] The writer is indebted to Dr. W. Baade, who has given valuable advice and assistance in the discussion of the individual objects and in the selection of group members.

TABLE 2.

Survey of the M 81 region.

Object	Type	m_{pg}	V	Remarks
NGC 3031	Sb	7.85	− 55 (H)	member
NGC 2366	Ir	12.6	+ 140 (M)	»
NGC 2403	Sc	8.79	+ 66 (H)	»
NGC 2976	Sc	10.73	+ 40 (M)	»
NGC 3034	Ir	9.20	+ 263 (H)	»
NGC 3077	Ir	10.57	− 130 (M)	»
IC 2574	Ir	10.91	+ 17 (M)	»
Ho I	Ir	13.27	—	prob. memb.
Ho II	Ir	11.14	—	» »
NGC 2523	S	12.7	+3410 (M)	background
NGC 2551		13.1	+2310 (M)	»
NGC 2633	SBc	12.6	+2260 (M)	»
NGC 2646		12.8	+3540: (M)	»
NGC 2655	Sa	11.6	+1299 (H)	»
NGC 2715	Sb	11.89	+1170 (M)	»
NGC 2742	Sc	12.5	—	»
NGC 2748	Sc	12.4	+1500 (M)	»
NGC 2768	E	12.0	+1408 (H)	»
NGC 2787	SBa	12.1	+ 627 (H)	»
NGC 2805	Sc	11.77	—	poss. memb.
NGC 2820	S	13.17	—	background
NGC 2880	E	12.9	+1520 (H)	»
NGC 2977	S	13.25	—	»
NGC 2985	Sb	11.8	+ 970 (H)	»
NGC 3027	Sc	12.31	—	poss. memb.
NGC 3043		13.2	—	background
NGC 3061	Sc	13.44	—	»
NGC 3065	E	12.9	—	»
NGC 3147	Sc	11.9	+2721 (H)	»
NGC 3183	SBb	12.59	—	»
NGC 3259	S	12.9	—	»
NGC 3329	E	12.9	—	»
NGC 3348	E	12.1	+2855 (H)	»
NGC 3359	S	12.2	+1080 (M)	»
NGC 3364	Sb	13.40	—	»
NGC 3403	S	12.9	—	»
NGC 3516	Sa	12.2	+2614 (H)	»
NGC 3735	Sc	12.6	—	»
IC 529	Sc	12.54	—	»
Ho III	Sc	12.94	—	»

Ho I: $\alpha=9^h36^m.0$, $\delta=+71°27'$ (1948)
Ho II: $\alpha=8^h13^m.9$, $\delta=+70°52'$ (1948)
Ho III: $\alpha=9^h\ 9^m.3$, $\delta=+74°28'$ (1948)

taken from the Harvard catalogue[1]. The radial velocities (fourth column) have been determined by HUMASON and by MAYALL.

Table 2 includes 40 nebulae, of which three (Ho I—III) are not listed in previous

[1] It has, on account of unfavorable weather conditions, not been possible to include the object NGC 2366 in the photometric program.

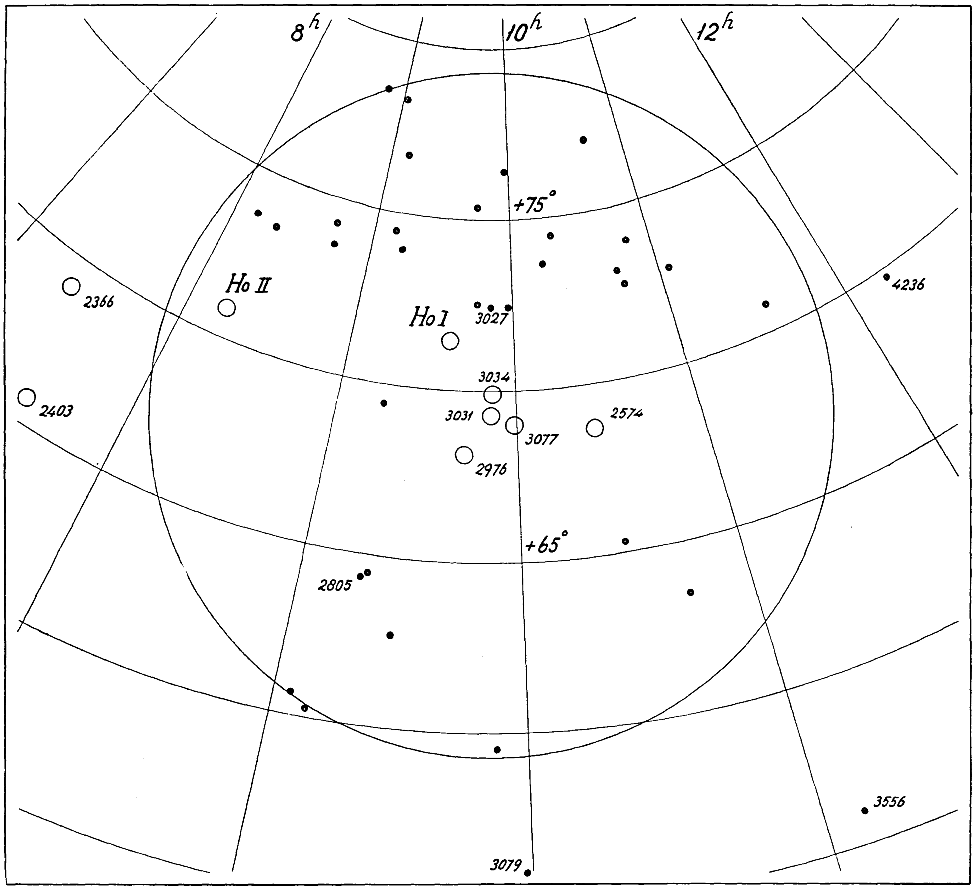

Fig. 1. The Messier 81 region (1950). Nebulae accepted as group members are denoted by open circles.

catalogues[1]. The distribution of the objects is shown in Fig. 1, where the nebulæ accepted as members of the group are denoted by open circles. Two of the latter objects, NGC 2366 and NGC 2403, fall outside the survey area but have been included in the list since they are undoubtedly physically associated with the group. There are three other conspicuous nebulæ, NGC 3079, 3556, and 4236, which are situated just outside the area. For one of these (NGC 3556) HUMASON reports a radial velocity of + 636 km/sec, which seems to exclude the possibility of membership.

Nine of the nebulæ included in Table 2, three spirals and six irregular objects, have been selected as members of the M 81 group. The radial velocities available for the seven nebulæ designated »members» seem to give a rather definite indication

[1] After the completion of this investigation it was found that the irregular system Ho I has been previously discovered by F. ZWICKY (Phys. Rev. II, 61, p. 489, 1942).

of physical membership[1]. The two nebulæ Ho I and Ho II, which have been classified as »probable members», may be assumed to be associated with the group on account of their high degree of resolution. They have diameters of about 5′ and 10′, respectively, and appear to be systems very similar to IC 1613. Two objects in the list have been classified as »possible members». These nebulæ are probably not group members, but a final decision has to await the determination of radial velocities. The remaining 29 objects apparently belong to the background field, as is indicated by their small angular dimensions, or by the high radial velocities. It may be remarked that three of the latter objects (NGC 2633, 2742, and 2748) are included in HUBBLE's list of resolved nebulæ[2]. There does not seem to be any doubt, however, that we in these cases have to deal with background nebulæ.

4. *The Messier 101 group.* The well-known spiral system M 101 (NGC 5457) seems, just as M 81, to be the principal object in a small group of nebulæ, the most conspicuous members being the spirals NGC 5474 and 5585, and the double object NGC 5194—95. The M 101 group will, as was mentioned above, be included in the statistical investigation.

In order to find possible faint group members an examination has, as in the previous case, been made of a circular area of a diameter of twenty degrees centered on M 101. The survey has been based on plates taken with the Mount Wilson refractor, and a working list has been prepared containing NGC- and IC-objects, together with the new objects discovered on the survey plates. The selection of members has been based on radial velocities and degree of resolution, the latter being studied on plates from the 60-inch and 100-inch telescopes.

The results of the survey, as regards nebulæ brighter than the thirteenth magnitude, are shown in Table 3. The different columns[3] of the table are explained in the same way as those of Table 2. The distribution of the nebulæ appears from Fig. 2, where the selected group members are denoted by open circles. Four conspicuous spiral nebulæ are, as appears from the figure, situated in the neighborhood of the investigated area. For three of these objects HUMASON reports radial velocities of +282 km/sec (NGC 4736), +500 km/sec (NGC 5055), and +553 km/sec (NGC 5907). The first nebula may, perhaps, be associated with the M 101 group.

Six of the objects listed in Table 3 have been classified as »members». Although the radial velocity of the double system NGC 5194—95 is somewhat larger than the mean velocity of the other four members, the agreement between NGC 5194 and NGC 5457 as regards magnitudes of brightest resolved stars (Cp. Table 12)

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[1] The mean velocity of the seven nebulae is equal to +49 km/sec. If this value is corrected for red shift (526 km/sec per 10^6 pars.) and for galactic rotation (275 km/sec) we get a final velocity of about — 100 km/sec, which represents the motion of the M 81 group with respect to the galactic system.

[2] Mt Wilson Contr. 548=ApJ 84, p. 158 (1936).

[3] The object NGC 5204 has, as appears from the table, not been included in the photometric program. The nebula was first classified as a »possible member». The radial velocity which has later on been determined at the Lick observatory seems, however, to leave no doubt about the membership.

TABLE 3.

Survey of the M 101 region.

Object	Type	m_{pg}	V	Remarks
NGC 5457	Sc	8.20	+ 285 (H)	member
NGC 5194	Sc	8.88	+ 438 (H)	»
NGC 5195	Ir	10.47	+ 542 (H)	»
NGC 5204	Sc	12.2	+ 270 (M)	»
NGC 5474	Sc	11.22	+ 230 (M)	»
NGC 5585	Sc	11.25	+ 300 (M)	»
NGC 5173		—	+2404 (H)	background
NGC 5198	E	12.9	+2482 (H)	»
NGC 5301	S	13.0	—	poss. memb.
NGC 5308	E	12.8	+2046 (H)	background
NGC 5322	E	11.6	+1902 (H)	»
NGC 5376	S	13.0	—	»
NGC 5377	Sa	12.8	+1830 (H)	»
NGC 5422	S	13.0	—	»
NGC 5430	SB	12.8	—	»
NGC 5448	S	12.5	+1970 (H)	»
NGC 5473	SBa	12.8	+1976 (H)	»
NGC 5480	S	12.6	—	»
NGC 5485	E	12.9	+1985 (H)	»
NGC 5631	Sa	12.5	+1979 (H)	»
NGC 5633	Sb	12.8	—	»
NGC 5660	Sc	12.3	—	»
NGC 5676	Sc	11.9	+2240 (M)	»
NGC 5678	Sc	12.1	—	poss. memb.
NGC 5687	S	12.7	+2119 (H)	background
NGC 5689	SBa	12.6	+2205 (H)	»
NGC 5820	E	12.8	+3269 (H)	»
NGC 5866	Sa	11.5	+ 598 (H)	poss. memb.
NGC 5879	Sb	12.1	+ 876 (H)	background
Ho IV	S	12.95	—	poss. memb.
Ho V	Sc	13.23	—	background

Ho IV: $\alpha = 13^h52^m.9$, $\delta = +54° \ 9'$ (1948)
Ho V: $\alpha = 13^h38^m.7$, $\delta = +54°36'$ (1948)

suggests distance moduli of the same order of size[1]. A careful examination of the survey plates has in the present case failed to reveal any faint group members of the irregular type. Four objects in the list have, on account of comparatively large angular dimensions, been classified as »possible members». Among these is one of the new nebulæ found by the writer (Ho IV), which seems to be a spiral system seen edgewise. The possible member NGC 5866 probably belongs to a separate and more distant group together with NGC 5879 and NGC 5907, as is indicated by the radial velocities. The remaining nebulæ of the list, including the spiral NGC 5633 which is resolvable according to the above-mentioned investigation by HUBBLE, probably belong to the background field.

[1] The mean velocity of the six group members is equal to +344 km/sec. After correction for red shift and galactic rotation the value is reduced to about +190 km/sec, which represents the motion of the group with respect to the galactic system.

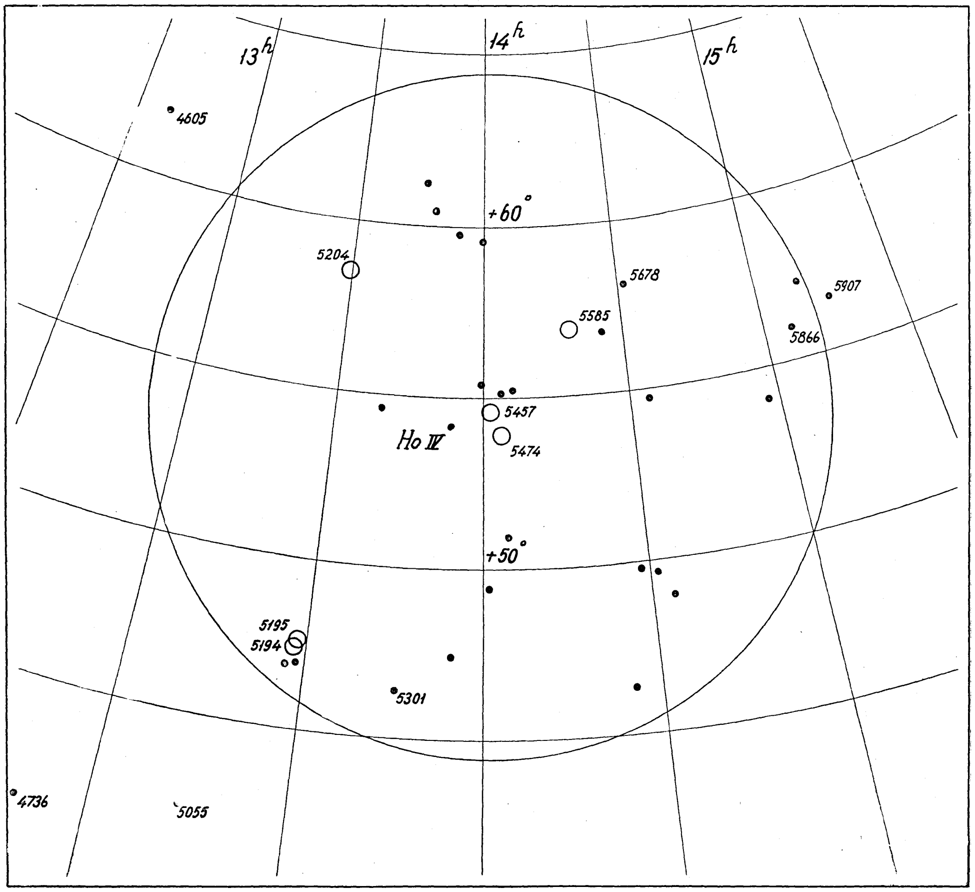

Fig. 2. The Messier 101 region (1950). Nebulae accepted as group members are denoted by open circles.

The comparatively large difference in radial velocity between NGC 5194—95 and the central group members raises the question of the maximum velocity deviations to be tolerated among the supposed members of a physical group. If we assume that the group has attained a state of equilibrium it is possible to derive, theoretically, the order of size of these deviations. Since we in the present case are dealing with a rather small group of nebulæ, it seems sufficient to confine the study to the simple case of a problem of two bodies.

The orbital motion of an outside object around a central group of nebulæ may be described by the equation,

$$av^2 = 4\pi^2 q \cdot M, \tag{1}$$

where a is the tangential distance (astr. units) between the outside object and the central group, and v is the differential radial velocity (astr. units per year). The quantity M represents the total mass (solar units) of all the systems involved. The projection factor q is defined by the equation,

$$q = \cos^2\varphi \cos^2\psi \cdot \sqrt{\cos^2\varphi + \sin^2\varphi \sin^2\psi}\,. \qquad (2)$$

The angle $(90 - \psi)$ gives the inclination of the orbital plane to the celestial plane, whereas φ is the angular distance of the orbital radius from the intersection line of these planes.

If q is put equal to 1, that is, if $\varphi = \psi = 0$, the differential velocity will attain a maximum value. If the angles φ and ψ are assumed to be distributed at random, which implies that the frequency functions of φ and ψ may be represented by a constant term and by $\cos\psi$, respectively, a numerical integration of the above trigonometric expression gives a mean value of q equal to 0.29. The quantity v^2 is, on the average, equal to about one third of the corresponding maximum value.

The formulæ given here presuppose a circular orbital motion. In the case of an elliptical orbit, the differential radial velocity may be larger (object near pericenter), or smaller (object near apcenter) than the velocity defined by equation (1).

Adopting a distance modulus of the M 101 group of 23.7 (Cp. Ch. III), a largest nebular mass of 10^{11} solar units, and a factor q equal to 1.0, we find for the orbital motion of NGC 5194—95 around the central group a maximum value of the differential radial velocity that is of the same order of size as the observed value. Thus, it does not seem possible to use the observed differences in radial velocity as an argument against the inclusion of the system NGC 5194—95 among the group members, especially if the accidental errors of the velocities are taken into consideration. For the three nebulæ NGC 5866, 5879, and 5907, which probably are physically associated, we find a corresponding difference in observed radial velocity that is more than twice as large as the maximum value derived from equation (1). These objects probably belong to the background field, as has been assumed above.

CHAPTER II

Determination of Integrated Magnitudes and Colors

5. *Photometric method and instruments.* The present photometric program comprises nebulæ belonging to the three metagalactic groups discussed in the previous chapter. Total photographic and photovisual magnitudes, on the international scale, have been determined for 22 of the 28 external group members. The inclusion of a number of suspected members and a few other interesting objects increases the material to a total number of 39 nebulæ.

The magnitudes have been derived by numerical integrations of the luminosity distributions of the nebulæ, as obtained from microphotometer tracings. The photographic plates have been calibrated by means of extrafocal star images, and the photometry is thus based on a comparison between two kinds of surface objects. Since the final magnitude of each nebula is derived from a large number of tracings of different cross-sections, the result is independent of any assumptions regarding the distribution of light in the object. It may be remarked that many of the nebulæ included in the program are no suitable objects for a photoelectric photometry on account of their large angular dimensions. The photographic method, although rather laborious and time-consuming, seems to give the most accurate results in these cases.

The present investigation is mainly based on a material of plates that has been secured by the writer at the Mount Wilson observatory. The observations, which extend from August 1947 to February 1948, have been performed partly with the 60-inch reflector (Newton focus) and partly with the 10-inch refractor. The latter instrument may be equipped either with a 10-inch lens ($f=50''$) or with a 5-inch lens (the Ross lens, $f=35''$), which are corrected for the photovisual and photographic regions, respectively. Later on (1949—1950), a number of additional plates have been obtained by means of the 60-cm and 100-cm reflectors (focal lengths = 3 m) of the Hamburg observatory. Most of the latter plates have been secured in order to study possible systematic errors in the magnitudes, as derived from the Mount Wilson material.

The procedure used in the calibration of the plates, which represents the most important step of the investigation, may be summarized as follows. (*a*) The relation between plate density and surface magnitude has, for all the plates, been derived from extrafocal exposures of a star field, the extrafocal distance being adjusted to give star images of suitable densities. Sensitometer calibrations have not been used. The results obtained from exposures of a number of the plates in a tube sensitometer indicate, that these »artificial» calibrations may be affected with systematic errors[1]. (*b*) The calibrations have been based on stars of the North Polar Sequence and, as regards the small-scale refractor plates, on neighboring stars included in the Mount Wilson catalogue[2] of stars north of $+80^{\circ}$. In order to attain the highest possible accuracy other standard regions, such as Selected Areas, have been avoided. (*c*) The nebula has usually been exposed on one half of the plate and the comparison field on the other. For practical reasons, no special rule has been followed as regards the order of the two exposures. On account of the low humidity at Mount Wilson no precaution has been taken to protect the plate for atmospherical moisture. At the Hamburg observatory, the plates have either been protected by a plate of clear glass mounted in the plate holder (100-cm telescope), or they have before the exposures been cut in two halves (60-cm telescope). As will be shown below, the final magnitude values are apparently independent of the order of the exposures. It may be remarked that variations in the seeing conditions from the nebular field to the polar region are of very small importance in the present case, since we are dealing with surface objects.

6. *The photographic plates and the method of measuring.* The present investigation has, exclusively, been confined to a determination of total magnitudes in the international photographic and photovisual systems. The photographic magnitudes have been derived from exposures on Eastman 103 $a-O$ plates, whereas the photovisual magnitudes have been determined from 103 $a-C$ plates exposed behind a Schott *GG* 11 filter (Mt Wilson and Hamburg reflectors), or a *GG* 7 filter (Mt Wilson refractor). In order to bring out the faint outmost parts of the nebulae, the exposures have usually been made as long as is permitted by the sky-fog, which at the 60-inch telescope means exposure times of 20 to 25 minutes (103 $a-O$), or 60 to 70 minutes (103 $a-C+GG$ 11).

Omitting short-exposed plates necessary to determine the distribution of light in the central parts of certain highly concentrated objects, the total number of exposures of nebulae amounts to 164. Each of these exposures is accompanied by another one of the same length, referring to the polar field. Table 4, which contains all the objects included in the present program, gives information about the number of plates secured for each nebula. In the fifth column, the first and the second figure give the number of exposures in the photographic and photovisual regions, respectively. Column 4 shows the type of instrument used in each case. For the 22 members of the metagalactic groups it was intended to secure at least 3 plates in the photographic region and 2 à 3 plates in the photovisual region, and it has, except for a

[1] The errors, presumably, depend mainly on differences in exposure time and in spectral composition of the source of light. For practical reasons, it has in the present case been necessary to limit the sensitometer exposures to about five minutes.

[2] F. H. Seares, F. E. Ross, and Mary C. Joyner, Mt Wilson Papers, Vol. VI (1941).

TABLE 4.

The plate material.

Object	Type	Diameters		Instr.	Plates	Sections
NGC 147	Ep	18′ × 12′		A	3+2	18
				C	2+2	18
NGC 185	Ep	14	12	A	3+2	17
				C	2+2	17
NGC 205	Ep	26	16	A	3+2	18
				C	2+2	19
NGC 221	E	12	8	A	3+3	19
NGC 224	Sb	197	92	B	3+3	27
NGC 247	Sc	28	10	B	2	12
NGC 598	Sc	83	53	B	3+3	16
NGC 891	Sb	15	3.8	A	2+2	22
NGC 2403	Sc	29	15	C	3	20
NGC 2715	Sb	7	2.8	A	1	14
NGC 2805	Sc	8	6	A	1	12
NGC 2820	S	4.7	1.4	A	1	10
NGC 2976	Sc	10	6	A	4+3	12
NGC 2977	S	2.9	1.3	A	1	8
NGC 3027	Sc	6	3.6	A	1	18
NGC 3031	Sb	35	14	A	2+1	23
				B	3+3	16
				C	2	22
NGC 3034	Ir	13	8	A	3+3	18
NGC 3061	Sc	2.6	2.6	A	1	8
NGC 3077	Ir	9	8	A	3+3	12
NGC 3183	SBb	3.6	2.5	A	1	10
NGC 3364	Sb	2.2	2.2	A	1	8
NGC 3623	Sa	12	5	A	2+2	18
NGC 4567	Sc	5	4	A	2+2	16
NGC 4568	Sc	7	4			
NGC 5194	Sc	14	10	A	3+3	18
NGC 5195	Ir	9	7	A	3+3	12
NGC 5457	Sc	28	28	B	3+1	26
				C	2	17
NGC 5474	Sc	7	7	A	3+2	13
NGC 5585	Sc	9	6	A	2+2	13
NGC 6822	Ir	20	20	B	3	19
IC 529	Sc	6	2.4	A	1	12
IC 1613	Ir	23	23	B	2+2	24
				C	4+3	14
IC 2574	Ir	16	8	A	3+1	16
				C	2	10
Ho I	Ir	5	5	A	4+2	12
Ho II	Ir	10	10	A	4+2	17
Ho III	Sc	4.0	4.0	A	1	10
Ho IV	S	7	2.7	A	1	12
Ho V	Sc	3.5	2.6	A	1	14
Wolf-Lundm.	Ir	13	6	A	1	20

A = Mt Wilson 60-inch reflector.
B = Mt Wilson 10-inch refractor.
C = Hamburg 60-cm and 100-cm reflectors.

few of the nebulæ, been possible to carry through this program. In some cases the number of plates is considerably larger than the required minimum.

The plates have been measured with a Moll self-recording, thermoelectric microphotometer belonging to the Physical Institute[1] of the Lund university. The registrations are, as usually, obtained on photographic papers, and a linear enlargement of about 7 times has been used for most of the plates. Since the registration papers have a length of 40 cm it is generally possible to get, on both sides of the measured nebula, background curves of a considerable extension, which is of large importance for an accurate definition of the background density of the plate. The zero-point of the galvanometer has been recorded before, after, and in the middle of each registration. The intensity of the photometer lamp has been adjusted to give a total galvanometer amplitude (zero point to plate background) of about 10 cm. The size of the photometer slit, as projected on the plate, has been varied according to the type of the nebula and the steepness of the density gradients. Most of the 60-inch plates have been measured with a slit having a size of 0.3×0.1 mm^2, corresponding to 8×3 square seconds of arc. For a smooth object of low surface brightness it seems permissible to use a comparatively large slit since, as will be shown below, an integration of plate density in this case is equivalent to a luminosity integration.

Most of the nebulæ have been measured in directions more or less perpendicular to the major axes. In some cases it has seemed practical to move the slit parallel to one of the edges of the plate (parallel to a right-ascension or declination circle), in other cases the plate has been oriented in a skew position by means of two stars situated on either side of the nebula. The measured cross-sections are distributed in the most suitable way over the nebular image, their number depending on the size and the smoothness of the object. According to the last column of Table 4, the number of measured sections is in most cases larger than 12. The total number of tracings of nebulæ analysed in this investigation amounts to about 2800. The major and minor diameters of the nebulæ[2], as obtained from the registrations, are given in the third column of the table. The final magnitudes thus represent the integrated light corresponding to the areas defined by these diameters.

The extrafocal exposures of NP have usually been centered on the polar star 18, and the calibration curves have, as regards the reflector plates, been based on 10 to 11 stars in this region[3]. For the refractor plates, which cover a more extended field, it has been possible to use a larger number of stars. By a proper adjustment of the extrafocal distance each plate has been provided with a sequence of star images of

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[1] The writer is indebted to the director of the institute, Dr. B. Edlén, for the permission to use this instrument.

[2] The diameters of the components of close double systems, such as NGC 4567—68 or NGC 5194—95, have been put equal to twice the corresponding radii, as derived for the undisturbed halves of the nebulae.

[3] The calibration stars are No. 16, 17, 18, 20, 21, 22, 7r, 8r, and 12r of the Polar Sequence. In addition to these, the stars 89°,33 and 89°,36 of the Mount Wilson polar catalogue have been used.

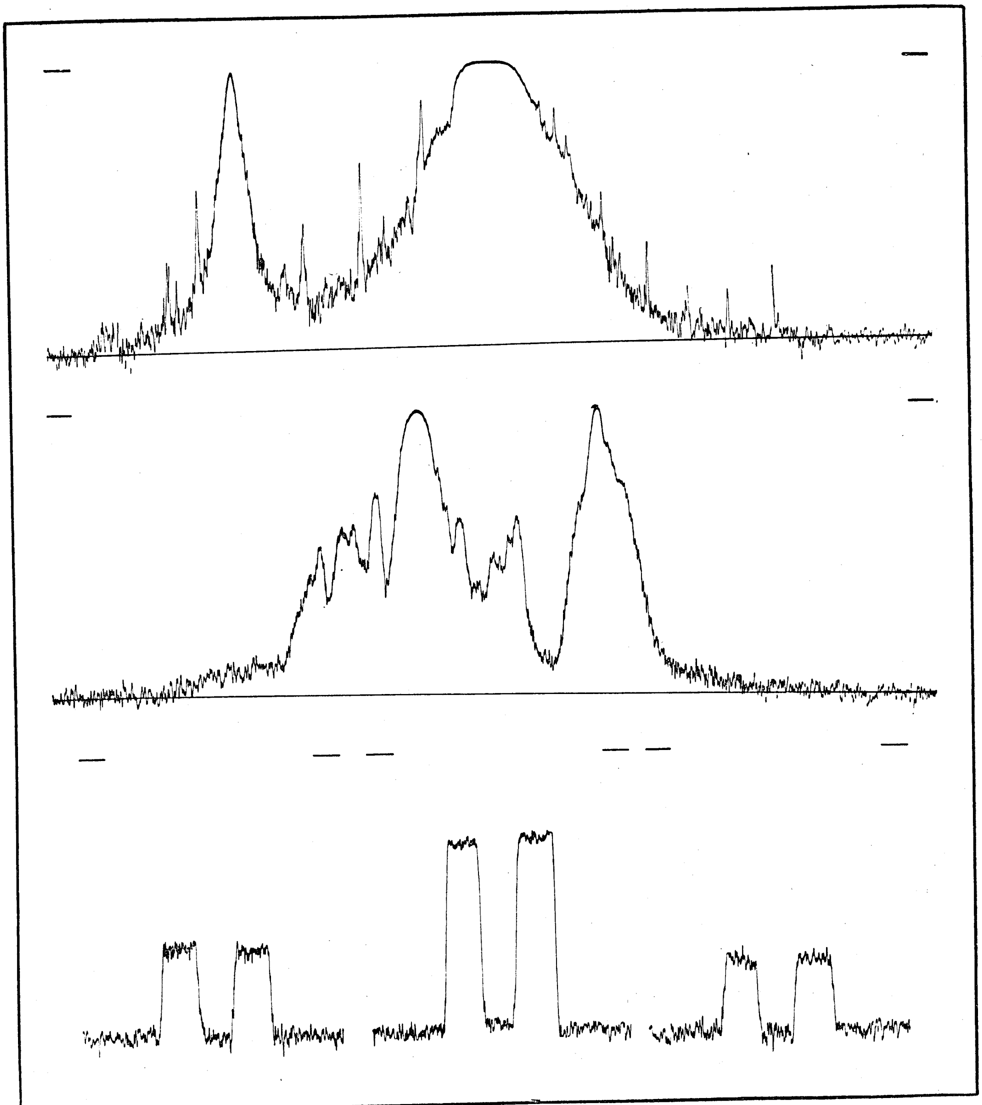

Fig. 3. Microphotometer tracings of NGC 205, 224 (10-inch refr. plate), of NGC 5194, 5195 (60-inch refl. plate), and of extrafocal star images (60-inch refl. plate). The tracings of the nebulae, which give the density distributions in cross-sections passing through the nuclei of the respective objects, indicate that the small companions are situated within the boundaries of NGC 224 and NGC 5194. It may be remarked that the original registrations include longer pieces of background curves. The tracings of the calibration stars show a good definition and permit a very accurate determination of relative density.

suitable densities, the diameters of the images in most cases ranging from 1 to 3 minutes of arc. It may be mentioned that the extrafocal images obtained with the 60-inch telescope are of very high quality from a photometric point of view.

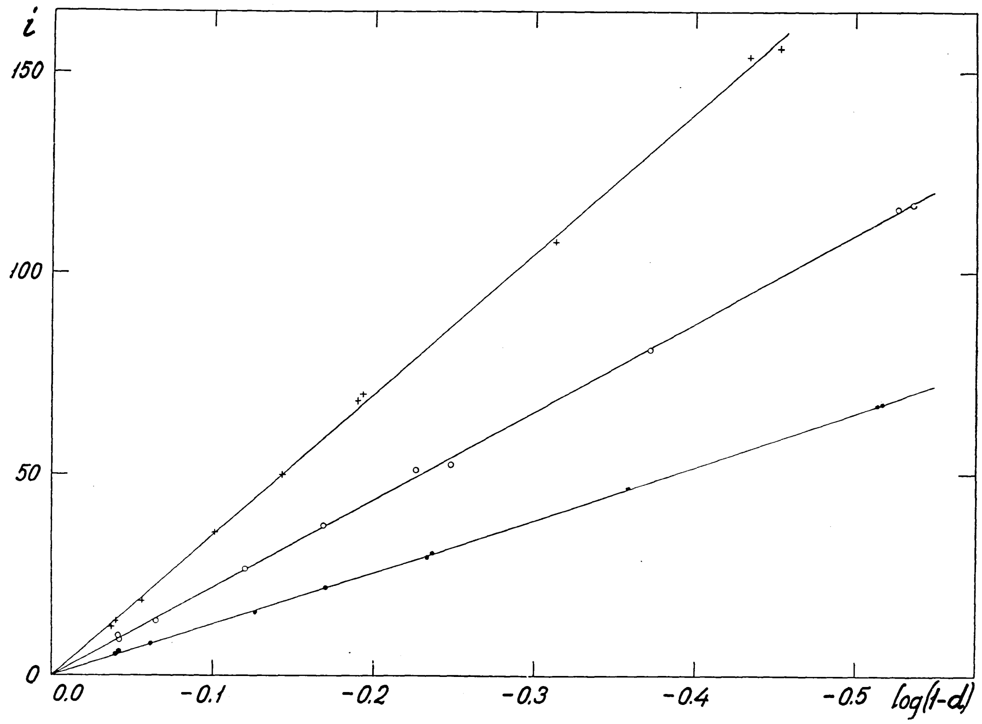

Fig. 4. Calibration curves for three *103 a—O* plates. The curves are based on different units of surface brightness.

The calibration stars have been measured in the photometer in exactly the same way as the nebulæ. The density of the star image, *d*, is in the present case defined as the galvanometer deflection (clear plate *minus* star) expressed in per cent of the total deflection (clear plate *minus* zero-point). This density is related to the surface brightness, *i*, as given by the quotient of stellar brightness, expressed in convenient units, and total area covered by the extrafocal image. There seems, for most of the plates, to be a practically linear relation between surface brightness and *log* $(1-d)$. We refer to Fig. 4, which reproduces the calibration curves derived for three 103 *a—O* plates (60-inch telescope). It may be remarked that the mean corrections (including color equation), as obtained from all the plates of the same type, have been applied to the magnitudes of the stars. The dispersion of the individual points around the adopted relation lines is exceedingly small. For the entire plate material, the mean dispersion is only slightly larger than $0^{m}.02$. All the calibration curves seem, as is indicated by the figure, to pass through the point $i=0$, $d=0$.

As regards the limiting magnitudes reached on the long-exposed plates, it may be mentioned that a density of one per cent on a 103 *a—O* plate (60-inch telescope, exp. 25 min.) corresponds to a surface brightness of about $26^{m}.7$ per square second of arc. The combination 103 *a—C* + *GG* 11 (exp. 70 min.) gives a photovisual limit of about $26^{m}0$ per square second. The photometer tracings, in extreme cases, permit the measuring of densities as low as 0.5 per cent.

By means of the calibration curves the density distributions obtained for the different cross-sections of a nebula are transformed into light distributions. Numerical integrations of the latter curves give the corresponding total luminosities, Σi. A second integration based on the curve defined by the Σi-values gives the luminosity of the entire system. The luminosity is expressed in the units of light adopted for the calibration stars, and is easily transformed into magnitudes. It may be remarked that the total extension of the Σi-curve, which cannot be accurately derived from the cross-section measures, has been determined from photometer tracings along the major axis of the object.

The integrated magnitudes have been reduced to the international system by means of the color equations found for the calibration stars. The following correction has been applied to the photographic magnitudes derived by means of the Mount Wilson and Hamburg reflectors:

$$\Delta m_{pg} = -0.11\,(C-0.6).$$

The corrections do not exceed $0^{m}04$. In those cases where no color index has been determined, C has been replaced by the mean color index corresponding to the type of the nebula (Table 7). In the photovisual region no color correction seems to be indicated. For the refractor plates the following corrections have been derived:

$$\Delta m_{pg} = +0.06\,(C-0.6),$$

$$\Delta m_{pv} = -0.11\,(C-0.6).$$

The extinction corrections adopted in the present case are the same as those used in earlier photometric investigations at the Mount Wilson observatory:

$$\Delta m_{pg} = 0.295\,(sec\,z_1 - sec\,z_2),$$

$$\Delta m_{pv} = 0.128\,(sec\,z_1 - sec\,z_2).$$

The quantities z_1 and z_2 represent, as usual, the zenith distances of the pole and the nebula. At the Hamburg observatory the atmospherical extinction is larger, and the above coefficients have to be replaced by 0.40 and 0.27, respectively[1]. As regards the Hamburg observations it may be remarked, that the nebulæ in most cases have been photographed in altitudes which only slightly differ from the altitude of the pole.

7. *The results.* Table 5 presents the results for those nebulæ where the magnitude determinations are based on more than one plate. All the objects, except three, have been investigated in both the photographic and photovisual regions. In order to

[1] The values have been taken from the investigation by W. Dieckvoss, published in AN 255, p. 117 (1935).

TABLE 5.

Individual values of total photographic and photovisual magnitudes.

The plates refer to the 60-inch telescope, except those denoted by an asterisk (Hamburg reflectors) or by an open circle (Mt Wilson refractor).

Nebula	Photographic magn.	Photovisual magn.
NGC 147	10.43 .37 .55 .52*.43*	9.69 .80 .71*.73*
NGC 185	10.18 .18 .25 .04*.21*	9.34 .52 .43*.45*
NGC 205	8.86 .97 .91 .86*.84*	8.20 .12 .18*.17*
NGC 221	9.01 .03 .14 — —	8.21 .07 .21 —
NGC 224	4.33°.36°.30° — —	3.47°.47°.46° —
NGC 247	9.41°.52° — — —	— — — —
NGC 598	6.24°.12°.21° — —	5.86°.74°.78° —
NGC 891	10.85 .85 — — —	9.98 .09 — —
NGC 2403	8.77*.76*.83* — —	— — — —
NGC 2976	10.68 .72 .74 .77 —	10.11 .08 .07 —
NGC 3031	7.79 .86 .83°.87°.90°	7.05 .01*.92* — .97°.02°.01° —
NGC 3034	9.15 .26 .18 — —	8.47 .36 .33 —
NGC 3077	10.55 .61 .54 — —	9.93 .91 .84 —
NGC 3623	10.16 .20 — — —	9.30 .43 — —
NGC 4567	11.98 .97 — — —	11.30 .38 — —
NGC 4568	11.63 .70 — — —	10.90 .89 — —
NGC 5194	8.90 .89 .86 — —	8.36 .41 .29 —
NGC 5195	10.54 .51 .36 — —	9.46 .57 .44 —
NGC 5457	8.17°.24°.19° — —	7.93°.86*.91* —
NGC 5474	11.19 .30 .17 — —	10.90 .92 — —
NGC 5585	11.28 .22 — — —	10.84 .96 — —
NGC 6822	9.20°.18°.25° — —	— — — —
IC 1613	10.11*.01*.96*.94* — .92°.09° — — —	9.58*.69*.59* — .65°.55° — —
IC 2574	10.95 .87 .90 — —	10.58 .60*.70* —
Ho I	13.15 .23 .34 .34 —	12.89 .07 — —
Ho II	11.13 .17 .06 .20 —	10.78 .82 — —

facilitate a study of possible systematic errors and their dependence on the focal length of the telescope, some of the nebulæ have been measured on a comparatively large number of plates.[1]

There are, for five of the nebulæ, magnitude determinations referring both to the Mount Wilson reflector and the two Hamburg reflectors. The sixteen magnitude values based on the latter telescopes agree very nicely with the Mount Wilson values:

$$\overline{m_H - m_W} = -0.02 \pm 0.02 \quad \text{(m. e.)}.$$

[1] The total magnitudes of NGC 5194 and NGC 5195 listed in Table 5 represent the integrated light on either side of the division line between the two nebulae that corresponds to minimum density (Cp. Fig. 3). The same definition has been used for NGC 4567 and NGC 4568. In the determination of the magnitudes of NGC 205 and NGC 221 the variations in background density caused by the Andromeda nebula have been taken into consideration.

A similar result is obtained, if the Hamburg reflectors are compared to the Mount Wilson refractor:

$$\overline{m_H - m_W} = -0.01 \pm 0.02 \quad \text{(m. e.)}.$$

The latter comparison is based on eleven Hamburg values. The good agreement between the three series of magnitudes seems to justify the conclusion, that the present material is free from any appreciable systematic errors depending on the type of telescope used.

The mean errors of the magnitudes have been derived from the dispersion among the individual values, which all represent independent measures. It has seemed justified to apply unit weights to most of the magnitude determinations. The Hamburg results are, as regards the size of the accidental errors, apparently comparable to those obtained at Mount Wilson. Reduced weights have been given to only a few objects, namely Ho I (60-inch plates), IC 1613 and IC 2574 (Hamburg plates), and NGC 247, NGC 6822, and IC 1613 (refractor plates). In these cases the magnitude measures may be of a somewhat lower quality, mostly on account of the unusually low surface brightness of the nebulæ.

For each object in Table 5 with at least two magnitude determinations (photographic or photovisual) of unit weight a mean error, ε, may be computed from the equation

$$\varepsilon^2 = \frac{\Sigma(m - \overline{m})^2}{n-1}, \tag{3}$$

where n represents the number of individual measures. By taking the weighted mean of all the values of ε^2, we arrive at the following final mean error of a plate of unit weight:

$$\varepsilon = 0^{\mathrm{m}}.056.$$

The photographic and the photovisual magnitude series give results which are practically identical. The mean error derived here is comparatively small, which in the first place may be explained as a result of the large number of cross-sections measured for each nebula.

Table 6 contains the definitive values of integrated magnitude and color index for all the nebulae included in the present program. The mean errors given in the table are based on the error derived above for a plate of unit weight. In the comparatively few cases where a reduced weight has been suggested, the weight has been put equal to 1/2. The same weight has also been applied to all the magnitude values which are based on only one plate. The latter magnitudes generally refer to faint objects of small angular dimensions.

The mean errors of the final magnitudes range from 0.02 (6 plates) to 0.08 (1 plate), and the mean errors of the color indices from 0.03 to 0.07. It may be remarked once more, that these errors represent the internal variations in the material investigated. Although different telescopes have been used, all the magnitudes are the

TABLE 6.

Definitive total magnitudes and color indices.

Nebula	m_{pg}		m_{pv}		C	
NGC 147	10.46±0.03		9.73±0.03		+0.73±0.04	
NGC 185	10.17	.03	9.43	.03	0.74	.04
NGC 205	8.89	.03	8.17	.03	0.72	.04
NGC 221	9.06	.03	8.16	.03	0.90	.05
NGC 224	4.33	.03	3.47	.03	0.86	.05
NGC 247	9.47	.06	—	—	—	—
NGC 598	6.19	.03	5.79	.03	0.40	.05
NGC 891	10.85	.04	10.03	.04	0.82	.06
NGC 2403	8.79	.03	—	—	—	—
NGC 2715	11.89	.08	—	—	—	—
NGC 2805	11.77	.08	—	—	—	—
NGC 2820	13.17	.08	—	—	—	—
NGC 2976	10.73	.03	10.09	.03	0.64	.04
NGC 2977	13.25	.08	—	—	—	—
NGC 3027	12.31	.08	—	—	—	—
NGC 3031	7.85	.03	7.00	.02	0.85	.03
NGC 3034	9.20	.03	8.39	.03	0.81	.05
NGC 3061	13.44	.08	—	—	—	—
NGC 3077	10.57	.03	9.89	.03	0.68	.05
NGC 3183	12.59	.08	—	—	—	—
NGC 3364	13.40	.08	—	—	—	—
NGC 3623	10.18	.04	9.37	.04	0.81	.06
NGC 4567	11.98	.04	11.34	.04	0.64	.06
NGC 4568	11.66	.04	10.89	.04	0.77	.06
NGC 5194	8.88	.03	8.35	.03	0.53	.05
NGC 5195	10.47	.03	9.49	.03	0.98	.05
NGC 5457	8.20	.03	7.90	.03	0.30	.05
NGC 5474	11.22	.03	10.91	.04	0.31	.05
NGC 5585	11.25	.04	10.90	.04	0.35	.06
NGC 6822	9.21	.05	—	—	—	—
IC 529	12.54	.08	—	—	—	—
IC 1613	10.00	.03	9.61	.04	0.39	.05
IC 2574	10.91	.03	10.62	.04	0.29	.05
Ho I	13.27	.04	12.98	.06	0.29	.07
Ho II	11.14	.03	10.80	.04	0.34	.05
Ho III	12.94	.08	—	—	—	—
Ho IV	12.95	.08	—	—	—	—
Ho V	13.23	.08	—	—	—	—
Wolf-Lundm.	11.13	.08	—	—	—	—

result of the same observation procedure, and we may expect that the true (external) mean errors are somewhat larger than the tabulated values.

The color indices, which in Table 6 are given for 23 objects, range from +0.29 to +0.98. The color seems to be related to the type of the nebulæ, as appears from the figures given in Table 7. The mean value of C changes from about +0.8 for elliptical objects and early spirals to $+0.3$ for the objects denoted by *Ir I*. The irregular nebulæ have been divided in two groups, *Ir I* and *Ir II*, referring to resolvable and irresolvable objects, respectively. The irresolvable objects seem to have color indices comparable to those of elliptical nebulæ, which indicates that we in these cases are dealing with stellar populations of the same type.

TABLE 7.

Color index and type.

Type	N	$\overline{C}$
E	4	+0.77
Sa	1	0.81
Sb	3	0.84
Sc	8	0.49
Ir I	4	0.33
Ir II	3	0.82
All	23	0.62

8. *Discussion of possible systematic errors.* In this section we will investigate certain sources of error, which to some extent may falsify the photometric results. Systematic errors may in the present case be introduced by incorrect extrapolations of the background curves of the plates, by differences in sky fog between polar region and nebular region, and by the use of a too large photometer slit. Furthermore, the magnitudes may contain errors related to the exposure times of the plates.

In the reduction of the density curve of a photometer tracing to a light distribution curve serious errors may result, especially in the case of a nebula of low surface brightness, from an incorrect definition of the background curve measuring the sky fog of the plate. According to the writer's experience, the background densities of long-exposed plates may generally not be represented by straight lines. If the sky fog increases towards the centre of the plate, which seems to be the rule, a linear extrapolation of the background curve across the area covered by the nebula will result in a total luminosity that is systematically too high.

In the present case the photometer registrations have been extended over large areas of the plates, and the tracings include, on both sides of each nebula (or calibration star), long pieces of background curves. The greatest care has been exercised in the extrapolation of these curves over the nebular region. The extrapolation is facilitated by the fact that the background curves of different cross-sections have more or less the same shape.

Systematic errors may also result from a difference in sky fog between nebular region and polar region. In some cases the two halves of a long-exposed plate exhibit noticeable differences in background density. Deviations of this kind may be found both on Mount Wilson and Hamburg plates.

Since the sky fog is usually less pronounced in the region of the pole, an application of the polar calibration curve to the nebular field has to be based on the following assumption:

$$f\left(\frac{a+\delta}{D+\delta}\right)-f\left(\frac{\delta}{D+\delta}\right)=f\left(\frac{a}{D}\right). \tag{4}$$

Here, the function $f(x)$ represents the calibration curve derived from the polar stars, that is, $f(x)$ gives the surface brightness corresponding to the density x, as defined above. The total galvanometer deflection is on the nebular half of the plate equal to D, and on the polar half equal to $D+\delta$. The quantity a represents the deflection obtained for a certain point in the nebula (background *minus* nebula). The left member of the equation gives the correct calibration curve to be applied to the nebular region, whereas the right member gives the curve that has actually been used.

It has been shown above, that the calibration function $f(x)$ derived for the different plates may, with a good degree of approximation, be represented by the expression $-log\ (1-x)$. Since the latter function satisfies equation (4), it seems justified to assume that possible differences in sky fog between nebular region and polar region will not cause any serious systematic errors in the present case.

In order to get smooth and well-defined background curves, the registrations of the nebulæ have usually been made with as large a photometer slit as possible. If the size of the slit is increased above a certain limit the results will, however, be more or less falsified, especially in the case of nebulæ with steep density gradients. If the gradients are small, or equal to zero (calibration stars), the size of the slit is apparently of minor importance.

TABLE 8.

Systematic errors introduced by the use of a too large photometer slit.

$\Delta=$	0.05	0.10	0.15	0.20
$d=0.1$	+0.5 %	+1.9 %	—	—
0.2	0.3	1.2	+2.7 %	+4.8 %
0.3	0.2	1.0	2.2	3.9
0.4	0.2	0.9	2.1	3.8
0.5	0.2	1.0	2.2	4.0
0.6	0.3	1.2	2.7	4.9
0.7	0.4	1.6	3.8	7.2
0.8	0.7	2.8	7.2	—

Let us assume that, within the area covered by the photometer slit, the densities of the different surface elements range from $d-\Delta$ to $d+\Delta$. The density variation will produce a systematic error in the surface brightness, as derived from the calibration curve, the size of the error depending on the distribution of the densities. If we assume that all density values within the given interval $d-\Delta$ to $d+\Delta$ are represented in the same proportion, the measured mean density will be equal to d, whereas the corresponding average surface brightness, $i+\delta i$, is given by the integral equation

$$2\Delta\,(i+\delta i)=\int_{d-\Delta}^{d+\Delta} f(x)\,dx. \qquad (5)$$

The function $f(x)$ represents, as before, the calibration curve.

By introducing the above calibration function, $-log(1-x)$, we are able to solve equation (5) numerically. The results are contained in Table 8, which gives the relative systematic errors, $\delta i/i$, corresponding to Δ-values ranging from 0.05 to 0.20. The errors, which always are positive, seem to be neglectable in the interval $\Delta = 0.0$—0.1, but increase rapidly as Δ grows larger. In the registration of a smooth nebula of low surface brightness, for which the maximum density in many cases does not exceed 20 per cent, even a very large photometer slit would, accordingly, not introduce any serious systematic errors. In other cases, the slit has to be reduced to such a size that the deviation Δ is kept sufficiently small.

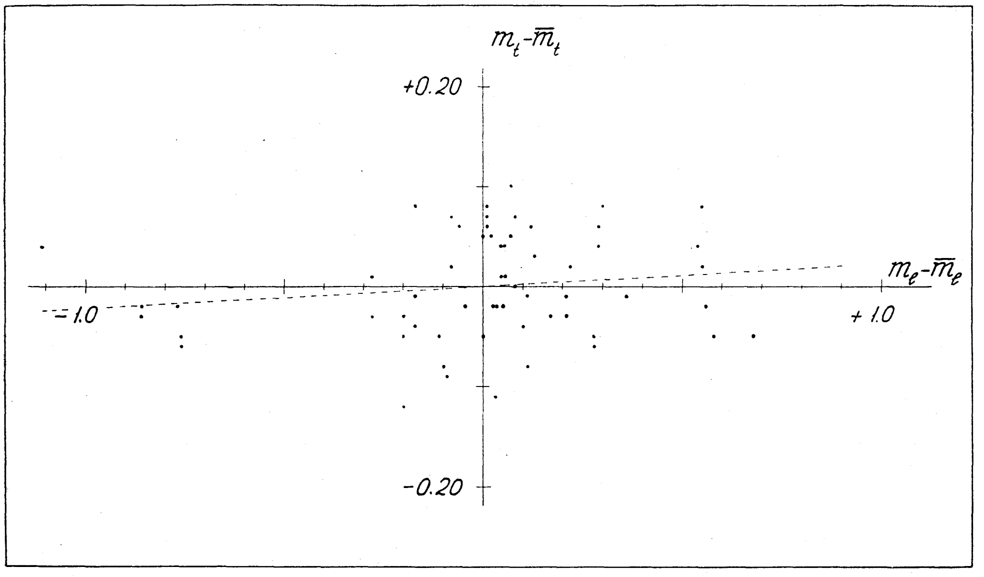

Fig. 5. Relation between total magnitude and limiting magnitude of the plate.

It is a well-known fact that the angular diameters of a nebula, as determined from a photographic plate, seem to increase with the exposure time. This increase in dimensions may be accompanied by a corresponding change in total magnitude. Most of the plates included in this investigation represent maximum exposures, as is shown by the resulting diameter values (Table 4) which are considerably larger than those usually adopted. It seems justified to assume, that a very small fraction of the total luminosity of a nebula is to be found outside the area defined by these diameters. The corrections to the measured magnitudes, if any, probably amount to only a few hundreds of a magnitude.

Fig. 5 illustrates the relation between total magnitude and limiting magnitude of the plate that is obtained from the present material. The figure includes those objects for which at least three independent determinations (60-inch telescope) of photographic or photovisual magnitude are available. The deviation of each individual magnitude from the adopted mean value (ordinate) is plotted against the corresponding deviation in limiting magnitude (abscissa). The latter magnitude, which is defined as the surface magnitude giving a relative plate density of one per cent, may differ somewhat from one plate to another on account of changes in zenith distance, sky fog, or exposure time. A least squares solution gives the result, that the total magnitude may increase slightly numerically as the limiting magnitude grows fainter. This correlation may be explained entirely by the fact that the accidental errors in the limiting magnitudes, which are derived from the calibration curves, are included also in the total magnitudes. The figure does not give any indication of the presence of systematic errors of the kind mentioned above.

In this connection we may also examine the total magnitudes with respect to the order of the two exposures of nebula and calibration stars. If the magnitude values (60-inch telescope) derived from plates where the first exposure refers to the nebula are denoted by m_1, and the magnitudes which represent the opposite order are denoted by m_2, the present material gives the mean deviation

$$\overline{m_1 - m_2} = +0.03 \pm 0.02 \quad \text{(m. e.)}.$$

The small difference indicates that no serious systematic effects are present.

9. *Comparison with other results.* In Fig. 6 the present magnitude determinations have been compared to the magnitudes of three previous lists.

In the upper part of the figure a comparison is made with the photoelectric magnitudes derived by A. E. Whitford[1] for a number of bright nebulæ by means of the Mount Wilson 10-inch refractor. The seven objects in common to both lists are contained in Table 9, where the second column gives the photoelectric measures. Each magnitude has been reduced to the international photographic system by applying a correction equal to $+0.18\,C$, the color index being taken from the present investigation. The diameter of the diaphragm, which is given in the third column, is usually not large enough to include the entire area covered by the nebula. Corrections have been derived by means of the distribution curves obtained from the photometer tracings, and in this way the final values contained in the fourth column are obtained. The resulting differences, W_{corr} *minus* Ho, have in the figure been

TABLE 9.

Comparison with Whitford's photoelectric magnitudes.

Nebula	W	Diaphragm	W_{corr}	$W_{corr} - Ho$
NGC 221	9.40	7'.5	9.31	+0.25
224	4.46	128	4.36	+ .03
598	6.83	30	6.46	+ .27
3031	7.71	24	7.69	− .16
3034	8.97	12	8.97	− .23
5194–95	8.94	12	8.90	+ .25
5457	8.73	24	8.60	+ .40

[1] Mt Wilson Contr. 543=ApJ 83, p. 424 (1936). On account of the small diaphragms used, the photoelectric magnitudes given in a later list by Stebbins and Whitford (Mt Wilson Contr. 577=ApJ 86, p. 247, 1937) cannot be compared to the present magnitudes.

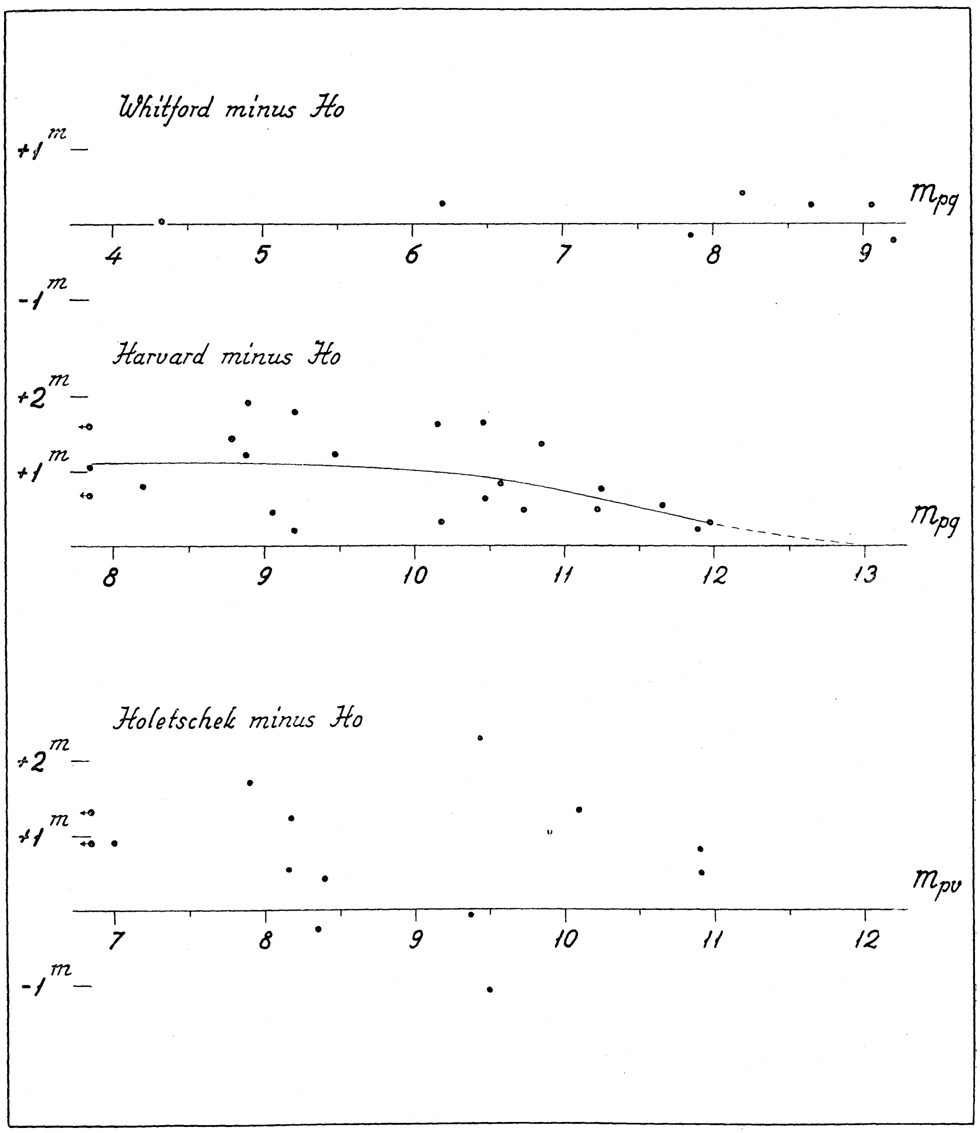

Fig. 6. Comparison of the writer's magnitudes with other determinations.

plotted against the magnitudes derived by the writer. Considering the comparatively large mean errors of the photoelectric measures, the agreement between the two magnitude scales seems to be satisfactory.

The lower part of the figure gives the results of a comparison between the writer's photovisual magnitudes and Holetschek's visual estimates, as revised by K. GRAFF[1].

[1] Wien Mitteil. 4, No. 4 (1948).

The fifteen objects in common to both lists give a mean difference, Holetschek *minus* Holmberg, equal to $+0^m.8$. It does not seem possible to reduce the very large dispersion in the magnitude differences, about 0.8 magnitudes, by introducing a color equation. The deviations may be explained by the assumption that the visual magnitudes refer only to the central, more condensed parts of the nebulæ.

Fig. 6 also includes a comparison with the Shapley-Ames catalogue[1], which has 23 objects in common with the present list. The Harvard magnitudes are all numerically larger than the magnitudes derived by the writer. The systematic deviation amounts to about $1^m.1$ for nebulæ brighter than the tenth magnitude (present scale), and seems to decrease as we approach the limiting magnitude 13.0. The large dispersion ($0^m.5$) of the individual points around the adopted relation curve indicates that the mean error of the Harvard magnitudes is larger than the previously assumed value, at least in the case of bright nebulæ.

It seems possible to explain the above deviations only by the assumption of a systematic error in the Harvard magnitude scale. The existence of such an error has been indicated by some previous investigations. We may, for instance, refer to the results obtained by R. O. REDMAN and E. G. SHIRLEY[2] in a photographic photometry of a number of elliptical nebulæ. The total magnitudes of these objects have been determined by an integration method similar to that used in this investigation. A comparison with the Harvard catalogue gives deviations which rather closely agree with the relation curve derived from the present material.

An application of corrections of the size indicated above to the magnitudes of the Harvard catalogue will have rather important consequences. Thus, the distribution of the corrected magnitudes will no longer correspond to a constant space density of nebulæ, but to a density that decreases with the distance. Such a variation in space density seems, however, to be questionable. It may be necessary to consider the possibility that the Harvard survey is more or less incomplete, especially as regards nebulæ of the two last magnitude classes. A comparison with the present material suggests, in fact, that the number of missing objects may be considerable: of the 22 nebulæ in Table 6 which have magnitudes in the interval $m_{pg}=10.0$—13.0, only 12 have been included in the Harvard catalogue. In view of the great importance of a complete survey of the brighter nebulæ it seems highly desirable that this problem is further investigated. The degree of completeness of the Harvard list may, for instance, be found to depend on galactic latitude, as is indicated by the abnormally high value of galactic absorption that is obtained from the latitude distribution of the nebulæ.

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[1] Harvard Ann. 88, No. 2 (1932).

[2] MN 96, p. 588 (1936) and MN 98, p. 613 (1938).

CHAPTER III

Absolute Magnitudes and Dimensions of Nebulae

10. *The material.* The study of absolute luminosities and dimensions of nebulae will be based on the results of the previous two chapters. The material comprises the 29 objects which have been selected as members of the local group and the groups associated with M 81 and M 101. Total apparent magnitudes (and diameters) have been determined by the writer for the majority of these nebulae.

The material chosen for this investigation refers to a given volume of space and is thus free from systematic errors due to statistical selection. The members of the three groups presumably form a representative sample collection of nebulae. It is true that we do not know whether or not the luminosities (or dimensions) of group nebulae are comparable to those of isolated objects in the general field. This question does not seem important, however, since most of the extragalactic nebulæ very likely are members of larger or smaller agglomerations. A more serious problem is presented by a possible incompleteness of the material. It seems quite probable that future discoveries will increase the number of dwarf systems especially in the local group.

Table 10 gives a list of the 29 nebulæ. The observed and the corrected distance moduli are contained in columns 3 and 4. The difference between the two values represents the galactic absorption (latitude effect). Columns 5 and 7 give the apparent total magnitudes and major diameters, whereas the absolute magnitudes and diameters, as corresponding to the adopted distance moduli, are reproduced in columns 6 and 8.

The distance moduli of nebulæ belonging to the local group have, with one exception, been taken from a recently published list by W. Baade[1]. The distance modulus of the Wolf-Lundmark system has been derived from a comparison of the bright end of the stellar luminosity curve with the curve obtained for IC 1613, and is thus based on the assumption that the brightest stars in both systems have comparable luminosities[2]. As regards the M 81 and M 101 groups, the distance moduli are based

[1] Mt Wilson Contr. 697=ApJ 100, p. 147 (1944). It may be remarked that H. Shapley (Harvard Repr. II, 29, 1949) in a later discussion of the cepheid light curve has derived a distance modulus (uncorr. for abs.) for LMC and SMC equal to 17.25.

[2] For permission to include this unpublished result in the present paper the writer's thanks are due to Dr. W. Baade.

TABLE 10.

Distance moduli, absolute magnitudes and diameters.

Object	Type	Modulus Obs.	Modulus Corr.	m_{pg}	M_{pg}	a	A
Local group:							
NGC 147	Ep	22.4	21.5	+10.5	−11.9	18′	1.0 kpc
NGC 185	Ep	22.4	21.5	10.2	12.2	14	0.8
NGC 205	Ep	22.4	21.8	8.9	13.5	26	1.7
NGC 221	E2	22.4	21.8	9.1	13.3	12	0.8
NGC 224	Sb	22.4	21.8	4.3	18.1	197	13.1
NGC 598	Sc	22.3	21.9	6.2	16.1	83	5.8
NGC 6822	Ir	21.6	21.0	9.2	12.4	20	0.9
IC 1613	Ir	22.0	21.8	10.0	12.0	23	1.5
LMC	Ir	17.1	16.7	1.2	15.9	720	4.6
SMC	Ir	17.3	17.0	2.8	14.5	480	3.5
Fornax syst.	Ep	21.0	20.8	9.1	11.9	50	2.1
Sculptor syst.	Ep	19.4	19.2	8.8	10.6	45	0.9
Wolf-Lundm. syst.	Ir	22.3	22.0	11.1	11.2	13	1.0
Galactic syst.	Sb (Sc?)	—	—	—	—	—	24.
Messier 81 group:							
NGC 2366	Ir	24.0	23.6	12.6	11.4	10	1.5
NGC 2403	Sc	24.0	23.6	8.8	15.2	29	4.4
NGC 2976	Sc	24.0	23.6	10.7	13.3	10	1.5
NGC 3031	Sb	24.0	23.6	7.8	16.2	35	5.3
NGC 3034	Ir	24.0	23.6	9.2	14.8	13	2.0
NGC 3077	Ir	24.0	23.6	10.6	13.4	9	1.4
IC 2574	Ir	24.0	23.6	10.9	13.1	16	2.4
Ho I	Ir	24.0	23.6	13.3	10.7	5	0.8
Ho II	Ir	24.0	23.6	11.1	12.9	10	1.5
Messier 101 group:							
NGC 5194	Sc	24.0	23.7	8.9	15.1	14	2.2
NGC 5195	Ir	24.0	23.7	10.5	13.5	9	1.4
NGC 5204	Sc	24.0	23.7	11.2	12.8	10	1.6
NGC 5457	Sc	24.0	23.7	8.2	15.8	28	4.5
NGC 5474	Sc	24.0	23.7	11.2	12.8	7	1.1
NGC 5585	Sc	24.0	23.7	11.3	12.7	9	1.4

on the results obtained by E. HUBBLE in an earlier investigation[1]. The moduli, which refer to M 81, M 101, and NGC 2403, have been derived from apparent luminosities of novae and irregular variables and are not comparable in accuracy to the distance moduli of local nebulæ. Furthermore, we have to assume that all group members are situated at the same distance. The uncertainties in the adopted distance moduli will naturally be reflected in the final results. In comparison with the very large dispersion in the absolute luminosities (or diameters) of the nebulæ possible errors in the distance scale seem, however, to be of minor importance. It may be observed that an error or half a magnitude in the modulus of the M 101

[1] Mt Wilson Contr. 548=ApJ 84, p. 158 (1936). In a private communication to the writer, Dr. Hubble has reported a revision of the distance modulus of M 101. The revised value is 24.0.

group will change the final mean magnitude by only 0.1, corresponding to one third of the mean error.

The apparent magnitudes and major diameters in most cases represent measures by the writer. The magnitudes are taken from Table 6. The diameters, which are found in Table 4, have been measured on plates with approximately the same limiting magnitudes and may be assumed to form a homogeneous material. For LMC, SMC, and the two systems in Fornax and Sculptor we have accepted the magnitudes and diameters adopted by Baade in the above-mentioned list. No measures are available for NGC 2366 and NGC 5204, which were not included in the photometric program. For the first nebula the Shapley-Ames catalogue gives a magnitude of 12.6, and this value may be accepted as a preliminary estimate. The second object has been given a magnitude of 11.2, since, as appears from the survey plates, the luminosity is comparable to that of NGC 5474, or NGC 5585. The diameters of both nebulæ have been estimated at 10′.

TABLE 11.

Mean values and dispersions for different types.

Type	N	$\overline{M}_{pg}$	$\sigma(M_{pg})$	$5\,\overline{\log A}$	$5\,\sigma(\log A)$
E	6	−12.2	1.1	+15.3	0.9
S	10	14.8	1.8	17.4	1.7
Ir I	9	12.7	1.7	16.1	1.3
Ir II	3	13.9	—	16.0	—
All	28	−13.5	1.85	+16.4	1.55

The members of the local group have, according to Table 10, a mean absolute magnitude of — 13.4 (gal. syst. excl.). For the M 81 and M 101 groups the mean magnitudes are — 13.4 and — 13.8, respectively. As regards the logarithm of the diameter, *log A*, we find the corresponding means 3.27, 3.29 and 3.25. The rather close agreement between the different mean values seems to indicate that the distance moduli adopted for the M 81 and M 101 groups are at least of the right order of size.

11. *Distributions of magnitudes and diameters.* The absolute photographic magnitudes of the nebulæ range from — 10.6 (Sculptor system) to — 18.1 (NGC 224), and the major diameters from 800 to about 13000 parsec (gal. syst. excl.). Table 11 gives mean values and dispersions as obtained for different types. The irregular nebulæ have been divided in two groups, *Ir I* and *Ir II*, comprising resolvable and irresolvable objects, respectively. *For the whole material we get a mean absolute luminosity of* —13.5 *and a dispersion of not less than* 1.85 *magnitudes.* The mean errors of the two quantities amount to 0.3 and 0.2 magnitudes, respectively. In order to make possible a comparison, the means and dispersions referring to the dimensions have been computed for 5 *log A*. The dispersion of the latter quantity seems to be somewhat smaller than the magnitude dispersion.

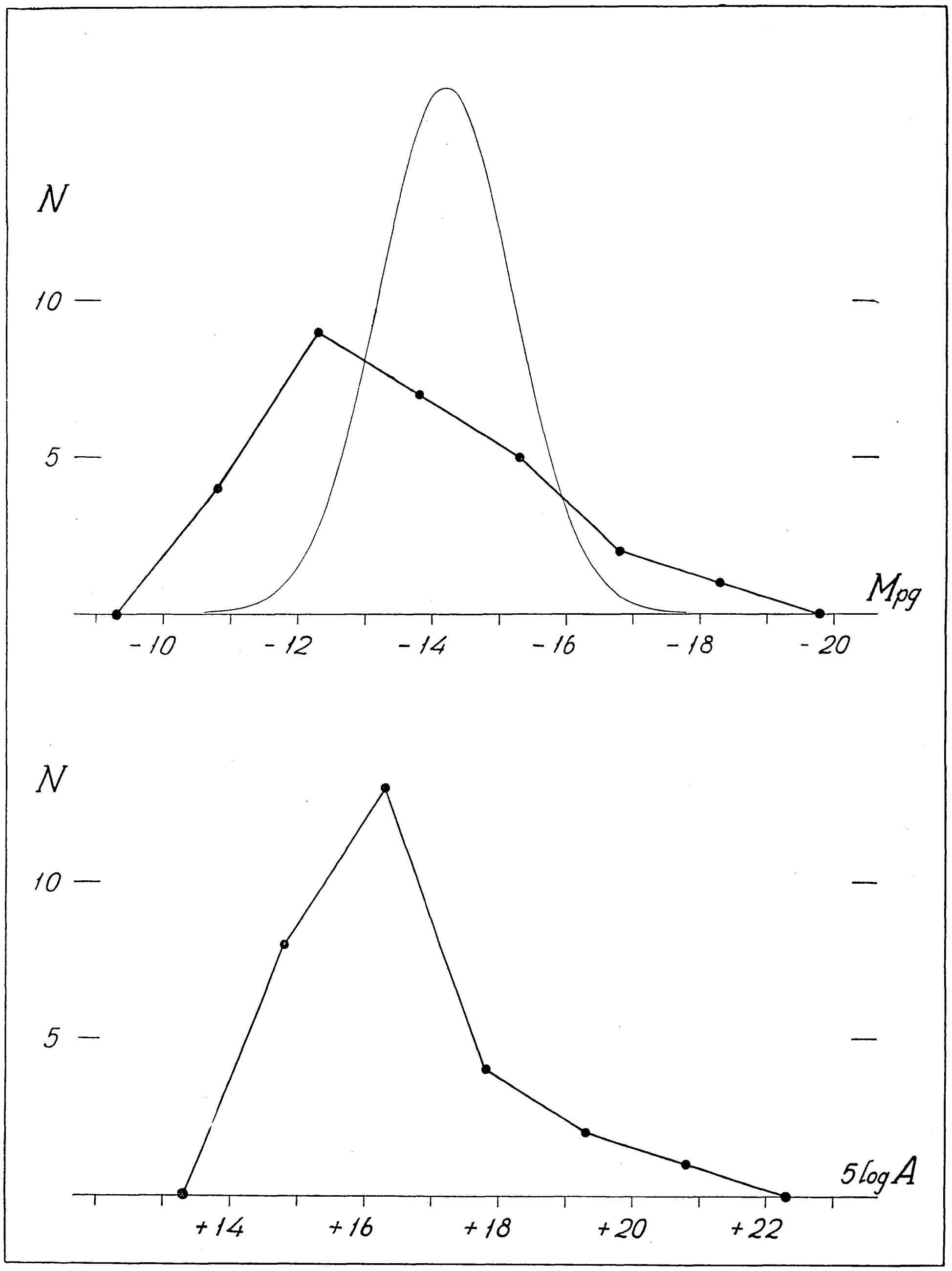

Fig. 7. Frequency distributions of absolute luminosities and dimensions of nebulae included in the present material. The smooth curve is a normal error-curve with a mean of $-14^{M}.2$ and a dispersion of one magnitude.

On account of the uncertainty as regards total magnitude the galactic system has not been included in Table 11. An omission of the system may, when we try to establish the mean luminosity of nebulæ, counterbalance a possible incompleteness of the material in the fainter magnitude classes.

Fig. 7 gives the frequency distributions of absolute magnitudes and dimensions of the 28 nebulæ. The frequencies correspond to a class-breadth, ΔM_{pg} or $5\Delta log\ A$, equal to 1.5. The smooth curve, which has been included for the sake of comparison, is a normal error-curve with a mean, $\overline{M}_{pg}$, of — 14.2 and a dispersion of one magnitude. Both distributions appear to be unsymmetrical, the maximum frequencies corresponding to luminosities, or dimensions, that are smaller than the respective means. If the absolute magnitude —10.5 is assumed to represent the lower limit for nebulæ, future additions of dwarf systems will only increase the skewness of the luminosity curve.

The dispersion in absolute magnitude found from the present material is considerably larger than that obtained by E. HUBBLE[1] in his investigation of resolved stars of nebulæ. The latter dispersion was derived from the differences between total magnitudes, m_t, of nebulæ and magnitudes, m_s, of brightest resolved stars. A statistical discussion based on 145 such differences gave a luminosity dispersion for the nebulæ of only 0.85 magnitudes.

The relation between the dispersion in $(m_s—m_t)$ and the dispersion in M_t is given by the equation,

$$\sigma^2(m_s-m_t) = \sigma^2(M_s-M_t) \doteq \sigma^2(M_s)+\sigma^2(M_t)-2r\,\sigma(M_s)\,\sigma(M_t), \qquad (6)$$

where r means the coefficient of correlation between M_s and M_t. By applying the same corrections for nebular type as those used by HUBBLE we find from the data collected by the writer in Table 12 a coefficient r of about $+0.95$, and a dispersion in M_s of approximately one magnitude. If we accept these values of the parameters in equation (6), HUBBLE's magnitude differences $(m_s—m_t)$ lead to a final dispersion in M_t that is more than twice as large as the above value, or 1.8 magnitudes. Since the writer's determination of the two parameters is based on a comparatively large and photometrically rather homogeneous material it seems likely that the latter value represents the real luminosity dispersion of the nebulæ.

The good agreement between HUBBLE's revised value and the dispersion obtained from the writer's material is noteworthy, especially when we take into consideration that the two determinations are independent of each other. It may be remarked that the nebulæ in HUBBLE's material have been selected on the basis of the apparent magnitude of the brightest resolved stars. This selection will not cause any systematic errors in the derived luminosity dispersion supposed that the space density of nebulæ is constant. A small positive correction should, however, be applied to the dispersion on account of the fact that the total magnitudes of most of the objects have been taken from the Shapley-Ames catalogue, the magnitude scale of which seems to be somewhat compressed.

[1] Mt Wilson Contr. 548=ApJ 84, p. 158 (1936).

CHAPTER IV

Magnitudes of Brightest Resolved Stars and their Relation to Total Magnitudes of Nebulae

12. *Introduction.* In 1936 an extensive and very important investigation of resolved stars in extragalactic nebulæ was published by E. HUBBLE[1]. The mean magnitudes, m_s, of the brightest resolved stars[2], derived from Mount Wilson photographs with the 60-inch and 100-inch telescopes, are here given for 145 spirals and irregular objects. The principal aim of the investigation was to try to use the brightest stars as a criterion of distance and in this way derive the luminosity function of nebulæ.

For 16 nebulae of intermediate spiral type (*Sb, SBb*) HUBBLE found a mean difference between the magnitude m_s and the total magnitude m_t equal to 8.57. For the 117 objects of late spiral type (*Sc, SBc*) the mean difference amounted to 7.84, whereas the 11 irregular objects gave a mean of 7.15. If the irregular nebulæ are chosen as standard objects, in accordance with the procedure to be followed below, the systematic deviation in $(m_s—m_t)$ will amount to about $+0.7$ for the late type spirals, and to $+1.4$ for the intermediate types.

The difference in $(m_s—m_t)$ derived for the three types of galaxies may be explained as a systematic effect relating either to the magnitudes of the resolved stars or to the total magnitudes of the nebulæ. According to HUBBLE, the intermediate and late spirals have comparable luminosities, but the resolved stars are in the first group fainter than in the second group. The irregular objects are, on the other hand, systematically fainter than the spiral nebulæ, although their resolved stars have a luminosity similar to that of stars in late spirals.

In this chapter an investigation will be made of the magnitudes of brightest resolved stars and their dependence on the total magnitudes of the nebulæ. In order to get a complete picture, the relation between M_s and M_t will also be studied for the solar neighborhood and for open clusters. It will be shown that there is a very

[1] Mt Wilson Contr. 548=ApJ 84, p. 158 (1936).

[2] The magnitude m_s is defined as the mean of the three or four brightest individual stars, with about the same luminosity, below which the numbers steadily increase. Single, outstanding objects are generally assigned to the foreground.

TABLE 12.

Magnitudes of brightest resolved stars.

Nebula	Type	m_s	Mod.	M_s	M_t
NGC 224	Sb	16.0 (H)	22.4	−6.4	−18.1
NGC 598	Sc	15.6 (H)	22.3	6.7	16.1
NGC 6822	Ir	15.8 : (H)	21.6	5.8 :	12.4
IC 1613	Ir	17.5 (B)	22.0	4.5	12.0
LMC	Ir	10.0 (H)	17.1	7.1	15.9
SMC	Ir	11.5 (H)	17.3	5.8	14.5
Wolf-Lundm. syst.	Ir	17.6 (B)	22.3 :	4.7 :	11.2 :
NGC 2366	Ir	19.0 : (H)	24.0	5.0 :	11.4
NGC 2403	Sc	18.0 : (H)	24.0	6.0 :	15.2
NGC 2976	Sc	20.2 : (H)	24.0	3.8 :	13.3
NGC 3031	Sb	18.5 : (H)	24.0	5.5 :	16.2
NGC 5194	Sc	17.7 (B)	24.0	6.3	15.1
NGC 5457	Sc	17.9 (B)	24.0	6.1	15.8
NGC 5474	Sc	19.5 : (H)	24.0	4.5 :	12.8
NGC 5585	Sc	19.8 : (H)	24.0	4.2 :	12.7

pronounced correlation between M_s and M_t in all stellar agglomerations which are comprised of or dominated by stars of type I, as defined by W. BAADE[1]. The results indicate that the systematic deviations observed by HUBBLE may be referred to the total magnitudes of the nebulæ, and that they are caused by differences in stellar content.

13. *Relation between M_s and M_t for nebulae.* The study of brightest resolved stars of nebulæ will be based on our previous material, which refers to the local group and the two groups around Messier 81 and Messier 101. Among the 29 objects listed in Table 10 we find nine irresolvable elliptical and irregular nebulæ.[2] The objects NGC 5204, IC 2574, Ho I, and Ho II have, as yet, not been investigated as regards resolved stars. The galactic system will not be included in the following discussion, the total luminosity of the system and the luminosity of the brightest supergiant stars still being rather uncertain. The remaining fifteen nebulæ are all to be found in Table 12.

The third column of the table gives the apparent photographic luminosity of brightest resolved stars, according to BAADE (B) and HUBBLE (H). The magnitude derived by BAADE[3] represents $\overline{m}_5$, or the mean magnitude of the five brightest stars.

[1] Mt Wilson Contr. 696=ApJ 100, p. 137 (1944).

[2] The term »irresolvable nebula» is no longer quite relevant and may, perhaps, be replaced by »type II nebula», meaning a system which contains a stellar population of pure type II. We may refer to the successfull observations by BAADE, which have shown that the elliptical companions of the Andromeda nebula are resolvable in red light.

[3] The writer is indebted to Dr. BAADE, who has communicated the results of his investigations in advance of publication.

Considering the uncertainties necessarily introduced by foreground stars the magnitudes defined in this way ought to be comparable to those given by HUBBLE which, according to his definition, refer to the three of four brightest stars. The latter magnitudes have been taken from the above-mentioned paper.

On account of uncertainties in the magnitude sequences HUBBLE's original values of m_s may need some slight corrections, and it has seemed appropriate to include them with half weight in the following discussion. Unit weights have, however, been given to NGC 224, NGC 598, LMC, and SMC. The magnitude scale used for NGC 224 has been checked by BAADE. Furthermore, the magnitude m_s seems to agree well with the results obtained by C. K. SEYFERT and J. J. NASSAU[1] from star counts in the Andromeda nebula. The magnitude scale for NGC 598 refers partly to the same comparison stars as those used for NGC 224. The magnitudes of brightest stars derived for the two Magellanic Clouds are based on Harvard investigations.

The fourth column of Table 12 gives the distance moduli, and the fifth and sixth columns the resulting values of M_s and M_t. The distance moduli and the total magnitudes are the same as those listed in Table 10. Since the modulus adopted for the Wolf-Lundmark system only represents a preliminary estimate, as has been mentioned above, half weight will be given to this nebula.

In the following discussion the irregular nebulae will be chosen as standard objects, and the total magnitudes of late and intermediate spirals will be reduced to the »normal» values valid for the irregular group. The magnitudes M_s obtained in the three groups apparently refer exclusively to the most luminous stars of type I, whereas the total magnitudes refer to a mixture of type I and type II populations. It seems justified to assume that stars of the former type predominate in the irregular (resolvable) nebulae, like in the solar neighborhood, and that the ratio of type I to type II stars decreases along the sequence *Ir* — *Sc* — *Sb* — *Sa* — *E*. For the types *Sa* and *E* the ratio apparently equals zero. The average color indices, as obtained for nebulae of different types (Table 7), give strong support to the assumption of a gradual change in stellar content. According to the writer's opinion, a study of the correlation between M_s and M_t should not be based on the observed total magnitudes, but on the total magnitudes of the type I populations. The corrections to M_t will very likely be small for irregular nebulæ, and may for these objects be put equal to zero. As is shown below, the corrections indicated for spiral nebulae will surpass one magnitude.

With the weights assigned above the irregular nebulae listed in Table 12 define a mean total magnitude of —13.3 and a mean magnitude of brightest stars of —5.6. For the seven nebulae of type *Sc* the corresponding means are —14.8 and —5.7. There is apparently a systematic difference in total luminosity between the two groups of objects. Anticipating the relation between M_t and M_s to be derived below and reducing the total magnitudes to a constant value of M_s, we find that the spirals are, on the average, 1.3 magnitudes brighter than the irregular nebulae. It is, of

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[1] ApJ 101, p. 179 (1945).

course, also possible to refer the systematic difference to the luminosities of the brightest stars. If the latter magnitudes are reduced to a constant value of M_t, they will differ by 0.8 magnitudes. This difference happens to agree closely with the mean difference in $(m_s—m_t)$ found by HUBBLE for the same types of nebulae.

The observed difference between spirals and irregular nebulæ will here be referred to the total magnitudes, and is interpreted as a disturbance caused by type II stars. If an average correction of +1.3 is applied to the integrated magnitude of a late spiral we will, accordingly, obtain the luminosity of the type I population in the system. Assuming that the main body of a spiral nebula is formed by type II stars and that the type I stars are mainly concentrated in the spiral arms, as the observed colors indicate, we may be able to support this assumption by observational evidence. The integrated magnitude of the spiral arms of a *Sc* (or *Sb*) nebula may be derived from microphotometer tracings of different cross-sections of the object. Such an investigation has, of course, to be restricted to nebulæ which possess well defined arms and which have small inclinations to the celestial plane. In Table 13 the results[1] are reproduced for three spirals included in the writer's material. The third column gives the correction that should be applied to the total magnitude of the nebula in order to get the magnitude of the spiral arms. The corrections obtained for the two late spirals are of the same order of size as that suggested above. For the intermediate spiral NGC 3031 the correction is about twice as large[2].

TABLE 13.

Luminosity of spiral arms.

Nebula	Type	Δm_t
NGC 3031	Sb	+2.5
NGC 5194	Sc	+1.2
NGC 5457	Sc	+1.5

Fig. 8, where the nebulae given unit weight are denoted by open circles, shows the final relation between total magnitude and magnitude of brightest resolved stars. The magnitudes of type *Sc* nebulae have been corrected by +1.3. The adopted regression line, which gives the average values of M_s corresponding to different values of M_t, has the equation[3],

$$M_s = +0.60\ M_t + 2.4. \tag{7}$$

The two intermediate spirals, NGC 224 and NGC 3031, have also been included in

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[1] On the tracings of the different cross-sections of the nebula the minima between the spiral arms have been joined by smooth curves, which are assumed to represent the main body. In some sections an accurate definition of the curve presents difficulties, and the final results may be subject to a certain personal equation.

[2] According to a report by H. SHAPLEY in *Galaxies* (1945) Harvard investigations of a number of spiral nebulae have given the result that not much more than twenty per cent of the total light comes from the spiral arms. The corresponding magnitude difference is of the same order of size as the mean difference derived for the three objects in Table 13.

[3] The relation derived here does not agree with the results obtained by HUBBLE in the above-mentioned investigation of resolved stars of nebulae. The disagreement may be entirely explained by the systematic deviations in the adopted values of m_s and m_t.

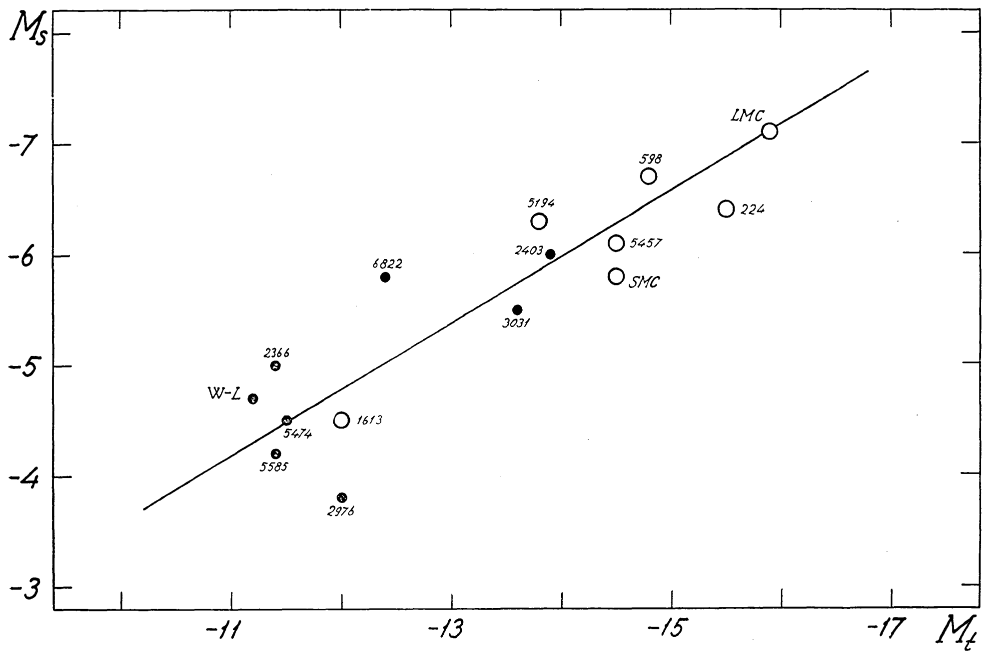

Fig. 8. Relation between M_s and M_t for the nebulae included in Table 12.

the figure. The total magnitudes of these nebulae have been given a provisional correction of $+2.6$, or twice the correction derived for type *Sc*.

Although the material discussed here is comparatively small, there does not seem to be any doubt that the magnitudes of the brightest stars are intimately related to the total luminosities of the nebulae. As will be shown in the following sections, the relation is of a statistical nature and applicable, as it seems, to all agglomerations of type I stars.

The pronounced correlation between M_s and M_t will, unfortunately, reduce the value of the apparent magnitude of brightest resolved stars as a distance indicator. Neglecting the effects of statistical regression, the above equation may be transformed into the following one,

$$M_t = -2.5\,(m_s - m_t) + 6.0. \tag{8}$$

If the absolute magnitude of a nebula is derived from the observed difference $(m_s - m_t)$ by means of this relation we will, on account of the large angle coefficient, get a result of comparatively low weight. A study of the correlation diagram of M_t and $(m_s - m_t)$ obtained from the present material shows that the array dispersion in M_t amounts to about one magnitude. It may be remarked, however, that this dispersion is still considerably smaller than the total dispersion in M_t as derived in the previous chapter.

14. *Relation between M_s and M_t for stars near the sun.* In this section an investigation will be made of the stellar populations in spheres of different radii around the sun. Since stars of type I apparently predominate in the solar neighborhood, it would be of great interest to compare the relation between M_s and M_t valid for these stars with the relation derived above for irregular (and spiral) nebulae.

For a study of the neighborhood of the sun we may conveniently use the last list of nearby stars, published by G. P. KUIPER[1], as a starting-point. The list gives photovisual magnitudes (international system), spectral classes and other data for 254 stars with parallaxes larger than or equal to 0".095, and seems to be essentially complete down to the eleventh absolute magnitude.

A computation of the total absolute magnitude (*pv*) of the 254 stars gives the result, $M_t = -0.81$. Although the material is complete only to the eleventh magnitude, we may safely assume that this value represents the integrated luminosity of *all* stars within the adopted distance limit. It has been shown by the writer in a previous discussion[2] of total magnitudes of galaxies that, with the stellar luminosity function generally adopted for the solar neighborhood, stars fainter than the eleventh absolute magnitude contribute practically nothing to the integrated magnitude of the system. According to KUIPER'S approximation of the luminosity curve in the interval $M = +11 - +17$, these stars would, in fact, give a correction of only about -0.001 to the total magnitude as derived from the brighter stars.

The magnitude of most luminous stars, M_s, will here, as well as in the following sections, be represented by M_5, or magnitude of fifth brightest star. The difference between M_5 and the magnitude used for the nebulae (mean of 3—5 brightest stars) will very likely be small, since in the determination of the latter magnitude exceptionally bright stars have usually been referred to the foreground[3]. Before the selection of the fifth star the absolute photovisual luminosities of the brightest stars have been reduced to the photographic system in the manner described below.

The results obtained for stars nearer than 10.5 parsec are reproduced in Table 14, where the second column gives the limiting photovisual magnitude and the third column the number of stars. The resulting total magnitude (*pv*) is contained in the fifth column. As appears from the table, integrated luminosities have also been derived for stars brighter than the fifth and the fourth magnitudes. These values will furnish corrections to be used for an extension of the investigation to larger volumes of space. The sixth column gives the difference between total luminosity of *all* stars within the volume investigated and luminosity corresponding to the adopted limiting magnitude, a difference which in the present case may be put equal to zero. The final values of total photographic magnitude and photographic magnitude of fifth brightest star are contained in the two last columns of the table.

The color indices necessary to reduce the magnitudes of the individual stars to

[1] Mc Donald Contr. 44=ApJ 95, p. 201 (1942).

[2] Festschrift, E. Strömgren, p. 114 (1940).

[3] In the different groups of nearby stars investigated here the brightest star is, on the average, 0.6 magnitudes brighter than the second brightest star, and 1.0 magnitude brighter than the third star. The mean luminosity of the stars 2—6, or 3—7, agrees closely with the adopted magnitude M_5.

TABLE 14.

M_t and M_s for stars near the sun.

Group	pv M_{lim}	N	C_t'	pv M'	Corr.	pv M_t	C_t	pg M_t	pg M_s
$\pi \geq 0''095$	(+11.0)	254	+0.18 (0.33)	−0.81	0.00	−0.81	+0.28	− 0.5	+2.5
	+ 5.0	23	+0.13 (0.27)	−0.69					
	+ 4.0	13	+0.11 (0.27)	−0.60					
$\pi \geq 0''060$	+ 5.0	112	+0.29	−2.50	−0.12	−2.62	+0.28	− 2.3	+1.0
	+ 4.0	59	+0.27	−2.39					
	+ 3.0	25	—	−2.22					
$\pi \geq 0''035$	+ 4.0	312	+0.19	−4.26	−0.22	−4.48	+0.28	− 4.2	−0.1
	+ 3.0	150	—	−4.10					
$\pi \geq 0''020$	+ 3.0	782	—	−6.04	−0.39	−6.43	+0.28	− 6.1	−1.1
$\pi \geq 0''010$	—	—	—	—	—	—	—	− 8.4	−3.1
$\pi \geq 0''005$	—	—	—	—	—	—	—	−10.4	−4.1

the international photographic system have been derived from the HD-spectra by means of the relation between color and spectral type given by F. H. Seares and Mary C. Joyner[1]. The color indices obtained in this way seem to have an accuracy sufficient for the present purpose.

By using the spectrum-color relation we are able to determine the total *photographic* luminosities of the stars. The resulting values of total color index, as obtained for the three magnitude groups, are contained in the fourth column of the table. The values within brackets are obtained if we omit the brightest star, α Lyrae, which has an unduly large influence on the results. The mean of the two color indices derived for the first group, +0.18 and +0.33, will be accepted as the most probable value of C_t for the nearest stars. A more reliable color index may, however, be obtained if we include the results from the next two parallax groups ($\pi \geq 0''060$ and $\pi \geq 0''035$), which will be further discussed below. By using the same procedure, we find for these groups total color indices of +0.29 and +0.19, respectively. The first value refers to stars brighter than $M = +5.0$ and needs a correction of about +0.05, as is indicated by the results obtained in the first parallax group. The second value, which only represents stars brighter than the fourth magnitude, needs a corresponding correction of about +0.07. The three values of C_t (0.25, 0.34, and 0.26) may be combined with equal weights, and we thus obtain a final color index of +0.28. This value has been used, as appears from column 8 of the table, to reduce the total photovisual magnitudes to the photographic system.

The color index derived here for the solar neighborhood agrees very well with the

[1] Mt Wilson Contr. 684=ApJ 98, p. 261 (1943).

mean color index of about +0.3 obtained for the irregular (resolvable) nebulae (Table 7). The agreement lends strong support to the assumption of a similar stellar content.

By means of the Yale catalogue[1] of stars brighter than 6.5 visual magnitude the above investigation may be extended to larger volumes of space. The catalogue gives mean parallaxes, as obtained from trigonometric, spectroscopic, and dynamic determinations, together with visual magnitudes and spectra, the latter quantities being taken from the Henry Draper Catalogue[2]. As regards parallaxes, the catalogue seems to be essentially complete down to $\pi = 0''020$, as will be demonstrated below. The adopted mean parallaxes are, naturally, of a rather heterogeneous quality. The systematic effects introduced by accidental errors in the parallaxes will be discussed later.

In Table 14 the results[3], as regards M_s and M_t, are given for three different parallax groups: $\pi \geq 0''060$, $\pi \geq 0''035$, and $\pi \geq 0''020$. The magnitudes M_s have been determined in the same way as above. Computations of M_t have been made for different limiting magnitudes of the stars, the value of M_{lim} ranging from +5.0 to +3.0. Since the parallax catalogue only extends to the apparent magnitude 6.5, the investigation naturally has to be confined to absolute magnitudes brighter than these limits. The computed magnitudes are reduced to total luminosities of *all* stars by means of the corrections given in the sixth column of the table. The size of this correction is, for a certain parallax group, indicated by the results obtained in the previous groups. Thus, the correction that has been applied to the total magnitude in group 4 ($\pi \geq 0''020$) represents the mean of the corresponding differences 0.40 and 0.38 which, as appears from the table, have been obtained for groups 2 and 3. Since the corrections are comparatively small, no serious errors are introduced in the final luminosities by a possible uncertainty in the adopted values. The total photovisual magnitudes, which range from —2.6 to —6.4, are reduced to the photographic system by adding the total color index, as derived above.

By making some assumption concerning the space density of stars of different luminosity classes, we may be able to extend the investigation to still larger volumes of space. Since, for distances over fifty parsec, a determination of M_t can no longer be based on the available parallax material, we have to find some other method for an estimation of total magnitude. For distances up to 100, or perhaps even 200 parsec it seems possible to extrapolate the total magnitude by assuming that the integrated luminosity of the stars is more or less proportional to the corresponding volume of space.

It is a well established fact that the space density falls off rather rapidly as the

[1] F. Schlesinger and Louise F. Jenkins, *Catalogue cf Bright Stars*, Yale university observatory (1940).

[2] The visual magnitudes (RHP) of the stars included in the present investigation have been reduced to the international photovisual system by means of the corrections derived by F. H. Seares (Mt Wilson Contr. 288=ApJ 61, p. 284, 1925).

[3] The components of visual binaries and multiple systems have been treated as single stars, if they are given separate numbers in the catalogue. Variable stars have been included with their mean magnitudes.

distance from the galactic plane increases. The decrease in density is accompanied by a change in the mean luminosity of the stars. According to J. H. OORT[1], the luminosity density (amount of photographic light per cubic parsec) is reduced by about 50 per cent as the distance from the galactic plane increases from 0 to 125 parsec. For spherical volumes with radii equal to 50, 100, and 200 parsec we find, by adopting OORT's values, relative mean luminosity densities of 1.0, 0.9, and 0.7, respectively. These results will be used to correct the extrapolated total magnitudes. In low latitudes our knowledge of the spatial arrangement of the stars is still rather limited on account of the disturbances caused by the interstellar absorption of light. A number of investigations of stars of different types indicate that our sun may be located in a spiral arm, and that the star density generally decreases as the distance from the sun grows larger. The results obtained by OORT[2] seem, on the other hand, to show that the sun is situated between two arms in a region of subnormal density. It is difficult to decide, for the present, which of the two contrary views is correct. For the comparatively small volume of space investigated here it seems possible to assume, as a first approximation, that planes parallel to the galactic plane represent equidensity surfaces.

The two last rows of Table 14 give, for the parallax groups $\pi \geq 0''.010$ and $\pi \geq 0''.005$, the extrapolated total luminosities. The values have been corrected for the density variation perpendicular to the galactic plane by means of the factors derived above. The extrapolations are based on the total magnitude derived for the stars in the group $\pi \geq 0''.020$. Very nearly the same results are, however, obtained if the extrapolations are based on the second, or the third parallax group. Thus, a total photographic magnitude of —10.4, or approximately the same luminosity as that obtained for the faintest elliptical and irregular nebulae, is indicated for the stars within a distance of 200 parsec[3].

The absolute luminosities of the fifth brightest star have, as before, been derived by means of the Yale catalogue. Since the most luminous stars are to be found among the *O* and *B* stars, the photographic magnitudes of which in many cases exceed —3.0, we may confine the study to those early type stars in the catalogue which are brighter than +3.5 apparent photovisual magnitude. It appears that the material will, besides a large number of *B* stars, contain only a few stars of type *O*. Since no reliable parallaxes can be obtained for the latter stars, they have to be omitted from the list. For the *B* stars a considerable number of spectroscopic parallaxes is available. If we accept the mean parallaxes of the catalogue, which for the majority of the

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[1] BAN 6, p. 249 (1932).

[2] BAN 8, p. 233 (1938).

[3] According to A. G. MOWBRAY (Pop. Astr. 48, p. 515, 1940), the surface luminosity of our part of the Galaxy as seen from an outside point is approximately equal to one star of absolute magnitude +2 per square parsec. With $M_t = -10.4$, the surface luminosity corresponding to the above spherical volume will be about half a magnitude fainter. If the latter luminosity is increased by the luminosity corresponding to stars situated at distances of more than 200 parsec from the galactic plane, it will agree rather closely with MOWBRAY's result.

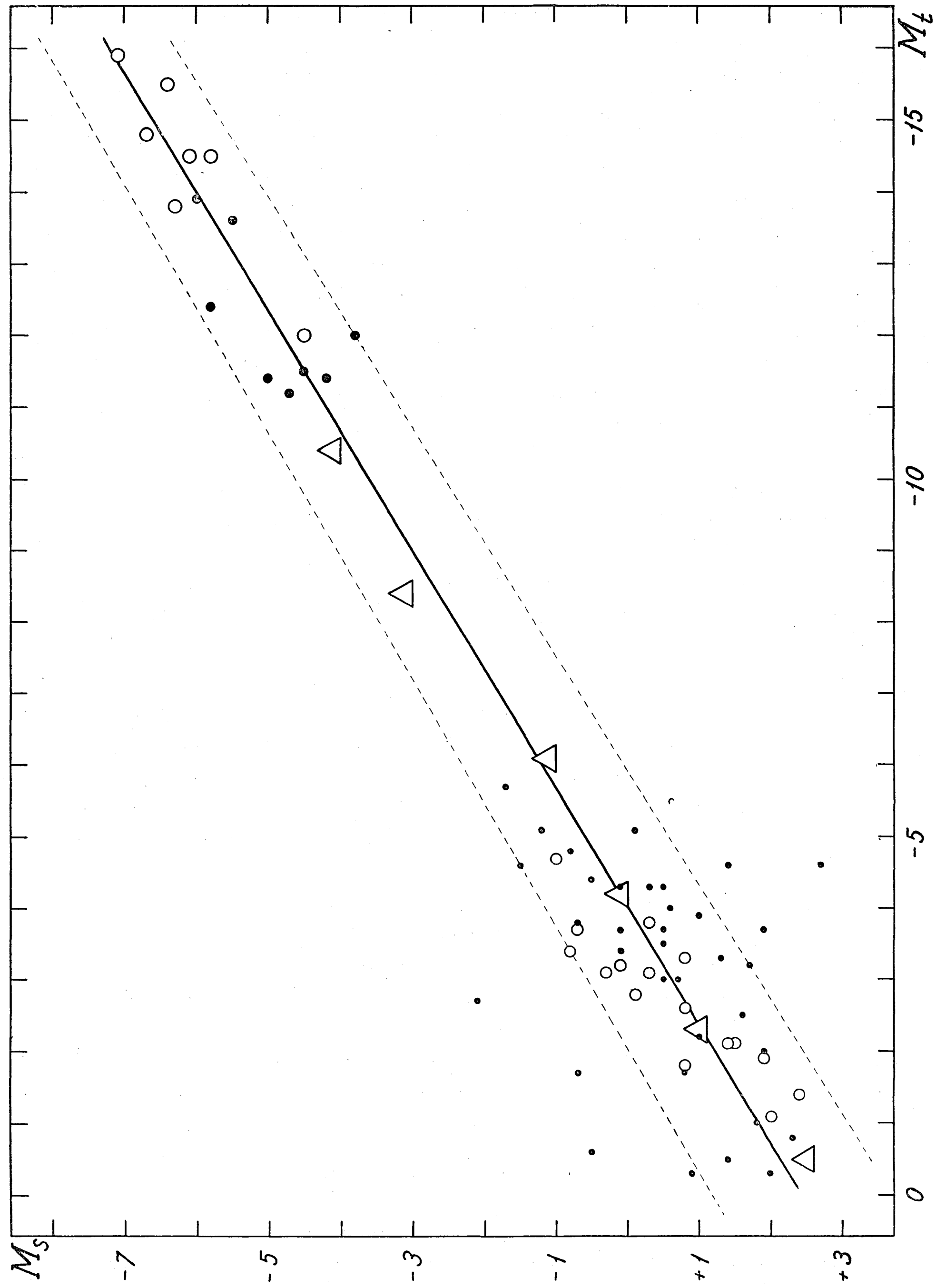

Fig. 9. Relation between M_s and M_t for nebulae (right), open clusters (left), and nearby stars (triangles).

stars are based on three or more individual determinations, we find for the two parallax groups photographic magnitudes M_5 equal to —3.1 and —4.1, respectively. It may be mentioned that the absolute magnitudes of stars 3—7 range from —3.5 to —3.0 ($\pi \geq 0''010$), and from —4.5 to —3.9 ($\pi \geq 0''005$).

The final results of the investigation of the solar neighborhood, as obtained for the different parallax groups, are reproduced in Fig. 9 (open triangles). A variation of the total magnitude from —0.5 to —10.4 gives a change in the magnitude M_s (M_5) from +2.5 to —4.1. In the upper right corner of the figure we find the extragalactic nebulae investigated in the previous section. The straight line represents the relation between total magnitude and magnitude of brightest stars derived for these objects. *The same line, if extrapolated, seems to define the relation between* M_t *and* M_s *for the solar neighborhood.* The good agreement between nebulae and nearby stars indicates that we, in both cases, have to deal with one and the same cosmical law and, furthermore, that this law represents an effect of statistical selection.

In conclusion we will discuss a number of sources of error and try to estimate their influence on the results derived above.

An examination will first be made of the completeness of the Yale catalogue as regards parallaxes. Any incompleteness in the parallax material would, naturally, result in more or less serious errors in the total luminosities derived for the different groups of stars. In Fig. 10 we have reproduced the logarithmic frequency distribution of the distance moduli for stars with absolute visual magnitudes brighter than or equal to +3.0, the distance moduli and the absolute magnitudes being computed from the mean parallaxes given in the catalogue. The distance modulus extends to +3.5, corresponding to a parallax of 0''020, and the material is thus the same as that used above in the group $\pi \geq 0''020$. The straight line, which seems to represent the observed frequencies very well, has an inclination corresponding to constant space density, that is, the angle coefficient is equal to +0.6. It seems justified to assume that the parallax material is essentially complete down to the adopted limit. Only in the last class, corresponding to $\pi = 0''020$—$0''021$, a certain deficiency is indicated. The missing number of stars is, however, rather small and would probably cause an error in the computed total luminosity of only a few hundreds of a magnitude. In the other two parallax groups ($\pi \geq 0''035$ and $\pi \geq 0''060$) the missing numbers of stars are certainly still smaller.

The accidental errors in the absolute magnitudes of the stars may, in the present case, give rise to systematic errors of a statistical nature. The mean correction[1] to be applied to the observed absolute magnitude is given by the expression,

$$corr. = -\varepsilon^2 \cdot D\left[ln\, N(m-M)\right] + \varepsilon^2 \cdot D\left[ln\, \varphi(M)\right], \quad (9)$$

where ε is the mean error of the absolute magnitude (or the distance modulus). The correction terms contain the derivative of the logarithmic (*log. nat.*) frequency

[1] The corrections given here are based on the theoretical investigations by K. G. Malmquist (Lund Medd. I, 100, 1922).

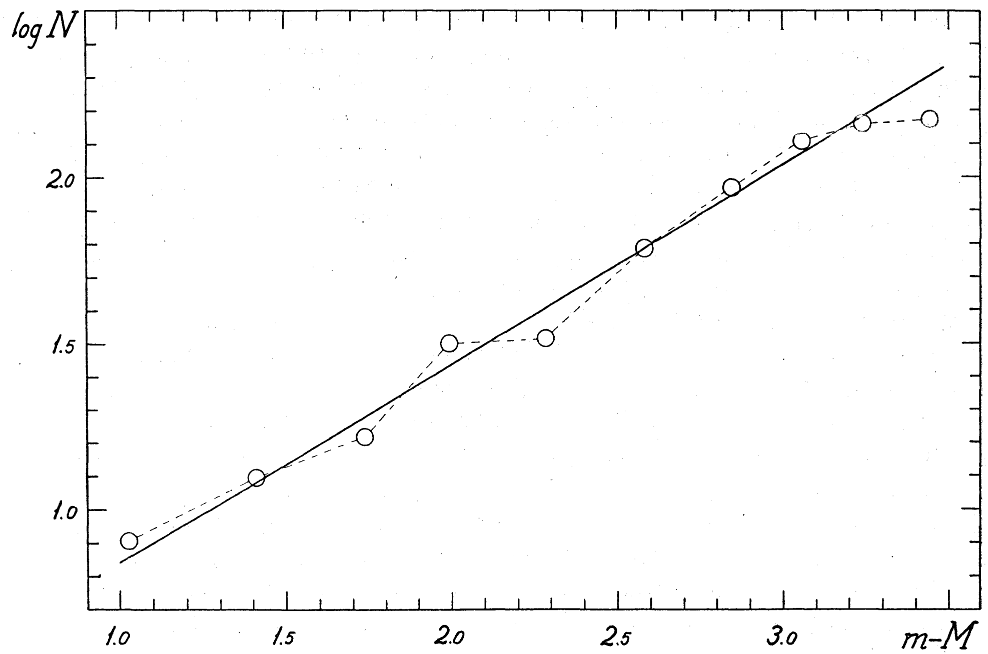

Fig. 10. Distribution of distance moduli of stars in the Yale parallax catalogue.

distribution of $(m - M)$ and the derivative of the logarithmic luminosity function $\varphi(M)$. Each of the two terms represents a systematic effect caused by statistical selection, the stars of the different parallax groups being selected (*a*) on the basis of the distance modulus, and (*b*) on the basis of the absolute magnitude.

With a constant space density $D[ln\ N(m-M)]$ becomes equal to $+1.38$. The derivative $D[ln\ \varphi\ (M)]$, which varies with the absolute magnitude, seems to have a value[1] slightly larger than $+1.0$ for stars brighter than $M = +1$ à $+2$. The mean errors of the absolute magnitudes are comparatively small in the first two groups ($\pi \geq 0''.095$ and $\pi \geq 0''.060$), where the distance moduli in most cases are based on trigonometric parallaxes. The distance moduli of stars in the other groups are mainly obtained from spectroscopic observations. Since the adopted parallaxes generally represent means of several individual determinations it seems, for these groups, possible to assume a mean error in M of about 0.4 magnitudes. Adopting the numerical values suggested here, we find that the above formula gives corrections (to M_s or to M_t) that are smaller than one tenth of a magnitude. Since the uncertainties introduced by other sources of error are probably larger, it seems permissible to neglect the statistical selection errors in this connection.

The interstellar absorption may produce a noticeable effect in the two last parallax groups. The absorption will make the parallaxes, as computed from the spectro-

[1] The distribution *log* $\varphi\,(M)$, as obtained for stars in the group $\pi \geq 0''.020$, is reproduced in Fig. 11.

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scopic absolute magnitudes, slightly too small, and the magnitudes of brightest stars, M_s, thus correspond to smaller volumes of space (and to smaller populations of stars) than those defined by the parallax limits 0."010 and 0."005. The extrapolated total magnitudes should, accordingly, be reduced to the same smaller volumes. If we adopt a mean photovisual absorption (mean for different latitudes) of $0^m.5$ per kiloparsec, the correction to total magnitude would, in the last parallax group, amount to +0.15. A correction of this size has been applied to the extrapolated value of M_t obtained for this group. In the group $\pi \geq 0''.010$ the correction would be only half as large and may be neglected.

15. *Relation between M_s and M_t for open clusters.* The study of the relation between total magnitude and magnitude of brightest stars will in this section be extended to open clusters in the galactic system. According to the H—R diagrams, the clusters are condensations in stellar populations of Baade's type I, and we may for these objects expect a relation between M_s and M_t that is more or less similar to that derived for nearby stars.

In P. Collinder's extensive paper on open clusters[1] we find integrated magnitudes for, in all, 471 objects. The magnitudes represent estimates by Collinder and by K. Lundmark obtained from the Franklin-Adams material (plates and charts), and the following study will be based on the mean of the two values which are usually given for each cluster. The magnitude of the fifth brightest star has been estimated at the Harvard observatory[2] for about 200 clusters.

Before proceeding further it seems necessary to check the values of m_t and m_5 in order to ascertain whether these estimated magnitudes are free from systematic errors. For this purpose we may use the photometric data for 25 open clusters, which have been collected in Table 15. The table is based on investigations published during the past twenty years and includes, without any demand for completeness, those clusters for which photographic magnitudes of individual stars are available and for which a separation of stars in members and non-members has been possible by means of one, or several criteria (prop. mot., rad. vel., spectra, colors). The estimated values of m_t and m_5 are reproduced in the second and third columns. The next two columns give the integrated photographic magnitude of the cluster and the photographic magnitude of the fifth brightest cluster star, as derived by the writer from the sources contained in the last column. The figures given here indicate volume and reference number in the *Astronomischer Jahresbericht*. Finally, the sixth and seventh columns give the diameter of the cluster (or diameter of investigated area), according to the quoted authority, and the number of selected cluster stars.

Although the criteria used for the selection of cluster members are not always quite reliable, the magnitudes m_t and m_5 derived from the above material probably represent the most accurate values to be obtained for the present. Excluding the exceptionally bright Hyades cluster, we find that the estimated total luminosities

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[1] Lund Ann. 2 (1931).

[2] H. Shapley, *Star Clusters* (1930).

TABLE 15.

Photometric data for 25 open clusters.

Object	Estimated		Measured				Authority
	m_t	m_5	m_t	m_5	d	n	
NGC 436	+9.3	+ 9.8	+9.0	+12.1	6′.0	22	Alter, 45,8703
NGC 663	7.5	9.8	6.8	10.0	16.4	49	Alter, 43,8701
NGC 752	6.6	9.6	6.0	9.8	50.	63	Ebbighausen, 41,8706 (cl. 1,2)
NGC 869	4.3	9.5	4.9	8.8	30.	369	Oosterhoff, 39,8718
NGC 884	4.3	9.5	5.3	9.3	30.	331	» , 39,8718
IC 1805	6.8	—	6.3	9.5	30.	27	Clasen, 39,8704
NGC 1027	7.4	9.8	7.1	10.5	24.	37	» , 39,8704
NGC 1039	5.8	—	5.4	8.5	35.	86	Brüggemann, 37,8711
Hyades	0.8	—	1.8	4.9	480.	100	Holmberg, 45,8714
NGC 1912	7.0	9.7	6.4	10.5	19.	110	Wallenquist, 34,8703
NGC 2548	5.5	9.4	5.5	8.9	30.	75	Ebbighausen, 41,8707 (cl. 1,2)
NGC 2632	3.9	—	3.4	6.8	110.	179	Ramberg, 43,8710
IC 2395	4.6	10.1	4.8	7.8	35.	67	Schmitt, 38,8707
NGC 2682	7.4	10.8	7.8	11.6	30.	107	Vanderlinden, 45,8724 (cl. 1,2)
Coma Ber.	2.9	—	2.8	5.4	420.	42	Trumpler, 40,8735
IC 4665	5.4	—	4.7	7.5	120.	25	Kopff, 45,8715
NGC 6613	8.0	10.9	7.7	10.7	9.8	18	Alter, 45,8702
NGC 6633	5.6	8.0:	4.8	8.9	120.	45	Kopff, 45,8715
IC 4725	6.3	9.3	5.6	9.3	32.8	125	Alter, 45,8702
NGC 6645	8.4	11.3	8.4	11.8	10.9	53	» , 45,8702
IC 4756	5.3	—	6.2	9.7	120.	35	Kopff, 45,8715
NGC 7092	5.3	6.5	4.7	7.6	30.	30	Ebbighausen, 42,8717 (cl. 1,2)
NGC 7209	8.0	10.1	7.1	10.7	30.	49	Mävers, 42,8723
NGC 7243	6.7	9.0	6.6	9.2	30.	32	» , 42,8723
NGC 7654	8.2	11.0	7.7	11.5	12.	154	Lundby, 45,8716

are, on the average, 0.2 magnitudes fainter than the measured values. The dispersion in the magnitude differences amounts to 0.5, and the main part of this dispersion probably represents accidental errors in the estimated values. The magnitudes m_5 given in the Harvard list seem to be systematically somewhat too bright, the mean deviation amounting to 0.3. If two exceptional differences are omitted, we also in this case find a dispersion in the magnitude differences of 0.5. The average deviations obtained from these comparisons will be used as provisional corrections to the estimated values of m_t and m_5. Thus, the former magnitudes are given a correction of —0.2, and the latter magnitudes a correction of +0.3.

The apparent magnitudes may be transformed into absolute luminosities by means of the distance moduli given by R. J. TRUMPLER[1] in his list of standard distances of open clusters. The distances, which have been derived from the H—R

[1] LOB 14, p. 154 (1930). It may be remarked that the spectroscopic parallaxes derived by CAROL A. RIEKE (Harvard Circ. 397, 1935) give a final result that is very nearly the same as that obtained from TRUMPLER's parallaxes.

diagrams of the brighter cluster stars, in some cases represent preliminary estimates since they are based on only a few stars. In the following discussion we will include only those objects in TRUMPLER's list which have observed distance moduli smaller than +11.0. To this limit, which corresponds to a distance of somewhat more than 1000 parsec, the material seems to be essentially complete, and the investigation is thus referred to a given volume of space. If more distant objects are included we will have to deal with rather serious selection effects, the more remote clusters in the list having considerably brighter mean magnitudes (M_t and M_5) than the nearby clusters.

The relation between M_t and M_s (M_5) for clusters with distance moduli smaller than +11.0 appears from Fig. 9 (lower left corner). The total number[1] of objects is 52, and of these 16 are included in Table 15. For the latter clusters, which in the figure are denoted by open circles, we have used the more accurate values of m_t and m_5 derived above.

The mean of M_t (—3.1) and of M_s (+0.5) obtained for the 52 clusters agrees very closely with the relation line derived in the previous sections for nebulae and nearby stars. In spite of the large dispersion of the individual values, it seems to be clearly indicated that the open clusters obey the same law as that found for the other groups of objects. The dispersion may be explained partly as a result of the mean errors in the apparent magnitudes and the distance moduli, and partly as an inherent cosmical dispersion dependent on the differences between clusters in spectral composition and on the chance variations due to small numbers of stars.

16. *Theoretical interpretation of the M_s—M_t relation.* It may be of some interest to examine the relation between M_s and M_t, as derived above for the different groups of stars, from a theoretical point of view. Since we apparently have to deal with a statistical law, expressing the tendency of the brightest stars to accumulate in the largest populations, we may expect that the relation is intimately connected with the form of the stellar luminosity curve.

The luminosity function, $\varphi(M)$, of the stars in a stellar system may be represented by the following linear expression,

$$\log \varphi(M) = a M + b, \tag{10}$$

where the parameters a and b are functions of M. For the bright end of the luminosity curve, that is, for stars brighter than a certain limiting magnitude M', the variations in a and b seem to be small and the parameters may in this interval be treated as constants. The quantity a then gives the relative increase in $\varphi(M)$ with increasing M, whereas b is proportional to the total absolute magnitude, M_t', of the stars brighter than the limit M':

[1] Total magnitudes have been given by COLLINDER for all clusters included in TRUMPLER's list, whereas the material contained in the Harvard catalogue is not fully complete. Those values of m_5 which are denoted as uncertain have been omitted.

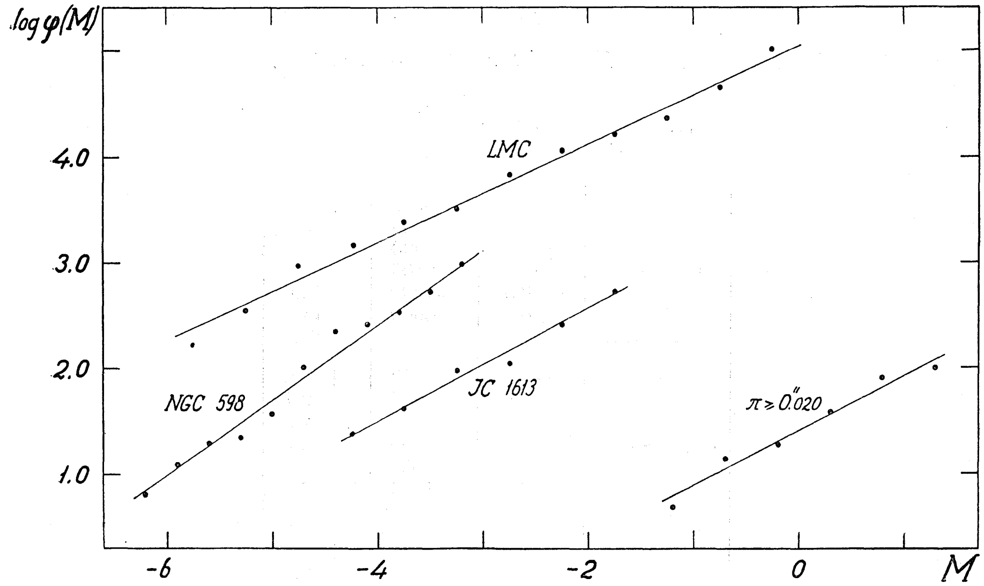

Fig. 11. Luminosity curves for nebulae and nearby stars.

$$10^{-0.4\,M'_t} = \int_{-\infty}^{M'} 10^{-0.4\,M} \cdot 10^{aM+b} \cdot dM = 10^b \cdot \frac{\log e}{a-0.4} \cdot 10^{(a-0.4)\,M'}. \tag{11}$$

Let us now introduce M_5, or the absolute magnitude of the fifth brightest star. The magnitude of this star apparently satisfies the following equation,

$$5 = \int_{-\infty}^{M_5} 10^{aM+b} \cdot dM = 10^b \cdot \frac{\log e}{a} \cdot 10^{aM_5} \tag{12}$$

By combining this equation with equation (11) we obtain the final relation,

$$M_5 = \frac{0.4}{a} \cdot M'_t + \text{const.}, \tag{13}$$

where the constant term is a function of the parameter a and the limiting magnitude M', which in the present case are assumed to have fixed values.

We have here obtained a linear relation between the magnitudes M_5 and M'_t, the angle coefficient being equal to 0.4/a. In order to exchange M'_t for M_t, or the total magnitude of the entire system, we may assume that the latter magnitude differs from the former one by an amount that is more or less constant for a certain type of nebulae. The difference depends, of course, on the form of the luminosity curve for the fainter stars. Whereas nebulae of the same type probably have similar luminosity distributions, we have to expect large deviations when we compare different types, especially on account of the varying ratio of type I to type II stars. The systematic differences in M_t (or in M_5) which have been found in section 13 for different types of nebulae, may thus be reproduced in the above equation.

In Fig. 11 the logarithmic luminosity functions are given for the brightest stars in

TABLE 16.

Extrapolation of the M_s—M_t relation.

Object	M'	N	ΔM_t
SMC ($M_t=-14.5$)	−4.3	230	+ 4.2
	−3.3	1600	6.3
	−2.3	3000	6.9
	−1.3	9100	8.2
	−0.3	51000	10.0
	+0.7	290000	11.9
NGC 6822 ($M_t=-12.4$)	−5.2	12	+ 1.0
	−4.6	36	2.1
	−4.0	72	2.9
	−3.4	150	3.7
	−2.8	538	5.1
	−2.2	1736	6.4
IC 1613 ($M_t=-12.0$)	−4.0	25	+ 1.7
	−3.5	67	2.8
	−3.0	162	3.8
	−2.5	274	4.3
	−2.0	535	5.1
	−1.5	1076	5.8

some nearby galaxies. The distributions are based on star counts made by H. Shapley[1] in the Large Magellanic Cloud, by E. Hubble[2] in NGC 598, and by W. Baade[3] in IC 1613. The distribution derived by the writer in section 14 for nearby stars with $\pi \geq 0''.020$ has also been included. The figure shows that it is, in a first approximation, possible to represent the luminosity curves referring to the brightest stars by linear relations, as suggested above. The inclination of the lines (parameter a) ranges from about 0.5 to 0.7, and the coefficient 0.4/a contained in equation (13) would thus obtain numerical values ranging from 0.8 to 0.6. These values are of the same order of size as the mean value (0.60) derived in the previous sections from the observational material.

The theoretical analysis may be completed with a demonstration of extrapolated M_s—M_t curves to be derived from the observed luminosity distributions. The demonstration is based on the hypothetical division of a stellar system into a number of separate parts and a study of the resulting changes in M_s and M_t.

Let us assume that in an irregular nebula there are N stars brighter than the absolute magnitude M'. If this nebula could be divided in $N/5$ equally luminous parts, each of the sub-systems would get a total luminosity 2.5 *log* $N/5$ magnitudes fainter

[1] *Galaxies,* The Blakiston Co. (1945).

[2] Mt Wilson Contr. 310=ApJ 63, p. 236 (1926).

[3] The star counts, which cover the entire system, have not yet been published. The writer is indebted to Dr. Baade for the communication of these results.

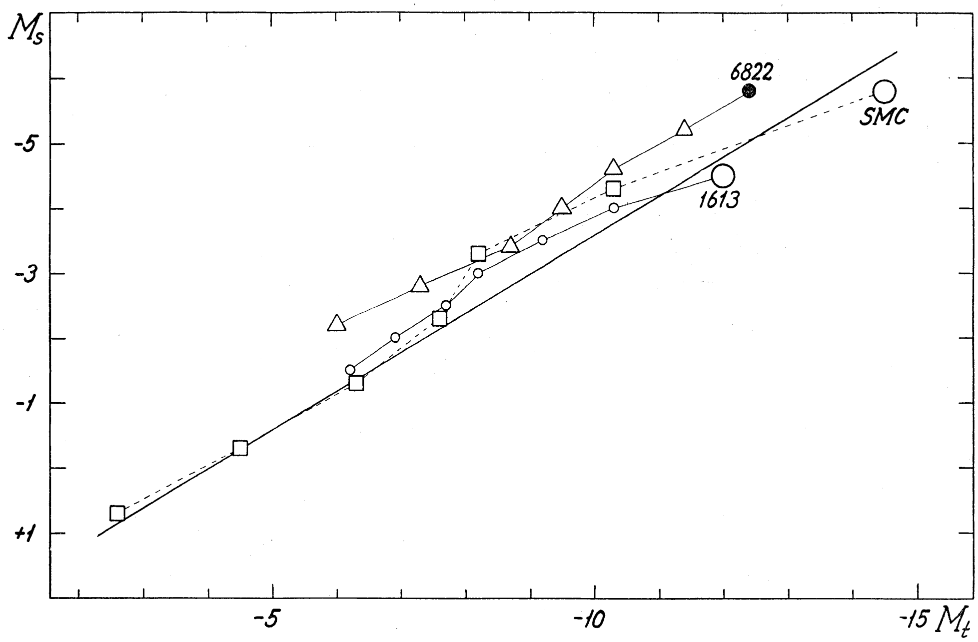

Fig. 12. Extrapolation of the $M_s - M_t$ relation.

than that of the mother system. In each group we will, on the average, find five stars brighter than the limit M', and this magnitude may, accordingly, be assumed to represent M_5 in the sub-systems. Actually, M' will agree closely with the median value of the individual M_5-values, supposed that the stellar populations of the different groups have similar luminosity functions[1]. By computing total magnitudes corresponding to different values of M', we are thus able to extrapolate the relation between M_5 and M_t in the direction of fainter magnitudes.

The procedure outlined here presupposes that at least the brighter end of the luminosity function is known for the stars in the mother system. In Table 16 some data are given for three irregular nebulae, which have been selected as objects of demonstration. The third column reproduces the number of stars brighter than the photographic limiting magnitude M' (second column), as derived by H. SHAPLEY[2] for SMC, by E. HUBBLE[3] for NGC 6822, and by W. BAADE for IC 1613. The numbers given for each nebula refer to the entire area covered by the object and have been

[1] If the N stars are distributed at random in the different sub-groups, the probability of finding x stars in a certain group is given by Poisson's law, $P(x) = e^{-b}\frac{b^x}{x!}$. The quantity b represents the mean number, which in the present case is equal to 5.

[2] HC 260 (1924). SHAPLEY has in the reduction of the counted star numbers to numbers corresponding to the entire system suggested values of 2 to 5 for the relative densities in the selected regions. In the present case we have used the mean of these values, which seems to give the best results.

[3] Mt Wilson Contr. 304=ApJ 62, p. 409 (1925). The reduction factor has been put equal to 2, as suggested by HUBBLE.

statistically corrected for foreground stars. In the fourth column of the table we find the corresponding changes in total magnitude (2.5 *log* $N/5$), as defined above.

The values of M_s (M_5) and M_t obtained for the different hypothetical sub-systems are reproduced in Fig. 12. The extrapolated curves agree very nicely with the straight line, which represents the relation between M_s and M_t derived in the previous sections for nebulae, nearby stars, and open clusters.

The results obtained in the last section seem to justify the conclusion that the observed correlation between total magnitude and magnitude of brightest stars is entirely an effect of statistical selection.

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