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LINE INTENSITIES AND THE SOLAR CURVE OF GROWTH

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ABSTRACT

The solar curve of growth is redetermined by combining the best available equivalent-width data (those of Allen and the Utrecht photometric atlas) with King's laboratory gf-values for lines of *Ti* I and *Fe* I. A comparison of laboratory intensities with theoretical intensities, calculated for LS coupling, shows that the latter may be seriously in error for complex atoms. Excitation temperatures for the reversing layer of the sun are found to be 4550° ± 125° K for *Ti* I lines and 4900° ± 125° K for *Fe* I lines; this difference appears to be real. Solar gf-values may be calculated for any atomic line with a known solar equivalent width if an excitation temperature of 4700° K is assumed.

The theory of the curve of growth for absorption lines in stellar spectra was first given by Minnaert and Slob[1] in 1931, when they showed how the intensity of a stellar absorption line, measured by its equivalent width, depended on the "number of active atoms" involved in the electron transition. The theory was extended in 1936 by Menzel[2] and again in 1938 when Unsöld[3] gave a complete review of the subject. Detailed, practical applications of the theory to the sun and stellar spectra have been attempted, among others, by Menzel, Baker, and Goldberg,[4] Greenstein,[5] Miss Steel,[6] and the author[7]; but in all cases the scatter of observed intensities about the mean curve of growth has been much greater than might have been desired. Three main contributing factors producing this scatter are: (*a*) The theory is very much oversimplified, largely because more probable assumptions concerning the passage of radiation through a stellar atmosphere involve mathematical equations which have not yet been solved. (*b*) Line intensities calculated from theoretical models of the atom (LS coupling) are not accurate for complicated atoms, such as iron and titanium; and extensive lists of laboratory intensities are available for the spectra of very few atoms. (*c*) The accurate measurement of stellar line intensities for this purpose requires a detailed study of high-dispersion spectra; a great many lines covering as wide an intensity range as possible are desired for each element, and, under such conditions, systematic errors in measurement may be introduced. It is the purpose of this paper to use the best available data for *b* and *c* for the sun and to show that certain difficulties still remain with regard to *a*.

The theory of the curve of growth indicates that for weak lines the intensity, measured by $\log (W/\lambda)$, is directly proportional to the "number of active atoms," N, according to the Doppler principle; and for strong lines it is proportional to the square root of N if produced by a combination of radiation and collision damping effects. The transition portion of the curve, which represents the sum of these effects as the line becomes saturated, has not yet been completely studied mathematically; the best representation of the curve has been given by Baker,[8] although his formula, given in equation (1), applies

* On loan to the Department of Physics, University of British Columbia, during the 1943–44 session.

[1] *Proc. kgl. Acad., Amsterdam*, **34**, 542, 1931.

[2] *Ap. J.*, **84**, 462, 1936.

[3] *Physik der Sternatmosphären*, pp. 264–285, Berlin: Springer, 1938.

[4] *Ap. J.*, **87**, 81, 1938.

[5] *Ap. J.*, **95**, 161, 1942.

[6] Unpublished.

[7] *Pub. A.A.S.*, **10**, 34, 1940.

[8] *Ap. J.*, **84**, 474, 1936.

only to the sun and very similar stars:

$$\log \frac{W}{\lambda} = \log X_0 \frac{v}{c} \sqrt{\pi} - \tfrac{1}{2} \log (1 + X_0) - \frac{\tfrac{1}{2} \log 4 \sqrt{\pi} \frac{v}{c} \frac{\nu}{\Gamma}}{1 + 15 e^{-2 \log X_0}}, \tag{1}$$

where W is the equivalent width, λ the wave length in Angstrom units, X_0 the optical depth at the center of the line, and c/v and Γ/ν are constants which must be derived from the observed curve of growth. As X_0 is not known at the beginning of an investigation, log (W/λ) must be plotted against known values which are proportional to log X_0. Theory indicates that, for a Boltzmann distribution of energy within the atom, X_0 is proportional to $Ss/\Sigma s\ e^{-Ei/kT}$ or to gf $\cdot \lambda \cdot e^{-Ei/kT}$, where $Ss/\Sigma s$ is the theoretical line strength on the basis of pure LS coupling, gf is the observed laboratory intensity, and E_i is the excitation potential of the lower level of the atomic transition. In this paper log (W/λ) has been plotted against log X_f as defined by

$$\log X_f = \log \mathrm{gf} + \log \lambda - \frac{5040}{T} E_i , \tag{2}$$

where T is the excitation temperature of the given atoms and may be determined from the curve itself. In previous work the stellar intensity has been plotted against the theoretical intensity for all lines in the same transition array, and the various segments have been shifted along the X_0-axis to form the best mean curve of growth; or the stellar intensities have been compared with the values of X_0' determined by Menzel[4] for the sun.

For complex atoms it is known that LS coupling does not represent the interaction of electrons well, but line intensities are not readily calculable for other models. Laboratory intensities should be more reliable if the necessary corrections for self-reversal, temperature, and frequency are made. Laboratory emission intensities have been published;[9] but only Seward[10] and Allen and Hesthal[11] have completely corrected the observed values. An attempt was made to use Seward's intensities for *Mn* I, and satisfactory results were obtained for lines in the region λλ 4000–4800; but when lines in the region λλ 3500–3900 were included, the scatter about the mean curve of growth was so greatly increased that the *Mn* I intensities have been omitted. The laboratory intensities upon which this investigation is based are the gf-values measured in absorption in the electric furnace for *Ti* I and *Fe* I by A. S. and R. B. King.[12]

In Figure 1, log $(Ss/\Sigma s)$ has been plotted against log (gf $\cdot \lambda$) for the transition arrays of *Ti* I and *Fe* I for which King and King have published laboratory intensities. The straight line in each diagram indicates a one-to-one correspondence between theoretical and laboratory intensities. As King and King consider that the probable error of their measurements is about 10 per cent, the failure of LS coupling in these atoms seems to be the most likely cause for the observed scatter in the diagrams. In general, it appears that the intensity rules hold fairly well within individual multiplets, but the theoretical multiplet strengths (S) are frequently in error. These diagrams indicate that large errors may be introduced if theoretical intensities are used for complicated atoms. Laboratory intensities should, therefore, be employed whenever possible.

In order to obtain the best available solar intensities, the values published by Allen[13] and measures made by the author from the *Utrecht Photometric Atlas of the Solar Spectrum*[14] have been used. Equivalent widths of about seven hundred lines in the atlas

[9] See list in Unsöld, *op. cit.*, p. 207.

[10] *Phys. Rev.* **37**, 344, 1931.

[11] *Phys. Rev.*, **47**, 926, 1935.

[12] *Ap. J.*, **87**, 24, 1938.

[13] *Mem. Comm. Solar Obs., Canberra*, No. 5, 1934; No. 6, 1938.

[14] By Minnaert, Mulders, and Houtgast, Amsterdam, 1940.

in the region λλ 3500–6700 were obtained by counting squares after corrections for blending with other lines and for the position of the continuous spectrum had been made. The agreement between Allen's data and the Utrecht values is very satisfactory. The probable error of any given equivalent width is about 7 per cent for very weak lines ($W < 0.050$ A) and slightly less than 6 per cent for all stronger lines. As no appreciable systematic differences have been detected, average values have been used wherever a line was available in each publication. Woolley[15] and Mulders[16] have also published tables of solar line intensities, but these have not been used because they cover only limited regions of the spectrum. Phillips[17] published solar intensities for the region λλ 3500–3900; but, since the scatter when compared with the Utrecht measures was great and since the Utrecht measures agreed well with those by Allen in other regions, Phillips' observations have been omitted.

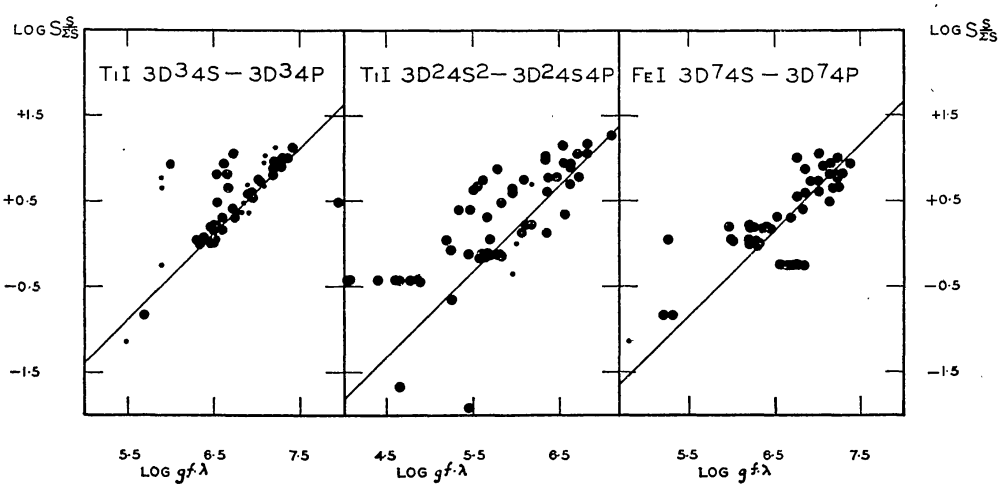

FIG. 1.—Comparison of theoretical intensities ($S\frac{s}{\Sigma s}$) and laboratory intensities ($gf\cdot\lambda$) for lines of *Ti* I and *Fe* I.

The solar curve of growth has been derived by plotting log (W/λ) against log X_f (eq. 2) for all available lines of *Ti* I and *Fe* I. All lines in the solar spectrum from which the blending of other lines could be eliminated were used, with weights ranging from 4 (a line almost free from blends) to 1 (a line of uncertain equivalent width). Both solar and laboratory intensities were available for 160 lines of neutral titanium and 97 lines of neutral iron. As a first approximation, the temperature, T (in eq. 2), was assumed to be 4400° K—a value obtained by Menzel[4] and also by King.[18] The titanium and iron observations were treated separately at first, and the best empirical curve was drawn through each set of points, adopting only the criteria that the slope of the lower Doppler region be 1.0 and that of the radiation-damping portion be 0.5. As the slopes of the transition portions of the two curves were very nearly equal and as the gf-values were only relative, the two curves were moved along the X_f-axis until the single curve with the least scatter was obtained. The observational points were grouped, and a smooth curve was drawn through the weighted mean values. Improved excitation temperatures were then calculated from the formula

$$Y = \log X_f - \log \mathrm{gf} - \log \lambda = L - \frac{5040}{T} E_i \, , \qquad (3)$$

[15] *Ann. Solar Phys. Obs., Cambridge*, **3**, Part II, 1933.

[16] *Zs. f. Ap.*, **10**, 297, 1935.

[17] *Ap. J.*, **96**, 61, 1942.

[18] *Ap. J.*, **87**, 40, 1938.

in which Y is defined from the first half of the equation, log X_f is taken from the curve for each value of log (W/λ), and L is a constant which should equal zero in this case if the correct excitation temperature has been assumed but which is proportional to the abundance of the atom in the solar reversing layer when the theoretical value of X_0 (as given in Baker's formula [eq. 1]) is used. Temperatures of 4550° for *Ti* I and 4850° for *Fe* I were derived from least-squares solutions of this equation.

In order to cover the transition portion of the curve more completely, W. Petrie's calibration[19] of King's eye-estimates[20] of emission-arc intensities, in terms of the laboratory gf-values, was used for lines of *Fe* I. Although laboratory intensities have been obtained only for lines with lower excitation potentials of less than 1.6 volts, the relation appears to be linear; and, in order to use the weak iron lines, Petrie's curves were extrapolated up to a lower excitation potential of 4.0 volts. These calibrated gf-values may be in error for individual lines, but the two hundred additional lines, each of weight 1, extended the transition portion of the curve for the iron lines 0.5 units along the log X_f-axis with very little increase in scatter about the mean curve.

Several attempts were made to reconcile the different temperatures obtained for *Ti* I and *Fe* I; but when a mean temperature of 4700° K was used to form the curve, the temperature calculated for *Ti* I lines became 4300° K, and that for *Fe* I lines was increased to 4950° K. The adopted curve of growth for the sun shown in Figure 2 is based on an assumed temperature of 4500° for *Ti* I, which leads to a calculated temperature of 4575° ± 125° and an assumed temperature of 4850° for *Fe* I, which leads to a calculated value of 4750° ± 150° for all iron lines and 4975° ± 100° for lines with measured gf-values. When Y is plotted against E_i, the mean points fit a straight line very well, and there is no appreciable change in slope with excitation potential, such as W. Petrie[21] has observed for chromospheric intensities.

The adopted curve of growth for the sun is the mean of the two curves obtained from the observations of neutral titanium and of neutral iron lines, as indicated above. The overlap of these curves is only on the flat transition portion, and, in order to obtain an objective fit, this section was considered to be straight, and the best straight line was drawn through the points on each curve. As the slopes were nearly identical it was readily found that a factor of 3.00 must be added to log X_f for the titanium lines in order to make the best fit with the relative gf-values for the iron lines. An additional factor (−3.73) has been added to all observations in order to make the scale agree with the absolute gf-values for neutral iron as determined by King.[22] Thus the present scale along the X_f-axis is based on the absolute gf-values for iron. The weighted mean values of the observed points for both titanium and iron are given in Table 1 and points taken from the smooth curve are given in Table 2. Log X_f may be obtained directly from this curve for any line whose solar intensity, log (W/λ), is known and absolute gf-values on the iron scale are readily derived by means of the equation

$$\log gf_{\odot} = \log X_f - \log \lambda + \frac{5040}{T} E_i . \tag{4}$$

The most uncertain quantity in this equation is T, which apparently should be 4550° for *Ti* I lines and 4900° for *Fe* I lines. As, however, the factor $(5040/T)E_i$ varies slowly with T for lines of low excitation potential, this discrepancy is small, and a mean value of $T = 4700°$ K for lines of atoms other than *Ti* I and *Fe* I might well be used.

In order to compare this empirical curve of growth for the sun with the theoretical curve, it was related to the family of curves computed by Menzel[23] to obtain the damp-

[19] *Pub. A.A.S.*, **10**, 257, 1942.

[20] *Ap. J.*, **37**, 239, 1913; **56**, 318, 1922.

[21] *J.R.A.S. Canada*, Vol. 38 (in press).

[22] *Ap. J.*, **95**, 78, 1942.

[23] *Pop. Astr.*, **47**, 74, 1939.

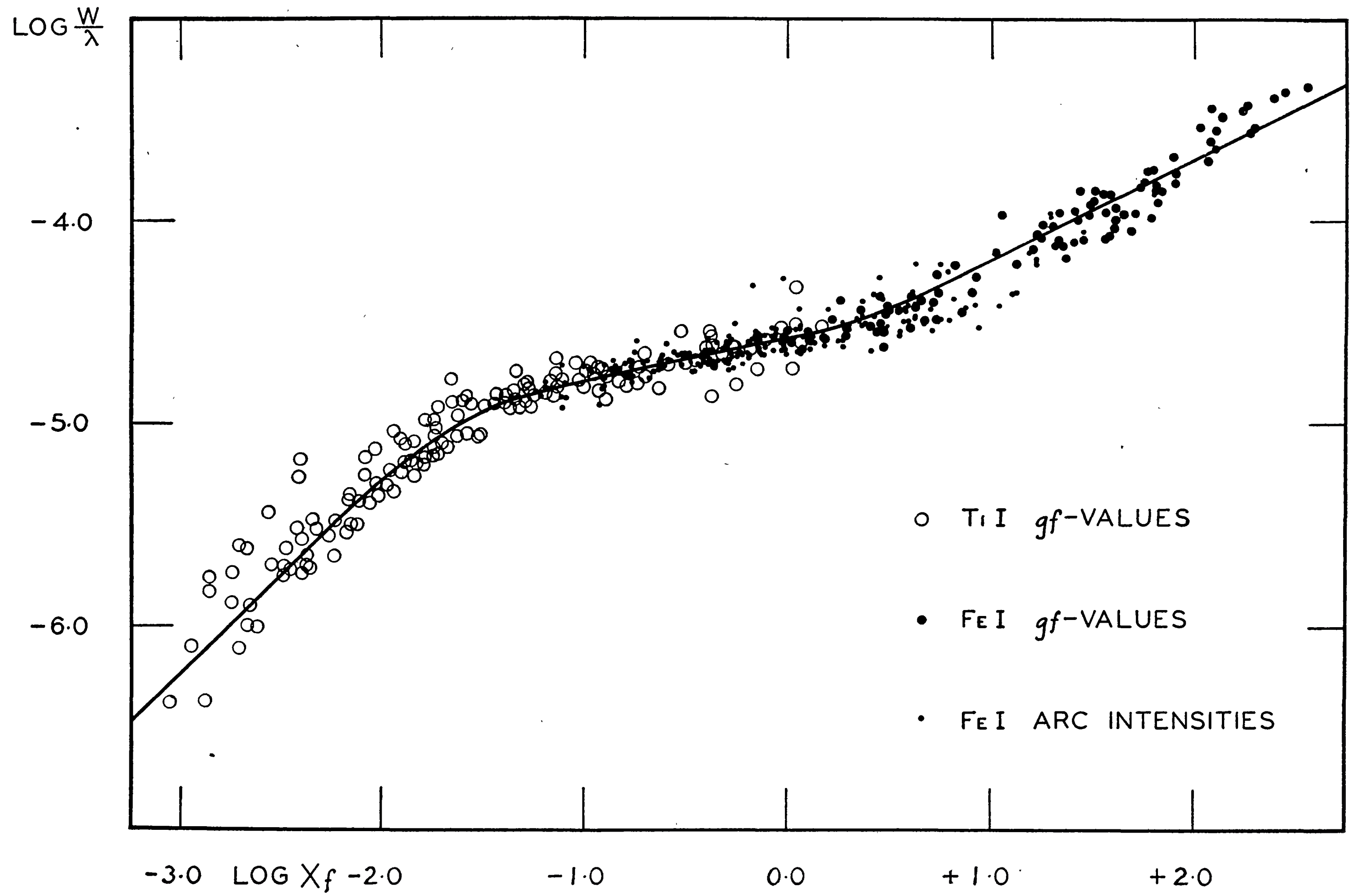

FIG. 2.—Curve of growth for the sun obtained from equivalent-width measurements (Allen and the *Utrecht Photometric Atlas*) and King's laboratory *gf*-values for lines of *Fe* I and *Ti* I. In drawing the mean curve the weights of individual points have been considered.

ing factor, Γ/ν, and the turbulent velocity, v. The best fit between empirical and theoretical curves is obtained by adding +1.81 to log X_f, and these values, together with the difference (Empirical − Theoretical) for a given value of log (W/λ) are given in columns 3, 4, and 5 and columns 8, 9, and 10 of Table 2. The empirical curve is definitely above the theoretical curve of best fit where the Doppler region changes to the transition por-

TABLE 1

MEAN VALUES OF OBSERVED POINTS ON SOLAR CURVE OF GROWTH

Ti I			*Ti* I			*Fe* I			*Fe* I		
log (W/λ)	log X_f	Wt.	log (W/λ)	log X_f	Wt.	log (W/λ)	log X_f	Wt.	log (W/λ)	log X_f	Wt.
−6.18	−2.92	6	−4.87	−1.33	53	−4.83	−1.13	8	−4.39	+0.80	32
5.99	2.68	17	4.81	−1.14	36	4.76	−0.90	12	4.07	1.28	48
5.63	2.50	15	4.77	−0.95	24	4.71	−0.70	31	3.94	1.61	33
5.58	2.35	17	4.78	−0.73	20	4.68	−0.43	36	3.79	1.84	25
5.44	2.15	28	4.67	−0.42	24	4.62	−0.19	48	3.56	2.10	24
5.25	1.95	43	4.65	−0.24	19	4.58	+0.03	35	3.49	2.27	10
5.12	1.76	44	−4.54	+0.06	15	4.50	+0.33	40	−3.36	+2.47	9
−4.96	−1.56	22				−4.45	+0.56	45			

TABLE 2

COMPARISON OF OBSERVED AND CALCULATED CURVES OF GROWTH FOR THE SUN*

LOG (W/λ)	LOG X_f	LOG X_0			LOG (W/λ)	LOG X_f	LOG X_0		
		Observed	Calculated	O−C			Observed	Calculated	O−C
−7.00	−3.77	−1.96	−1.96	0.00	−4.70	−0.58	+1.23	+1.17	+0.06
6.50	3.27	−1.46	−1.45	− .01	4.60	−0.08	1.73	1.59	.14
6.00	2.76	−0.95	−0.93	.02	4.50	+0.31	2.12	1.96	.16
5.50	2.24	−0.43	−0.37	.06	4.40	+0.58	2.39	2.26	.13
5.25	1.96	−0.15	−0.04	.11	4.25	+0.90	2.71	2.64	.07
5.00	1.60	+0.21	+0.37	.16	4.00	+1.41	3.22	3.19	+ .03
4.90	1.40	+0.41	+0.58	.17	−3.50	+2.41	+4.22	+4.22	0.00
−4.80	−1.05	+0.76	+0.83	−0.07					

* Note added April 5, 1944: In a recent letter Dr. Menzel has pointed out that line strengths based on $(Ss/\Sigma s)$ values ar more important in theoretical discussions than gf-values. Therefore, the scale for the X_f-axis given in columns 3 and 8 of Table may prove more useful than the data given in columns 2 and 7. As these values differ only by the additive constant (+1.81 the same notation may be used.

tion, and it is below the theoretical curve where the transition portion changes to th damping region. This curve corresponds to a turbulent velocity, v, of 0.9 km/sec an a damping factor, $\Gamma/\nu = 2.61 \times 10^{-6}$; Menzel and Rubenstein's solar curve[24] give corresponding values of 0.6 km/sec and 1.7×10^{-6}.

A comparison of the present curve of growth for the sun with those obtained previou ly shows that the scatter of individual points about the mean curve has been conside ably decreased by the use of accurate solar intensities combined with reliable laborato

[24] *Ap. J.*, **92**, 114, 1940.

intensities. Observations of weak lines ($0.004 < W < 0.030$ A) indicate that the theory for the Doppler portion of the curve agrees well with observation. The well-defined shoulder of the empirical curve between the Doppler and the transition portions indicates that at that point log (W/λ) is greater than provided by Baker's formula. The status of the damping portion of the curve is uncertain. King[25] observed that the strong iron lines fitted a curve with $T = 4800°$ better than one with $T = 4400°$, but he attributed the discrepancy to variations in the damping factor. As King and Minkowski[26] have pointed out, a study of the natural widths of these lines indicates that this factor is much smaller than that observed for lines in stellar spectra and that it should vary considerably from line to line. However, they conclude that differences in line width plus pressure effects do not account for the observed scatter about the curve of growth. The best straight line through the observed points on the damping portion of the empirical curve has a slope of 0.57 rather than the theoretical value of 0.50, but in the absence of satisfactory justification for the higher value, the theoretical slope has been adopted in this paper.

This communication presents the further results of a paper read before the American Astronomical Society in May, 1943. The author wishes to thank his colleagues at the Dominion Astrophysical Observatory for many helpful discussions, Dr. W. Petrie for the use of his calibration of King's arc intensities, and Lt. Commdr. D. H. Menzel, U.S.N.R., for his continued interest in the work.

[25] *Ap. J.*, 95, 78, 1942.

[26] *Ap. J.*, 95, 86, 1942.