By JORGE BOBONE and CARLOS G. TORRES
Plates taken with the astrographic telescope of the Argentina National Observatory

|  | U. T. | $\alpha$ | $\delta$ | Exp. | $p_{\alpha} \Delta$ | ${ }_{p}{ }_{\delta} \Delta$ | Reference Stars |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dec. | 1942 | (1942.0) |  | Comet Whipple 2 |  |  |  |
|  |  |  |  |  |
|  | 14. 15019 | $8^{\text {h }}$ OI $^{\text {m }} 29{ }^{\text {s }}$ :56 | +17 $7^{\circ} 29^{\prime} 4^{\prime \prime \prime}$. 0 |  |  |  | ${ }^{1} 5{ }^{\text {m }}$ | -0 ${ }^{\text {s }} 392$ | $-5 \prime$ " 85 | AG Berlin A 3174, 3175, 3197. |
|  | 15.14720 | $803 \quad 27.42$ | +175408.3 | 10 | -0.397 | -5.86 | AG Berlin A $3187,3197,3201$. |
|  | 19.15060 | 8 II 37.08 | +19 4113.8 | 10 | -0.381 | -6.06 | AG Berlin A 3227, 3257, 3271 ; B 3290, 3296, 3326. |
|  | 29.15612 | 83423.06 | +25 1745.8 | 5 | -0.354 | $-6.60$ | Yale 9 4582, 4584, 460I, 46I 4, 4629, 4642. |
| Comet Oterma $=$ Comet Stephan (1867 I) |  |  |  |  |  |  |  |
| Nov. | .14.11710 | $4^{\mathrm{h}} 09^{\mathrm{m}} 5^{2}{ }^{\text {s }} 15$ | $+2^{\circ} 56^{\prime} 34.18$ | $30^{\text {m }}$ | -0 ${ }^{\text {s }} 265$ | -4 ". 88 | AG Albany 1209, 1225, 1237. |
|  | 14.15900 | 40951.43 | + 25725.0 | 15 | -0.145 | $-4.93$ | AG Albany I210, 1218, 1241. |
|  | 28.13711 | 40539.88 | + 83442.6 | 20 | -0.087 | -5.6I | AG Leipzig II 1509, 1529, 1540. |
|  | 1.07592 | 40437.37 | + 95825.0 | 20 | -0.245 | $-5.63$ | AG Leipzig I 1203, 1211, 1217; II 1506, 1513, 1522. |
| Dec. | 5.06536 | 40315.94 | +115712.2 | 20 | -0.243 | $-5.83$ | AG Leipzig I 1100, 1195, 1200. |
|  | 12.09133 | 4 O1 25.00 | +1535 57.1 | 20 | -0.104 | -6.38 | AG Berlin A $1064,1077,1083$. |
|  | 29.11595 | $4 \quad 03$ OI. 44 | +2420 46.3 | 15 | +0.130 | $-7.17$ | Yale 9 2006; 10 I304, I309, I318, I322, I341. |
| Jan. | $\stackrel{1943}{8.05853}$ | $40936.60^{\text {(10 }}$ | +0) +284844.1 | 20 | +0.012 | $-7.62$ | Yale 9 2000, 2010, 2015, 2022, 2025, 2034. |

Plates Nos. I, 2, 5, 7 and io were taken and reductions made by J. Bobone, the remainder by C. G. Torres. Stars to which proper motion was applied are indicated by italics.

National Astronomical Observatory,
Córdoba, Argentina,
1943 March.

PHOTOGRAPHIC OBSERVATIONS OF THE COMET 1942a (WHIPPLE)
By JORGE BOBONE

| 1942 U. T. | $\alpha \quad$ (1942.0) |  | Exp. | $p_{\alpha}{ }^{\Delta}$ | ${ }^{0}{ }_{\delta}{ }^{\Delta}$ | Obs. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Oct. 2.03293 | $22^{\text {b }} 29^{\text {m }} 55{ }^{\text {s }} 66$ | $-83^{\circ}$ I $^{\prime} 5^{\prime \prime} .4$ | $10^{\text {m }}$ | - I ${ }^{\text {s }} 466$ | $+6^{\prime \prime} .47$ | D |
| 7.06690 | 215535.62 | -80 23 51.6 | 5 | +o. 299 | $+6.60$ | P |
| 10.00473 | 214520.21 | -78 41 00.4 | 5 | -0.496 | +6.32 | P |
| 30.02492 | 213021.50 | -67 39 39.6 | 5 | +o.440 | +4.80 | D |
| 30.03392 | $2 \mathrm{I} 302 \mathrm{I} \cdot 5 \mathrm{I}$ | -67 3923.0 | 5 | +0.509 | +4.65 | D |
| Dec. 2.0457 I | 21.5048 .94 | $\begin{array}{llll}-53 & 19 & 58.8\end{array}$ | ${ }^{1} 5$ | +0.682 | +o.76 | D |
| 15.04622 | 22 O1 57.71 | -48 5309.2 | 30 | +o.687 | -0.59 | D |
| 1943 U. T. | (1943.0) |  |  |  |  |  |
| Jan. 8.04796 | 222338.26 | $\begin{array}{llll}-42 & 02 & 10.4\end{array}$ | 14.5 | +o.669 | $-2.84$ | D |

The plates were taken with the 154 cm reflector of the Bosque Alegre Observatory, by Ricardo P. Platzeck (P) and Martín Dartayet (D).

National Astronomical Observatory, Córdoba, Argentina,
1943 March.

## ON THE CHOICE OF CONFIGURATIONS OF REFERENCE STARS IN LONG-RANGE ASTROMETRIC PROBLEMS

By PETER VAN DE KAMP
I. Introduction and Summary. In conventional photographic parallax determinations made with long-focus instruments, the material covers only a few years and, even if the central star has a large proper motion, it has always been satisfactory to use one single configuration of reference stars; one single dependence set and center have usually been found adequate. However, in investigations on stars of large proper motion, which are photo-
graphed for several decades, it has appeared desirable to introduce successive dependence sets and correspondingly successive dependence centers which are reduced to the scale, orientation and origin of an adopted standard frame, based on the configuration of the reference stars. The details of the latter procedure have been discussed in a recent article by the author. ${ }^{1}$

The purpose of the present note is to discuss in some
detail the choice of a satisfactory reference system for the case of a long focus astrometric problem involving an appreciable displacement of the central star because of proper motion. This leads to further considerations when the path of the central star reaches the same order of size as that of the reference system. Since the latter is limited by the size of the telescopic field and the photographic plate, any configuration has only a limited period of usefulness within which maximum efficiency is reached. In any long-term astrometric problem therefore, it is important to study carefully any contemplated choice of reference stars, so that one will not be faced with early obsolescence, but will have a well planned foundation for the present and possibly future configurations of reference stars.

Not only the size and shape of the configurations but also their stability deserves consideration. For a central star with considerable proper motion the effect of the separate reference stars on its reduced position changes with the time; thus secular effects due to magnitude, color, and proper motion are introduced. The first two should be kept at a minimum by limiting the range in magnitude and spectrum for the reference stars; the dispersion effects can be minimized by photography in the longer wavelengths. The spurious acceleration effect due to the proper motions of the reference stars can be corrected for afterwards, if the proper motions of the reference stars are known. ${ }^{2}$

The chief consequence of the central star's motion is the varying accuracy of its geometrical fixation due to its changed location within the configuration. It should be remembered that greatest astrometric accuracy is reached by having as small as possible a configuration of reference stars. In case of a limited problem such as a conventional parallax determination, nothing therefore is more desirable than a small configuration, with approximately equal dependences; the choice from any available stars will of course depend on the exposure time and limitations due to required magnitude compensation.

It is well to emphasize at this point the value of using double plates, i.e., pairs of reversed exposure sets as described in a recent article by Dr. Gustav Land. ${ }^{3}$ This procedure results in partial cancellation of the emulsion shifts for the relatively proximate exposure sets of the central star; it is especially desirable for larger configurations where this source of error is to a great extent responsible for the decreased positional accuracy of single sets of exposures.

For the general problem, the following considerations are presented:

For any investigation spread over a restricted time interval greatest accuracy is, of course, reached if the position of greatest dependence accuracy is about the middle of the interval (section 2).

In any case it is wise to provide for a possible temporal extension of the investigations. The transition to a reference system involving the inclusion of one extra star is most conveniently carried out about the time of vanish-
ing dependence value for that star. - The importance of a reference star in a long-range problem should be judged not only by its present dependence value but by the annual dependence variation as well. A star which always would have a negative dependence would generally be a liability to any configuration. But, on the other hand, a star may be a valuable asset in establishing a configuration even though at one end of its own period of usefulness the dependence would be small or even negative (section 3).

An illustration is given for the case of Luyten's star (section 4); formulae are given for the background transition involving the inclusion of one additional star (section 5).
2. Dependence sets and their accuracy. The secular changes of the dependences are caused by the proper motion of the central star. The annual parallax can be accounted for by considering the heliocentric dependences and their secular changes. Let the positions $\left(x_{o}, y_{o}\right)$ of the $n$ reference stars defining the standard frame of reference be given relative to their mean, i.e. $\left[x_{0}\right]=\left[y_{o}\right]=0$. For this standard frame and an initial heliocentric position $X_{o}, Y_{o}$ of the central star the dependences can be expressed by the following formulae:

$$
\begin{equation*}
D_{i}=f_{i} X_{o}+g_{i} Y_{o}+\mathrm{I} / n \cdots(i=\mathrm{I}, \cdots n) \tag{I}
\end{equation*}
$$

Here

$$
\begin{align*}
f_{i} & =\frac{x_{o i}\left[y_{o}^{2}\right]-y_{o i}\left[x_{o} y_{o}\right]}{\left[x_{o}^{2}\right]\left[y_{o}^{2}\right]-\left[x_{o} y_{o}\right]^{2}} \\
g_{i} & =\frac{y_{o i}\left[x_{o}^{2}\right]-x_{o i}\left[x_{o} y_{o}\right]}{\left[x_{o}^{2}\right]\left[y_{o}^{2}\right]-\left[x_{o} y_{o}\right]^{2}} \tag{2}
\end{align*}
$$

The annual dependence changes are given by

$$
\begin{equation*}
\Delta D_{i}=f_{i} \mu_{X}+g_{i} \mu_{Y} \tag{3}
\end{equation*}
$$

Here $\mu_{X}, \mu_{Y}$ are the rectangular components of the yearly proper motion of the central star.

We shall neglect the possible variation in intrinsic positional accuracy of each reference star with its location on the photographic plate. The geometrical accuracy of the reduced position of the central star thus depends on the distribution of the dependences for the reference stars. In case of a central star of appreciable proper motion these dependences change and result in a corresponding change in accuracy for the changing dependence center. In that case, the error squared, or inverse weight, of the position measured on the dependence background is proportional to

$$
\mathrm{I}+\left[D^{2}\right]
$$

Writing $D_{t}=D_{o}+t \cdot \Delta D$, this error function becomes

$$
\begin{equation*}
\mathrm{I}+\left[D_{o}^{2}\right]+2 t\left[D_{o} \cdot \Delta D\right]+t^{2}\left[\Delta D^{2}\right] \tag{4}
\end{equation*}
$$

reaching a minimum value for

$$
\begin{equation*}
t_{m}=\left[D_{o} \cdot \Delta D\right] /\left[D^{2}\right] \tag{5}
\end{equation*}
$$

Calling the dependences at this epoch $D_{m}$, the error function at any other time is given by

$$
\begin{equation*}
\mathrm{I}+\left[D_{m}^{2}\right]+\left(t-t_{m}\right)^{2}\left[\Delta D^{2}\right] \tag{6}
\end{equation*}
$$

It is clear, of course, that for any investigation spread over a restricted time interval, greatest accuracy is reached if the position of greatest dependence accuracy is reached about the middle of that interval. If the interval is not more than a few decades it is generally not difficult to find an appropriate configuration for which the dependences vary not too much over the interval and would in any case remain positive.

From the expressions (1) and (2) for the dependences it follows that the absolute minimum value of [ $D^{2}$ ] in the configuration ( $x_{o}, y_{o}$ ) exists for the origin defined by $\left[x_{o}\right]=\left[y_{o}\right]=0$, where all dependences equal $\mathrm{I} / n$. For any central star, therefore, to insure a satisfactorily small [ $D_{m}{ }^{2}$ ] it is important to choose a configuration whose origin will not lie too far off the path of the star.
As to the number of reference stars, it is a well-known fact that even for a central star at the origin, the accuracy does not increase much with the number of reference stars. This is illustrated for the case $n=3, \cdots 12$, in Table I. Considering the extra work involved, not much

|  | Inverse <br> Weight | Relative Weight . <br> I. 250 | Relative Error |
| :---: | :---: | :---: | :---: |
| $n$ | $\mathrm{r}+\left[D^{2}\right]$ | $\overline{\mathrm{I}+\left[D^{2}\right]}$ | $.895 \sqrt{\mathrm{I}+\left[D^{2}\right]}$ |
| 3 | I. 333 | 0.938 | 1.031 |
| 4 | I. 250 | I. 000 | 1.000 |
| 5 | I. 200 | I. 040 | . 981 |
| 6 | I. 167 | I. 07 I | . 967 |
| 7 | I. I43 | I. 094 | . 957 |
| 8 | I. 125 | I. III | . 948 |
| 9 | I. 111 | I. I25 | . 943 |
| 10 | I. 100 | I. 136 | . 939 |
| II | I.091 | I. 146 | . 935 |
| I2 | 1.083 | I. I54 | . 931 |
| $\infty$ | 1. 000 | 1. 250 | 895 |

accuracy is generally gained by using more than 4 reference stars.

Since $\mu_{X}, \mu_{Y}$ are fixed, the only parameters for studying the minimum value of $\left[\Delta D^{2}\right]$ are $f_{i}$ and $g_{i}$. From the expression (2) for these quantities, it appears that, other things being equal, they are inversely proportional to the linear extent of the configuration. In so far as the useful life of a configuration is determined by the size of the $\Delta D$, a large configuration is therefore preferable. But since a large configuration leads to larger errors inherent in the background (see section 1), this matter deserves careful exploration in each case.
3. Transition from one configuration to another. When planning measurements for a region which is to be observed for a long time, one therefore should make a survey of potential configurations. The proper motion of the central star should be sufficiently known, so that its future path can be traced with adequate accuracy. After preliminary consideration of magnitude compensation and available exposure time, a few possible configurations are likely to be found, all of them satisfying the condition that at some time in the future the path of the central star will pass close to the center of the configuration. Now, if the problem is definitely going to be a short term
one, it will probably be desirable to use a relatively small configuration with the corresponding advantage of high astrometric accuracy. The spurious proper motion and parallax terms ${ }^{2}$ would be relatively large, but they would not play any role in a short-term problem anyway. Even in this case, however, it would seem wise to provide for a possible extension toward a long-range problem. This could of course be done by starting with a larger "longrange" configuration, even though it may be relatively inaccurate now. An alternative approach is to make a transition from a smaller configuration to a larger one whenever future developments would seem to warrant such procedure.

As is well known, the proper motions of the reference stars impose an accelerated motion on either background as compared with a fixed reference system. ${ }^{2}$ Hence, any reduction from one background to the other will prove to be of an accelerated character. Since we want to lose as little accuracy as possible in such a transition, we shall for the present limit ourselves to transitions involving only the inclusion of one extra star. This would increase the size of the configuration, resulting in an overall decrease of $\Delta D$ and a correspondingly longer active life for the individual reference stars and hence for the configuration.


It is clear that the additional star might be of little or no value at present, as would be demonstrated by the small or even negative dependence value. The useful procedure would be to realize the potential future importance of such a star, but for reasons of economy to refrain from measuring this star until its dependence attains an appreciably positive value. By measuring and reducing a sufficient number of common backgrounds at that time, any reductions of earlier plates to the new frame could be evaluated with adequate accuracy from the quadratic run of difference in background; the epoch of identical dependences, at which this difference vanishes, can be
known with adequate accuracy. With enough foresight, knowledge of this function can, however, be appreciably strengthened by measuring and reducing a few common backgrounds for which the new reference star still has a large negative dependence. Some further considerations are given in section 5 .
4. Illustration for Luyten's star ( $B D+5^{\circ} \mathrm{I} 668$ ). Luyten's star has a large proper motion, $3^{\prime \prime \prime} 76$ in $171^{\circ}$. The accompanying Figure I shows five reference stars which after some elimination were found eligible. For a shortterm problem the configuration (123) is adequate, while for long-term problems configurations (1234) or (I2345) are preferable.

The relevant data for the three corresponding standard frames are given in Table II (units of $x_{o}, y_{o}, X_{o}, \dot{Y}_{o}, \mu_{X}$, $\mu_{Y}$ are mm ). From Table III it appears that in the (I234)

$$
\begin{aligned}
& \text { TABLE II. LUYTEN'S STAR. STANDARD FRAMES WITH 3, 4, AND } 5 \\
& \text { REFERENCE STARS. HELIOCENTRIC DEPENDENCES FOR I940.0 } \\
& \text { AND THEIR ANNUAL VARIATIONS } ; \mu_{X}=+.030 \mathrm{~mm} \text {, } \\
& \mu_{Y}=-.195 \mathrm{~mm}
\end{aligned}
$$


configuration the dependence of star 4 became positive in 1914. In the (12345) configuration, the change of sign occurred in 1890, but the dependence of star 5 is now negative and will not become positive until 1955 . In

Table IV are given data relevant to the accuracy of the dependence background for the different configurations. The data contained in Tables II, III, and IV are illustrated in Figures 1, 2, and 3. In Figure 2 the early obsolescence of the 3 -star configuration is demonstrated by the precipitous dependence courses for stars I and 2. The heavy lines represent the 4 -star dependence courses between 1914 and 1955, which are used at present.


Figure 2. Luyten's Star; heliocentric dependences at different epochs for configurations of 3,4 , and 5 stars.


Figure 3. Luyten's Star; inverse weight, $1+\left[D^{2}\right]$, for configurations of $3(-), 4(-)$, and $5(-$ - - $)$ stars.

Further leveling off and subsequently longer life obtains for the full drawn 5 -star dependence courses beyond 1955 .

Since, generally, the dependences for all the reference stars of a set change with the time, the interval of usefulness of different reference stars may start or stop at widely different epochs. With this fact in mind the problem of the validity of the configuration should be approached.

Measurements for Luyten's star were started in 1937. At that time the accuracy of the (I234) configuration was hardly superior to that of the (I23) configuration. Looking into the future, however, it appears that the accuracy of (I23) reaches a maximum in 1946 and then decreases while the (1234) accuracy will increase till 2007. Since the dependence of star 4 was already +.05 in 1937 (and increases +.00233 per year) there seemed to be little reason for preferring the (123) configuration, if a continuation of observations appears now at all likely.

The only alternative remains the (12345) configuration, which reaches maximum accuracy in 2055 and would thus be useful for about a century longer than the (I234) configuration. However, the dependence of star 5 in this configuration will not become positive till 1955, its amount in 1937 being -. 03 and not reaching the value +.05 until about 1980 . Even if we should decide now to use the (I2345) configuration, it would be a waste to measure star 5, since its dependence is just about to change sign. This simply means that the two dependence backgrounds, (I234) and (I2345), are almost identical, differing only by a quadratic term which could easily be evaluated at some future time when the adoption of the (12345) system should be wanted. It was decided, therefore, to remain on the (1234) system; after 1955 there will be ample time to consider the most desirable epoch at which to shift to the (I2345) reference system.
5. Formulae for background transition. Although the problem is still very much of an academic nature now, we shall give the formulae for the transition from one background to another in case one star is added to the configuration. To be specific we shall consider the addition of a fifth reference star to a four-star configuration $(I V)$. For the latter the dependence reduction ${ }^{1}$ is:

$$
\begin{align*}
X_{I V} & =X^{\prime}-\left[D x^{\prime}\right]_{I V}+\left[D_{o} x_{o}\right]_{I V} \\
Y_{I V} & =Y^{\prime}-\left[D y^{\prime}\right]_{I V}+\left[D_{o} y_{o}\right]_{I V} \tag{7}
\end{align*}
$$

A corresponding formula exists for the five-star configuration ( $V$ ):

$$
\begin{gather*}
X_{V}=X^{\prime}-\left[D x^{\prime}\right]_{V}+\left[D_{o} x_{o}\right]_{V},  \tag{8}\\
Y_{V}=Y^{\prime}-\left[D y^{\prime}\right]_{V}+\left[D_{o} y_{o}\right]_{V} .
\end{gather*}
$$

In (7) and (8), $X^{\prime} Y^{\prime}$ and ( $x^{\prime}, y^{\prime}$ ) refer to the measured positions of central and reference stars, while the relative coordinates $\left(x_{o}, y_{o}\right)$ refer to the adopted positions of the respective standard frames. The transition from the fourstar to the five-star background is thus given by:
$X_{V}-X_{I V}=\left[\left(D_{I V}-D_{V}\right) x^{\prime}\right]+\left[D_{o} x_{o}\right]_{V}-\left[D_{o} x_{o}\right]_{I V}$,
$Y_{V}-Y_{I V}=\left[\left(D_{I V}-D_{V}\right) y^{\prime}\right]+\left[D_{o} y_{o}\right]_{V}-\left[D_{o} y_{o}\right]_{I V}$,
or, in the more conventional form used in the plate reductions:

$$
\begin{align*}
X_{V}-X_{I V} & =\xi_{V}-\xi_{I V}+\left[D_{o} x_{o}\right]_{V}-\left[D_{o} x_{o}\right]_{I V}  \tag{io}\\
Y_{V}-Y_{I V} & =\eta_{V}-\eta_{I V}+\left[D_{o} y_{o}\right]_{V}-\left[D_{o} y_{o}\right]_{I V} .
\end{align*}
$$

Here $\xi_{I V}, \eta_{I V}$ and $\xi_{V} ; \eta_{V}$ are the "plate solutions" for the case of the four- and five-star configurations respectively.

We shall now write the measured positions of the reference stars as follows:

$$
\begin{align*}
& x^{\prime}=x+\mu_{x} t,  \tag{II}\\
& y^{\prime}=y+\mu_{y} t .
\end{align*}
$$

Here ( $\mu_{x}, \mu_{y}$ ) are the proper motions of the reference stars; $t$ is the time which will be counted from the time that the dependence of the fifth star is zero, i.e. from the epoch at which the two dependence backgrounds are identical. Since the differences in the plate-solutions in (Io) or of the corresponding bracketed quantities in (9) are not affected by plate-constants, they can now be expressed as follows:

$$
\begin{align*}
& \xi_{V}-\xi_{I V}=\left[\left(D_{I V}-D_{V}\right)\left(x+\mu_{x} t\right)\right],  \tag{12}\\
& \eta_{V}-\eta_{I V}=\left[\left(D_{I V}-D_{V}\right)\left(y+\mu_{y} t\right)\right] .
\end{align*}
$$

The heliocentric dependences $D_{I V}$ and $D_{V}$ are linear functions of the time, their differences are also, and may be written:

$$
\begin{equation*}
D_{I V}-D_{V}=t \cdot \Delta D_{I V-V} \tag{I3}
\end{equation*}
$$

Partial substitution of (13) in (12) leads to:

$$
\begin{align*}
& \xi_{V}-\xi_{I V}=\left[\left(D_{I V}-D_{V}\right) x\right]+t^{2}\left[\Delta D_{I V-V} \cdot \mu_{x}\right]  \tag{14}\\
& \eta_{V}-\eta_{I V}=\left[\left(D_{I V}-D_{V}\right) y\right]+t^{2}\left[\Delta D_{I V-V} \cdot \mu_{y}\right]
\end{align*}
$$

The small errors in the quadratic term caused by rounding off the dependences are negligible and non-systematic.

Now the interval over which one and the same set of dependences is used, should always be so chosen that the quadratic term in (14) is below a negligible level. ${ }^{1}$ Thus, for any two corresponding dependence sets, the transition (14) may be considered a constant. For a series of corresponding successive dependence sets the second term is proportional to $t^{2}$, while the first one may be written:

$$
\begin{align*}
& {\left[\left(D_{I V}-D_{V}\right) x_{o}\right]+\left[\left(D_{I V}-D_{V}\right)\left(x-x_{o}\right)\right],}  \tag{15}\\
& {\left[\left(D_{I V}-D_{V}\right) y_{o}\right]+\left[\left(D_{I V}-D_{V}\right)\left(y-y_{o}\right)\right] .}
\end{align*}
$$

Or, since the right-hand portions of these expressions are very small, (14) may be written as:

$$
\begin{align*}
\xi_{V}-\xi_{I V}= & {\left[\left(D_{I V}-D_{V}\right) x_{o}\right] }  \tag{I6}\\
& +t\left[\Delta D_{I V-V}\left(x-x_{o}\right)\right]+t^{2}\left[\Delta D_{I V-V} \cdot \mu_{x}\right] \\
\eta_{V}-\eta_{I V}= & {\left[\left(D_{I V}-D_{V}\right) y_{o}\right] } \\
& +t\left[\Delta D_{I V-V}\left(y-y_{o}\right)\right]+t^{2}\left[\Delta D_{I V-V} \cdot \mu_{y}\right],
\end{align*}
$$

whence:

$$
\begin{align*}
& \xi_{V}-\xi_{I V}-\left[\left(D_{I V}-D_{V}\right) x_{o}\right]=A_{x} \cdot t+B_{x} \cdot t^{2},  \tag{17}\\
& \eta_{V}-\eta_{I V}-\left[\left(D_{I V}-D_{V}\right) y_{o}\right]=A_{y} \cdot t+B_{y} \cdot t^{2} .
\end{align*}
$$

The background transition thus involves terms varying with the time in a quadratic way, and vanishing for the epoch of identical background.

For any particular epoch the transition constant can be determined through a number of common reductions; from (12) it follows that the accuracy of the difference of two corresponding plate solutions is given by the probable error:

$$
\begin{equation*}
\epsilon \sqrt{\left[\left(D_{I V}-D_{V}\right)^{2}\right]}, \tag{I8}
\end{equation*}
$$

where $\epsilon$ is the probable error of a measured position $x^{\prime}$ or $y^{\prime}$. For all practical purposes (18) may be written as:

$$
t \in \sqrt{\left[\left(\Delta D_{I V-V}\right)^{2}\right]},
$$

and thus it is seen that the probable error of the measured transition is proportional to the time counted from the epoch of identical backgrounds.

REFERENCES
I. A. J. 49, I49, 1942 .
2. A. J. 44, 73, 1935.
3. A. J. 50, 5 I, 1942.

Sproul Observatory,
Swarthmore College,
Swarthmore, Pa.,
Swarthmore
I 943 May.

THE DOUBLE STAR Cor 193
1900: $16^{\mathrm{h}} 01.1^{\mathrm{m}},-37^{\circ} 46^{\prime}$
By HAROLD L. ALDEN

The following list of measures of this double star (HD i44503; 7.7, G5) has been supplied by Dr. van den Bos who thought that the motion was such as to make a trigonometric determination of the parallax desirable.

| Date | p | d | Nights | Telescope | Observer |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1875.6 | $72^{\circ} \cdot 7$ | 9 " 1 I | I | Mer. Cir. | Cor. |
| 1920.30 | 80.5 | 6.48 | 4 | 17 in . | Tapia |
| 1929.26 | 83.1 | 5.68 | 4 | 26 | B |
| 1930.54 | 83.2 | 5.5I | 2 | 24 ptg . | V |
| 1931.25 | 83.4 | 5.44 | 2 | 24 ptg . | V |
| 1933.55 | 83.6 | 5.10 | 5 | 15 | Wallenquist |
| 1936.33 | 84.7 | 4.90 | 3 | 24 ptg . | V |
| 1936.58 | 85.6 | 4.88 | 4 | 26 | B |
| 1941. 59 | 87.9 | 4.38 | I | 26 | B |

To these should be added the measures by the writer of 68 exposures on 20 plates taken in the years 1939-1942 with the Yale telescope. The position angle is referred to the equator of 1941.56.

$$
\begin{aligned}
& p=87.09+0.36(t-1940.74) \\
& \pm .04 \pm .04 \\
& \begin{aligned}
d= & 4 \text { ". } 483-\quad \text { о".112 }(t-1940.74) \\
& \pm .005
\end{aligned}
\end{aligned}
$$

The probable errors are given under the quantities to which they refer.

The hypothetical parallax computed from these figures or from the measures listed is of the order of 0 ". 20 . The
components are nearly equal, each being of visual magnitude 8.5. The hypothetical parallax gives an absolute magnitude of io.o, which is unusually faint for stars of spectral type G5. This leads to the suspicion that the system is purely optical and that the relative motion is the result of differential proper motion. This suspicion is confirmed by the small trigonometric parallaxes derived from the 20 plates taken here. The results are:

|  | SP Component NF |  |
| :--- | :--- | :--- |
| Rel. parallax | $+0.007 \pm .007$ | $-0 " .004 \pm .005$ |
| Rel. P. M. in R.A. | $+0.061 \pm .006$ | $-0.048 \pm .005$ |
| Rel. P. M. in Decl. | $-0.035 \pm .012$ | $-0.046 \pm .012$ |
| P. E. I | $\pm 0.026$ | $\pm 0.020$ |

The relative proper motion in declination was derived from one pair of plates having an interval of 2.93 years, the probable error being estimated.
A plot of the observations indicates that the companion is moving in a straight line relative to the primary in position angle $247^{\circ}$ and that the distance will be a minimum about the year 1980 when the separation will be 1 " 6 in position angle $156^{\circ}$.
Yale University Observatory,
Southern Station,
Johannesburg, S. Africa,
1942 December.

$$
\begin{aligned}
& \text { THE ORBIT OF WOLF } 358 \\
& \left(1900:-10 \mathrm{~h} 45.8 \mathrm{~m},+7^{\circ} 22^{\prime}\right) \\
& \text { By HAROLD L. ALDEN }
\end{aligned}
$$

This star of large proper motion and parallax (Cin. 20, 591) was found at the Van Vleck Observatory ${ }^{1}$ to give residuals with a variation having a period of 3.75 years. Twenty-seven plates with seventy-seven exposures have been taken on this star with the Yale telescope. Three of these were in 1930, and the remainder in the years 1939 to 1943. The latter cover a complete period and, since the early plates provide a good determination of
the proper motion, orbital motion should be detected if present.

The plates were measured first in right ascension in order to establish the reality of the periodicity before undertaking the measurement in declination. The writer has found so many cases where residuals could be represented by periodic terms, most of which have not been confirmed by subsequent observations, that he has grown

