ON STELLAR EVOLUTION By WILLIAM DUNCAN MACMILLAN

There are two questions at the present time which are of fundamental interest to astronomers and physicists. The first question is, What becomes of the enormous flood of energy which is poured forth so lavishly by the sun and by the stars? Does it travel unendingly through the depths of space until it strikes some material object, or does it not?

The second question is, What is the source of the enormous subatomic energies which have been revealed in recent years by the radioactive elements, and which by implication exist in all of the other elements?

In the first question we ask, What becomes of this energy? In the second, Where does this energy come from? Surely such a situation is not so embarrassing as it would be if we had but one of these questions, for an infinite source or an infinite sinkhole of energy is scarcely to be thought of. The two questions seem mutually to answer one another, and it seems reasonable to conjecture that the energy which disappears from the sun and stars into space reappears sooner or later in the subatomic energies of the atoms.

One may suppose that the physical universe is finite or that it is infinite, for it is not possible to verify either supposition. The idea that the physical universe is finite is doubtless repugnant to most minds that have dwelt upon the subject, and we therefore reject this supposition. The distribution of matter in space may be roughly uniform or it may be distinctly non-uniform. Again we are at liberty to make either supposition, for neither can be verified. But if we assume the universe to be infinite, then unless the distribution of stars is non-uniform of a special type the entire sky should glow with a brightness equal to that of the sun's disk. Certainly this would be true if radiant energy is not extinguished in its course through space.

It is quite possible to distribute infinitely many stars in such a manner that the total quantity of light received from them should be anything we please. For example, imagine a series of concentric spheres of radius 1, 2, 3, \ldots n, \ldots ; and on the surface of each sphere is placed a number of stars, the number being equal to the integral part of the square root of the radius of the sphere. If the amount of light received from the star on the first sphere be taken as unity, then the entire amount of light received from all of the stars would be less than $\mathbf{I} + \frac{\mathbf{I}}{2^{3/2}} + \frac{\mathbf{I}}{3^{3/2}} + \frac{\mathbf{I}}{4^{3/2}} + \frac{\mathbf{I}}{5^{3/2}} + \ldots$ which is finite, but the number of stars in the system would be infinite. In any such distribution, however, the average stellar density approaches zero as the distance becomes sufficiently great. While such distributions of stars are possible, they seem so highly im-

such distributions of stars are possible, they seem so highly improbable that we reject them and seek some other explanation of the blackness of the night sky. There is no recourse save in the hypothesis that radiant energy

is extinguished in its course through space. If we assume that there is a uniform distribution of stars and that the stars are all alike, there should be four times as many stars in any given magnitude as in the magnitude next brighter. The actual star-counts, however, show that while this ratio is maintained between stars of magnitude one and magnitude two it falls off steadily until between magnitudes sixteen and seventeen the ratio is only 1.8 instead of 4. Is the decline in the number of stars due to the extinction of light in traversing these enormous distances? It is a simple matter to assume that a certain percentage of radiant energy is lost in traveling through space and to test the hypothesis by an appeal to the star counts. Obviously the stars do not all emit the same amount of light; that is, they are not all of the same absolute brightness. Thus the star AOe(N) 17,415 is only 0.004 times as bright as the sun, while Canopus cannot be less than 10,000 times as bright as the sun (absolute magnitudes, of course, being understood). Thus between the faintest known star and the brightest known star there is a ratio of 2,500,000 or sixteen magnitudes (absolute). Assuming that the stars are distributed over fourteen magnitudes (absolute) in accordance with the law of probability, and that I per cent of light

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is extinguished in traveling 4.11 parsecs (13.6 light-years), the following table has been computed showing the number of stars of the various relative magnitudes on the hypothesis of uniform distribution of the stars in space. The actual star-counts of Chapman and Melotte of the Royal Observatory at Greenwich are given for comparison.

Mag.	Star-Counts	Computed	Mag.	Star-Counts	Computed
6	2,026	2,201	12	961,000	960,200
7	7,095	7,135	13	2,023,000	2,080,000
8	22,550	21,770	14	3,964,000	4,219,000
9	65,040	62,230	15	7,824,000	8,034,000
o	172,400	166,200	16	14,040,000	14,420,000
. I	426,200	413,400	17	25,390,000	24,510,000

Certainly there is nothing in these figures to forbid us from supposing that the blackness of the sky is due to the extinction of light in its journey through space; and the amount of the loss (1 per cent in 13.6 years) does not seem excessive.

But what becomes of the energy which is lost? Is it permissible to suppose that the light is intercepted by dark material scattered through space? It is clear that the effectiveness of dark material in cutting off light is increased by supposing it in a finely divided state. If it is supposed that the dust of space consists of particles one one-hundredth of an inch in diameter it is found that one such particle to every 560 cubic miles of space would be sufficient to account for the 1 per cent of loss mentioned above. This does not seem to be an excessive amount of dust particles, and yet a continuation of the computation shows that in the 40 cubic parsecs which, according to the foregoing figures, is the sun's share of space, there is $6\frac{3}{4}$ times as much material as there is in the sun itself, and if the particles average one-tenth of an inch in diameter there is 67.5 times as much material as in the sun.

It may indeed be true that such dark material exists in space, but nevertheless it cannot account for the blackness of the sky, because the energy which it intercepts is either retained or radiated. If it is radiated, then there is no change in the total amount of radiation; at most merely a change of wave-length, since the

amount radiated is the same as the amount intercepted. So far as the total quantity of energy is concerned the result is the same as though the dark material were transparent. If the energy is retained, then the dark material would eventually become hot and would itself be bright. One concludes, therefore, that dark material in space cannot account for the blackness of the sky.

The accepted notion that radiant energy suffers no loss in transmission through a dust-free ether is not analogous to other physical processes, for in the physical world "perfection" does not seem to be attained. Perfection is an intellectual ideal, comfortable only so long as it represents the known facts with an approximation sufficient for our purposes. If we confine ourselves to a sufficiently small portion of the earth's surface we may be well satisfied with the hypothesis that the earth's surface is a plane, for the facts encountered are in close agreement with our hypothesis; but in a larger field of operations the curvature of the earth's surface is thrust upon us and cannot be ignored. So with the transmission of radiant energy it may be quite accurate enough to assume that there is no loss in such distances as are encountered in the solar system, but appreciably wrong when the distances encountered are of interstellar dimensions. According to Kapteyn the average distance of the first magnitude stars is 75 light-years. We have a right to be cautious in extending our hypothesis of "perfection" in the transmission of radiant energy into regions in which 75 lightvears is the unit of distance.

If dark material seems inadequate to diminish the total amount of radiation, we may have recourse to the absorption of energy in the ether. But the energy cannot be absorbed without doing work, and in casting about for some sort of work which this lost energy might do there occurs the possibility that it is here that the foundations of the atoms are laid, and perhaps also the completed structure.

Let us assume that absorption does occur and attempt to construct a model to illustrate how the kinetic energy of the ether-waves might be converted into the potential energy of an organized system.^I

¹ It is not essential, perhaps, to suppose that there is an ether. Some other process would answer our purpose; but it seems preferable to use the current concepts of physics.

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Imagine a number of spheres floating on the surface of the ocean. Imagine further that on these spheres there are springs, and that at the bottom of each spring there is a hook. As these spheres are tossed about by the waves there will be frequent collisions, followed in general by an immediate separation. Occasionally, however, two spheres will collide in such a way that when the springs are compressed the hooks are engaged, and separation does not follow. The two spheres are locked in tight embrace, and we have the beginning of an organized system. The energy of the compressed springs was absorbed from the energy of the ocean waves, though the amount of energy absorbed was perhaps relatively small. The two spheres thus joined would, in the course of time, unite with other spheres, and thus an organized system would be built up and the internal energy of the system would have been derived from the ocean waves. It is not necessary, indeed, to dwell upon the details of such a process. Through the agency of chlorophyll it is known that the radiant energy of the sun is absorbed and locked up in the organized systems of the vegetable world, thoughthe mechanical details of the process are quite unknown. In a manner analogous to the organic molecule, and by a process the details of which are quite unknown, we may suppose that the ordinary atom comes into being and that the familiar properties of inertia and gravitation are due to the energies locked up within. Disrupt the atom and set its energies completely free and the properties of mass and gravitation at once disappear.

Important consequences follow the admission that atoms are built up in this manner. It would follow that space contains much material of atomic or even molecular dimensions, and that regions long undisturbed by stellar objects would tend to become more or less crowded with atoms and molecules on account of the ceaseless passage of radiant energy through it. In this manner we see the genesis of a nebula with its enormous gravitational and subatomic energies. A sufficiently large mass in its journey through space would gather in this atomic and molecular material and feed upon its substance and energies. It would be a nucleus around which material would gather. If this nucleus were relatively small and dark, such as the earth, its growth would be slow; the

subatomic energies would persist as subatomic energies, and the mass would increase. In the course of time the internal pressure, density, and temperature would increase, and one can imagine that a critical situation would eventually be reached in which the subatomic energies can no longer wholly persist as subatomic. The atoms begin to break down and give up their stores of energy. In the event of a complete dissolution of the atom we would expect the complete disappearance of its mass and a complete restitution of the energy by which it was organized. If the dissolution were but partial we would have the familiar radioactive phenomena and a partial restitution of the subatomic energies. The energies thus released would raise the temperature of the nucleus and presumably hasten the process of disintegration. The density would decrease until, if the mass were large enough, the increased molecular energies would convert the once solid nucleus into a gaseous sphere. If the mass continued to grow after a completely gaseous state had been reached, the increased gravitational pressure would cause the density again to increase, and this increase with a growing mass would continue until eventually again a critical state would be reached of heat and pressure, and the release of subatomic energies would be so great that the gaseous mass would begin to glow. A further increase of mass would again hasten the process of dissolution accompanied by a rise in temperature and a second decrease in density, and the process could continue, as far as we can see, until the tenuity of a nebula was attained. If the various bodies in our own solar system, with whose masses and densities we are familiar, be arranged according to their masses, it is found that they do conform to these ideas, as is shown in the diagram (Fig. 1). Thus all of the planets and satellites which are smaller than the earth are in a solid state and their densities increase with an increase of mass. Somewhere between the mass of the earth and the mass of Uranus, which is fourteen times the mass of the earth, there would exist a mass of maximum density beyond which a solid mass cannot persist as wholly solid, and there begins a transitional state between the solid and the wholly gaseous condition, and in this transitional state we find the planets Uranus and Neptune. Saturn, with a mass equal to 94 times the mass of the earth, seems to

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have attained a wholly gaseous condition and has the smallest density of any object in the solar system. The mass of Jupiter is 3.3 times the mass of Saturn and its density is about twice that of Saturn. There are no bodies in our solar system intermediate between Jupiter and the sun, but it is not difficult to imagine that somewhere in between the increased mass and increased density would again produce an internal condition in which the production of heat would be so great that the mass would begin to glow and that relief from this state of excessive energy would be found in a decreased density. Thus the sun, which is in the very midst of stellar conditions, has a density but little in excess of that of Jupiter notwithstanding its enormous mass.

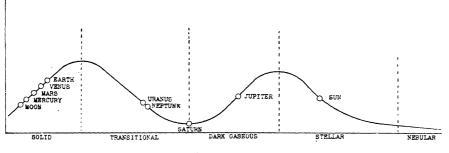


FIG. 1.—Density as a function of mass

When the stellar condition had been reached by a growing gaseous mass, the radiation of its energies into space would afford relief to the imprisoned atoms, tending to check the disintegrating process. Eventually there would be an equilibrium between the energies furnished by the process of dissolution and the energies expended in radiation and gaseous expansion against gravitation. If the process of gathering up atomic and other material from space were discontinued, the mass of the star would diminish, its volume would shrink, and its density would increase. The temperature eventually would fall and the critical state would be passed again, but this time in the direction of a return to the pre-star state. There would be relief from the excessive pressure and temperature which had brought about the release of subatomic energies, and a return to the dark state would be possible.

It is not necessary, however, to suppose that the ingathering process is stopped. If it continued at a suitable constant rate an equilibrium would be attained between the income and outgo of energy, the energies of radiation would be continued, the mass would remain constant, and it would endure as a star forever. If in its wanderings the star passed through a region unusually rich in material, its mass again would grow and its temperature would rise until it attained the white brilliancy of star of spectral type A or B or in the extreme case pass into the nebular state, in which the internal energies are gravitational-potential rather than the kinetic energies of heat. If it is assumed, as is natural, that in such a vigorous process of dissolution there would be a large residue of hydrogen and helium, we can account for the peculiar character of the spectra of stars of these types, for, owing to the lightness of these gases, they would rise to the surface and form an extensive envelope surrounding the brilliant star itself.

In this connection W. W. Campbell has made the following observations:¹

The class B stars and the stars containing bright lines are where the planetary and irregular nebulae exist. Going further into detail: wherever there is a great nebulous region either in or near, or outside of the Milky Way you will find the class B and earlier types of stars abnormally plentiful; and the chances are fairly strong that some of the stellar spectra will contain bright lines. This is true of great regions in the Milky Way; it is true of the Orion and Pleiades regions, which we see at some distance outside of the Milky Way structure, though they are doubtless within our system. If you see a wisp of nebulosity near a bright star, look up the star's spectrum and you will probably find it an early class B, as in the case of Gamma Cassiopeiae, a secondmagnitude star, with nebulous structure near it whose spectrum contains both dark and bright lines of hydrogen and helium. If you see an isolated bright star enmeshed in an isolated patch of nebulosity, such as the one shown in Fig. 37, and the books say the star (BD-10°4713) is yellow, or of class G, communicate your suspicions that the books are mistaken about the star's spectrum to Professor Pickering, and he will probably reply that the star is in reality a very blue one of early class B. That is what happened a fortnight ago about this particular nebula and the star near its apparent center. If you find a red or yellow star of normal type do not look for a nebula in apparent contact with it. Nebulae and red stars do not coexist. You will find about

¹ "Address of the Retiring President of the American Association for the Advancement of Science," *Science*, **45**, 545, 1917.

the same number of red stars in the Milky Way that are visible in similar areas far from the Milky Way. You will find an occasional red star in the region of the Orion nebula and of other large nebulae, but the red stars will not appear there in greater numbers than their approximately uniform distribution over the sky requires.

The connection between the nebulae and the bright line stars and between nebulae and the early class B stars is close, both as to their types of spectra and as to their geometric distribution.

Just as the kinetic theory of gases shows that there is a lower limit to the mass of a planet which can retain an atmosphere, so the present considerations suggest a lower limit of mass to a star which can emit light, and possibly also an upper limit to the mass of a star beyond which the star passes into a nebula. An exact correlation between the mass of a star and the intensity of its radiation would be expected only for those stars in which there was equilibrium between the energies released and the energies radiated. A star growing in mass with relative rapidity might lag in temperature, owing to a possible time element in the release of subatomic energies; and likewise a star decreasing in mass might remain for a long period relatively too hot. But on the whole one would expect an increase in temperature with an increase of mass until the kinetic energy of the molecules became so great that the star tended to pass from the gaseous to the nebulous stage. This would mean a decline in internal pressure and in radiation, although the internal energies, increasingly of the potential form, were exceedingly The increase in the mean free path of the molecules and great. the decline in internal pressure would tend to check the release of subatomic energy, and in the extreme nebular state this release may virtually cease. In this manner one is led to imagine a maximum mass beyond which a star, as such, cannot exist. If nebular radiation be left out of consideration, a maximum of stellar mass would imply that there exists a maximum of stellar radiation. Although different stars vary enormously in the amount of their radiations their variations in mass are not excessive, at least if we may judge from the few masses which are definitely known.

If in its early stellar stage a star is of red color, one would expect to find a class of red stars of relatively small mass, viz., those masses which have been slowly growing toward starhood. On the other

hand, stars which are condensing from a nebula of large mass would present a class of red stars of large mass. The kinetic theory of gases would lead us to doubt the possibility of a nebula of small mass, or a nebula of very low density, even though the mass be large, ever condensing into a star through its own gravitational attraction. Thus, if all the mass in the solar system were a spherical nebula 10 times as large as the orbit of Neptune, its velocity of escape would be less than the velocity of escape on the moon, and it is well known that the moon cannot retain an atmosphere. It would seem that such a nebula would dissipate rather than concentrate. A third class of red stars would be those which were approaching extinction or passing into the dark gaseous state. If the main source of stellar radiation is the subatomic energy,¹ and if these energies are completely given up so that the atomic mass disappears, then one would expect those stars which are approaching extinction to be of small mass. If the various chemical elements have different critical conditions of temperature and pressure for the release of their subatomic energies, the mass of a fading star would depend upon its chemical constitution, and so also would the mass of a young star. While this variation of composition would permit a variation in the mass of such stars, on the whole one would expect them to be small and of high density. One would expect, further, red stars of small mass to show considerable variability in their luminosity, owing to the cataclysmic nature of the process of changing from one physical state to another. These anticipations with respect to the red stars are quite in harmony with our present knowledge of this class of stars.

It is natural to suppose that atoms which are formed by the flow of energy through space would have little or no velocity at the time of their formation, and that the recently formed, irregular nebulae would have low velocities. On the other hand, nuclei of stellar types which have long been of stationary or of decreasing mass would have relatively high velocities, owing to the differential

^r According to the hypothesis of the present paper the energy, or heat, obtained from gravitational contraction is merely energy which has been absorbed from the star itself on some previous occasion during a process of expansion against gravitation. In the long run no energy is obtained from this source, though it serves admirably as a reservoir of energy which can be drawn upon during times of famine.

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gravitation of all the stars exerted over enormous periods of time. During the process of its growth, however, a star would be increasing its mass without increasing its momentum, since the momentum of the added material would be approximately zero, and therefore its velocity would be decreasing. If the surmise that a growing mass means an increase in the rate of the release of subatomic energy is correct, then the growing star would push its way through the various spectral types toward class B with an ever-decreasing speed, which is quite in harmony with our knowledge that stars of class B have low velocities and that higher velocities are associated with the stars of deeper color. It harmonizes also with the knowledge that the stars of class B are on the whole the massive stars.

So also a star of class B which was produced from a recently formed nebula would have a low velocity, and this velocity would increase through the ages, owing to the gravitational action of other stars. But in the meantime, as it radiated away its energies and decreased in mass its spectral type would change in the direction from B toward M, so that again there would be an association of the deeper colors with higher velocities. Obviously, the same star may at one time be increasing in mass and decreasing in speed, and at another be decreasing in mass and increasing in speed, the spectral type changing correspondingly, and these changes may be repeated indefinitely. In view of these possibilities, then, we cannot assign an upper limit to the duration of the life of a star; nor indeed could we say that a star has but one life, for it is quite conceivable that its life may be extinguished and renewed many times.

One can imagine that a wandering star finds its way into a nebula of sufficiently enormous expanse and has its velocity so decreased that it is unable to escape. Another star, and still another, is entangled in its filmy substance, and finally a whole group of stars are brought to rest within its borders. If these stars come from all directions at random, the moment of momentum of the group would be small. Since the moment of momentum of the nebula itself would be small, there would be little tendency for the system as a whole to rotate, but under their mutual attractions these stars would take the form of a globular cluster. In the course of time they would sweep up the nebulous material which had bound them together; and this material for many ages would furnish the energy for their lavish radiation. But eventually it would be exhausted and the stars would decline in mass. As they did so, the gravitational control of the group on the individual members would be relaxed. The cluster would expand, and finally, one by one, the stars would escape and pursue their lonely journeys in search of new adventures.

An unusual epoch in the existence of a star will occur when it happens to pass through the immediate neighborhood of another star, an event which is almost certain to occur in a sufficiently extended period of time. The results of such an encounter are studied in the well-known researches of Chamberlin and Moulton on the planetesimal hypothesis of the development of our own planetary system. In this hypothesis the fundamental assumption is that at some remote epoch in the past our sun, even at that time a star, passed close by another star and that our planetary system has grown up and developed from the material which was torn from the sun by the tidal and disruptive action of the second sun. If the life of our sun is limited to some such period as a thousand millions of years it must be admitted that the chance for its encounter with another sun during this relatively short interval is small, and this objection has been urged. But if the time limit on the duration of the sun's life be removed, such an encounter becomes very probable, indeed almost a certainty, and our confidence in the planetesimal hypothesis is strengthened, or, perhaps better, our skepticism is somewhat weakened.

But if the sun is living upon material which is drawn in from surrounding space, the planets, too, at a smaller rate, must be adding to their masses and therefore growing, since they have not yet reached a stellar condition. In the course of time they too will grow to the full stature of a sun unless in the meantime the mutual perturbations of the planets shall have brought about the destruction of one or more of them through collisions among themselves or with the sun. Obviously the growth of the planets in such a manner would result in greatly contracting the dimensions of the planetary orbits, not only through the gravitative effect of increased

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mass, but also because the ingathering process would have much the same effect as friction, since the process would add nothing to the momentum of the planets and would therefore decrease their speeds. It seems quite likely that the road to stellar condition would not be traveled very far before the terrestrial planets would be swallowed up by the sun, and, if the sun also were growing, by the time it had arrived at spectral type B the sun and Jupiter only would be left to form a binary star of short period—a typical type B binary. At the present time Jupiter is too small in mass to be a star. If the sun is now passing through a period in which its mass is decreasing, the mass of Jupiter will be growing, or at the very worst will be stationary, and therefore gaining relatively to the sun. One can fancy the sun decreasing toward extinction while Jupiter is slowly growing toward stellar conditions. In this manner Jupiter and the sun might approach equality in mass. At a later time, if the solar system passed through a region rich in matter the sun and Jupiter would grow equally, and our solar system, if not reduced to two members, at least would present the interesting phenomenon of a system of two dominating suns of approximate equality. In the early stages of this rejuvenation the sun and Jupiter would form a binary star of long period and reddish color. As they grew in mass gathered from surrounding space their colors would brighten, they would draw closer together, and their period would shorten. Since the effect of a resisting medium is to make the orbits circular, it can be shown that the product of the period and the fifth power of the sum of their masses would remain constant during this process of growth, and so also would the product of the mean distance and the cube of the sum of the masses remain constant. In other words, the period would vary inversely as the fifth power of the sum of the masses, and the mean distance inversely as the cube of the sum of the masses. If the masses of the sun and Jupiter were increased by gathering in atomic material to five times their present masses, the distance of Jupiter would be reduced from 500,000,000 miles to 4,000,000 miles, and its period would be reduced from about twelve years to approximately thirty-three hours. If the masses were of approximate equality their spectra would change in the direction from type M toward type B, and if their masses were sufficiently great to have a spectrum of type B we should certainly have a short-period binary with the circular orbit which is characteristic of this class of spectroscopic binaries. On the other hand, if we follow such a binary in our imaginations as through the ages it decreases in mass, we see the reverse process taking place, the distance between the stars increasing and the eccentricity increasing likewise, but the temperature decreasing and the color tending through the yellow toward the red until we see finally a typical visual binary.

An indefinite prolongation of a star's life undoubtedly would vastly increase the significance to be attached to a close approach of two stars, since such a close approach is merely a matter of sufficient With the relatively short span of life hitherto assigned to a time. star such an event is highly improbable until many acons after the star has become cold and dead. The approach of two cold and solid stars would certainly have to be extremely close to have any further effect on the stars than to change their speeds and their paths, and it is very doubtful if anything short of actual collision would disrupt them. A quite different state of affairs would arise, however, if the stars were massive, and very hot and active. These are the conditions postulated in the planetesimal hypothesis, and the consequences of such conditions must play a very common rôle in the life-history of the stars. From the consequences in the sun-Jupiter system, in which it has been shown how Jupiter might become as massive as the sun and the sun become a binary star, a possible mode of genesis of these interesting objects is obtained. An extension of the planetesimal hypothesis seems quite competent to account for the existence of many binary stars, from the typical yellow, visual binaries of high eccentricity and long period to the typically short-period, white binaries of spectral type B with their almost perfectly circular orbits. The coalescence of the many members of a planetary system into a system of two or three members would furnish many occasions for the flashing up of a star into an intense but temporary brilliance, such as is exhibited in the relatively frequent temporary stars. It is not necessary, however, to suppose that all binaries are formed in the same manner, for it is quite conceivable that if a nebula has two centers of con-

densation a binary of long period and high eccentricity should result through the process of condensation. It is quite possible too that there exist other processes which have not yet been formulated.

Two processes are here recognized by which a star comes into existence. The first is a *possible* condensation from a nebula, though this does not seem to be as inevitable a process as it has generally been regarded. The second is the growth of a nucleus a fragment perhaps from some disrupted mass, a witness of some titanic cataclysm—by the accretion of atoms and small particles to such a mass that the release of subatomic energies transforms it into a radiating star. By collision, or very close approach, only can we account for a star passing out of existence, in the first of which two masses are united into a single one, and in the second a single solid mass is disrupted into many fragments. But during the continuance of its existence a star is essentially a singular point in an infinite field of energy. Through these singular points the energy ebbs and flows. When the flow exceeds the ebb the star grows in mass and radiating power and character of spectrum. When the ebb exceeds the flow the star declines in mass and radiation, at times even to the point of extinction. But even during the period when its radiation fails, the singular point persists, and through it again flows the tide of energy when the conditions are suitable. Tust as the atom and the molecule are permanent forms of physical existence, so also is the star a permanent form of physical existence, notwithstanding that the individual may pass from birth to its There is no necessary limit to its age, and though dissolution. the star itself may rise and fall, the universe as a whole is not essentially altered. The singular points may change their positions and their brilliancy, but it is not necessary to suppose that the universe as a whole has ever been or ever will be essentially different from what it is today.

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