COMPUTATION OF TIMES OF RISING AND SETTING OF THE MOON

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Perhaps it might be of general interest, in view of the tables of moonrise and moonset, sunrise and sunset, which are given in each number of the Publication and in many popular almanacs, to explain the method by which these tables are calculated.

The method used for the Publication is a combination and modification of the method given by Newcomb in "A Compendium of Spherical Astronomy" together with that of Prof. Frisby, U. S. N., as given by Albert S. Flint in "Popular Astronomy," Vol. 19, pp. 261-275, 363-365.

Essentially the problem of finding the time of rising or setting of any celestial object consists in determining the hour angle of the rising or setting point of the object. In other words, we need to determine the angle at the pole between the meridian and the hour circle of the object as the object is on the horizon. We can readily find the zenith distance of an object on the horizon. From the latitude of the place for which the rising and setting times are desired we can find the distance from the poles to the zenith. From the American Ephemeris and Nautical Almanac we can find, if not directly, at least by interpolation, the declination and hence the polar distance of the object. Thus we have a spherical triangle of which we know the three sides, and we wish to find one of the angles.

In an oblique spherical triangle, where a, b and c are the sides and A is the angle opposite the side a, the following formula holds:

$$\cos a = \cos b\ c + \sin b \sin c \cos A$$

Solving for cos A:

$$\cos A = \frac{\cos a}{\sin b \sin c} - \cot b \cot c$$

Therefore in the spherical triangle Pole, Zenith, Object, we have:

$$\cos t = \frac{\cos \zeta}{\sin (90° - \delta) \sin (90° - \phi)} - \cot (90° - \delta) \cot (90° - \phi)$$

where t is the hour angle of the object, ζ is its zenith distance, δ is its declination, and ϕ is the latitude of the place on the earth's surface. Simplifying the equation, we obtain:

$$\cos t = \frac{\cos \zeta}{\cos \delta \cos \phi} - \tan \delta \tan \phi$$

Or:

$$\cos t = \tan \delta \tan \phi \left(\frac{\cos \zeta}{\sin \delta \sin \phi} \right) \quad (1)$$

This is the general formula for the hour angle of any object in terms of its declination, its zenith distance, and the latitude of the observer.

Now when any object is on the horizon it appears to be 90° from the zenith. It is, however, apparently raised by the refraction of the earth's atmosphere. Therefore, if r be the angle of refraction, the true zenith distance of the object is 90°+r. To an observer on the moon, the earth's radius would subtend a considerable angle—this angle is called the horizontal parallax of the moon. The effect of this parallax, since it measures the effect consequent upon the observer's position not being in line with the center of the earth, is apparently to lower the moon. Consequently when the moon's center is on the horizon—that is, when the moon is rising or setting—its zenith distance is $90°-\pi+r$, where π is the moon's horizontal parallax. In formula (1) let us now substitute the zenith distance of the moon as above obtained:

$$\cos t = \tan \delta \tan \phi \left(\frac{\cos (90° - [\pi - r])}{\sin \delta \sin \phi} - 1 \right)$$

$$\cos t = \tan \delta \tan \phi \left(\frac{\sin (\pi - r)}{\sin \delta \sin \phi} - 1 \right) \quad (2)$$

Or:

$$\cos (180° - t) = \tan \delta \tan \phi \left(1 - \frac{\sin \delta \sin \phi}{\sin (\pi - r)} \right) \quad (3)$$

These are the general formulae for the hour angle of the moon's rising or setting points.

For any given station ϕ is constant, and r at the horizon is assumed to be constant and equal to 36′. Hence we have an equation in three variables. Of the two independent variables δ has a range of from about +30° to —30°, while π has a range of from about 62′ to 52′. Here we follow Professor Frisby's method. We take arbitrary values of π and make tables for t by using δ as the variable.

Consider equations (2) and (3). If $\pi = r = 36'$, the equations reduce to:

$$\cos t = -\tan \delta \tan \phi \qquad (4)$$

Or:

$$\cos (180° - t) = \tan \delta \tan \phi$$

Let us now return to equation (1). The value of ζ for sunrise or sunset is $90°+r+s$, where $s = 16'$ is the sun's semi-diameter, for, in the case of the sun, parallax is negligible and sunrise or sunset is reckoned as the time when the sun's upper limb is on the horizon. Hence the general equation of the hour angle of the rising or setting sun is:

$$\cos t' = -\tan \delta \tan \phi \left(\frac{\sin (r+s)}{\sin \delta \sin \phi} + 1 \right) \qquad (6)$$

Substitute $\pi - r$ for $r+s$, and change the sign of δ:

$$\cos t' = \tan \delta' \tan \phi \left(1 - \frac{\sin (\pi - r)}{\sin \delta' \sin \phi} \right) \qquad (7)$$

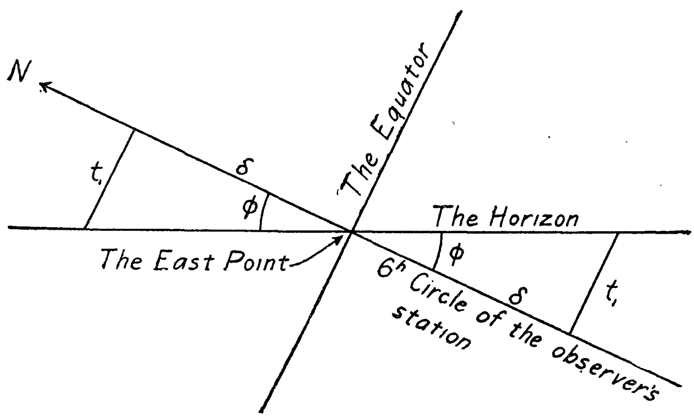
Evidently, by comparison of equations (3) and (7), $t' = 180° - t$, when $\delta' = \delta$ and $\pi - r = r + s$ or $\pi = 2r+s = 2 \times 36' + 16' = 88'$.

Professor Frisby suggests that, if tables for the sun are available, if the sign of the solar declination be changed, and if π be assumed equal to 88′, the supplementary hour angles may be read as the hour angles of the rising or setting moon. He then takes the average values of the hour angles between the values for $\pi = 88'$ and $\pi = 36'$ as the values for $\pi = 62'$. He then adds one-tenth of the tabular differences between the values for $\pi = 88'$ and $\pi = 36'$ to the values for $\pi = 62'$. The results are the values for $\pi = 57'$. He tabulates the values of the hour angles for $\pi = 62'$ and $\pi = 57'$, and is thus able to interpolate or extrapolate for any actual values of π as given in the Nautical Almanac.

Although we had no tables for the sun, we computed the hour angles by Professor Frisby's method. That is, we took $\pi = 88'$ in equation (3) and $\pi = 36'$ in equation (5) and interpolated for $\pi = 62'$ and $\pi = 57'$. Tables were calculated for every 0°.5 of declination between +30° and −30°.

Interpolation in a table is at best a slow and rather unsatisfactory process; especially is it so when a double interpolation is necessary, as in this case. To render more rapid the reading of hour angles corresponding to any given declination and parallax we have plotted curves from the tables of hour angles. In this graph one set of coördinates represents declinations, the other set hour angles. The two values of π give two curves between which the hour angles for given values of π may be interpolated for any given value of δ. By the use of the graph, not only are the results of the tabular computation easily seen, but for **any** declination the hour angle may be read correctly and quickly to the tenth of a minute.

The method of construction and use of the curve for the hour angles of the rising or setting moon when the declination is given have been shown. But the moon has a rapid and varied motion in declination. Hence it is necessary to compute the declination of the moon before the curve can be used.



In the Nautical Almanac in the Washington Ephemeris are given the declinations of the moon at the upper and lower culminations for the meridian of Washington and the corresponding differences in declination for one hour of longitude. The declination appropriate to be substituted in the curve is the declination of the moon at the time of rising or setting. Hence it is desired, for purposes of correcting the declination by means of the difference for one hour of longitude, to find the approximate meridian on the earth's surface over which the moon is culminating as it rises or sets at the observer's station.

As one measures eastward from the observer's station the hour angle of the rising point is $6^h+t'$. From the observer's antimeridian position eastward the hour angle is $6^h+t'$. For purposes of approximation the triangles shown above may be regarded as plane. For a given position ϕ is a constant, hence t' varies only with δ. Upon this basis hour angles measured eastward, for rising from the Washington meridian, for setting from the Washington antimeridian, are equal if δ be taken with opposite signs. That is, tables for t' are labeled Rising North-Setting South and Rising SouthSetting North. In practice t' is added to the complement of the hour angle of the observer west from Washington. The tabulation of these approximate hour angles east from the Washington meridian or antimeridian expressed in hours and decimals is called Table I.

By interpolation to the hundredths of an hour in Table I is obtained the factor by which the difference in declination for one hour of longitude is multiplied in order to obtain the correction for the declination of the moon at the time of rising or setting. This correction is subtracted from the declination for the Washington culmination, because the hour angle from Table I is measured eastward. Thus is obtained the declination of the moon at the time of rising or setting. This is the value of the declination to be used in the curve. Since changes in π are small the effect of its variation during a few hours of absolute time is negligible. Hence we take the value of π as given in the almanac.

Such is the first part of the computation. Its object is the finding of the declination of the moon at the time of rising or setting and from that declination the hour angle of the rising point from the observer's station. Next we reduce that hour angle to the hour angle eastward from the Washington meridian or antimeridian by means of the appropriate correction for difference in longitude of the station and the Washington meridian or antimeridian.

The Nautical Almanac gives the Washington mean time of the Washington upper and lower culminations and the difference in the local times of culminations for one hour of longitude. The product of the hour angle east from the Washington meridian or antimeridian, and this difference in time of culmination accompanying one hour of longitude is then subtracted from the Washington time of the Washington culmination. This gives the local time of culmination at the rising or setting point. If the setting point is west of the International Date Line twelve hours must be added to the Washington time of the Washington antimeridian culmination.

We have now determined the absolute time of the rising or setting of the moon at the station. By subtracting the hour angle of the rising point from the local time of culmination of the rising point, we obtain the time (local to the station) of moonrise. By adding the hour angle of the setting point to the local time of culmination of the setting point we obtain the time (local to the station) of moonset. If an eastward crossing of the International Date Line is involved, twenty-four hours must be subtracted to obtain the local setting time. All that then remains is to convert, if desired, the local time into the standard time of the region or into the civil time.

The actual work of the computation for Claremont, as performed for the Publication, is tabulated under heads in the following order:

A. Date. Every other day in the computation for the Publication. Rising or Setting. Change from rising to setting at new moon and from setting to rising at full moon........N. A.*

B. Declination at the Washington culmination. Take "U" values for rising, "L" values for setting..................N. A.

C. Difference in declination of the moon at culmination for one hour of longitude...................................N. A.

D. Approximate hour angle of the {rising / setting} point east from the Washington {meridian / anti-meridian} ....B in Table I**

E. Correction for declination in minutes. C×D÷60.

F. Corrected declination. B—E. (Consider their signs).

G. Horizontal parallax of the Moon (π)...................N. A.

H. Hour angle of the {rising / setting} point {east / west} from the Claremont meridian. Taken from the curve for values of F and G.

I. Hour angle of the {rising / setting} point east from the Washington {meridian / anti-meridian} {Rising: H—2^h $42.^m6$ / Setting: 12^h—H—2^h $42.^m6$}

I¹. I reduced to hours and decimals.

J. Mean Washington time of Washington culmination. U for Rising, L*** for Setting....................N. A.

K. Difference in local time of culmination for one hour of longitude ....N. A.

L. Correction for the local time of culmination at the rising (or setting) point. $I^1 \times K$.

M. Mean local time of culmination at the rising (or setting) point. J—L.***

N. Mean local time of rising (or setting) at Claremont. M—H for rising, M+H—12 for setting.***

O. Standard Pacific Time (astronomical) of rising (or setting) at Claremont. N—9.m13.

P. Standard Pacific Time† (civil) of rising (or setting) at Claremont.

PICKERING'S MONTHLY REPORT ON MARS

ELVA A. HENRY

Is there rational life on Mars, and if so can we communicate with the inhabitants? These are questions which have grown in interest ever since the discovery that our Sun is the center of a system of worlds of which the earth is almost the most insignificant, and that our Sun itself is but an unimportant factor of the Universe in comparison with other suns, each with its attending system of worlds. But these outlying worlds of space are too distant for us to more than theorize about, so, naturally, we have confined our investigations to other worlds of our own system, and, more than any other, to our nearest celestial neighbor, Mars.

Still, the questions have not been answered—at least not to the satisfaction of all. So it is with the deepest interest that we have read the series of monthly Reports on Mars by Prof. William H. Pickering in Popular Astronomy.

Six of these articles have been published, beginning in the January, 1914, issue. Prof. Pickering has been stationed at the Harvard College Observatory station at Mandeville, Jamaica, W. I., and has issued regular monthly reports of his observations of

*N. A. stands for the American Ephemeris and Nautical Almanac.

**Table I is computed for Claremont as t + 3h.29.

***The mean Washington anti-meridian time of the Washington lower culmination is 2 hours later than the time given in J. For the same reason M for setting should be J, as given, —L+12h. The Claremont local time of this event will be found by subtracting 24h from the Washington anti-meridian time, since the International date line is crossed eastward. But the whole of this process amounts simply to the subtraction of 12 hours from N, if the other 12-hour corrections are disregarded.

†These times are computed not for the visible, but the true horizon. The actual computation is carried to the tenth of a minute throughout, hence the results are certainly accurate to the minute, as published. Greater accuracy would be useless.