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ERRORS IN LONGITUDE, LATITUDE AND AZIMUTH DETERMINATIONS — I.

By F. A. McDiarmid

A POINT on the surface of the earth is defined geographically by its longitude and latitude, and the direction of one point from another is measured by its azimuth. In the determination of these three quantities, longitude, latitude and azimuth, all possible errors must be eliminated. In astronomical observations the measured quantities, seconds of time or arc, are small in magnitude, yet when transferred to units of length on the surface of the earth are of considerable dimensions. At latitude 45° one second of time is equivalent in longitude to 1077·9 feet, or one-tenth of a second means 107·8 feet. An error of this size might easily enter if extreme care were not exercised ; it might come from errors in reading the level, from change in personal equation of the observer or from many other sources. If in the time sets the azimuth is poorly defined, an error of considerable dimension will exist in the clock correction. If the chronometer or clock used in the determination has an irregular rate, there might be a large error in the computed clock correction for any instant. Also errors of considerable size may arise from the exchange of time signals between the two observers engaged in the longitude work. These sources of error will be fully discussed later. In latitude determinations the errors arise from three sources, inaccurate level reading, faulty micrometer setting and poorly determined declinations of the stars employed. In the azimuth observations the errors come from faulty setting on either star or mark, inaccurate time determinations, poor level determination, and from lateral refraction.

Accurate astronomical observations are required for establishing boundary lines, for checking land surveys, for giving base points for triangulation surveys, as well as for comparisons with the data given by a geodetic survey, for the purpose of determining the true longitude and latitude of a point on the surface of the earth, and for the determination of the shape of the earth. A small error in either longitude or latitude coupled with a small error in azimuth may carry with it the loss of valuable property. It may place large and valuable mineral deposits in possession of a people not the rightful owners. The valuable copper claims at the head of the White River were claimed by both Canadian and American prospectors ; and it was therefore vitally important that the 141st meridian which forms the boundary line between the Yukon territory and the district of Alaska, should be accurately determined.

The errors in astronomical observations may be classified under three heads: instrumental, personal and catalogue. The errors of the catalogues can only be reduced or eliminated by more fundamental observations at the fixed observatories. The stars used in time work are those on which many observations have been taken at the fixed observatories, and their defined positions have very small errors in either right ascension or declination, and consequently the errors either in time or longitude due to faulty star places are very small indeed. In azimuth observation, the stars employed are generally α Ursæ Minoris, λ Ursæ Minoris, 43 Hev. Cephei, δ Ursæ Minoris and 39 Hev. Cephei. The places of all these stars have been well defined by a great number of observations at the fixed observatories, and consequently the errors coming from defective star places are small indeed. It is in latitude work that large errors come directly from errors of catalogue. These will be discussed later in the consideration of latitude observations. The personal and instrumental errors entering into astronomical field work will in the course of these papers be discussed, and an attempt will be made to derive methods of eliminating as far as possible all these errors.

In this paper it is the intention to discuss longitude determinations, treat briefly of the development of the present day methods, show how errors have been eliminated and attempt to derive some results which will be as accurate as possible. Latitude and azimuth observations will also be discussed in later papers with the same end in view.

LONGITUDE

The longitude of a place is the distance east or west of some fixed meridian. The meridian through Greenwich is the base for all our longitudes; the longitude of Ottawa is the distance Ottawa is west of Greenwich, or, reckoning in time, the longitude of Ottawa is the time which the Greenwich clock will show at Ottawa midnight, or the longitude of Ottawa is the difference between Ottawa and Greenwich times. The difference of longitude between any two places is therefore the difference of their times. The problem of accurate longitude determination resolves itself into the problem of accurate time observation at the two stations whose difference is required, and a comparison of these local times.

Previous to the introduction of telegraphic methods of longitude determination, longitude was determined from eclipses of the sun and occultations of the stars, by the methods of moon culminations, lunar distances, and by measurements of the moon's altitude or azimuth. All these methods depend upon the same general principle, *viz* : the moon has a comparatively rapid motion of its own, in consequence of which it makes a revolution about the earth in 27·3 days. The elements of its orbit together with the effects of the various perturbing forces being known, it is possible to determine the position of the moon at any instant of time ; thus in the almanacs will be found the right ascension and declination of the moon several years in advance for every hour of Greenwich time. If at a place whose longitude is required the position of the moon is determined by observation ; the local time being noted, the ephemeris above mentioned gives either directly or through computation the Greenwich time corresponding to that position. A comparison of the Greenwich time with the observed local time gives the difference of longitude required.

Some of these methods give fairly accurate results, but there is one difficulty which precludes the attainment of an accuracy commensurate with that obtained by the telegraphic method.

The angular velocity of the earth on its axis, which is the measure of time, is twenty-seven times greater than the angular velocity of the moon in its orbit ; it follows therefore, that any errors in determining the moon's position, or of the ephemeris will produce errors in the longitude twenty-seven times as great. So, if the errors to be anticipated in determining the position of the moon are of the same order as those of determining and comparing clocks by electric telegraph, we might expect to reach an ultimate degree of precision by the latter method twenty-seven times greater than by the former.

In the year 1888, and again in 1896 Mr. William Ogilvie employed the method of longitude determination by moon culminations for the purpose of establishing the 141st meridian, which forms the boundary line between the Yukon territory and the district of Alaska. In 1906 a second determination was made by the telegraphic method ; and the two results only differed by about 367 feet, or by $^{s}\cdot 56$ of time. This is well within the limits to be expected.

LONGITUDE BY THE ELECTRIC TELEGRAPH

With the construction of telegraph lines to all parts of the world, there was placed at the disposal of the observers a means of accurately determining differences of longitude. The only error that can enter into the actual comparison of clocks over the telegraph wire is due to a difference in the transmission time between the eastern and western signals. In the year 1906 during the Vancouver-Boundary (141st meridian) longitude campaign an opportunity was afforded of testing the transmission times in opposite directions through repeaters.

The distance from Vancouver to Boundary is nearly two thousand miles, and the wire, an iron one of number eight Birmingham, weighing 360 pounds to the mile, passes nearly the whole of its course through a thinly settled country, in fact, by far the greater part through a wilderness. Through the woods a "right of way" was cut some years ago, and the wire is supported on trees from which the branches have been cut. On the line between Vancouver observatory and Boundary there were four sets of repeaters, one each at Atlin, Hazelton, Ashcroft and Vancouver C.P.R. telegraph office. These repeaters, of the Weiny-Phillips type, were all alike, and their adjustments all similar, so that a good opportunity was afforded of making a test to ascertain if the transmission time under such circumstances would be the same in either direction. On the afternoon of September 4th the following experiment was carried out: (1) exchange of arbitrary signals as usual, 20, 40, and 20 signals; (2) repeaters reversed; (3) repeaters reversed, poles reversed; and (4) as usual, same as (1). The last, (4) condition assured, compared with the first, (1), the determinations of the differential rate of the chronometers for application to (2) and (3). It may be pointed out that there was no change made in the adjustment of the points of the repeaters in the various experiments.

From these observations the transmission time going east, *i.e.*, going from Boundary to Vancouver is $^{s}\cdot 0225$ less than going in the opposite direction. We say apparently, for the inter-agreement of (1) and (4), (2) and (3), which should be identical, is of a magnitude of that quantity. It is, therefore, not certain whether the difference of transmission times is apparent or real. It is in any case a very small quantity.

At about the time that this longitude campaign was being carried out, a series of experiments for the determination of transmission time through repeaters was made at the Dominion Observatory by Mr. R. M. Stewart. The first experiment consisted in measuring the time of transmission in both directions under different conditions of adjustment, due care being taken that all the adjustments were normal, and such as might easily occur in actual work. The times of transmission east and west are given below for each adjustment :—

	East s	West s
First adjustment	·015	·054
Second adjustment	·016	·069
Third adjustment	·014	·034
Fourth adjustment	·039	·008

Each of these values is the mean of twenty signals; the probable error of each value works out about ^{s}·002 or ^{s}·003. The effect on a longitude determination ranges from ^{s}·026 to – ^{s}·015, an amount by no means desirable in primary longitude work.

Whatever method of longitude is employed, it is necessary to determine accurately the errors of the chronometers. The researches of the past few years have added much to the accuracy of time determinations. Formerly the observer obtained his clock correction by the "eye and ear" method of observation. The observer was obliged to carry the clock beats in his head and estimate to fractions of a second the instant that a star crosses the fixed lines in the reticule of the eye-piece. Errors of considerable dimensions were likely to enter, the largest that of the observer himself, commonly called his personal equation. The personal equation between two observers was in large part eliminated by its determination, both before and after a longitude campaign, or by an exchange of stations when the work was half completed. Personal equation or personal error between two observers depends upon the physical condition, the experience and temperament of the observers, as well as on the instruments employed. The existence of a large personal error is by no means a mark of a poor observer. The two eminent observers, Bessel and Struve, found in 1814 their personal equation was zero ; in 1821 it was ^{s}·8 and in 1823 it was an entire second ; in 1823 the personal equation between Bessel and Argelander was 1^{s}·2. Bessel found when he used a clock beating half-seconds he observed transits ^{s}·49 later than when he employed a clock beating seconds. The changing of personal equation indicates a gradual formation of a fixed habit of observing. The "eye and ear" method was employed in the early work in this country and splendid results are connected with the names of Dr. King and Dr. Klotz.

With the invention of the chronograph, the "eye and ear" method of observation was replaced by the key method, and lately the key has been replaced by the travelling wire micrometer. A sheet of paper is fastened to the drum of the chronograph, and any interruptions of the chronograph circuit are recorded on the chronograph sheet. When the key was used, the observer, on seeing the star image cross the fixed lines of the reticule, tapped the key placed in the chronograph circuit, and there was recorded on the chronograph paper the instants of the star transits. This process was carried out much more easily and accurately than by the old method, but there still remained in many cases a large personal equation. From values of personal equation obtained at the Dominion Observatory it appeared that the personal equation in key observations is neither a fixed quantity nor necessarily a small one. In May of 1905 Mr. R. M. Stewart and the writer determined their personal equation to be ^{s}·096 ± ^{s}·007. In November of the same year a second determination showed the personal equation to be ^{s}·325 ± ^{s}·011. Mr. Stewart during these observations was using the key, and the writer the travelling wire micrometer. During the winter of 1905-06 Mr. Stewart made elaborate tests to determine his own personal equation, as well as the advantages, if any, of the travelling wire micrometer over the key as a means of making star observations. The results of his observations showed a difference in recording transits by key and micrometer of ^{s}·445 ± ^{s}·010.

The existence of this large personal equation led to the introduction of the travelling wire micrometer or transit micrometer as it is usually called.

The important claim for the transit micrometer is that it greatly reduces the size of the personal equation, and accordingly removes a great deal of the uncertainty of our time determinations. Observations with the transit micrometer for personal equation have been made at the Dominion Observatory during the past years. Five observers were engaged in the work in 1908 and 1909, and two observers in 1910 and 1911 and 1912. A table of values of the personal equations is given below :—

Observer	1908	1909	1910	1911 (Apl.)	1911 (Sep.)	1912
R. M. S.	·035	·080				
C. C. S.	·000	·000				
D. B. N.	- ·009	- ·014				
W. C. J.	·059	·070	s·040	s·038	s·087	s·021
F. A. M.	·022	·060	·000	·000	·000	·000

In the comparisons for 1908 and 1909 all the observers are referred to C. C. S., whose absolute personal equation seems to be nearly zero. The values of C. C. S., D. B. N. and W. C. J. are nearly the same for two years, while those of R. M. S. and F. A. M. while agreeing between themselves have changed relatively to the other three observers. In the figures for 1910 and 1911 the observer W. C. J. is referred to observer F. A. M. There is considerable change in the value of the personal equation between April, 1911, and September, 1911. The probable errors of the April and September determinations are of nearly the same size, and there appears no reason for the change.

The instruments employed at the Dominion Observatory are the ordinary reversible type of Cooke transit with object glass of three inches aperture and of focal length three feet. Until a few years ago our time sets consisted of two clamps of seven or eight stars each, one polar and six or seven time stars. After applying the correction due to level and diurnal aberration, we have for each star an equation of the form $Aa + Cc + \triangle T = T$, where a and c are the azimuth and collimation errors respectively, $C = sec\ \delta$, $A = sine\ (\phi - \delta)$, $sec\ \delta$ and $\triangle T$ is the clock correction. The fourteen or sixteen equations are reduced by least squares, giving equal weights to the separate observations, and hence are deduced the values of c, a, and $\triangle T$. The probable error of $\triangle T$ is formed from the residuals of the separate stars. It has been the usual custom to "balance" the set, that is, so select the stars that $\xi A = 0$ and $\xi C = 0$.

Those who have been engaged in time work have noticed certain discrepancies which frequently show themselves in the results. In an extended series of observations for personal equation, the different values obtained, even on the same night, may sometimes, if not frequently, differ by one-tenth of a second of time or even more ; while the probable errors of the individual sets do not exceed one-hundredth of a second. Also in longitude determinations, a fair average of the extreme differences during a few nights' work would probably be one-tenth of a second or more ; and the differences in clock error obtained by the same observers on the same night from successive determinations are often of about the same order of magnitude. From a comparison of the magnitude of these frequent discrepancies with that of the corresponding probable errors, and from the fact that they do not follow the same law, and seem to have no connection, it is at once evident that the discrepancies are not the result of truly accidental errors, but are systematic in their nature, that is to say, that of two sets taken on the same night, one may be affected by a certain systematic error, the other by a different error, also systematic. An explanation which has been given and accepted is that there has been a change in the observer's personal equation. If this were so, then with observations taken with the transit micrometer we would expect these discrepancies to disappear, or to be greatly reduced. Mr. Stewart in his researches mentioned above found the range of discrepancies in the case of the key observations to be as high as $^{s}\cdot 129$ with an average of $^{s}\cdot 049$; in that of the micrometer observations the range was as high as $^{s}\cdot103$ with an average of $^{s}\cdot045$; the average probable error of each set of observations is $^{s}\cdot011$. It is well to note that on the night when the highest discrepancy with the micrometer occurred the probable errors of the two sets were $^{s}\cdot009$ and $^{s}\cdot011$, while the discrepancy of $^{s}\cdot129$ with the key occurred between sets whose probable errors were $^{s}\cdot007$ and $^{s}\cdot011$.

The conclusion forced upon us is that these effects are practically independent of the instrument used, and therefore not due to personal equation. We must evidently look for some source of error which from its nature would not show itself in the residuals, which might systematically affect the result of a time set by a considerable quantity, and which may vary from set to set.

A faulty determination of azimuth seemed to be the most likely explanation. This was also strengthened by the fact, that when the azimuths of the two clamps are reduced separately, they frequently differ by a considerable amount. It was also found that by observing more polars in each time set, and by making the reduction separately for each pair of polars, the resulting clock correction depended largely on the polars chosen. From an examination of the discrepancies the conclusion is reached that nearly all the differences between the computed clock errors is due to azimuth only. Accordingly an increase in the number of polars in each set would better determine the azimuth error and give a more accurate clock correction.

The transit instruments, Cooke Nos. 1, 2 and 3, have been reconstructed so that all the stars may be observed twice, once on each side of the line of collimation. The polars may be observed twice on each side of the line of collimation. The two observations are made over the same part of the field, and hence the error of collimation will disappear in the mean of the two observations. Also the weight of the combined observations is about 1·5 times that of a single observation.

The correction for level is applied directly to the star's time of transit, and in many cases is a more or less uncertain quantity.

The pier on which the instrument rests is a block of concrete about six feet long, three feet of which are in the ground. But often the pier or instrument changes irregularly during the progress of a night's work. When the foundation is in a heavy clay, the pier generally acts satisfactorily, but sometimes there are changes which are difficult of explanation.

The pier used at Winnipeg during the longitude campaigns of 1909, 1910 and 1911 behaved in a very peculiar manner. The level showed changes which followed no general law, and which were different on different nights. The bottom of this pier is over six feet below the surface of the ground, and every care was taken in its construction to insure good results. During the observations of 1909 and 1910 the changes were very large and irregular, whereas in 1911 the pier was fairly steady and the level readings consistent. The only explanation that presents itself to the observers is that in 1911 the ground around the pier was in a more constant state than in the previous years. When the observations were made in 1909 and 1910 there was a great deal of evaporation taking place during the day and a cooling down at night, while in 1911 the pier was practically sitting in a pond of water and there was little or no change due to the earth drying out. Also in 1911 the time observations were taken much later in the evening than in 1909 and 1910. Great care should always be taken in the selection of a site for an observatory to insure a solid foundation, and to guard as well as possible against any changes due to cooling of the earth's surface. It is also not well to have the pier near the west or north slope of a hill. The pier at Gateway, B.C., was on a hill facing the sun; on nights following bright clear days it was always found that there were large changes of level, while on nights following dark cloudy days the changes of level were invariably small. A rapid cooling down of the instrument or pier is undoubtedly one cause of this irregular level reading.

On the following page is a copy of the computation of a time set taken at Winnipeg, on September 9th, 1911. In the column headed Star is given the *Berliner Astronomischen Jahrbuch* star

Station........ Winnipeg .....
Date.......... Sept. 9th, 1911.
Instrument..... C . . No. 2 ....

Observer, F. A. M.
Computor, F. A. M.

TRANSIT OBSERVATIONS

Clamp	Star	T	B	Bb	Aberr.	(T-T$_o$) r$_h$	α	T	l	A	Al	A^2	Aa	T	− l	v
		h m s					h m s	s								
	817	21-41-13·70	2·85	·23	− ·04		21-40-40·07	− 33·82	− 1·89	− 1·10	2·08	1·21	− 1·90	·13	1·89	·12 p
	823	49-34·21	1·00	·09	− ·02		48-03·21	− 31·07	·86	·46	·40	·21	·79	·13	− ·86	·06
	79D	52-21·94	3·19	·24	− ·05		51-47·88	− 34·25	− 2·32	− 1·38	3·20	1·90	− 2·38	·13	2·32	·07 p
	S442	58-35·87	3·11	·24	− ·05		57-62·00	− 34·06	− 2·13	− 1·31	2·79	1·72	− 2·26	·13	2·13	·00 p
	831	22-03-25·46	1·00	·09	− ·02		22-02-54·57	− 30·96	·97	·46	·45	·21	·79	·13	− ·97	− ·05
	833	05-50·59	1·14	·10	− ·02		05-19·46	− 31·21	·72	·35	·25	·12	·60	·13	− ·72	·01
	835	06-35·63	1·14	·10	− ·02		06-04·50	− 31·21	·72	·35	·25	·12	·60	·13	− ·72	·01
	S450	10-37·16	1·27	·11	− ·02		10-05·96	− 31·29	·64	·24	·15	·06	·41	·13	− ·64	− ·10
	843	17-41·62	·80	·07	− ·01		17-10·87	− 30·81	1·12	·63	·71	·40	1·09	·13	− 1·12	·10
	S456	26-31·20	1·12	·09	− ·02		25-60·01	− 31·26	·67	·36	·24	·13	·62	·13	− ·67	·08
	226C	31-20·98	3·66	·25	− ·06		30-46·41	− 34·76	− 2·83	− 1·78	5·04	3·17	− 3·07	·13	2·83	− ·11 p
	852	35-49·72	1·25	·10	− ·02		35-18·51	− 31·29	·64	·25	·16	·06	·42	·13	− ·64	− ·08
	857	39-23·28	1·08	·08	− ·02		38-52·29	− 31·05	·88	·40	·35	·16	·69	·13	− ·88	− ·06
	859	42-48·08	·97	·07	− ·01		42-17·19	− 30·95	·98	·49	·48	·24	·85	·13	− ·98	·00
	862	46-15·93	·99	·08	− ·02		45-45·03	− 30·96	·97	·48	·47	·23	·83	·13	− ·97	− ·01
									·00	− 1·10	17·02	9·94				

NORMAL EQUATIONS

For Clock Time T$_o$ = 22h 20m
Assumed $\triangle$ T$_o$ = − 31s·93
r$_h$ = . . . δ T = + s·13 + ·004
a = + 1·726
$\triangle$ T = − 31s·796

a	T	l
	·111	1·712
9·94	− 1·10	17·02
− 1·10	15·00	·00
	14·88	1·88

number of the stars used ; under T is the mean of the scalings of the chronograph breaks ; under B is given the value of cos $(\phi - \delta)$, *sec.* δ for each star ; under Bb is the level correction and under *Aberr.* is the correction due to diurnal aberration. The column headed $(T - T_o)\ r_h$ is for the correction for clock rate ; α is the column for the right ascension of the star, and T is the clock correction including the azimuth error ; the column l is the difference between the individual T'*s* and the mean of the T'*s* ; A is the value of *sin* $(\phi - \delta)$. *sec.* δ ; T_o is the mean of the T'*s* of the time stars ; and $\triangle\ T_o$ is the mean of the T'*s* of all the stars ; δT is the value of the clock error as given by the least square solution ; a is the azimuth error of the instrument as shown by the least square solution ; and V is the residual from each star. The total clock error is $\triangle\ T_o + \delta\ T$, or $\triangle\ T$. The stars with the letter p are the polar stars for the determination of azimuth.

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OTTAWA, CANADA,