ratio N_{NaI}/N_{CaII} were suggested, but were not analyzed quantitatively.

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Sen, Hari K. and Donald H. Menzel. Matrices for birefringent filters.

The authors have given an alternative development of the Jones matrix calculus¹ for the treatment of optical systems. They have proved by the Poincaré sphere² as well as by the matrix method the following general theorem on the conditions under which a system of retardation plates behaves rigorously as a rotator.

A system of retardation plates, oriented with respect to each other at half the face angles, respectively, of a polyhedral angle O, and with retardations equal to the supplements, respectively, of the dihedral angles of O, behaves rigorously as a pure rotator. The rotation is equal to half the excess of four right angles over the sum of the face angles of O.

The Jones theorem³ that we may produce a rotator by inserting a retardation plate between two crossed quarter-wave plates in such a position that its axes are at 45° with respect to the axes of the quarter-wave plates, and that the angle of rotation will be one-half of the phase retardation of the inserted plate, follows as a particular case of the theorem enunciated above.

The authors have also shown that the Jones calculus (*loc. cit.*) can be extended, to the order ϕ^2 , to slightly inclined off-axis rays (θ , ϕ) by replacing the phase retardation, n_0 , for the ray normal to the crystal surface, by n , where

$$n_0 = \frac{2\pi}{\lambda} d(\gamma - \alpha), \quad (1)$$

$$n = n_0 \left[1 + \phi^2 k \left(\frac{\cos^2 \theta}{\gamma} - \frac{\sin^2 \theta}{\alpha} \right) \right], \quad (2)$$

$$k = \frac{\alpha\gamma - \beta^2}{2(\gamma - \alpha)\beta^2}, \quad (3)$$

in which λ is the wave length, and α , β , γ are the smallest, intermediate and greatest principal indices of refraction of a bi-axial crystal, of thickness d , cut perpendicular to the β -axis.

The formula (2) is due to Evans⁴ who, however, did not consider the inclination of the displacement vectors to the α and γ axes, for an off-axis ray. The authors have shown that the formula holds good, to the order ϕ^2 , even under this more general circumstance.

In their attempt to simplify the Lyot birefringent filter⁵ of n pairs of polarizers and retardation plates, by the general theorems on optical systems enunciated and proved by Jones by the matrix calculus,⁶ the authors find that the Lyot arrangement is the simplest possible to achieve the desired object. They believe that the illusory nature of the simplifications implicit in the general theorems is due to the restriction of the Jones matrices to monochromatic light and that the removal of this restriction will be an important step forward. Possible bases of approach may be a perturbation method, when the incident light is sensibly monochromatic, and the Jones time algebra, as described by N. G. Parke III.⁷

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Shapley, Harlow, Virginia McKibben Nail and William G. Tift. On the superluminous variable stars in the Large Magellanic Cloud.

The supergiant red variable stars in the Small Magellanic Cloud have been reported in *Harvard Reprint* No. 346, 1951. The survey of stars of top luminosity and of recognized variability in that system was carried down to the fifteenth apparent magnitude. The brightest variables at maximum have absolute magnitudes around -5 ; of the twenty brightest, sixteen are Cepheids and four are irregular red variables. The brightest long-period variables of the Small Cloud have absolute photographic magnitudes around -3.0 .

For both Clouds the elimination of superposed stars of the various variable and non-variable classes was accomplished through the study of stars of many comparison fields in similar galactic latitudes.

Our complete survey of variables of the Large Magellanic Cloud has been taken at this time only to apparent magnitude 13.1, absolute magnitude -4.3 and brighter. We have dealt, therefore, only with the superlative luminosities. A considerable number of non-variable stars of classes O, B, and P Cygni are found among these