n^{th} order, defined by

$$K_n(\tau) = \int_1^\infty \frac{e^{-x\tau}}{x^n} dx, \quad (4)$$

and

$$\gamma = \frac{1}{1 + (\cos \beta) \ln \tan \frac{\beta}{2}}. \quad (5)$$

The coefficients, A_j , can be evaluated from a system of linear equations involving the C_{nj} 's tabulated in Paper I. We have calculated the emergent intensity corresponding to the solution of four simultaneous equations and compared our values with those obtained from the Chandrasekhar-Hopf formula:²

$$I(\mu, \mu_0) = \frac{1}{4} F \frac{\mu_0}{\mu_0 + \mu} H(\mu) H(\mu_0), \quad (6)$$

where $H(\mu)$ is the solution of Ambartsumian's functional equation:

$$H(\mu) = 1 + \frac{1}{2} \mu H(\mu) \int_0^1 \frac{H(\mu') d\mu'}{\mu + \mu'}, \quad (7)$$

and

$$\mu_0 = \cos \beta. \quad (8)$$

The two sets of values of the emergent intensity agree to within one part in a thousand, on the average, and somewhat less for $\beta = 0^\circ$, on account of a singularity in the solution, cf. (5).

Solution of more equations does not consistently improve the agreement. We believe that for higher accuracy we need improve the trial function (3).

1. *Ap. J.* **110**, 1, 1949.

2. *Ap. J.* **106**, 143, 1947.

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Oke, J. B. and R. E. Williamson. A theoretical Hertzsprung-Russell diagram for red dwarf stars.

The best model which has yet been suggested for stars of small masses is one first devised by Cowling in which a radiative envelope is fitted to a convective core. If the opacity in the star is represented by the law $\kappa = k\rho^{1-\alpha}T^{-3.5}$, where k is a function of the chemical composition, then it is found that α varies from 0.43 to 0.25 as the mass of the star goes from 0.2 to 1.0 of the mass of the sun. If the carbon cycle is assumed to be the source of energy in stars less massive than the sun, a Hertzsprung-Russell diagram can be constructed. It is found that the main sequence

should have a considerable width if reasonable variations in chemical composition are allowed. If several stars have a constant chemical composition, then they should fall on a single curved line in the diagram, their positions on the line being governed by their masses.

If the proton-proton reaction is assumed to be the only source of energy, then it is found that only two-thirds of the stars of known mass, luminosity, and radius conform to the proton-proton reaction models. This is so even when considerable errors are allowed for in the radius and luminosities of these stars. For the late-type main sequence stars, the Hertzsprung-Russell diagram based on the proton-proton reaction models does not appear to fit the observational diagram quite so well as that based on the carbon cycle.

Rather surprisingly, it is found that the sun may derive all of its energy from the proton-proton reaction if the hydrogen plus helium content is more than 90 per cent. If the hydrogen plus helium content is 98 per cent, the proton-proton reaction gives for the sun a central temperature of 16×10^8 °K, a central density of 124 grams per cc, a hydrogen content of 74 per cent, and a helium content of 24 per cent. The heavy material is assumed to be Russell mixture.

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O'Keefe, John A. A new determination of the lunar parallax.

The lunar parallax has been measured by determining by photoelectric measurements the times of occultation of the same star at several widely separated points in the United States. The essential tool not used in previous determinations of the lunar parallax is the geodetic triangulation of the United States, which appears to be accurate to 1 part in 200,000 on individual loops. It is a further consequence of the use of geodetic triangulation that what is determined is the moon's distance, rather than its parallax; consequently it is not necessary to make an assumption about the earth's radius in order to convert the results to kilometers.

The technique is to place the observers along the line covered by the shadow of a chosen lunar

feature. With a 12-inch Cassegrain reflector feeding into a 1P21 photocell and thence to a Brush Recorder, which also records the time signals emitted by WWV, the observers measure the instant of occultation. In the computation, the results are substituted into the following equation:

$$\Delta\sigma = \bar{a}\Delta\lambda + \bar{b}\Delta\phi + \bar{c}\Delta\alpha + \bar{d}\Delta\delta + \bar{e}\Delta\pi + \bar{f}\Delta k + \bar{g}\Delta H.$$

One equation of this kind is formed for each station. The quantities $\Delta\lambda$, $\Delta\phi$ and ΔH can be set equal to zero, since the U. S. triangulation, after transfer to the Hayford ellipsoid, is essentially error-free. The quantities $\bar{c}$, and $\bar{d}$, and $\bar{f}$ are the same at both (or all three) stations, since they depend on the star's declination and the position angle of the contact, which do not change. The quantities $\Delta\alpha$, $\Delta\delta$ and Δk , error of the moon's radius, are also the same at all stations; consequently these terms drop out in forming the differences. Thus the correction to the parallax becomes, for two stations, S_1 , and S_2 ,

$$\Delta\pi = \frac{\Delta\sigma_1 - \Delta\sigma_2}{e_1 - e_2}.$$

The value found was

$$\Delta\pi = -0.056 \pm 0.033.$$

The errors arise largely from the libration of the moon during the time of the observation, and cannot be estimated with any degree of precision. The residuals $\Delta\sigma$ at individual stations indicated an error of 0".0052 for an individual observation. The resulting value of the moon's distance is:

$$384,410 \text{ international kilometers,}$$

an increase of about 6 kilometers from Brown's value. This value is to be regarded as preliminary, since it may be possible to correct it when the librations can be taken into account, as, for example, by the aid of the new tables under construction at the Naval Observatory.

A byproduct of the investigation is the discovery that the star 228 B Aurigae, No. 996 of the Zodiacal Catalog, is double. The separation is 0".053 resolved along a position angle of $255^\circ 53'$; the other component of the separation is unknown. The difference in brightness between the components is 0.5 mag. It will be possible to observe this star again in the spring of 1951, and it is hoped that observers with photocell equipment will get it.

The instrumental equipment for this observation was designed by Mr. D. D. Mears, under the supervision of Prof. A. E. Whitford, of the University of Wisconsin. Mr. Mears also supervised the field parties. The office computations were performed under the supervision of Miss Nora B. Moser. The work was done under the general supervision of Mr. F. W. Hough, Chief, Geodetic Division, Army Map Service. Thanks are due to the Commanding Officers of the Army Map Service, Col. W. H. Mills and Col. John G. Ladd, for their kind interest and support of the program, and to the Naval Observatory for much technical advice and assistance.

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Omer, Guy C., Jr. A hypothetical cosmological model.

Recent observational work indicates that Hubble's¹ reductions of surveys of red shifts and counts of nebulae may have to be revised. The six-color photometry of Stebbins and Whitford² has shown that the radiation from extragalactic nebulae is not that of a black body at 6000°K. Mr. Merle F. Walker has calculated from the six-color photometry the magnitude corrections due to red shift. This work has not yet been published but was communicated privately. In all cases these values are appreciably greater than the 4z approximation used by Hubble. The two-color photometry of Stebbins and Whitford² gives color indices for the E-type nebulae in the clusters which are linear with red shift but whose values are greater than would be expected from red shift alone. M. Schwarzschild has suggested that this may be due to a change with time of the stellar composition of these nebulae. Without further information about the consequent change of absolute magnitude with time for these nebulae, a definite model of the universe cannot at present be deduced from the observational data.

Let us consider a hypothetical cosmological model which we shall assume to be non-static but homogeneous with a present average density of 10^{-29} gm/cc. If the only important interactions between the nebulae are gravitational and if the curvature of space is small, it can be shown that the red shift should be essentially linear with distance for the present observational material. On the assumption that the change of absolute magnitude of E-type nebulae is proportional to