Menzel, Donald H. A symbolic solution of the equation of radiative transfer.

The basic equation of radiative transfer,

$$\mu \frac{\partial I}{\partial \tau} = I - B, \quad (1)$$

has the symbolic solution

$$\left. \begin{aligned} I^+ &= \frac{1}{1 - \mu D} B \\ I^- &= \frac{1}{1 - \mu D} B - e^{\tau/\mu} \left[\frac{1}{1 - \mu D} B \right]_{\tau=0} \end{aligned} \right\}, \quad (2)$$

where I^+ and I^- represent the specific intensities of the outgoing and incoming beams. D is the symbolic differential operator, μ is the cosine of the angle between the beam and the outward vertical, τ is the optical depth and B is the "source function." The equation of radiative equilibrium,

$$\int_0^1 I^+ d\mu + \int_{-1}^0 I^- d\mu = \int_{-1}^1 B d\mu = 2B, \quad (3)$$

when combined with (2), gives the equation for B :

$$\begin{aligned} \int_0^1 \frac{\mu^2 D^2}{1 - \mu^2 D^2} B d\mu \\ = \frac{1}{2} \int_0^1 e^{\tau/\mu} \left[\frac{1}{1 - \mu D} B \right]_{\tau=0} d\mu. \end{aligned} \quad (4)$$

Setting

$$B = a + b\tau + \int_0^1 e^{-\tau/y} \varphi(y) dy, \quad (5)$$

and taking

$$\varphi(y) = \sum_0^\infty A_j y^j, \quad (6)$$

we obtain a set of simultaneous linear equations for determining the coefficients a and A_j in terms of b . The general solution is

$$B = a + b\tau + \sum_0^\infty A_j E_{j+2}(\tau), \quad (7)$$

where

$$E_n(\tau) = \int_1^\infty \frac{e^{-\tau x}}{x^n} dx. \quad (8)$$

Three terms of the summation give ample accuracy for most purposes. Equations (7) and (2) determine the angular distribution of the radiation intensity. Placzek has tabulated the functions E_n .

*Harvard College Observatory,
Cambridge, Mass.*

Millman, Peter M. and D. W. R. McKinley. Three-station radar and visual triangulation of meteors.

This paper presents one phase of the combined Dominion Observatory-National Research Council program for the observation of meteors. Since the summer of 1947 the following observational data have been collected:

- 100 meteor photographs,
- 5,000 visual meteor plots,
- 50,000 photographic recordings of Doppler meteor whistles,
- 1,500,000 photographic records of meteor radar echoes.

The Doppler and radar work has been carried out in the 30 megacycle band. In previously reported results we have indicated the dependence of the form of the echo on the distance of the meteor from the radiant, i.e. on the angle between the meteor path and the line of sight. We have also shown, in agreement with others, that the great majority of meteor echoes are produced in a fairly restricted region of the upper atmosphere centered somewhere between 90 and 100 km above sea level. Since the radar employed was essentially non-directional, all height data depended on a correlation of visual and radar observations.

To investigate the possibility of height determination by radar alone, three stations located at Ottawa, Arnprior and Carleton Place respectively, were put into operation in August, 1948. The base lines between stations were 36, 41 and 57 km in length. Photographic, visual and radar observations were carried out at all three stations. It was found that a large percentage of the meteors producing enduring echoes were detected by all radar sets. Where visual observations were also available from two or more points this made possible the computation of heights in three ways:

- (a) by the use of the three radar ranges,
- (b) by the combination of one radar range with a visual plot,
- (c) by the use of two or more visual plots.

To study the relation between heights determined by the three methods detailed computations have been carried out for the night of Aug. 10/11, 1948. Since the computation of meteor heights by radar range alone becomes insensitive for objects at long ranges the most reliable

results are obtained from meteors with sub-points less than 150 km distant from the centroid of the three observing stations. A total of 41 meteors was found which satisfied the above condition, had radar echoes from all stations lasting over one second, and were visually observed from at least one point. The values of mean radar height for each meteor varied from 79 to 112 km with an average height of 94 km. The corresponding values for the same 41 meteors determined by combining radar ranges with visual plots gave heights from 68 to 142 km with an average height of 100 km. The addition of 21 meteors of long range or with very short duration echoes did not materially alter these figures. Heights computed purely from visual plots showed a much greater spread and were not accurate enough to be of value individually. The mean height of 26 meteors computed from visual observation alone was 98 km, agreeing well with the other results.

It would appear that, for the August period, the heights computed from radar alone average somewhat lower than those found from a combination of visual and radar data. This indicates that there is a tendency for the radar echo to occur near the end of the visual trail—for the Perseids at least. Confirmation of this fact, and a check on the reliability of the correlation of the radar echoes with visual meteors, was found by computing from the radar ranges alone the position of each echoing source in the sky and then comparing this with the visual meteor plot. The mean angular distance on the celestial sphere of all radar points from the corresponding visual meteor plots was only 7 degrees, a good agreement since this figure includes the summed effect of at least five separate sources of error. Over twice as many radar points were plotted near the end of the visual trails as near the beginning.

Additional radar heights were computed for meteors appearing on the same night but not visually observed. A compilation of all radar heights determined, corresponding to a total of 94 different meteors, gave a mean height of 95 km. The moving type of echo (*b* or *h* characteristic in our classification) was observed for 16 of these meteors. Mean height of appearance of this echo was 110 km and mean height of disappearance 95 km.

It seems clear from the above results that meteors produce ionization clouds which may be satisfactorily detected on the 30 megacycle band from widely separated points on the earth's surface. Triangulation of these clouds by range

measures alone is practical and yields results which compare favourably with those obtained by any other methods. This technique, when applied to individual meteor echoes of certain types, makes possible the accurate determination of daytime meteor paths and orbits. It may also be noted that these results confirm our earlier conclusions concerning echo heights and the relation between the echo form and distance from the radiant.

*Dominion Observatory,
Ottawa, Ont.,
and National Research Council of Canada.*

Morgan, H. R. A possible lag in the nutation.

In a recent paper¹ on the lunar nutation Dr. Jeffreys asks for an examination of observations for a possible lag in the nutation. The Washington observations of circumpolar stars, 1903-1925, recently discussed for the nutation constant,²

CORRECTIONS TO THE NUTATION CONSTANT, $N = 9''.210$

From the Right Ascensions

Star	ΔN	<i>W</i>	$\Delta\theta$	<i>W</i>
43 H Cep	−.076	7	+°.49	10
α UMi	+ .060	22	+ .26	16
Grmb 750	− .027	11	+ .25	11
Grmb 944	+ .004	8	+ .62	10
51 H Cep	− .086	17	+ .20	15
4 B UMi	− .079	8	+ .36	7
1 H Dra	+ .092	16	− .23	13
30 H Cam	+ .035	9	− .77	8
6 B UMi	− .032	11	+ .58	11
57 B UMi	− .137	10	+ .44	10
ϵ UMi	− .133	6	+ .13	7
δ UMi	− .005	9	+ .12	9
λ UMi	+ .041	7	+ .23	6
76 Dra	− .072	6	+ .25	6
39 H Cep	− .011	10	+ .32	12
Means	−.016 ± .013		+°.22 ± .06	

From the Declinations

Star	ΔN	<i>W</i>	$\Delta\theta$	<i>W</i>
43 H Cep	−.149	6	+°.50	5
α UMi	+ .054	12	− .07	14
Grmb 750	+ .007	6	− .07	6
Grmb 944	− .117	11	− .73	9
51 H Cep	+ .047	9	− .08	11
4 B UMi	+ .020	8	− .32	10
1 H Dra	+ .039	10	− .38	12
30 H Cam	+ .110	5	− .29	6
6 B UMi	+ .077	5	+ .61	5
57 B UMi	− .047	8	− .24	8
ϵ UMi	+ .033	13	+ .62	11
δ UMi	+ .019	15	+ .08	15
λ UMi	− .008	6	− .64	6
76 Dra	− .030	5	+ .86	7
39 H Cep	− .116	5	+ .93	5
Means	.000 ± .012		.00 ± .08	