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ON THE FOCAL LENGTH OF THE 36-INCH REFRACTOR OF THE LICK OBSERVATORY, AND THE EQUIVALENT, IN SECONDS OF ARC, OF REVOLUTION OF THE MICROMETER SCREW

BY

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(1) MEASUREMENT OF THE FOCAL LENGTH

The focal length of a telescope is the fundamental dimension governing its power of magnification, and a knowledge of it is therefore essential to an understanding of the instrument's performance. Notwithstanding this fairly obvious fact, it is probable that the focal lengths of relatively few refracting telescopes have been determined with a high degree of precision. This is because it is generally sufficient, in the making of telescopic measurements, to know the equivalent, in seconds of arc, of a revolution of the micrometer screw with which the telescopic image, either as viewed directly or as reproduced on a photographic plate, is, in practice, compared. This equivalence, commonly denoted by R , does not depend explicitly on the focal length, but on the ratio to it of the "pitch" of the screw, and this ratio can be conveniently obtained, with an accuracy sufficient for most purposes, by measuring a known angle, such as that subtended by two stars of given position. R is related to the pitch of the screw, p , and to the focal length, f , as in the expression

$$R = \frac{p}{f} \csc. 1'' \quad (1)$$

so that if any two of the quantities involved are known, the other may be simply calculated. It is a common practice to determine R by calibration as above, to measure p , and thus to derive f .

This procedure is permissible provided the first two quantities have been determined with the precision to which it is desired to know the third. There are no espe-

cial difficulties attending the measurement of p against a reliable metric standard, but the evaluation of R by calibration is not so simple a matter, for the reason that dependable standards of angular measure are not usually so readily available as are corresponding standards of length. If the standard angle adopted is that between two stars, it is to be observed that the errors of the instrument with which the separation of the stars was originally determined, or with which the positions of the stars were separately measured, as the case may have been, will be carried into the value of R , as will any uncertainty in the relative displacement of the two stars since the time of their earlier observation. It is therefore preferable, where the highest precision is sought, to measure the two linear quantities p and f directly, and to calculate R by equation (1). This is substantially the conclusion to which Bessel came after considering at length a number of ingeniously conceived alternative procedures.¹ Bessel's method of determining the focal length of the Königsberg heliometer was to measure the distance between conjugate focal points of minimum separation (equal forward and back foci), a distance which is approximately four times the focal length. Another plan has been adopted here, though the result has been checked by measuring conjugate foci.

Discrepancies in published data relating to the focal length of the 36-inch refractor have been the source of some confusion. In Volume 1 of the *Publications of the Lick Observatory*, page 61, under date of September, 1886, it is stated—undoubtedly on the authority of the makers, for the telescope had not then been installed on

¹ *Astron. Untersuchungen*, 1, 145, 1841.

Mount Hamilton—to be 674 inches (56 ft. 2 in.), while in Volume 3, page 1, in a note dated February, 1894, it is given by Professor Holden as 694 inches. While the latter is close to the true value, as determined by recent measurements, it is worthy of remark that neither it, nor the less value just referred to, is in agreement with the one calculated by the usual formulae² from the dimensions and refractive indices of the component lenses. The dimensions supplied by Clark are given in the first of the text references of this paragraph, and the indices of refraction in a later paper by Professor Hastings.³ The calculated focal length for the sodium line D_2 , based on these data, is 719 inches, or 45 inches in excess of the value given by Clark, and about 25 inches greater than the true focal length. It speaks rather well for the general interest in the subject of geometrical optics, that the Lick Observatory frequently receives inquiries from students and from others, relative to these disparities. It is with the purpose of answering such inquiries, as well as that of making a record of the result of a recent examination, that the following memoranda have been prepared.

It would be difficult at this time to explain the altogether incorrect value of 674 inches given by the makers. The error may be a purely typographical one, due to the accidental substitution of 7 for 9 in the approximately correct figure of 694, or there is the possibility that it was intended to represent the "back focus," a term sometimes applied to the distance from the last surface of the lens to the focal plane, the actual value of which is about 672.5 inches. The origin of the value 694 inches, given by Holden is, on the other hand, easy to account for. Several years previous to the publication of his paper the filar micrometer used with the telescope had been calibrated by Professor S. W. Burnham,⁴ who obtained the value

$$R = 9''.907$$

and the focus was evidently calculated by means of equation (1), under the slightly erroneous assumption that the pitch of the screw is exactly thirty threads to the inch. This gives 694.0 inches. Had the true pitch of the screw, namely 30.06 threads per inch, been used, the result would have been 692.6 inches, in close agreement with the recent measurement. In considering these correspondences it is to be remembered that the focal length changes appreciably with temperature, and, in the normal variation between summer and winter, suffers an alteration of about an inch.

Specific need for a reliable value of the focal length arose recently from the requirements of a special problem, namely, the measurement of the diameter of the

² The formulae employed are those given in Southall, *Principles and Methods of Geometrical Optics*, p. 285, Macmillan, 1910.

³ *Publ. Lick Obs.* 3, 133, 1889.

⁴ *Publ. A. S. P.*, 2, 196, 1890. The method used was that of measuring the differences in declination of pairs of stars. For a revision of Burnham's reductions see page 60 of this Bulletin.

Sun at partial eclipse. Photographs of the eclipse of April 28, 1930, had been taken with the refractor, and, as it was desired to carry the reduction of the measurements to a fraction of a second of arc, a knowledge of the focal length to closer than 1 part in 10,000, or as it works out arithmetically, to $\frac{1}{10}$ of an inch, was necessary. This is the equivalent of a unit in the third decimal place in the expression of R , and as there was little assurance that the value of that quantity in current use ($9''.907$) was dependable to that degree of precision, the measurement of the focal length was undertaken.

As is generally known, the focal length of a lens is the distance from the second principal, or Gaussian, point, to the focus.⁵ It was at first proposed simply to measure, with a steel tape, the distance from the focus to the near surface of the lens, and to calculate the comparatively short distance from there to the principal point. The latter distance, however, turned out to be a matter of 20.9 inches, and it was at once apparent that if the discrepancy between calculated and observed values of the focal length ($719 - 594 = 25$ in.) was to be taken as a measure of the dependability of the computations, the calculated position of the principal point might be in error by $\frac{3}{4}$ of an inch, a quantity twelve times the adopted tolerance of $\frac{1}{10}$ of an inch. It therefore seemed desirable to remove the lens from the telescope for the purpose of determining the position of the principal point by experiment, using the usual test, which involves the rocking of the lens on a transverse axis, in a beam of parallel light.⁶ With the approval of Director Aitken the lens was therefore removed from the telescope, and the observations undertaken.

There are certain practical advantages in the adopted method of procedure, especially where the focal length to be measured is a long one. They originate both in the

⁵ Of the three "cardinal" points, namely the two principal points and the optical center, the second principal point is the only one that directly affects the performance of an astronomical lens, for the reason that the objects observed are so far away that their appearance from all parts of the optical system are the same. It is therefore deemed unnecessary, for the purposes of this paper, when referring to this one of the principal points, continually to qualify it as the second. The term "principal point," without further specification, will therefore be understood to mean the second of the two Gaussians.

⁶ The test, which is described in a number of optical laboratory manuals, is comparatively simple. The lens is placed on edge, and provision made for rotating it about a vertical axis which intersects the optic axis, here assumed to be horizontal. Means are provided for moving the vertical axis in a direction perpendicular to the lens. If, as in the present case, the second principal point is to be located, the back of the lens is toward the observer. Parallel light is admitted through the front of the lens and forms an image at the focus. When the lens is rotated or "rocked" about the vertical axis this image moves to the right or left unless the axis happens to pass through the principal point, in which case the image remains stationary. To determine the position of the principal point this critical position of the axis is found by trial, and its distance from one of the surfaces of the lens measured. For handling the 36-inch lens, which with its cell weighs 537 pounds, unusual arrangements had of course to be made, but upon these it is unnecessary to particularize.

circumstance that the principal measurement—that from the micrometer wires to the lens—is made with the lens in place on the tube, and in the consideration that the dimension determining the size of the image is not in fact the focal length, but, in the event the wire is not precisely in focus, the actual distance of the plane of the wire, which we may call the plane of observation, from the principal point. It will be convenient, for the purposes of this discussion to have a name for the distance just mentioned, and the term “working focal length” is proposed with the idea that its use will be restricted to the present group of papers. We therefore define the *working focal length* (f_w) as the distance of the actual plane of observation (the plane of the micrometer wires, or of the photographic plate) from the second principal point of the objective.

It is well to remember that f_w is not in reality a focal length. It is a dimension depending on the instrumental setting, which controls the *observed* size of the image. It will, of course, usually differ very little from the actual focal length.⁷ The method at present under consideration provides this dimension, or distance, for a given reading of the scale on the focusing draw tube, from which distances corresponding to other readings of the scale can readily be obtained. It is therefore unnecessary, in the reduction of observations, to know the dependence of the focal length upon temperature or upon the spectral characteristics of the body under observation, since the influence of these factors is felt upon the the scale setting.⁸ These considerations rest upon the assumption that the principal point remains fixed with respect to the lens. The assumption is valid only within limits of tolerance, because the position of the principal point, like that of the focus, is influenced both by the temperature of the lens and the color of the object observed. The distance of the principal point from the lens is, however, so small in relation to the focal length⁹ that its variation under these influences may safely be disregarded, especially as it is estimated to be within the range of uncertainty of the experimental determination of the point's position. It is to be observed that f_w differs from the focal length only by errors in the focal setting, so that if it is desirable to know the focal length for a given color and temperature, it can be obtained by making focal settings as exactly as possible, under known conditions with respect to the variables mentioned, and reading the focal scale. The focal length will be the value of f_w corresponding to this reading.

In all the tests of the lens, light from a sodium lamp

⁷ The distinction drawn may, at first sight, appear trivial, and it is questionable whether it would be worth emphasizing in relation to ordinary measurements of small angles with the filar micrometer. In photography, however, where the angles measured are likely to be large, and where the lens is sometimes diaphragmed down (thus increasing the difficulty of focusing), the consideration is an important one.

⁸ For the expression of R in terms of the scale reading see equations (3) and (4).

was employed. This lamp, which was kindly presented to the Lick Observatory by the General Electric Company, provides a light in which the D lines of sodium are dominant. The parallel beam used in experimentally locating the principal point was obtained by reflection from a parabolic mirror of 11 inches aperture, so the tests refer specifically to the circular area of that diameter centrally situated on the lens. In preliminary experimentation it was found possible to rock the lens about 1°5 on either side of the normal position without destroying the definition, and this amount of motion was employed in the tests.

The examination showed the principal point to be 10.84 inches in advance of the front surface of the lens, or 20.24 inches forward from the back surface, with an estimated possible error of 1/32 of an inch. This result is small, with respect to the value calculated from the data provided by the maker, in about the same proportion that the measured focal length is less than the one calculated.⁹ The cause of these discrepancies is not known. In an effort to determine it the lens was taken apart, and the separation and dimensions of the elements, including their radii of curvature, were measured as well as possible with the limited means available. The focal length calculated from the new measurements is 706 inches. This reduces the inequality between calculated and observed values to half the former amount.¹⁰ The disagreement is of course without significance in the present inquiry, which has for its principal purpose only the experimental determination of the optical constants of the lens.

The distance of the principal point from the fourth lens surface being known, it remains to add to it the distance from that surface to the micrometer wires. The principal part of the latter distance was measured with a steel tape. The measurement was made with the telescope vertical, and the tape was used under the standard tension of 10 lbs. The Lick Observatory is indebted to the United States Bureau of Standards for calibrating the particular divisions of the tape that were employed in the measurement. It has been convenient to reduce the measurement to a standard temperature of 62°0 Fahr. = 16°7 Cent. At this temperature the reading of the focal scale, when a star is in focus on the crosswires, is approximately 66mm. With that setting of the scale,

⁹ The quantities emerging from the formulae are in the one instance, the distance between the second principal point of the combined lens and that of its second element, and, in the other, the focal length of the lens. These distances as calculated are 20.35 in. and 719.4 in., and as observed, 19.67 in. and 693.0 in. The calculated values are seen to be respectively 3.4% and 3.8% too large.

¹⁰ There seems to have been some difficulty in identifying the glass actually used in the construction of the lens (see *Publ. Lick Obs.*, 3, 133) but a review of the evidence makes it appear unlikely that an error was made in that respect. It seems within the range of possibility, however, that the samples of glass investigated by Professor Hastings, may not have been truly representative of the blanks used in making the lens.

and at the given temperature, the distance of the wires from the fourth surface of the lens is 672.53 inches. Adding to this the distance of the principal point from the same surface, namely 20.24 inches, we obtain

$$f_w = 692.77 \text{ in.} = 17.5962 \text{ m.}$$

In view of the care exercised in determining the scale setting for focus at the mentioned temperature, this may be regarded as the focal length of the lens for integrated starlight, at the temperature 62° Fahr. The probable error of the determination is estimated to be approximately 1 mm.

The focal length of the refractor is quite dependent upon temperature, the effect of a rise in temperature being to increase it. The increase exceeds the thermal expansion of the telescope tube, and, as a consequence, the focal scale reading varies in the same sense as the temperature. The quantitative relation between the two has not yet been accurately determined for the visual focus. Many years ago Dr. Campbell, in his work with the Mills spectrograph, found that for violet light (4341A), the focal setting increases 0.56 mm per degree Centigrade.¹¹ It would, however, hardly be permissible to assume that the rate is the same for the rays for which the lens is corrected. Dr. Aitken has recently made visual focal settings at three temperatures which yield the approximate relation (5), from which it is seen that the focal scale setting increases about .67 mm per degree. Assuming the coefficient of thermal expansion of the tube to be 1.16×10^{-5} , its increase in length, per degree, is 0.20 mm, making a total lengthening of the focus, per degree Cent. of 0.87 mm. The following approximate expression, which is subject to revision in the light of more complete knowledge of the influence of temperature on the focal setting, results from the foregoing considerations:

$$f = 17.582 + 0.00087t_c \text{ (meters)} \quad (2)$$

(2) MEASUREMENT OF THE PITCH OF THE MICROMETER SCREW AND DERIVATION OF THE VALUE, IN SECONDS OF ARC, OF A SINGLE TURN

The pitch was measured on the Gaertner universal measuring engine, which was in turn calibrated by measuring Scale "A" of the Repsold engine. This scale has been examined on two occasions.¹² In the reductions the data furnished by the more recent examination were used.

Two separate measurements of the screw were made. In the first a small progressive increase in the pitch was indicated. In the second this increase was not found, though both sets showed relatively small values at the extreme left of the run, at a position corresponding to

readings of the micrometer head more than 32 revolutions less than that for coincidence of the wires. The second series of measurements is regarded as the better, and is the only one considered in this discussion.

For the purposes of these measurements the micrometer box was removed from its slide and placed upon the table of the Gaertner Engine, under precautions insuring its proper position and stability. The micrometer was then set for a certain reading, and measurements were taken on the "traveling" wire¹³ for that reading. Measures were made at intervals of 10 revolutions of the micrometer head, insofar as possible, and were repeated twice, using adjacent zero readings of the head, that is to say, the first set of readings was made at 90, 100, 110, . . . the second at 89, 99, 109, . . . and the third at 91, 101, 111, . . . The means were struck for touching values and the result regarded as that for the central value. It is to be remarked that inasmuch as the revolution counter of the micrometer is likely to be put out of step from time to time, the significance of the integral numbers of the revolutions depends upon the reading for coincidence between the travelling and fixed wires. In these measurements the coincidence was approximately at 12.650.

In the second column of the following table are the values of the pitch of the screw in mm, and in the third the corresponding values, in seconds of arc, of a revolution of the micrometer head for a focal setting of the mm scale on the telescope draw tube of 66, based on a focal length of 17.5962 meters (692.77 in.). From equation (1), $\lg R = \lg p + 1.069006$. Both pitch and focal length measurements are reduced to the temperature 62° Fahr. = 16.7 Cent.

Micrometer Head (coincidence=12.6)	p	R
83 to 90 { -27 to -20	0.8446 mm	9'901
90 to 100 { -20 to -10	84476	9024
100 to 110 { -10 to 0	84487	9037
0 to 10	84503	9055
10 to 20	84491	9041
20 to 30	84491	9041
30 to 40	84516	9070
40 to 50	84489	9039
50 to 60	84492	9042

With the exception of those in the first line, the above values show no certain variation throughout the investigated portion of the screw. A least squares reduction of the observations was therefore undertaken. It provided the residuals given in the column 2 of the following table. They are the observed minus calculated positions of the micrometer wire for the several settings of the head. (The original measures, which yielded the residuals, are not recorded in this report.)

¹³ That is to say the wire that is moved by the screw, as distinguished from the fixed wire.

¹¹ *Ap. J.*, 8, 138, 1898.

¹² *Publ. Lick Obs.*, 3, 159, 1894, and *Publ. A. S. P.*, 45, 92, 1933.

Micrometer Head	Residuals ($O-C$) in mm	
	All observations	Omitting obsn. (-27)
-27.000	-0.0025	
-20.	- 4	-0.0015
-10.	+ 13	+ 3
0.	+ 18	+ 11
10	+ 8	+ 3
20	+ 9	+ 6
30	+ 11	+ 10
40	- 13	- 12
50	- 9	- 6
60	+ 9	- 3

The first of the residuals is unquestionably larger than the error of measurement, and, inasmuch as the previous examination of the screw, referred to above, gave a discrepancy of like sign and amount, it was assumed that the fault lies with the screw, and the observation was accordingly discarded. A second reduction, based on the remaining nine measurements, yielded the following value of the pitch:

$$P = 0.844946 \pm 0.000028$$

corresponding to:

$$R = 9''.9045 \pm 0''.00033.$$

In respect to the accuracy of this result, it is to be stated that the probable error of R , as just given, is derived solely from the measures of the pitch of the screw, and does not take into consideration the uncertainty in the determination of the focal length of the telescope. An estimate of the accuracy of that determination can be made only arbitrarily, but it is judged that the chances of the error being greater or less than $1/32$ of an inch are about even. This signifies an uncertainty of about 1 part in 20,000, which is somewhat in excess of the probable error of the pitch. In view of these considerations the value of one revolution of the micrometer screw, for focal setting 66, based on the focal length given above, is:

$$R = 9''.9045 \pm .0005 \quad (3)$$

The correction to R corresponding to any other reading, s , of the mm focal scale is

$$\Delta R = -.000563 (s - 66) \quad (4)$$

If the scale reading s , at which an observation was made, is not known, but the temperature is available, s should be approximately obtainable from the following rough relation, based on three observations by Dr. Aitken:

$$s = 55 + 2/3 t_c \quad (\text{approximate only}) \quad (5)$$

where t_c is the temperature (centigrade); or combining (4) and (5)

$$\Delta R = .0062 - .00037 t_c \quad (\text{approximate only}) \quad (6)$$

Equations (5) and (6) should eventually be corrected on the basis of a more complete investigation of the relation between s and t . It is to be remarked that the value of R requires no adjustment for the change, with temperature, of the length of the telescope tube, since it, and the micrometer screw, being both of steel, presumably expand and contract alike. The temperature correction here considered arises from the circumstance, already noted, that the focal length of the lens increases with temperature more rapidly than does the tube, and it is this excess that is cared for by equation (6).

For the part of the screw corresponding to readings more than 30 revolutions *under* the reading for coincidence of the wires, the value of R appears to be less than given by relation (3). In the other direction the screw appears to be reliable as far as it was measured. The screw was not examined for periodic errors.

Note.—Several determinations of the value of R have been made since the 36-inch refractor first came into use.¹⁴ The value that has generally been employed is $9''.907$, resulting from early observations by Burnham. That value was obtained by measuring differences in declination of pairs of stars whose positions were assumed to be known. After the completion of the foregoing discussion, Burnham's reduction of his observations was examined, with the result that it appeared advisable to rediscuss his measurements with the aid of more reliable star positions than were available at the time of his work. The new reductions have been performed by Mr. Clifford Smith, who describes them in the note following. The revised value is:

$$R = 9''.9022 \pm 0.0021.$$

It will be observed that the revision moves the value from the positive to the negative side of the one derived in this paper. As has been pointed out, R is subject to change with temperature. Using equation (6) it is found that the revised Burnham value would correspond to a temperature of 22° Cent., on the system of the present paper. Burnham did not record the temperatures at which he observed, but it is doubtful whether they averaged as high as 22° . The discrepancy is, after all, within the limits of tolerance suggested by the probable errors.

It is quite obvious that, in relation to the small angular distances usually measured with the filar micrometer, the value $R = 9''.907$, which has generally been employed in the reduction of observations, is sufficiently close for ordinary purposes. The difference between calculations based upon it and upon $9''.9045$ amounts to less than $0''.1$ for angles smaller than 7 minutes of arc.

¹⁴ *Publ. A. S. P.*, 2, 196, 1890; *Publ. Lick Obs.*, 5, 11, 1901.