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A STUDY OF THE SPECTRA OF ECLIPSING BINARIES

BY

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INTRODUCTION

Our knowledge of the eclipsing binary stars has made rapid progress in recent years. According to Prager's *Katalog und Ephemeriden Veränderlicher Sterne für 1934*, more than 800 variables are known to be eclipsing binaries. New ones are being discovered at the rate of 75 a year. Although photometric orbits have been computed for about 300 systems, it would perhaps be too optimistic to consider more than two-thirds of them very reliable.

The accumulation of spectroscopic data has been far in arrear of the photometric measures. The variables for which the spectral types are well determined number less than 200. Half of these spectral estimates can be said to refer to the bright components, because of the faintness of the secondaries; the rest are rendered more or less uncertain by the composite nature of the spectra. The most valuable spectroscopic work, namely, the computation of the absolute orbits of both components, is necessarily difficult. The faintness of the stars, the shortness of the periods, and the diffuse character of the spectral lines, all combine to hinder progress in this important work.

There are relatively few systems in which both components have been observed spectroscopically. Heretofore these have represented, in general, a selected group in which the primary and secondary are of the same order of photographic brightness. It seemed worth while, therefore, to observe spectroscopically both components of as many eclipsing binaries as possible, to determine the spectral types, and to examine them with regard to the other known properties of the systems.

INSTRUMENTS

The 2-prism spectrograph of the 36-inch refractor is admirably suited for such an investigation. The $3\frac{1}{2}$ -inch

camera (f 1.9) used with this instrument gives a linear dispersion at $H\gamma$ of 130 Å/mm. The definition is excellent over the entire instrumental range (about λ 3900 to λ 4900 for an early-type star). With an Imperial Eclipse plate (H. & D. 850), a well-exposed spectrogram of a 12th magnitude star can be obtained in three hours.

For some of the brighter stars the 6-inch camera was used in the 2-prism spectrograph. With this combination a dispersion of 75 Å/mm at $H\gamma$ is realized, and the spectra are in good focus throughout the region mentioned above.

Alternatively, frequent use was made of the 2-prism quartz slitless spectrograph of the Crossley reflector. This is only slightly faster than the first instrument indicated above.

In the selection of an emulsion, definition had to be sacrificed for speed, in view of the faintness of the stars, and also because of the limit set on the exposure time by the light variation. Eastman 40 plates were employed when conditions permitted; otherwise the exposures were made on Imperial Eclipse plates.

OBSERVATIONAL MATERIAL

It would have been desirable to make the observing list representative of eclipsing binaries as a whole. Unfortunately this was impossible, for obvious reasons. Actually, the selection was made largely from Algol variables with flat minima. The larger component, which was generally the fainter of the two in total brightness, was observed at principal minimum, when the small, bright star was completely hidden throughout the exposure. The bright star was ordinarily observed at secondary minimum, at which time the light of the faint star was generally inappreciable. When the two components differed in total brightness by considerably more than a

magnitude, however, the bright star was observed during the maximum of the light curve. Experience has shown that under these conditions the effect of the secondary is negligible.

In addition, a few binaries with partial or grazing eclipses were included, when it seemed clear that the primary was practically hidden during the exposure at principal minimum. Most stars of the β *Lyrae* and *W Ursae Majoris* types were considered to be ill-adapted to the investigation, either because of their faintness and shortness of period, or because of the lack of accurate light curves.

Of prime importance is a precise knowledge of the epoch and period of light variation. These elements were taken from Prager's catalogue for 1933. Dr. McLaughlin kindly furnished the elements of *RS Ari*.¹

Two spectrograms of each component were obtained whenever possible. While this was accomplished for most of the primaries, it was found impossible for about half the secondaries, on account of the longer exposures and the interference of the Moon. Since most of the faint companions had to be observed while the Moon was below the horizon, the opportunities for observation were relatively few.

Details of the times of the individual exposures are appended in Table Ia at the end of this paper. Since subsequent photometric observations may disclose inaccuracies in the present light-elements, and thereby influence the interpretation of the spectrograms, it is thought advisable to publish these details. Column 1 of Table Ia gives the name of the star; columns 2 and 4 the time of mid-exposure (Julian Day, G.M.T.) for the bright and faint components respectively; and columns 3 and 5 the exposure time in minutes for the bright and faint components respectively. It will be noted that the exposure time gives no indication of the brightness of the star, because different spectrographs and different plate emulsions have been used according to individual circumstances.

SPECTRAL CLASSIFICATION

The spectra were classified according to the Mount Wilson system, which does not differ systematically from the Harvard system except at the two ends of the spectral sequence. Twenty-two stars of the Mount Wilson lists,² ranging from B0 to K5, were photographed with each instrument, and the spectra were used for comparison. The region from G0 to K5 included both giants and dwarfs according to the Mount Wilson classification. To supplement the use of standard stars, frequent reference was made to the Harvard descriptions of stellar spectra.³

The chief criteria of classification were based on the intensities of the following lines:

¹ Min. = J.D. 242 6326.822 + 8^h80327 E.

² *Ap. J.* 53, 13, 1921; 56, 242, 1922; 57, 294, 1923.

³ *H. A.* 91, 6-9, 1918.

Spectral Class

LINES

B	<i>He I</i> 4026, 4143, 4388, 4471; <i>Mg II</i> 4481; <i>Si IV</i> 4089, 4116.
A	K-line; enhanced metallic lines; <i>Ca I</i> 4227.
F	K-line; <i>Ca I</i> 4227; G-band; enhanced metallic lines.
G-K	<i>Ca I</i> 4227; G-band; enhanced metallic lines.

In the G- and K-type spectra, consideration was also given to the spectral energy distribution, especially in the classification of slitless spectrograms. The hydrogen lines were not taken as standards, first, because there is evidence that they vary in intensity with the absolute magnitude of the star, and second, because the Balmer series is abnormal in several of the faint components of eclipsing binaries.

All spectrograms were classified twice independently, so that four estimates were obtained for each star (or two estimates when only one plate was taken). The adopted spectral type was the mean of all the determinations. The agreement of the different estimates was gratifying, the average deviation from the mean (regardless of sign) being 0.06 of a spectral class. Of course no such accuracy is claimed for the spectral classification. The scatter would certainly be much larger if we were to compare the estimates of two different observers, or those of one observer using various degrees of dispersion. It is considered that the spectral determinations are in no case in error by more than half a spectral class, and generally they should be correct to two or three tenths. This point has an important bearing on the subsequent discussion of the surface brightnesses of stars.

No attempt was made to derive the absolute magnitudes of the stars from their spectra. With low-dispersion slit spectrograms it was found possible to distinguish only roughly between giant and dwarf stars of types G and K; this was done whenever practicable by noting the intensities of the lines $\lambda 4215$ and $\lambda 4077$ of *Sr II*. The other lines that have been used at Mount Wilson in the determination of absolute magnitude were too badly blended to be serviceable with such low dispersion. The distinction between dwarfs and giants was not perceptible on the slitless spectrograms taken under average conditions: the strontium lines were not clearly marked, and the exposures were too short to bring out the cyanogen band at $\lambda 3883$.

DATA

Table I contains the spectral types of 25 eclipsing binaries observed by the writer, together with other data essential to the discussion. Table II contains similar information for 28 systems studied spectroscopically by other investigators. The first six columns give the number and designation of the star, the spectral type of each component, the period, and the difference in visual magnitude of the two components as determined from the light curve. The letter "b" refers to the component that contributes more than half the visual light of the

system, while "f" indicates the other component. This convention is adopted for uniformity, and is observed throughout the discussion. Column 7 gives the ratio of visual surface brightness expressed in magnitudes, ΔM_J ; the values are derived from the light curves. Column 8 contains the same quantity computed from the spectral types. This calculation is described in the section on *Ratio of Surface Brightness*. In columns 9 and 10 are listed the mean densities of the bright and faint components respectively. Hypothetical densities are given in

brackets. The method employed in their computation is described in another part of this paper. The footnotes to Tables I and II give source references to the photometric and spectroscopic orbits that have been used in the present study. The hypotheses of uniform disks and limb darkening are noted by the letters "U" and "D" respectively. Remarks concerning the spectra of some of the stars in Table I are found in the footnotes.

The writer is indebted to Mr. A. H. Joy for the unpublished spectral types of *i Boo*, *S Cnc*, and *U Cep*.

TABLE I

DATA CONCERNING 25 ECLIPSING BINARIES OBSERVED BY THE WRITER

No.	Star	Sp _b	Sp _f	P	ΔM_{vis}	ΔM_J (f-b)	ΔM_{Jsp} (f-b)	ρ_b ($\odot=1$)	ρ_f
1	SY And	A0	K1	34.9	1.18	+3.4	+3.8	[0.10]	[0.0021]
2	TW And	dF0	G6	4.12	1.53	2.6	1.8	[.12]	[.018]
3	WW And	A5	F3p	23.3	0.62	1.2	0.9	[.031]	[.0088]
4	RS Ari	F9	G5	8.80	0.44	$\geq +1.4$	+0.8	[.091]	[.047]
5	SS Cam	gG1	dF5:	4.82	0.62	-1.7	-0.9	[.0045]	[0.073]
6	SV Cam	dG5	dG3	0.593	0.06	-1.4	-0.1	[.40]	[3.0]
7	RU Cnc	dF9	dG9	10.2	1.25	+3.1	+1.3	[.23]	[0.0095]
8	RZ Cnc	K0	K4	21.6	1.01	1.2	1.2	[.0014]	[.0011]
9	RW Cap	A3	A4	3.39	1.42	1.4	0.2	[.067]	[.030]
10	TW Cas	B9	A0	2.86	0.36	0.4	0.1	[.21]	[.16]
11	XX Cas	B4n:	B6n:	3.07	0.77	1.2	0.3	[.038]	[.014]
12	UZ Cyg	A3	K1	31.3	1.67	3.2	3.3	[.016]	[.00089]
13	TW Dra	A6	K2	2.81	2.22	3.6	3.1	[.21]	[.012]
14	RX Gem	A2	A3p	12.2	1.97	≥ 3.5	0.2	[.039]	[.0014]
15	RY Gem	A2	K2p	9.30	1.82	+3.5	3.7	[.074]	[.0036]
16	AD Her	A4	K2	9.77	2.04	≥ 3.1	+3.4	[.042]	[.0039]
17	AW Her	K2	G4	8.80	0.16	≤ -2.0	-1.1	[.0085]	[.13]
18	TT Hya	A3	dG6p	6.95	1.51	≥ 3.2	2.7	[0.52]	[0.0061]
19	SW Lac	G3p	G3p	0.321	0.18	+0.3	0.0	[1.6]	[1.2]
20	AR Lac	K0	G5	1.98	0.06	-1.0	-0.6	0.048	0.21
21	UX Mon	A5	dG1p	5.90	1.00	+2.0	+1.8	[.031]	[.0035]
22	RZ Oph	G0	gK5p	262.	0.13	2.7	3.2	[.00075]	[.000034]
23	RY Per	B6k	dF8	6.86	2.24	2.9	2.8	[.042]	[.0031]
24	V356 Sgr	B9	A1	8.90	0.46	≥ 1.1	0.3	[.011]	[.0032]
25	RW UMa	dF9	dG9	7.33	0.53	+2.8	+1.3	[0.38]	[0.014]

REFERENCES AND NOTES

- 1 D Shapley, *Pr. Cont.* No. 3, 1915.
- 2 D Dugan, *Pr. Cont.* No. 14, 1933.
- 3 D Wyse, Table III. Light curve: Leiner, *A. N.* 229, 273, 1927. In the spectrum of the secondary, $H\beta$ is weak, $H\gamma$ is very sharp, while $H\delta$ and $H\epsilon$ are strong and broad.
- 4 Wyse (without light curve). The primary spectrum is intermediate between giant and dwarf.
- 5 D McDiarmid, *Pr. Cont.* No. 7, 1924. Although the spectrum is composite, with the small, faint component contributing only about 60 per cent of the photographic light at secondary minimum, it definitely shows the dwarf characteristics, whereas the larger star is a giant. The uncertainty of the secondary spectrum is due to the presence of the primary spectrum on the same plate.
- 6 U Dunst, *A. N.* 249, 213, 1933.
- 7 D Wyse, Table III. Light curve: Nijland, *A. N.* 242, 65, 1931.
- 8 U Hertzsprung, *B. A. N.* 4, 156, 1928.
- 9 D Shapley, *Pr. Cont.* No. 3, 1915.
- 10 D McDiarmid, *Ap. J.* 42, 412, 1915.
- 11 D Wyse, unpublished. Light curve: Leiner, *A. N.* 222, 137, 1924.
- 12 D Shapley, *Pr. Cont.* No. 3, 1915.
- 13 D Shapley, *Pr. Cont.* No. 3, 1915. The secondary spectrum is intermediate between giant and dwarf.
- 14 Wyse (without light curve). In the spectrum of the secondary, $H\beta$ is weak, $H\gamma$ is very sharp, while $H\delta$ and $H\epsilon$ are strong and broad.
- 15 D Wyse, Table III. Light curve: Beyer, *A. N.* 234, 88, 1928. The spectrum of the secondary is intermediate between giant and dwarf. $H\beta$ is a weak emission line, and the spectrum is continuous at $H\gamma$.
- 16 Wyse (without light curve).
- 17 Wyse (without light curve).
- 18 D M. Shapley, *Harv. Bull.* No. 847, 1927. The spectrum of the secondary contains bright hydrogen lines. On the plate taken J.D. 2427521.873, $H\delta$ is very weak both in emission and absorption, the emission line appearing on the less refrangible edge of the absorption line. The same is true of $H\epsilon$, except that the lines are stronger and clearer both in emission and absorption. The emission at $H\gamma$ is very doubtful, and the absorption line is blended with $\lambda 4338$ of *Ts II*. On the plate taken J.D. 2427542.805, $H\beta$ appears clearly as an emission line in the normal position; the spectrum is continuous at $H\gamma$, and $H\delta$ is an absorption line. Observations at principal minimum should be continued.
- 19 D Schilt, *B. A. N.* 2, 175, 1924. The lines of both components are very diffuse. The spectrum is classified at Mount Wilson as G2p, *Pub. A. S. P.* 36, 139, 1924.
- 20 U Schneller & Plaut, *A. N.* 245, 387, 1932. Spectroscopic orbit by Harper, *R. A. S. C.* 27, 146, 1933. The K0 star is the fainter component in photographic light; corrected for color index, it is the brighter of the two in visual light. The K0 spectrum has sharp emission lines H and K of *Ca II*, superposed on the broad absorption lines.
- 21 D I. Woods & M. Shapley, *Harv. Bull.* No. 854, 1928. The iron and hydrogen lines in the spectrum of the secondary are weak relative to $\lambda 4227$.
- 22 D Shapley, *Pr. Cont.* No. 3, 1915. In the secondary spectrum $H\beta$ is bright and fairly strong; the spectrum is continuous at $H\gamma$.
- 23 D Shapley, *Pr. Cont.* No. 3, 1915.
- 24 U Wyse, unpublished. Light curve, Voûte, *Lemb. Ann.* V2, 4e, p48, 1932.
- 25 D Shapley, *Pr. Cont.* No. 3, 1915.

TABLE II
DATA CONCERNING 28 ECLIPSING BINARIES OBSERVED SPECTROSCOPICALLY BY OTHER INVESTIGATORS

No.	Star	Spb	Spf	P	ΔM_{vis}	ΔM_J (f-b)	ΔM_{Jsp} (f-b)	ρ_b	(C=1)	ρ_l
1	S <i>Ant</i>	A8	A8:	0.648	0.77	+0.2	0.0	0.31		0.38
2	σ <i>Aql</i>	B8n	B8n	1.95	0.32	0.3	0.0	.15		.12
3	TT <i>Aur</i>	B3	B3	1.33	0.82	0.5	0.0	.11		.12
4	WW <i>Aur</i>	A7	A7	2.53	0.28	0.2	0.0	.21		.21
5	β <i>Aur</i>	A0p	A0p	3.96	0.00	0.0	0.0	0.14		0.14
6	i <i>Boo</i>	G2	G2	0.268	0.00	0.3	0.0	[2.2]		[2.2]
7	S <i>Cnc</i>	A0	G5	9.48	1.96	3.4	+3.0	[0.13]		[0.0055]
8	RS <i>CVn</i>	F4n	dG8	4.80	0.87	3.5	1.6	.45		.012
9	SX <i>Cas</i>	A6	G6	36.6	0.18	1.8	2.8	[.0045]		[.00048]
10	TV <i>Cas</i>	A0	A0	1.81	1.83	2.1	0.0	.12		.045
11	AO <i>Cas</i>	O8.5n	O8.5n	3.52	1.11	0.2	0.0	.0027		.009
12	U <i>Cep</i>	A0	G8	2.49	1.79	2.8	3.4	[.22]		[.018]
13	U <i>CrB</i>	B3	B3	3.45	0.77	1.3	0.0	.18		.016
14	Y <i>Cyg</i>	O9.5n	O9.5n	3.00	0.35	0.4	0.0	0.16		0.16
15	GO <i>Cyg</i>	B9n	A0n	0.718		1.4	0.1			
16	YY <i>Gem</i>	dM1e	dM1e	0.814	0.26	0.0	0.0	1.4		1.8
17	Z <i>Her</i>	F2	F2	3.99	0.30	1.6	0.0	0.29		0.037
18	RX <i>Her</i>	A0	A0	1.78	0.37	0.1	0.0	.28		.34
19	TX <i>Her</i>	A2	A2	2.06	0.62	0.6	0.0	.52		.45
20	u <i>Her</i>	B3	B3	2.05	0.56	0.6	0.0	.030		.012
21	RT <i>Lac</i>	G9	K1	5.07	0.37	0.4	0.3	.0089		.017
22	U <i>Oph</i>	B5n	B5n	1.68	0.15	0.2	0.0	.20		.18
23	AG <i>Per</i>	B3	B3	2.03	1.05	0.3	0.0	.10		.25
24	V <i>Pup</i>	B1	B3	1.45	0.44	0.2	0.3	[.054]		[.042]
25	U <i>Sge</i>	B9n	gG2	3.38	2.66	3.2	3.1	0.091		0.012
26	W <i>UMa</i>	F8p	F8p	0.334	0.00	0.0	0.0	2.8		1.9
27	Z <i>Vul</i>	B3n	B3n	2.45	1.93	1.8	0.0	0.056		0.032
28	RS <i>Vul</i>	B8	B9	4.48	1.53	+2.1	+0.2	0.061		0.0086

REFERENCES

- 1 U Joy, *Ap. J.* 64, 287, 1926.
 2 U Wylie, *Ap. J.* 56, 232, 1922.
 Jordan, *Allegheny O.* 3, 199, 1916.
 3 U Joy & Sitterly, *Ap. J.* 73, 77, 1931.
 4 U Dugan, *Pr. Cont.* No. 10, 1930.
 Joy, *Pub. A. S. P.* 30, 253, 1918.
 5 U Stebbins, *Ap. J.* 34, 112, 1911.
 Baker, *Allegheny O.* 1, 163, 1910.
 6 Schilt, *Ap. J.* 64, 215, 1926.
 Joy, MS.
 7 D Shapley, *Pr. Cont.* No. 3, 1915.
 Joy, MS.
 8 U Sitterly, *Pr. Cont.* No. 11, 1930.
 Joy, *Ap. J.* 72, 41, 1930.
 9 U Dugan, *Pr. Cont.* No. 13, 1933.

- 10 D McDiarmid, *Pr. Cont.* No. 7, 1924.
 Plaskett, *Pub. D. A. O.* 2, 141, 1922.
 11 U Pearce, *Pub. D. A. O.* 3, 275, 1926.
 12 0.6D Dugan, *Pr. Cont.* No. 5, 1920.
 Joy, MS.
 13 Dugan, MS.
 Plaskett, *Pub. D. A. O.* 1, 137, 1920.
 14 D Shapley, *Pr. Cont.* No. 3, 1915.
 Redman, *Mon. Not.* 90, 754, 1930.
 15 Pearce, *R. A. S. C.* 27, 62, 1933; spectroscopic orbit only.
 16 U Joy & Sanford, *Ap. J.* 64, 250, 1928.
 17 D Shapley, *Pr. Cont.* No. 3, 1915.
 Adams & Joy, *Ap. J.* 49, 192, 1919.
 18 U Sanford, *Ap. J.* 68, 51, 1928.
 19 U Baker, *Laws Obs. Bull.* 3, 1, 1921.
 Plaskett, *Pub. D. A. O.* 1, 207, 1920.
 20 D Baker, *Laws Obs. Bull.* 2, 209, 1917.

- 21 D Fowler, *Ap. J.* 52, 257, 1920.
 Joy, *Ap. J.* 74, 101, 1931.
 22 U Shapley, *Pr. Cont.* No. 3, 1915.
 Plaskett, *Pub. D. A. O.* 1, 137, 1919.
 23 D Huffer, *Washb. Pub.* 15, 192, 1931.
 Plaskett, *Pub. D. A. O.* 3, 188, 1925.
 24 D Shapley, *Pr. Cont.* No. 3, 1915.
 Maury, *H. A.* 28, 177, 1901.
 25 U Shapley, *Pr. Cont.* No. 3, 1915.
 Joy, *Ap. J.* 71, 336, 1930.
 26 U Shapley, *Pr. Cont.* No. 3, 1915.
 Adams & Joy, *Ap. J.* 49, 189, 1919.
 27 U Baker, *Laws Obs. Bull.* 2, 173, 1916.
 Plaskett, *Pub. D. A. O.* 1, 251, 1920.
 28 U Dugan, *Pr. Cont.* No. 6, 1924.
 Plaskett, *Pub. D. A. O.* 1, 141, 1919.

Limiting values of the photometric elements have been computed for *RS Ari*, *RX Gem*, *AD Her*, and *AW Her*, by making use of the data in Prager's catalogue for 1934. The assumption of 90° for the inclination of the orbit planes results in minimum values of the ratio of surface brightness, J_b/J_f , and consequently for ΔM_J in column 7. The latter statement is also true of *V356 Sgr*.

The photometric orbits of *WW And*, *RU Cnc*, *RY Gem*, and *XX Cas*, referred to in the notes to Table I, are not definitive, because the observations are not of the highest accuracy. The orbits of the first three of these, however, are necessary to the subsequent discussion, and although they are only provisional, it is thought desirable to give a brief summary of the results. The orbital elements of *WW And*, *RU Cnc*, and *RY Gem*, both uniform and darkened, are listed in Table III. The methods of Russell,⁴ and of Russell and Shapley,⁵ were used in

⁴ *Ap. J.* 35, 315, 1912. ⁵ *Ap. J.* 36, 239, 1912.

the computation. Figure 1 gives the comparison of the theoretical light curves (darkened) with the observations, together with diagrammatic representations of the relative orbits (darkened).

WW And.—The orbit is based on 90 eye-estimates made by Leiner by the Argelander method. The light-range is taken as $1^m.1$ from Prager's catalogue. The observations are poorly distributed, and the orbital elements are quite uncertain in some respects. Principal minimum can be satisfactorily represented as the result of the eclipse of a small, bright star by a larger, fainter one. The orbit agrees fairly well with that of Gaposchkin,⁶ who used the same observations. Gaposchkin's value of 0.990 for the light of the bright star is doubtless a typographical error.

RU Cnc.—Shapley's⁷ and Fetlaar's⁸ orbits differ so

⁶ *Veröff. Berl.-Bab.*, Bd. 9, Hf. 5, p. 40, 1932.

⁷ *Pr. Cont.* No. 3, 1915. ⁸ *B. A. N.* 6, 29, 1930.

widely that the computation was repeated by the writer. The observations are probably the same as those used by Fetlaar, namely 282 eye-estimates by Nijland. The re-

sults of the uniform solution are almost identical with Fetlaar's orbit. It should be mentioned that Shapley had only one-fourth of these observations available in 1915.

TABLE III

ELEMENTS OF *WW Andromedae*, *RU Cancri*, AND *RY Geminorum*

	<i>WW Andromedae</i>	<i>RU Cancri</i>	<i>RY Geminorum</i>	
B. D. Number		+24°1959	+15°1573	
α	23 ^h 39 ^m 9	8 ^h 31 ^m 6	7 ^h 21 ^m 7	
δ	+45°8'	+23°54'	+15°51'	
Spectral class of bright star	A5	dF9	A2	
Spectral class of faint star	F3p	dG9	K2p	
Maximum	10 ^m 3	9 ^m 94	8 ^m 35	
Principal minimum	11 ^m 4	11 ^m 47	10 ^m 33	
Epoch	J.D. 2422207.14	2422650.720	2418015.324*	
Period	23 ^d 28564	10 ^d 172988	9 ^d 30090	
Nature of eclipse at principal minimum				
	Total		Total	
Hypothesis	U	D	U	D
k , ratio of the radii	0.690	0.750	0.370	0.415
r_b , radius of bright star	.071	.078	.063	.071
r_f , radius of faint star	.103	.104	.171	.171
$\cos i$, cosine of the inclination	.000	.012	.000	.000
L_b , light of bright star	.64	.64	.76	.76
L_f , light of faint star	0.36	0.36	0.24	0.24
J_b/J_f , ratio of surface brightness	3.7	3.1	23.	18.
ρ_b , hypothetical density of bright star ($\odot=1$)	0.042	0.031	0.33	0.23
ρ_f , hypothetical density of faint star ($\odot=1$)	0.0090	0.0088	0.0095	0.0095

* The epoch is that given by Beyer, *loc. cit.*

Fig. 1. Theoretical light curves and orbital models of *WW And*, *RU Cnc*, and *RY Gem*. The normal magnitudes of *WW And* and *RU Cnc* are those published by the observers. Those of *RY Gem* were formed by the writer from Beyer's individual observations, and were corrected in phase by +0°005. The smallest dots represent single observations, and the larger ones normal magnitudes. The unit of phase in the diagram is 1°0 for *WW And*, and 0°5 for *RU Cnc* and *RY Gem*.

The light curve is rather insensitive to variations in the ratio of the radii, and hence this constant cannot be determined with certainty without the most refined observations. The adopted value of k is close to the maximum value permissible, and therefore the ratio of surface brightness, J_b/J_f , is near its minimum possible value.

RY Gem.—The observations consist of 77 eye-estimates by Beyer, well distributed throughout the principal eclipse. It was found necessary to decrease Beyer's epoch of principal minimum by 0°005 in order to make the ascending and descending branches of the light curve equidistant from the point of mid-totally. In Figure 1 the phases of all the observations have been increased by 0°005.

HYDROGEN EMISSION LINES

The eclipsing binaries on the whole appear to have perfectly normal spectra. This applies to the faint companions as well as to the primaries. Exception is noted in an unexpected tendency of the secondaries to display bright hydrogen lines. *RY Gem ft*, *TT Hya ft*, and *RZ Oph ft* show $H\beta$ clearly in emission. In *RZ Oph*, $H\beta$ is bright at all phases; but it is sufficiently stronger at principal minimum to make certain that the emission belongs chiefly if not entirely to the spectrum of the faint component. In the secondary spectra of *WW And* and *RX Gem* the Balmer lines are progressively weaker

toward the less refrangible region, indicating that $H\alpha$ is probably bright. Thus out of the 25 secondaries in Table I, five have the characteristic of hydrogen emission. The interpretation of this phenomenon is not at all evident. One does not ordinarily expect to find bright hydrogen lines in A-, F-, G-, and K-type spectra.

The hypothesis that we are dealing with the flash spectrum of the bright component offers interesting possibilities. If we limit ourselves to direct observational evidence, we know little or nothing about the chromosphere of any star except the Sun. It is likely that stars of low density have much more extensive chromospheres than the Sun, and one might therefore expect that the chromosphere of the primary of an eclipsing binary would, in some cases, reach out far beyond the photosphere of the secondary at the times of second and third contact. The light of such a chromosphere would be too small to influence the light curve seriously. Concentrated in a few bright lines, however, the chromospheric radiation might well be of sufficient intensity to record itself on plates taken during total eclipse near second or third contact, provided the primary was considerably brighter than the secondary.

It must be remembered that the limb of the eclipsing body is not sharp like the limb of the Moon. At second or third contact, when the photosphere of the primary is just hidden behind the photosphere of the secondary, any chromospheric radiation from the primary must, before reaching the observer, pass through the outer ring of the secondary's apparent disk, i.e., through part of the secondary's chromosphere. Although some of the bright line radiation will be absorbed here, and re-emitted, for the most part, in the Lyman frequencies, it is not to be expected that the loss will be very great. Another factor, compensating at least partially for the depletion, will be at work. Atoms in the outer ring of the secondary's disk (in the chromosphere) will be excited by the high-frequency radiation of the primary's photosphere, and some of them will emit bright-line radiation in the direction of the observer. That is, the secondary will emit a kind of flash spectrum due to the presence of the primary, and this spectrum *may* be comparable in importance to the flash spectrum of the primary itself.

Whether or not these bright-line radiations will be blotted out by the normal absorption spectrum of the secondary depends upon conditions of which we are ignorant at the present time. It is not impossible that they may be noticeable in the most favorable cases.⁹

⁹ A one-hour exposure of *RW Tau* (8^m1-11^m5, A0, period variable) was taken on 1933 Nov 21 with the 2-prism spectrograph (3½-inch camera). The predicted time of eclipse, corrected for light time, was J.D. 242 7398.870 (G.M.T.), and the time of mid-exposure was four minutes later. The plate showed an A-type spectrum, on which were superimposed the bright lines $H\beta$, $H\gamma$, $H\delta$, and probably $\lambda 4481$ of *Mg II*. The emission lines were displaced to the violet edge of the absorption lines. The star was observed to increase slightly in brightness toward the end of the exposure, and it later became evident that the absorption spectrum was caused by the light of the primary. The 11^m5 secondary

Further light on the subject can be obtained from a comparison of stars showing hydrogen emission with similar stars showing no emission. *RZ Oph* can be dismissed from the discussion: $H\beta$ is bright at all phases, and hence its origin cannot be ascribed to a peculiar geometrical configuration. *WW And*, *RX Gem*, *RY Gem*, and *TT Hya*, all of which have hydrogen emission or at least a tendency in that direction, are shown in Figure 2, in the orientation in which the secondaries were observed. *TW Dra* and *RY Per*, which have no bright lines, are also shown. The small circle in every case represents the primary, located at the center of the relative orbit. The two large circles represent the secondary at the beginning (left) and at the end (right) of each exposure. The diagrams are based on darkened solutions (except *RX Gem*), and on Prager's light-elements. Although the photometric elements of *WW And*, *RY Gem*, *TT Hya*, and especially *RX Gem*, are not very certain, the quantity that concerns us here is rather well defined. This quantity is the smallest distance between the limbs of primary and secondary, in terms of the radius of the primary. A considerable change in the inclination and in the ratio of the radii affects this quantity only slightly.

In *WW And*, the primary is only 0^m6 brighter than the secondary, a fact that casts grave doubt on the flash spectrum hypothesis. Furthermore, if the period is correct, the primary began to appear during the middle of the exposure. To the writer it seems impossible that the primary's chromosphere could have any noticeable effect under such conditions. *TW Dra* would seem to be a more favorable case for bright lines, but in this star they do not appear.

In *RY Gem* the limbs of the primary and of the secondary are without doubt fainter than the 12th magnitude photographically, and is therefore too faint to register its spectrum in one hour.

It was next proposed to take a series of short exposures during principal minimum and immediately thereafter. The desirability of making the exposures as short as possible is obvious. From the intensities of the bright lines, it was judged that $H\beta$ and $H\gamma$ would probably appear in 15 minutes. Accordingly on 1933 Dec 16 seven consecutive 15-minute exposures were made. The four plates during minimum light showed no trace of a spectrum. The three that were taken while the bright star was emerging from eclipse showed perfectly normal absorption line spectra. Incidentally, these plates furnished the material for an ephemeris correction of approximately -0^d013, or -19^m.

The first plate was now repeated, at the same phase and under the same conditions, 1934 Jan 10. Strangely enough, no bright lines were visible, although the absorption spectrum was present with the same intensity as before. Unfortunately there has been no opportunity to continue observations of this star.

The possibility that the star was misidentified at the time the first plate was taken is exceedingly remote. The observed star was of the brightness to be expected, and appeared to increase slightly in brightness at the time predicted by the corrected ephemeris. A search of the field on two spectrograms taken with the slitless spectrograph of the Crossley reflector on 1933 Nov 10 and 23 failed to disclose any stars with bright lines.

In the present state of affairs, the only possible conclusion is that the bright lines are variable, and that they cannot be interpreted as the flash spectrum of the bright component. The chromosphere of the secondary may vary as a whole, giving rise to variable emission lines. Further spectrographic observations of *RW Tau* should be made, and a thorough search of the field is highly desirable. Even if the bright-line spectrum should prove to belong to another star, it would be an interesting discovery.

dary are apparently never very far apart at their nearest point. A glance at the figure, however, and at Table I, shows that *RY Per* is a more favorable case for a flash spectrum than *RY Gem*. The densities and magnitude differences in the two systems are very similar, whereas the spectral types of *RY Per* are somewhat earlier than

Fig. 2. Orientations of *WW And*, *TW Dra*, *RY Gem*, *RX Gem*, *TT Hya*, and *RY Per* at the phases at which the secondaries were observed. The small circles represent the primaries at the centers of their relative orbits. The large circles represent the secondaries at the beginning (left) and at the end (right) of each exposure. The diagrams are based on darkened solutions (except *RX Gem*), and on Prager's light-elements.

those of *RY Gem*. The B-type primary of *RY Per* has an abundance of ultra-violet radiation to provide the mechanism for the formation of bright hydrogen lines, and yet the three plates of *RY Per* show no trace of emission. The same argument can be applied to the cases of *RX Gem* and *TT Hya*, regardless of what inclinations are assumed for the orbit planes. In this connection the reader is referred to the section on the *Ratio of Surface Brightness*, in which it is shown that *RX Gem* is probably a three-body system.

Although the evidence is by no means exhaustive, it seems difficult to interpret the hydrogen emission of the above four binaries as arising either from the chromo-

sphere of the primary, or from the outer ring of the secondary's chromosphere, which is excited by the primary. The hypothesis should be reserved as a possibility, however. The most promising line of attack would be to take a series of exposures during totality, and to search for any variation in the appearance of the bright lines as the two stars change their relative positions.

The spectra of several other eclipsing binaries contain bright hydrogen lines. Notable cases in Table II are *SX Cas br* and *YY Gem*.

DIFFERENCE IN SPECTRAL CLASS BETWEEN THE TWO COMPONENTS

The bright lines and a few other peculiarities are noted beneath Table I, but in general, the spectra of the eclipsing binaries are, as has already been remarked, quite similar to ordinary spectra, and we shall treat them as such.

The spectral types of the secondaries are plotted against those of the primaries in Figure 3. It is seen that only four of the faint companions are earlier in the spectral sequence than their primaries, while 29 are later.

Fig. 3. Relation between the spectra of the primaries and secondaries of the eclipsing binaries.

The spectra of the two components of the remaining 20 pairs are of the same class. The paucity of stars in the first group is to be expected on the basis of observational selection. If, for example, a bright G-type star is coupled with a fainter A-type companion, the former, being a giant, will undoubtedly have a much larger radius than the latter. In this case the loss of light is comparatively small at both eclipses, and the probability of discovery is correspondingly remote. This is the same argument that was set forth by Shapley¹⁰ to explain similar facts relating to the light curves.

¹⁰ *Pr. Cont.* No. 3, 113-114, 1915.

The number of pairs with components having practically identical spectra is surprisingly large. A study of Tables I and II indicates that the spectral difference is associated with the orbital period, and it will therefore be well to examine this question more closely.

RELATION BETWEEN PERIOD AND DIFFERENCE IN SPECTRAL CLASS

In Figure 4 the difference in spectral class is plotted against the period for the 53 eclipsing binaries in Tables I and II. The figure shows a marked division at $P = 2^d.5$. The binaries with periods less than two and a half days show no differences greater than half a spectral type.

Fig. 4. Spectral difference between the two components plotted against period. The abscissa is the orbital period in days, on a logarithmic scale. The unit interval for the ordinate is one spectral class.

When the period exceeds this value, the spectral difference ranges from zero to more than three spectral classes. This relation is borne out qualitatively by the well known fact that the eclipsing binaries with very short periods are of the *W Ursae Majoris* type, with the two components about equal in color and surface brightness.

Closer scrutiny, however, reveals that Figure 4 is not representative of eclipsing binaries in general. In default of more spectroscopic data, it is possible to make use statistically of the ratio of surface brightness, J_b/J_f , which is reliably determined for some 200 systems. After plotting the quantity $(2.5 \log J_b/J_f)$ against the logarithm of the period, one concludes that the division mentioned above actually occurs at about $P = 0^d.5$. That is, no binaries with periods less than one-half day have differences of more than half a spectral class between the two components. If the period is over half a day, the difference may be larger. Furthermore, the transition is gradual in the group as a whole, and not abrupt as indicated in Figure 4.

Another point worth mentioning is the frequency of binaries having similar spectra, and having periods of from one to four days. A plot such as the one mentioned in the previous paragraph shows that this concentration is not real. Such stars have been observed in virtue of the fact that their spectra and periods are suitable for a determination of the absolute orbits. Selection is the predominating factor. Other causes are discussed elsewhere.

DENSITY

The densities of 22 eclipsing binaries in Table II, and of one in Table I, are known from the masses and radii. For the remaining 30 systems in Tables I and II,¹¹ only hypothetical densities can be computed. These involve only one assumption, namely, that of the mass-ratio, and there can be little doubt, therefore, that they are a fair approximation to the true densities.

The computation is based on the well-known relations,

$$\rho_b = \frac{1}{74.4 P^2 r_b^3} \cdot \frac{M_b}{M_b + M_f},$$

$$\rho_f = \frac{1}{74.4 P^2 r_f^3} \cdot \frac{M_f}{M_b + M_f},$$

where ρ = density ($\odot = 1$);

P = period in days;

r = radius in terms of the mutual distance as unity;

M = mass.

The equations require for their solution an assumption of the mass-ratio. From the various possible methods of approximating the ratio of the masses, the following was selected. For the 23 systems with known absolute dimensions, the bolometric magnitude difference of the two components was plotted against the logarithm of the mass-ratio. A straight line was drawn which best represented all the points.¹² For the other systems, this line was used to derive the logarithm of the mass-ratio from the known difference in bolometric magnitude. The resulting hypothetical densities are listed in brackets in Tables I and II, together with the known densities of the other systems.

In Figure 5 the logarithm of the density is plotted against the spectral class. Primaries are represented by open circles, and secondaries by full circles, and the primary and the secondary of each system are connected by a straight line. Included in the figure for comparison are a few representative densities of visual binaries belonging to the main branch of the Russell-Hertzsprung diagram, namely, *Sirius*, *Procyon*, *a Cen A*, *70 Oph A*,¹³ and *ζ Boo*.¹⁴ It is not such a simple matter to obtain the

¹¹ Except *GO Cyg*. There is no available light curve accurate enough for the computation of the radii.

¹² The equation of the line is $\Delta M = 4.24 \Delta \log M$. The slope is very nearly equal to the slope of Eddington's mass-luminosity curve for stars of high luminosity, i.e., of absolute magnitude -4^m . The empirical relation, however, takes no account of the temperature correction (Eddington, *The Internal Constitution of the Stars*, p. 137), which would have the effect of increasing the slope of the line, and thus making it comparable to the slope of Eddington's curve at some lower point. The reasons for using the empirical method are briefly stated: (1) it has not been shown that Eddington's mass-luminosity curve is applicable alike to primaries and secondaries of eclipsing binaries; and (2) in order to use this curve for the determination of the mass-ratios, it is necessary to make a guess at the masses or absolute magnitudes of the stars.

¹³ Russell, Dugan, and Stewart, *Astronomy*, Vol. 2, p. 740.

¹⁴ Bernewitz, *A. N.*, 213, 1, 1921.

densities of typical late-type giants. *Capella*¹³ is the only one for which the masses have been measured. Super-giants like *Antares* and *Betelgeuse* are not typical; furthermore, their parallaxes are uncertain, while their mass-estimates depend on doubtful extrapolations. *α Boo*,¹³ however, and the giants in the *Taurus* cluster¹⁵ are probably typical K-type giants. The parallaxes, and hence the radii,¹⁶ are well determined, and the only uncertainty is in the masses. The latter, taken from Eddington's mass-luminosity relation,¹⁷ should be approximately correct. We have, then, five yellow giants, presumably typical, viz., *Capella*, *Arcturus*, γ *Tau*, δ *Tau*, and ϵ *Tau*. The densities are doubtless correct within an error of a factor two, and therefore these stars are included in Figure 5.

Fig. 5. Densities. The abscissa is the spectral class, and the ordinate is the logarithm of the density in solar units.
○ Primary; ● Secondary.

The figure contains four groups of stars according to the method of computation of the densities: (1) Twenty-three eclipsing binaries for which the absolute dimensions are known. These represent, together with *Algol* and one or two others, the only stellar densities that have been determined from direct measurement. (2)

¹⁵ γ *Tau*, δ *Tau*, ϵ *Tau*, and θ^1 *Tau*. θ^1 *Tau* is omitted on the ground that its velocity is variable.

¹⁶ Through the ordinary relation,

$$M_{vis} = \frac{29,500}{T} - 5 \log R - 0.08$$

where M_{vis} = visual absolute magnitude; T = effective temperature; R = radius in solar units.

¹⁷ *Loc. cit.*

(3) Twenty-nine eclipsing binaries for which the densities depend on an assumption of the ratio of the masses. (4) The bright components of five visual binaries, of which the masses have been measured with some degree of certainty, and of which the radii have been computed from the effective temperatures and the absolute magnitudes. (5) One visual binary (ζ *Boo*), and four K-type giants, of which the masses and radii have been calculated by indirect methods.

Some of the features of Figure 5 have a real significance, while others are largely the result of observational selection. In the first category are the following: (1) In every spectral group there is a large range in density. The range is largest in the later spectral classes, the high temperature stars avoiding both high and low densities. (2) The densities of the eclipsing binaries of early spectral class are in accord with the accepted densities of early-type stars in general. (3) Of the stars later than F0, five pairs are typical dwarfs, *i Boo*, *SV Cam*, *YY Gem*, *SW Lac*, and *W UMa*. Similarly, the primaries of *RS CVn*, *Z Her*, *RW UMa*, and *RU Cnc* have characteristic dwarf densities.

As contrasted with the above, Figure 5 contains several other interesting points, which are, however, of little or no *fundamental* significance. (1) Among the later spectral classes the eclipsing binary stars with giant densities are rather numerous: *RZ Cnc*, *SS Cam br*, *SY And ft*, *S Cnc ft*, *SX Cas ft*, *UZ Cyg ft*, *RY Gem ft*, *AD Her ft*, *UX Mon ft*, and *RY Per ft*. Both components of *RZ Oph* have unusually low densities, and the luminosities are high, notwithstanding the calculated masses of $0.2 \odot$,¹⁸ which should receive little weight. Giant eclipsing binaries are rare, of course, in a volume of space, just as giant stars in general are rare. Although they are more numerous than the dwarf systems down to a given apparent magnitude, they are actually much less numerous in space than the dwarfs. (2) The remaining late-type stars are intermediate in density between giants and dwarfs. The variation from star to star is remarkably gradual; in fact, there is almost no region in the diagram in which stars do not occur, except that there are no early-type stars with very high or very low densities. The frequency of intermediate densities in the figure is a conspicuous example of the manner in which the material has been chosen. The light variation strongly favors secondaries of intermediate density, and of intermediate position in the Russell-Hertzsprung diagram (cf. Figure 6 below). As stated above (p. 43), the discovery chance is small for normal giant A0-G0 pairs. For analogous reasons, dwarf A0-G0 pairs are not likely to be discovered. But if the secondary happens to be fainter than a giant and larger than a dwarf, that is to say, if its density and luminosity are intermediate, the system has a favorable chance to be discovered as an eclipsing binary. Such stars are rare in a volume of space, and rare

¹⁸ Cf. McLaughlin, *A. J.* **38**, 21, 1927; Gaposchkin, *loc. cit.*

even down to a given limit of apparent magnitude. Great caution should be exercised in making generalizations about the masses, densities, and radii of stars on the basis of the eclipsing binaries. (3) When the spectral difference between the two components is considerable, the secondary generally has the later spectrum and the lower density. This is to be expected as the result of the selection factors discussed above. The only marked exceptions are *SS Cam*, *AW Her*, and *AR Lac*, in which the ranges in light variation are relatively small ($0^m.6$, $0^m.9$, and $0^m.8$, respectively).

McLaughlin¹⁹ has shown that the primaries of the eclipsing binaries of known absolute dimensions fall, on the average, about a magnitude above Eddington's mass-luminosity curve. He concludes from the size of some of the residuals that the luminosity is not a single-valued function of the mass. It should be emphasized, however, that many of the large deviations apply to stars for which the orbits are uncertain. More recent investigations of the orbits of *U Sge*, *RS CVn*, and *SW Lac* show that the primaries of these systems are in close agreement with Eddington's mass-luminosity curve; the same is true of *AR Lac*, which is not on McLaughlin's list. There still remain several outstanding cases: *S Ant*, *TV Cas*, *YY Gem*, *RX Her*, *RT Lac*, *AG Per*, and perhaps a few others. To what extent the disagreement can be attributed to the range of solution of the photometric elements is an important question, which will require a detailed investigation of every one of the large residuals before final conclusions can be drawn.

Lundmark²⁰ has recently pointed out correlations between mass and radius, and between mass and density, for the primaries of 39 eclipsing binaries. Although he does not list the stars used, it is known that practically all the bright components of eclipsing binaries for which the masses have been reliably determined belong to the main sequence of the Russell-Hertzsprung diagram, and also that they lie statistically on a mass-luminosity curve, with perhaps a systematic departure from that of Eddington. As a consequence of the mass-luminosity relation it follows that, at least for statistical purposes, the mass is a single-valued function of the luminosity and temperature, or of the temperature and radius, for stars in the main sequence. And since the temperature and radius vary together continuously for these stars, it follows as a corollary that there should be a correlation between mass and radius, and therefore obviously between mass and density. If a large number of late-type giants could be included in the discussion, the correlation coefficient would be much smaller. Before passing from eclipsing binaries to stars in general, it should be remembered that the relation between mass and radius is not fundamental except for stars of a particular group, such as the main sequence, whereas the relation connecting

mass, luminosity, and effective temperature seems to have a more essential significance.

Before leaving the subject of densities, we may mention a curious connection, or rather a lack of connection, between the spectra and densities of certain stars. Some of the stars listed in Tables I and II are classed as giants, dwarfs, or intermediate, according to the strength of the *Sr II* doublet $\lambda\lambda 4215, 4077$. In stars of a given temperature the intensities of these lines vary with the electron pressure. In general they are strong in stars of low density, and weak in stars of high density. Yet this rule is not without exception, such, for example, as the faint component of *RS CVn*. In discussing this star, Joy says:²¹ "The secondary star is certainly peculiar and cannot readily be classified among other stars for which we know the physical characteristics. Its absolute magnitude and spectrum would place it among the dwarfs, but the size and mass are exceptionally large, although not nearly large enough to compare with the normal giants of its spectral class. The density is, correspondingly, intermediate between dwarfs and giants." The secondaries of *SS Cam*, *RU Cnc*, *TT Hya*, and *RW UMa* seem to belong to the same category, while *UX Mon* and *RY Per* have dwarf spectra and giant densities. The weakness of lines $\lambda 4215$ and $\lambda 4077$ in these stars of low density may be due to a deficiency of strontium. A more likely reason is the fact that we have not yet considered all the factors contributing to line intensities in stellar spectra. The reader is referred to Miss Payne's discussion²² of the anomalous behavior of the strontium lines in the spectra of certain A-type stars.

ABSOLUTE MAGNITUDE AND SPECTRAL CLASS

The relation between spectrum and absolute magnitude of the bright components of eclipsing binaries has been discussed by McLaughlin,²³ whose treatment includes most of the stars in Table II, along with numerous others. He finds that nearly all the eclipsing binaries of known absolute magnitude belong to the main sequence of the Russell-Hertzsprung diagram.²⁴ Similar conclusions have been reported by Gaposchkin,²⁵ whose results, however, are somewhat weakened by the inclusion of stars of which the absolute dimensions are unreliable. So few new parallaxes of eclipsing variables have become

²¹ *Ap. J.* **72**, 45, 1930.

²² Miss Payne, *Stellar Atmospheres*, p. 169 ff.

²³ *Loc. cit.*

²⁴ Of the exceptions, *RS CVn* is erroneously classed as a giant. It is now known that the brighter star is the smaller; this brings the hypothetical parallax into agreement with the spectroscopic, so that the primary belongs to the main sequence. Another noteworthy exception is *SX Cas*, for which the absolute magnitude is given as $-3^m.5$. The more recent work of Joy and of Dugan (*Pr. Cont.* No. 13, 1933) shows that the absolute magnitudes of these two giants are close to zero. From Dugan's values of the radii, and Russell's temperature scale (see note 28 below), the writer finds for the absolute magnitudes $-0^m.9$ and $+0^m.2$.

²⁵ *Loc. cit.*

¹⁹ *Loc. cit.*

²⁰ *Lund Obs. Circ.* No. 5, 105, 1932.

available in recent years that it is at present impossible to extend our knowledge of this subject.

We can, however, compare the spectral difference with the magnitude difference in a given binary system. The following binaries have rather large differences in spectral class: *SY And*, *TW And*, *WW And*, *S Cnc*, *RU Cnc*, *RS CVn*, *U Cep*, *UZ Cyg*, *TW Dra*, *RY Gem*, *AD Her*, *TT Hya*, *UX Mon*, *RY Per*, and *RW UMa*. In this group the average magnitude difference per unit difference in spectral class is $0^m.7$. Therefore the mean slope of the lines connecting primary and secondary on the Russell-Hertzsprung diagram is $0.7 \text{ mag./sp. class}$. The slope of the main sequence of the diagram for stars in general is $1.7 \text{ mag./sp. class}$. Thus the above systems depart from the main sequence by one magnitude per unit spectral difference.

According to the densities it appears that the primaries probably belong to the main sequence, and that the secondaries fall above. This assumption may be made arbitrarily in order to illustrate the facts set forth in the previous paragraph. For the thirteen binaries the primaries are plotted on the main sequence in Figure 6; the relative positions of the secondaries are determined from the observational data.

Fig. 6. Russell-Hertzsprung diagram for fifteen eclipsing binaries. ○ Primary; ● Secondary. The primaries are arbitrarily placed along the main sequence.

The combined evidence of luminosity and density shows that most of the secondaries in Figure 6 are intermediate between giants and dwarfs. This conclusion is not a new one, since it has been known to be true for a large number of Algol-variables. A certain degree of definiteness, however, is added by the knowledge of the spectral types of the secondaries.

Another way of looking at the matter is the following. The ratio of surface brightness, J_b/J_f , is defined by the relation,

$$\frac{J_b}{J_f} = \frac{L_b}{L_f} \cdot \frac{r_f^2}{r_b^2},$$

where L_b and L_f are the light of the bright and faint star respectively, and r_b and r_f are their respective radii.

If ΔM is the magnitude difference (faint minus bright), and

ΔM_J is the ratio of surface brightness expressed in magnitudes (faint minus bright),

it follows that

$$\log \frac{r_f}{r_b} = -\frac{1}{5} (\Delta M - \Delta M_J).$$

Calculations made on the basis of the Russell-Hertzsprung diagram, together with Eddington's table of surface brightness,²⁶ show that if both components of an eclipsing binary belong to the main sequence, the primary having an A0 spectrum, for example, then the spectrum and radius of the secondary will exhibit the following relation:

Spt	r_f/r_b
F0	0.8
G0	0.6
K0	0.4

Since the secondaries are invariably larger than the primaries in Figure 6, it follows conclusively that either the primaries fall below the main branch of the Russell-Hertzsprung diagram, or else the secondaries fall above. Consideration of the densities indicates that the secondaries are too bright, although the departure may involve both primaries and secondaries.

One secondary in Figure 6, *SY And ft*, is nearly bright enough to be called a giant. Of the stars not shown in the figure, *U Sge*, *SX Cas*, *RZ Oph*, *RZ Cnc*, *SS Cam*, and numerous others of early spectral class are stars of high luminosity. *RT Lac* and *AR Lac* are intermediate in density and luminosity, as is shown from the determinations of their absolute orbits. Similarly, *RS Ari* and *AW Her* are doubtless intermediate, and for this reason it is important that they be observed photometrically. The remaining stars seem to belong to the main branch of the Russell-Hertzsprung diagram.

RATIO OF SURFACE BRIGHTNESS

Knowledge of the spectral types of both components of the eclipsing binaries should lead to a fairly sensitive test of the temperature scale, since the ratio of surface brightness can be computed both from the light curve and from the spectra. This ratio, J_b/J_f , is usually well determined from the light curve. It is converted into magnitudes through the ordinary relation,

$$\Delta M_J = 2.5 \log J_b/J_f.$$

The value of ΔM_J for visual light is given for each star in column 7 of Tables I and II.

In a few cases the published values of J_b/J_f have been computed from photographic light curves. These have been reduced to visual light by the writer, by means of King's table of color-indices.²⁷

²⁶ Eddington, *The Internal Constitution of the Stars*, p. 139.

²⁷ *Handbuch der Astrophysik*, Bd. 5, Hf. 1, p. 173: Table 9 (c).

The calculation of ΔM_J from the spectra is quite simple. Assuming the stars to radiate like black bodies, we can obtain the difference in bolometric magnitude per unit surface area from the spectral types of the two components, and then reduce this to visual light. This procedure is the converse of that commonly employed to ascertain from the light curve the spectral type of the faint companion. In the present computation, temperatures have been obtained from spectral types according to the scale adopted by Russell, Dugan, and Stewart.²⁸

Fig. 7. Ratio of surface brightness. The abscissa is the ratio of visual surface brightness in magnitudes as obtained from the light curve. The ordinate is the same quantity computed from the spectral types of the two components. Groups A and B are referred to in the text.

When there was reason to consider the secondary as an intermediate star, the writer adopted a temperature midway between the giant and dwarf temperatures of that spectral class. Eddington's table²⁹ was used to derive the visual surface brightness from the temperature. The resulting spectroscopic values of ΔM_J are listed in column 8 of Tables I and II.

The two values of ΔM_J are plotted in Figure 7 for 53 eclipsing binaries. Abscissae are the light-curve determinations, ordinates the spectroscopic. If the assumptions are correct, the points should all fall on the straight line representing perfect agreement between the two calculations. The uncertainties of the spectral types are probably the largest source of observational error in most cases. There is also some inaccuracy in the determination of J_b/J_f from the light curves. It is estimated that a residual of one magnitude is a generous allowance for all uncertainties of observation and computation.

It is apparent from Figure 7 that, although three-fourths of the binaries show satisfactory agreement, no

less than twelve deviate far beyond the limit of error. The departure is always in the sense that the spectra of the two components are too nearly alike to correspond to the observed ratio of surface brightness. Plaskett³⁰ and others have called attention to a few of the discordant cases. On the whole, the points with large residuals are fully as accurately determined as the rest. Let us therefore examine in turn the various causes that might give rise to conflicting results.

a) *Overlapping of Spectra*.—(The argument applies only to stars observed at conjunction.) If the time of predicted minimum were in error, or if for any other reason the primary were not completely hidden throughout the exposure of the secondary, the results would be at least partially vitiated. The periods are considered sufficiently accurate in every case to eliminate any error of this nature. A few binaries should be mentioned where composite spectra may confuse the issue.

1) *SV Cam*.—If the spectral class of the large star is correctly determined during principal minimum as G5, that of the smaller one should be A8 according to the light curve. At secondary minimum the large star contributes nearly 40 per cent of the photographic light. But even when allowance is made for this, it does not seem possible that the spectrum of the small star can be earlier than G0. There is almost no difference between the spectra at principal and at secondary minimum. Assuming the spectral types to be G5 and G0, we find $\Delta M_{Jsp} = -0.3$, and the residual is still 1.1 .

2) *RW Cap*.—According to Shapley the eclipse is grazing on the uniform hypothesis, partial on the darkened hypothesis. On either assumption the bright star contributes less than one-fifth of the total light during an exposure of the faint component (100 minutes). With an A3 primary, the secondary should be F5 according to the light curve. This is the one discordant star in which a slight error in the period could have a serious effect. It seems sufficiently accurate, however. Unless either the light elements or the photometric orbit are found in the future to require considerable change, we must conclude that the light of the secondary was preponderant when it was observed at principal minimum.

3) *GO Cyg*.—The discordance exhibited by this system should not be taken too seriously at the present time. No observations adequate for the computation of the photometric orbit have been published. The ratio of surface brightness was estimated by the writer from Kukarkin's light curve,³¹ after a rough correction for ellipticity. The spectral types were estimated by Pearce from plates taken at the two minima. The eclipses are partial and shallow. Until we know more about the system, we cannot judge how serious the discrepancy really is. The star merits further attention of photometric observers.

²⁸ Russell, Dugan, and Stewart, *Astronomy*, Vol. 2, p. 753.

²⁹ *Loc. cit.*

³⁰ *Pub. D. A. O.* 1, 191, 1920.

³¹ *Verein Freund. Phys. u. Astron.* Vol. 4, No. 1 (37), p. 19.

b) *Reflection Effect*.—(The argument applies only to stars observed near the nodes.) Eddington³² has discussed the spectroscopic effect of inter-radiation between the two components of an eclipsing binary. He concludes that if a bright, hot star, S_1 , and a faint, cool star, S_2 , are in proximity, the observed spectra must be very similar. For “the radiation from S_1 will ionize and excite atoms in S_2 very much as it does in S_1 itself, and these will absorb their appropriate lines in the outflowing radiation from S_2 The result is not merely a reflected spectrum of S_1 superposed on that of S_2 ; all the radiation from the bright hemisphere of S_2 is subjected to the absorption, and it may be relieved from the absorption by unionized atoms which would have occurred if S_2 were isolated.”

The writer is not prepared to investigate this matter in detail. The argument may be sufficient to account for the major part of the discrepancies of *TV Cas* and *RS Vul*. In these peculiar systems the hemisphere of the secondary facing the primary is twice as bright as the hemisphere turned away. Certainly it is to be expected that the presence of the primary should make the spectrum of the secondary earlier than it would otherwise be. To what extent it will make the two spectra indistinguishable is a question that will require further theoretical investigation.

Z Vul and *Z Her* are not such extreme cases. According to Baker the bright and faint sides of the secondary of *Z Vul* are of comparable brightness (respectively 0.072 and 0.057 of the total photographic light of the system). If the faint hemisphere has a spectrum considerably later than B3, it is difficult to see why its effect is not noticeable. The argument is still more cogent when applied to *Z Her*. Here the faint star emits 43 per cent of the total light of the system, and the reflection effect is less important. The lines are of good quality, even on double-lined spectrograms, and the uncertainty of the spectral types is comparatively small.

Dr. Dugan has generously permitted the writer to make use of his provisional results on the orbit of *U CrB*. From his figures the spectrum of the secondary should be A2 if that of the primary is B3. It is quite possible that Eddington's argument would account for the appearance of the helium lines in the radiation of an A2 star under these conditions.

There may be serious systematic errors in the estimation of spectral types on double-lined spectrograms, especially when one of the spectra is much weaker than the other. Thus most of the pairs in *Group A* of Figure 7 are reported to have identical spectra, whereas they may differ by a few tenths of a spectral class. The magnitude of the error should vary with the spectral class, with the linear dispersion, and especially with the judgment of the investigator. It is probably less serious in early spectra than in those of the solar type. This source of error

³² *Mon. Not.* 86, 325, 1926.

cannot be invoked to explain the presence of helium lines in an F-type spectrum such as that to be expected from *RS Vul ft*. But it might be combined with the reflection effect to account for some of the foregoing anomalies.

The discordant systems not mentioned in this section were observed at principal minimum, when the faint companions were visible only on the dark side. Consequently the reflection effect does not pertain to the question.

c) *Departure from Black-Body Radiation*.—The temperature scale is derived in part from spectral energy distribution curves. With the large amount of line absorption in the advanced spectral types, it is to be expected that the observed surface brightness of a late-type star will be less than that computed from the ionization temperature. This is in the right direction to diminish the residuals of *RS CVn*, *RU Cnc*, and *RW UMa*. But the agreement of the stars in *Group B* indicates that the departures from black-body radiation introduce no serious consequences. The fact is that the temperature scale is based largely on other methods, which tend to eliminate, so far as this discussion is concerned, any errors introduced by the assumption of black-body radiation. Consider, for example, the color-index method of determining the temperature scale. The excess of line absorption in the violet and ultra-violet regions over the green region makes the temperatures of the late-type stars too low. The line absorption throughout the visual region makes the observed visual surface brightness too low, and the two effects tend to cancel each other. Although this argument may be given some weight, the main point is the fact that most departures from black-body radiation would be expected to affect stars of the same temperature in the same way. Hence, even if the effect is present, it cannot cause the scatter in Figure 7.

d) *Errors in the Temperature Scale*.—The computation represented in the figure has been repeated on the basis of two other temperature scales, one by Miss Payne³³ and the other by Brill.³⁴ The former scale raises the points in *Group B* by two magnitudes, and alters the other points correspondingly. The latter scale raises *Group B*, as well as *RU Cnc*, *RS CVn*, *SX Cas*, and *RW UMa*, by almost half a magnitude, without changing the rest of the figure conspicuously. The result is that the residuals of *RY Gem*, *RZ Oph*, *SX Cas*, and *U Cep* are increased to a magnitude or more, while those of *RU Cnc*, *RS CVn*, and *RW UMa* are still seriously large (1^m.3 to 1^m.5). In view of the large residuals from any temperature scale, it is impossible from the present data to obtain a conclusive check on the relation between temperature and spectral class. It is certain that no alteration of the temperatures can remove the discrepancies.

e) *Effect of Limb Darkening*.—Our only conclusive observational evidence on limb darkening pertains to a

³³ Miss Payne, *Stellar Atmospheres*, p. 33.

³⁴ *Veröff. Berl.-Bab.* 7, 12, 1927.

G-type dwarf, the Sun. Milne's formula³⁵ may apply to other types of stars as well, but it is to be accepted with some reserve until it has been substantiated by observation. Studies of the effect of limb darkening on the light curves of eclipsing variables have not been based on accurate, homogeneous observations of a large number of stars. The problem is complicated by the presence of other factors in the light variation that have not been satisfactorily explained. The evidence indicates that the darkened solutions generally represent the light curves better than the uniform ones, and therefore the former have generally been used when available. The uniform solutions nearly always give larger absolute values of ΔM_J than the darkened solutions. The result of applying the uniform in place of the darkened hypothesis is to increase most of the residuals in Figure 7 by amounts varying from zero to one magnitude.

f) *Presence of a Third Body.*—The possibility must always be borne in mind that we may be dealing with three-body systems. In this event, we observe at principal minimum not the secondary spectrum alone, but the combined spectra of the close and the distant companions. (The question of a third body does not concern *TV Cas*, *U CrB*, *Z Her*, *Z Vul*, *RS Vul*, where the spectra were observed near the nodes. It probably does not involve *RS CVn* or *GO Cyg*.) If we postulate a third body, we see at once that it has two effects on the calculation. The one is to make the spectra of the close pair more nearly alike than they really are. The other is to decrease the depths of the two minima, both by the same factor.³⁶ Hence if J_b/J_f is computed from the principal minimum alone, its value is closer to unity than the true value. This tends to cancel the concomitant change in the observed spectral difference. To what extent the two changes will annul each other depends on the spectra and the relative brightness of the three stars. If, on the other hand, J_b/J_f is found from the relative depths of principal and secondary minima, its value will not be altered by the presence of a third body. The only effect of the third body, so far as Figure 7 is concerned, is to reduce the apparent difference in the spectra of the close pair.

The possibility of disturbance from a third body comparable in brightness to the secondary is precluded in *SV Cam* and *RW Cap* by the natures of their light curves. The effect would tend to be canceled in *RS CVn*, *RU Cnc*, *RX Gem*, and *RW UMa*, for the reason explained above. It is not necessarily eliminated, however.

Let us assume for the moment that *RX Gem* is a three-body system, with the A3p spectrum arising from the third body, and an invisible K-type spectrum coming from the second. It follows that in the imaginary light curve of the close pair, the true principal minimum

³⁵ *Mon. Not.* **81**, 361, 1921.

³⁶ This is true of a light curve in which the ordinate is I , the percentage of the total light of the system that is visible at any time.

must be at least one photographic magnitude fainter than it is observed, since the K-type spectrum is not visible. In the close pair, therefore, $J_b/J_f > 52$, and the secondary spectrum is K5 or later. The influence of the K5 spectrum at principal minimum might account for the weakness of $H\beta$ in the spectrum of the third body. The abnormal *progression* in the strength of the Balmer lines from $H\beta$ to $H\epsilon$, however, cannot be explained as the result of the superposition of a K5 spectrum, because in such a case the latter would be strong enough to be visible on the plates.

If we include in our plausibility argument the probable low density of the secondary, it is reasonable to conclude that *RX Gem* is a three-body system, although the question cannot be settled conclusively from the facts at hand. In any case, the behavior of the Balmer lines must be considered as "peculiar" in the spectrum at principal minimum.

The question of the presence of a third body comparable in brightness to the secondary can often be settled by the study of an accurate light curve. Such a companion may be disclosed from a variation in the period, or from the combined loss of light at the two minima. It is very improbable, though not impossible, that more than one or two of the large residuals in Figure 7 can be explained on this hypothesis.

g) *Density.*—It is a well established fact that the spectral type of a star depends not only upon the effective temperature, but also upon the pressure in the reversing layer. The difference in temperature between giants and dwarfs of the same spectral class has been established with considerable certainty. As stated above, whenever the density and total brightness of the secondary indicated that it was intermediate between dwarf and giant, an intermediate temperature was adopted. Whether or not this is a justifiable procedure may be open to question, but it is certainly not involved in the discrepancies in Figure 7. For every binary with a large residual, there are others of similar densities and magnitude-differences, that show good agreement in the figure. It seems impossible at the present time to reconcile the observed facts on the basis of stellar densities.

h) *Conclusion.*—The two independent methods of computing the ratio of surface brightness of an eclipsing binary should give results that are in harmony, not only statistically, but individually. The two computations are in agreement within the limits of error for 41 of the 53 systems. The large residuals of the remaining 12 binaries have led to a detailed investigation of their possible causes. After due allowances have been made, the writer is of the opinion that many of the discordances are real, in whole or in part. *SV Cam*, *RU Cnc*, *RS CVn*, *Z Her*, and *RW UMa* are the most refractory cases. The facts set forth in this section might be considered as lending support to the conclusion reached by Struve³⁷

³⁷ *Ap. J.* **78**, 73, 1933.

and others, that stars with the same atmospheric temperature and pressure do not necessarily have identical spectra. Our knowledge of the physical characteristics of the stars is so meager, however, that the present observations may have to wait some time for an explanation.

SUMMARY

Spectral types are determined for both components of 25 eclipsing binaries, most of the plates having been taken at conjunction. These stars, together with 28 systems studied by other investigators, constitute the material for the discussion.

The spectra of the eclipsing binary stars are in general very similar to ordinary stellar spectra. Four secondaries and two primaries have bright hydrogen lines, while two additional secondaries have a tendency in that direction, indicating that $H\alpha$ is probably bright. The hypothesis that some of the emission lines arise from the chromosphere of either the primary or the secondary, or both, in the nature of a flash spectrum, is discussed. Although the evidence is not conclusive, the general validity of the hypothesis seems rather doubtful. The problem deserves further investigation. A single reversal of the H and K lines of ionized calcium is found in *YY Gem* and *AR Lac br*. The secondaries are generally later than, or very similar to, the primaries in spectral type, an effect that shows the influence of observational selection. No large spectral differences are found in binaries of short period.

Preliminary orbital elements are computed from published photometric data of eight eclipsing binaries. The elements of *WW And*, *RU Cnc*, and *RY Gem*, uniform and darkened, are given in Table III. Accurate photometric observations are urgently needed of *RS Ari*, *RU Cnc*, *RX Gem*, *AW Her*, *TT Hya*, and a few others, because of their characteristics of peculiar interest.

Hypothetical densities are computed for the stars whose absolute dimensions are not known. Hypothetical and observed densities both show a wide range for every spectral group. The early-type stars have normal densities for their spectral class, and several of the late-type stars have characteristically dwarf densities. Many of the lower-temperature stars, especially the secondaries, have low or intermediate densities, and their luminosities are frequently intermediate between dwarfs and giants. Several lines of evidence show that the densities of the eclipsing binaries are, in some respects, not typical of stars in general.

The ratio of surface brightness as derived from the spectral types is compared with the value obtained from the light curve. While three-fourths of the stars show satisfactory agreement, the exceptions are numerous and noteworthy. Several possible sources of disagreement are examined, viz., overlapping of spectra, reflection effect, departures from black-body radiation, errors in the temperature scale, effect of limb darkening, presence of a third body, and density effects. It is concluded that many of the discrepancies are real, at least to some extent, but that no explanation is possible until we know more about the physical properties of the stars in question.

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APPENDIX

TABLE Ia
DETAILS OF EXPOSURE TIMES FOR STARS LISTED IN TABLE I
(See page 38)

Star	Bright Component		Faint Component	
	Date (G.M.T.) J.D. 2427000+	Exp. Time	Date (G.M.T.) J.D. 2427000+	Exp. Time
<i>SY And</i>	308.900	70 ^m	395.778	180 ^m
	308.961	40		
<i>TW And</i>	263.890	60	315.898*	70
	276.948	90		
<i>WW And</i>	327.806	54	306.781	190
	342.927	60		
<i>RS Ari</i>	367.750	50	400.816	110
	369.976	35		
	370.007	50		
<i>SS Cam</i>	369.878	70	357.899	50
	369.917	40	391.789	60
<i>SV Cam</i>	344.950	30	367.786	24
	356.810	30		
<i>RU Cnc</i>	388.927	40	432.009	200
	443.744	50		
<i>RZ Cnc</i>	348.973	24	403.010	40
			403.050	65
<i>RW Cap</i>	334.648	20	337.736	100
	334.660	12		
	350.683	30		
<i>TW Cas</i>	320.858	50	350.847	8
			350.855	12
<i>XX Cas</i>	334.783	12	356.768	60
	334.796	24		
<i>UZ Cyg</i>	299.856	60	389.728	300
	304.982	60		
<i>TW Dra</i>	262.885	30	334.706	86
<i>RX Gem</i>	330.993	70	395.006	120
	357.942	30	480.657	90
<i>RY Gem</i>	357.963	15	371.981	100
	357.979	30	372.037	60
<i>AD Her</i>	284.897	90	544.948	195
	296.796	80		
<i>AW Her</i>	338.682	45	388.634	30
	338.731	60		
<i>TT Hya</i>	536.787	4	521.873	35
	536.793	8	542.805	40
<i>SW Lac</i>	366.822	30	344.853	30
<i>AR Lac</i>	266.933	54	263.939	45
	276.883	60		
<i>UX Mon</i>	398.951	12	388.978	65
	425.951	20	394.931	40
<i>RZ Oph</i>	356.685	60	274.901	140
			278.831	350
<i>RY Per</i>	330.885	18	310.972	60
	330.906	36	365.786	40
			365.832	90
<i>V356 Sgr</i>	286.729	16	317.714	80
	286.756	16	326.644	32
<i>RW UMa</i>	447.826	40	480.792	150
	447.849	25		

* Exposure interrupted to reverse telescope. The actual exposure, 70 minutes in all, was as follows: J.D. 2427315.865— .893; .910—.931.

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