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OCTOBER, 1930.

PREFACE

The following report, with a discussion of results, of a subsurface gravity determination expedition undertaken in the United States submarine *S-21* was prepared through the collaboration of Prof. Dr. F. A. VENING MEINESZ, of the Netherlands Geodetic Commission, and Dr. F. E. WRIGHT, of the Carnegie Institution of Washington. Miss E. A. LAMSON, of the astronomical staff of the United States Naval Observatory, checked all computations in the report and prepared the appendix indicating the computing methods employed in reducing the observations at the various gravity stations occupied.

The expedition itself was conducted under the joint auspices of the Navy Department and the Carnegie Institution of Washington. Valuable assistance in securing results was rendered by the United States Coast and Geodetic Survey.

C. S. FREEMAN,

Captain, United States Navy, Superintendent.

UNITED STATES NAVAL OBSERVATORY,

December, 1929.

III

THE GRAVITY MEASURING CRUISE OF THE U. S. SUBMARINE S-21

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WITH AN APPENDIX ON COMPUTATIONAL
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THE GRAVITY MEASURING CRUISE OF THE UNITED STATES SUBMARINE S-21

CHAPTER 1

INTRODUCTION

Measurements of gravity over land areas have been made for many decades in this country and abroad. From them the shape of the earth and the fact that it is a figure of equilibrium have been deduced. The land covers, however, only a quarter (28 per cent) of the area of the surface of the globe and measurements of gravity at sea in different parts of the world are urgently needed for a more accurate determination both of the figure of the earth and of the state of equilibrium of its surface features.

Gravity measurements at sea are difficult to make because of the unsteadiness of the ocean surface; and, until recently, no satisfactory method was available by which this difficulty could be overcome. For gravity measurements on land the standard apparatus is a free-swinging pendulum of known length. On board ship the horizontal and vertical disturbances due to the ship's movements are so great that satisfactory records from a single pendulum can not be obtained. To meet this situation Dr. F. A. VENING MEINESZ conceived the plan of using a pair of pendulums swinging in the same vertical plane. Any horizontal disturbance or acceleration then affects the angular position of each pendulum equally, so that, if the difference between the angular positions of the two pendulums is taken, the behavior of the combined pair is that of a simple pendulum free from the effect of the horizontal disturbances. This is the fundamental improvement which was introduced and which has made possible the use of a pendulum apparatus at sea for gravity measurements. Obviously it can be used on the surface only in calm weather. If measurements are to be made at any desired point, irrespective of the weather, rough seas must be avoided; this can be done on a submerged submarine. Dr. VENING MEINESZ proved on three different voyages aboard Dutch submarines that his new method is feasible and yields reliable results. His voyages encircled the world and enabled him to occupy over 200 stations at sea.

The importance of gravity measurements at sea was realized by the United States Navy and in particular by Capt. C. S. FREEMAN, United States Navy, Superintendent of the United States Naval Observatory. An invitation was extended to Dr. VENING MEINESZ to visit the United States with his new apparatus for the purpose of making gravity measurements in the Gulf of Mexico and the Caribbean Sea aboard a submarine of the United States Navy. In this undertaking the Carnegie Institution of Washington and the National Academy of Sciences of the United

States cooperated; Dr. ARTHUR L. DAY, director of the Geophysical Laboratory, Carnegie Institution of Washington, was largely instrumental in developing the plan of cooperation and in working out the details. Dr. VENING MEINESZ accepted the invitation and reached New York on September 23, 1928. The Navy designated the U. S. submarine S-21 for the task and gave it orders to be ready to sail from Washington, D. C., about October 1. Mr. ELMER B. COLLINS, of the Hydrographic Office of the United States Navy, and Dr. F. E. WRIGHT, of the Carnegie Institution of Washington, were chosen members of the expedition to assist Dr. VENING MEINESZ in the measurements and to study the details of the method for use in future expeditions.

THE ITINERARY

Dr. VENING MEINESZ reached Washington on September 25 and proceeded at once to set up his apparatus at the gravity station in the basement of the United States Coast and Geodetic Survey. This is the base station for gravity work in the United States and has been compared repeatedly with European base stations. On completion of these measurements the apparatus was installed in the submarine S-21, which set sail on October 2 for Norfolk and thence to the Gulf of Mexico and the Caribbean Sea. Upon the return of the expedition to Washington measurements were again made at the gravity base station.

The submarine S-21, Lieut. J. L. FISHER, United States Navy, commanding, was accompanied by the two ships, U. S. Eagle boats 35 and 58, Lieut. Commander T. L. NASH, United States Navy, and Lieut. Commander L. R. MOORE, United States Navy, respectively, commanding, to serve as tenders in case of an emergency. Lieut. Commander NASH was in command of the expedition.

At Norfolk a gravity measurement was made on October 3 at Pier No. 2 of the naval base. On October 4 the expedition left for Key West, Fla., arriving there on the evening of October 8; three gravity stations at sea were occupied on the way. (Chart No. 1.) At Key West a gravity measurement was made. The period October 10 to 14 was spent at sea between Key West and Galveston, Tex.; measurements were made along a profile leading off the Florida coast to the Sigsbee Deep and thence in a northerly direction to Galveston, Tex.; on this cruise seven sea stations were occupied. At Galveston a gravity determination was made on October 14 at Pier No. 20. Leaving Galveston on October 19 we returned to Key West by way of the Mississippi Delta, arriving at Key West on October 23; seven stations were occupied during this voyage. At Key West the boiler tubes of the Eagle boats were cleaned, an operation that required a week to complete. At each port the photographic records were developed, and the necessary measurements and computations were made to determine the value of gravity for each station occupied during the voyage just completed. From each port a complete list of data for all stations was forwarded to the Naval Observatory in Washington for record and for transmission to the section of geodesy of the United States Coast and Geodetic Survey to be used in the isostatic reductions which Dr. W. BOWIE, chief of that section, kindly offered to have made. These reductions were accomplished so expeditiously that on the return to Washington, the final values of the isostatic reductions for most of the stations were available. It is a pleasure to express our grateful appreciation of the courtesy of the United States Coast and Geodetic Survey in this matter.

On the evening of October 29 we left Key West for the Bartlett Deep, south of Cuba. Two lines of measurements were taken across the east end of this deep, which is remarkable for its exceedingly steep rocky northern slope. Arriving at Guantanamo Bay on November 2, we were received by the commandant, Commander C. S. McWHORTER, United States Navy, who aided the work in every way possible. Leaving Guantanamo Bay on November 5, we skirted the north coast of Haiti to the Mona Passage and thence north for 250 miles, then east for 50 miles, and thence south to San Juan, P. R., and east to St. Thomas, Virgin Islands, where on November 10 we were received by Governor EVANS, who aided the work most effectively during the visit. On November 15 our course led over the small Virgin Island Deep off St. Croix in a southwesterly direction to latitude $15^{\circ} 47' N.$ over the deeper part of the Caribbean Sea south of Haiti. After occupying a station south of the Bartlett Deep and north of Jamaica we returned to Guantanamo Bay on November 19. On November 21 we started on the last leg of the cruise and steered almost due north along the 74° meridian. The greater part of this voyage was over rough seas, so rough that at a depth of 80 feet the submarine rolled 10° on a side. On the surface, rolls of 30° , 40° , and 45° on a side were observed. We arrived in Washington on November 27, thus completing the 7,000-mile cruise. At no time during the trip was there trouble either with the operation of the submarine or with the gravity-measuring apparatus. This bespeaks capable officers and an efficient crew to maintain in good working order the mass of complex machinery that is crowded into a submarine.

In planning the itinerary of the cruise of the *S-21* the effort was made to select areas that promised results of interest, such as the continental shelf, the Sigsbee Deep and the Mississippi Delta of the Gulf of Mexico, the Nares Deep, the Bartlett Deep, the deep portions of the Caribbean Sea and of the Atlantic Ocean. Gravity measurements at sea are valuable and thus far only a few measurements have been made; many of the earthquakes that affect land areas originate in adjacent submarine areas; measurements at sea are, moreover, less affected than measurements on land by local attractions that tend to produce local anomalies difficult to interpret. On the sea surface over depths of a mile or more these local anomalies are less pronounced and hence the anomalies observed represent more faithfully the condition of the earth's crust.

During the cruise 32 days were spent at sea and 25 days in port; 46 gravity stations were occupied at sea; 49 dives were made and approximately 40 hours were spent submerged. Over the Bartlett Deep 5 dives were made in 18 hours. On one dive an 18-minute gravity measurement was made during a 22-minute submergence; in other words, only 4 minutes were taken to submerge, to trim ship, to set the gravity apparatus in operation and to return to the surface. In calm seas submergence only to periscope depth (42 to 44 feet) was the rule; otherwise the submergence was to depths up to 80 feet. Even at the maximum depth a roll of 5° or 6° was not uncommon in rough weather. At still greater depths the roll due to the ocean swell would have been less; but this greater submergence was unnecessary because the apparatus functions satisfactorily with a roll of this magnitude. At no time was a submergence exceeding one and a half hours made.

On making a submergence no special sensation is felt by the men on board. The pitch of the ship at the beginning of a dive is about 6° and this slope is attained so gradually that it is hardly noticed. The Diesel engines are stopped and the electric motors are started. All measurements beneath the surface were made at a speed of 4 knots (actual speed 3.85 knots); at this speed the vibrations from the driving motors are not serious and do not affect the pendulums. At higher speeds the vibrations are perceptible and may seriously shift the pendulums on the support. On an ocean liner the vibrations from the engines are more pronounced and may prevent the use of a pendulum apparatus for gravity measurements. This is likewise true if the apparatus be used in calm seas aboard a sailing vessel or on a large vessel propelled at slow speed.

During each submergence the course of the submarine was maintained constant, as indicated on the gyro compass, and was recorded. During submergence depth soundings were taken at 5-minute intervals with the aid of the sonic depth finder. Depths were also taken while cruising on the surface at intervals of an hour or less, depending on the configuration of the sea bottom. The sonic depth finder functioned for the most part satisfactorily even over the greatest depths. Over certain areas good signals or echoes were not obtainable and the readings were not so accurate as those over other areas in which sharp, clear echoes were received.

When it is realized that during the past 35 years approximately 300 stations have been occupied in the United States and that during the cruise of the *S-21* nearly 50 stations were occupied within a period of 2 months, or 46 stations at sea occupied in 32 days, the effectiveness of the new method for measuring gravity is obvious. Inasmuch as the value of gravity measurements increases with the number and spread of gravity stations over the surface of the earth, especially over the ocean surfaces, the contribution of the *S-21* cruise to the theory of isostasy and to fundamental geological theory is of lasting value and should form the starting point for other measurements of similar nature over more extended areas. The regions surveyed in a reconnaissance way should be explored in detail with both the gravity apparatus and the sonic depth finder in order to obtain accurate information on the physiography and the distribution of the gravity anomalies and of the stresses present in the earth's crust. It is believed that much of the detailed gravity and sonic depth-finding work in the Caribbean area can be done on a submarine temporarily assigned for a few weeks at a time to the task.

Life aboard a submarine is interesting, but it is fraught with discomfort; notwithstanding this, the crew and officers on the *S-21* at no time complained, but were always ready and eager to aid the scientific work wherever possible and to dive at any time of day or night if necessity required it. The efficiency of the Navy in undertaking a project of this nature and carrying it through without a hitch of any sort formed a most impressive exhibit of preparedness for an emergency.

To the scientists who made the gravity measurements during the cruise the association with the officers and crew of the *S-21* was a constant pleasure. On a submarine the quarters are crowded and each officer and each member of the crew has definite responsibilities. Each one must do his task efficiently and willingly, otherwise something may happen and there is little chance to correct errors of any sort. The result is that only thoroughly capable men, willing to adapt themselves to the situation, remain for any length of time aboard a submarine. The men that

do remain are proud of their branch of the service and are attracted to it not only because of the increased pay, but also because of the informality that exists aboard and the mutual respect and good feeling that is engendered as a result of overcoming difficulties and carrying out a program that is not easy.

Special mention should be made of Lieut. J. L. FISHER, United States Navy, who commanded the *S-21*; he entered enthusiastically into the project and did everything possible to make the expedition a success. At St. Thomas he suffered an attack of appendicitis, but was unwilling, contrary to the advice of the naval surgeons at the hospital, to be operated upon there, chiefly because that would delay the expedition for several weeks. He postponed the operation, which fortunately proved most successful, until after his arrival in Washington and at the completion of the trip. Lieut. (Junior Grade) F. S. HALL, United States Navy, was in charge of engineering, Lieut. (Junior Grade) F. D. HAMBLIN, United States Navy, of ordnance and gunnery and of the mess, Lieut. (Junior Grade) A. R. SODERGREN, United States Navy, of navigation. Lieutenant HAMBLIN served also as historian of the cruise, and prepared an interesting account of the impressions received during the trip. Each one of these officers accomplished his tasks well and thoroughly and contributed much to the success of the expedition.

The crew of the *S-21* numbered 39; to this number were added 4 officers and 3 scientists; from St. Thomas Lieut. L. F. FARRELL, United States Navy, of the Medical Corps, was with us, making a total of 47 on the submarine.

In spite of the novelty and interest of life aboard a submarine, it soon became evident to the scientists that the crowded conditions aboard, the lack of adequate ventilation, and poor air resulting in part from the Diesel engine gases coupled with tropical temperatures, are not conducive to intensive intellectual work and that computations and the working up of results are not possible at sea. For this reason the operation of the gravity apparatus is made as simple as possible, so that the observer makes each measurement almost mechanically, starts the apparatus, records the temperature, the barometric pressure, and so on, and has only to take care that no essential step is omitted. The single difficult task is the determination of the rates of the chronometers by comparisons with the radio time signals. This operation, which is quite independent of the gravity measurements, requires a high degree of skill on the part of the observer. It is a task that should be learned by practice ashore in preparation for the cruise. It is possible that a method can be devised for making this comparison mechanically with the aid of a radio set-up, thus eliminating the personal factor in the method.

For the benefit of future observers who may make gravity measurements abroad a submarine, it may be stated that khaki clothes are suitable for wear during the cruise. There is so much machinery and oil about that it is not possible to keep wearing apparel clean. Khaki is easily washed and serves the purpose well. An adequate supply of underwear should also be taken to provide for frequent changes in the tropical humid zone.

Acknowledgment should be made of the many courtesies extended by Capt. R. W. McNEELY, United States Navy, commandant of the naval base at Key West, by Commander C. S. McWHORTER, United States Navy, commandant of the naval base at Guantanamo Bay, and by Capt. WALDO EVANS, United States Navy,

Governor of the Virgin Islands. Realizing that life aboard a submarine in port is less pleasant than on shore, Commander and Mrs. McWHORTER, and Governor and Mrs. EVANS entertained the scientists as house guests and added greatly to their comfort and pleasure.

SIGNIFICANCE OF GRAVITY MEASUREMENTS AND GRAVITY ANOMALIES

To one unaccustomed to gravity work the question arises: What is the significance of gravity measurements? They are made at considerable expense. Of what value are they? Is it worth while for the Navy to assign a submarine to the task now and then and to divert it from its routine duties? Is the measurement of gravity a matter of primary scientific interest? It is realized that on a gravity-measuring cruise the crew of a submarine gains experience in the handling of the ship and in making dives under different conditions; but does that offset the fact that, for the time being, it is detached from its division? In order to answer these questions adequately the purpose of gravity measurements must be set forth in a general way. The methods for measuring gravity will be considered in the next chapter.

The law of gravitation, first formulated by NEWTON two and a half centuries ago, states that two bodies attract each other in proportion to their masses and inversely to the square of the distance between their centers of mass.

Let us consider a homogeneous sphere and its gravitative attraction for points on its surface. If we were to measure the gravitational attraction of different points of unit mass on the surface of the sphere we would find the same value for each point, because all points on the surface are at the same distance from the center of the sphere.

If now the sphere be set rotating about an axis, the effect, especially if the sphere is rotating rapidly, is to give to the particles along the equatorial belt on the surface a tendency to fly off. This outward acting tendency, called centrifugal force, opposes the gravitational force, with the result that if we measure the attraction between the rotating sphere and different mass points on its surface we find it to vary from point to point and to decrease in a definite manner from the axis to the equator. The change in the value of the centrifugal force with latitude can be computed and the resultant of this force and the force of gravitation for any given point thus ascertained. For a sphere of volume equal to that of the earth (radius = 6,371.221 km. = 3,958.886 miles) and rotating with the same angular velocity, namely, once each sidereal day, the variation, with latitude, of the vertical component of the centrifugal acceleration is given in Table 1.

TABLE 1.—*Centrifugal acceleration in c. g. s. units*

Latitude	Rotating sphere cm/sec ²	Earth cm/sec ²	Latitude	Rotating sphere cm/sec ²	Earth cm/sec ²
°			°		
0	3.388	5.178	50	1.400	2.146
10	3.286	5.023	60	0.847	1.300
20	2.992	4.575	70	0.396	0.609
30	2.541	3.889	80	0.102	0.157
40	1.988	3.045	90	0.000	0.000

In the third column of Table 1 the actual variation in gravity, at sea level, with latitude (ϕ) is given according to the latest gravity formula of BOWIE: $g = 978.039 \cdot (1 + 0.005294 \sin^2 \phi - 0.000007 \sin^2 2\phi)$. Comparison of these values with those of the second column shows that something in addition to the decrease due to centrifugal acceleration must be present; the explanation is that the earth is not a sphere, but that it is an ellipsoid of rotation or spheroid whose polar axis is shorter than the equatorial axis. Only by lengthening the equatorial axis relative to the polar axis and thus decreasing the value of gravity at the equator can the distribution of observed gravity values be obtained on the earth rotating with its known speed.

If the earth were a sphere and had, for example, at the North Pole an acceleration of gravity, 982.000 dynes, the change in gravity with latitude would be that given in column 2 (Table 2); for comparison the values of gravity on the earth for different latitudes are listed in column 3 (BOWIE formula):

TABLE 2.—*Variation of gravity with latitude*

Latitude	Rotating sphere cm/sec ²	Earth (Bowie formula) cm/sec ²	Latitude	Rotating sphere cm/sec ²	Earth (Bowie formula) cm/sec ²
0	978.612	978.039	50	980.600	981.071
10	978.714	978.194	60	981.153	981.917
20	979.008	978.642	70	981.604	982.608
30	979.495	979.328	80	981.898	983.060
40	980.012	980.172	90	982.000	983.217

The geodesist is able from the values of gravity obtained from stations scattered over the earth's surface to find, by computation, the shape of the particular spheroid that yields gravity data that fit most closely the observed data. The dimensions of the spheroid can be ascertained by measurements of arc. Thus he has found the length of the polar axis to be approximately 6,356.112 km., or 3,949.994 miles, and the length of the equatorial axis, 6,378.338 km., or 3,963.308 miles. The ellipticity or ratio of the difference between these axes to the longer axis is $1/297$. The value found by NEWTON with the meager data then available was $1/229$, a value remarkably close to the present standard value. The above spheroid is the mathematical surface that most nearly coincides with the earth's actual sea-level surface or geoid. By increasing the number of gravity stations we shall in time be able to determine the small deviations of the geoid from the spheroid. The primary purpose of gravity measurements is therefore the accurate determination of the figure or shape of the earth.

Up to the present time gravity measurements have been made chiefly on land. Many attempts to measure gravity at sea have been made in the past, especially by HECKER,¹ by DUFFIELD,² and by BRIGGS³; but the only satisfactory measurements

¹ O. HECKER, Zentralbureau der International Erdmessung. Bestimmung der Schwerkraft auf dem Atlantischen Ozean, 1903; Bestimmung der Schwerkraft auf dem Indischen und Grossen Ozean, 1908; Bestimmung der Schwerkraft auf dem Schwarzen Meere, 1910.

² W. G. DUFFIELD, British Association Reports, Section A. Determination of Gravity at Sea, Manchester, 1915; Determination of Gravity at Sea, Newcastle, 1916; Determination of Gravity at Sea, Bournemouth, 1919.

³ L. J. BRIGGS, A New Method of Measuring the Acceleration of Gravity at Sea, Proc. Nat. Acad. Sci., vol. 2, 1916, 399-407.

giving the desired degree of precision have been made with the Vening Meinesz gravity apparatus aboard Dutch submarines and the S-21 of the United States Navy. To obtain a more accurate value of the figure of the earth it is essential that many more gravity determinations be made at sea and that the gravity stations be more uniformly distributed over the surface of the earth than is the case at present.

The fact that the earth's surface is irregular, with great mountain masses such as the Himalayas rising to heights of 29,000 feet in certain areas, and great ocean deeps extending to 32,000 feet below sea level in others, introduces factors of local attraction that have to be taken into account in actual gravity considerations. They cause the geologist to wonder why these immense structures fail to collapse and flatten out because of the yielding of the rocks under the shearing stresses thereby developed.

The forces produced in a rotating body the size of the earth are very large; computations show that if the earth were homogeneous in composition and had the strength of steel, even then its surface would be fairly smooth and devoid of great mountains or ocean deeps. Its figure would be one of gravitational equilibrium resulting from the rotation of a homogeneous mass the size of the earth turning at its present speed. If the rotational speed were to change, the shape would change accordingly. If it were to cease to turn, its figure would become spherical. The response of the earth to the tidal pulls exerted by the sun and moon show that its strength is about that of steel.

Geological observations prove that the surface and outer crust or skin of the earth is not homogeneous, but consists of rocks and materials of different densities. The oceans cover a large part of the earth's surface; over the continents rock masses of different kinds and densities occur. If the earth's figure is one of equilibrium, we should expect to find that, where high elevations occur, the average density of materials directly underneath is less than that below lower lying areas and especially below the oceans. This response in the figure of the earth to local differences in density and consequent departure from the exact spheroid form is in keeping with the principle of equilibrium. It was called *isostasy* by C. E. DUTTON and has had a profound influence in geodesy and geology.

The proof that the shape of the earth is in a condition of equilibrium or isostatic balance with respect even to features the size of a mountain mass can be adduced only indirectly. There is no direct method of separating the total gravity pull of the earth from that of local attractions. The procedure, adopted to determine how nearly the isostatic balance is attained, is first to ascertain the approximate shape of the earth's spheroid by using gravity values obtained by measurement at different points over the earth's surface, and then to deduce for each station the value it would have were it at sea level and free from local attractions. The problem is greatly simplified because the figure of the earth approximates very closely to that of a spheroid of rotation of slight polar flattening. From the gravity values, reduced to sea level for all stations, a gravity formula is derived showing variations in gravity with latitude that fits best the available gravity measurements; this formula is tentative only and subject to correction as more gravity data are obtained.

The next step is to consider in detail for each gravity station the corrections for local attractions. These corrections include not only that for elevation above sea level which causes a decrease in the earth's attraction; but also for the added attraction of topographic features above sea level and for corresponding compensating deviations of density below sea level, including depths of the ocean.

The difference between the observed value of gravity and the computed value of gravity for the latitude of the station together with the foregoing corrections is called the gravity anomaly.

From the size of these anomalies for widely scattered stations both the degree of approximation with which the formula represents the shape of the earth and the accuracy of the assumptions made in computing the corrections for local attractions are tested. The values of gravity are commonly stated either in dynes (units of force) or in accelerations in centimeters per second per second. The values range between 978.0 and 983.4 dynes approximately; thus the total change in gravity in passing from the equator to the North Pole is only one half per cent of its total value. Gravity anomalies are commonly expressed in thousandths of a dyne; for convenience the unit is called millidyne (1 millidyne = 0.001 dyne).

In computing the correction for elevation of a gravity station above sea level, the simple formula for the attraction of a sphere on a point is used. This means a decrease in the value of g of 1 millidyne for each increase of 3.24 meters (10.6 feet) in elevation. Thus the value of gravity in an aeroplane at 25,000 feet directly above a seaport is approximately 2.36 dynes less than at the seaport. An aviator weighing 200 pounds at sea level would weigh half a pound less at 25,000 feet elevation.

The local gravitational attraction of mountain masses and all other masses above sea level at any given station can be ascertained by different methods of computation, providing the densities and shapes and sizes of the different masses are known. The average density of the rock masses is commonly assumed to be 2.67; the shapes and sizes are read off directly from topographic maps. As a first approximation in fairly flat country, the attraction of a horizontal plate indefinitely extended and having a thickness equal to the elevation of the station is sometimes used. The attraction of a plate of this kind 8.93 meters thick (29.30 feet) or approximately 30 feet is 1 millidyne. The attraction of the plate does not depend on its distance below the observing station. More accurate methods of computation take into account the curvature of the earth and the more exact shapes and sizes of the attracting masses. The computation of the attractions of these topographic features, extended over the entire earth's surface, presents no serious difficulty, except that of estimating the densities of the attracting masses; it can be carried to any desired degree of refinement.

On the assumption that the topographic features are extra loads on the earth's surface, supported by the earth without deformation, the gravity anomalies show a definite tendency to depend on the elevation of the observing station and to increase with the height. On the other hand, if the assumption is made that below the elevations there is a deficiency of density to offset or to compensate for the extra load, in other words, that the elevations are there because of a deficiency of density in the rock masses directly underneath, just as an iceberg projects above a water

surface because ice is lighter than water, the gravity anomalies are much smaller when this compensating effect is taken into account. This is the proof of isostasy.

Different assumptions have been made as to the manner in which the compensation is distributed below sea level. The test for any given assumption relative to compensation below sea level is the smallness of the gravity anomalies for a large number of stations located in different parts of the world; also the lack of dependence of the gravity anomaly on the physiographic type of country adjoining the station, whether mountains, plains, shores, or ocean.

Up to the present time two hypotheses have been proposed to account for the compensation, the one by AIRY in 1855 and the second by PRATT at about the same time. In the AIRY hypothesis the mountains and other elevations are considered to be floating on a denser, slowly yielding substratum. In the PRATT hypothesis the compensation is considered to be distributed down to a certain depth; at this depth the masses of the columns extending from the earth's surface down to this depth are all supposed to be equal. In both hypotheses any change in the distribution of load at the surface gives rise to a flow of the subjacent denser, viscous, or plastic mass in the opposite direction. According to the AIRY view, an increase of load at the surface means a sinking of the mass into the denser substratum just as an extra load on an iceberg or on a ship causes it to sink deeper into the water. In the PRATT hypothesis it is assumed that changes take place in the densities of the materials in the column to offset the mass of the superimposed load.

Up to the present time no definite decision has been reached as to which hypothesis is the better. It is recognized that the earth's crust has strength and is therefore competent to support loads of a certain magnitude distributed over a somewhat extended region, but that this range of competency can not extend out indefinitely; also that, at the margins of such loaded areas, movements in the crust are especially likely to occur. In other words, it is realized that the load, due to any topographic feature, is not supported alone by the column immediately below it, but that adjoining areas below sea level help bear the load. Mechanically the distribution of stress in this case is well shown by observing the strains in a plate of glass supporting a load applied at a point at its surface; in such a plate the distribution of strain can be studied by observing its effect on polarized light transmitted through the glass normal to the direction of the applied load. The glass surface is depressed slightly and therefore stretched a little in the vicinity of the point of application. Directly below the point and extending down into the glass, as a cone, is an area of compression that gradually decreases in intensity with depth and laterally. Along the sides of this cone the glass shows no strain; above this zone of no strain a region of tension is observed that extends to the surface. The load applied may greatly exceed the elastic limit of the glass if the area of the point of application only be considered and yet neither rupture nor permanent deformation occurs. But if the load be applied suddenly, as by a hammer blow, there is no time for the distribution of the load and fracture does occur.

This feature of regional distribution of load, resulting from the strength of the earth's crust, is an important factor in the theory of isostasy. If we knew definitely

the actual distribution of the load in the vicinity of any given station, the gravity anomalies due to this lack of exact local compensation could be computed and the corresponding stresses in the earth's crust could be found.

In addition to this feature of regional compensation which may cause apparent anomalies, it is probable that real isostatic anomalies occur, caused by departures of isostatic equilibrium of the earth's crust. These departures in equilibrium can only be caused by stresses in the earth's crust or in the subcrustal layers, which prevent the crust from adjusting itself in floating equilibrium. If, for instance, the crust is subject to tangential stress (horizontal compression), its position may be on a higher level than that which corresponds to free floating equilibrium because of the curvature of the earth's surface. This gives rise to an excess of mass that reveals itself in positive isostatic anomalies. A formula may be deduced giving a relation between this positive anomaly and the tangential stress which is the cause of it; thus it is possible to ascertain the magnitude of the stress if the anomaly is known.

If, therefore, we succeed in determining the true isostatic anomaly for a certain part of the earth's crust, we may obtain data on the trend and the magnitude of the tectonic stresses existing in that region. In areas subject to orogenic movement, stresses of this nature are to be expected; gravity measurements in such areas are especially important and serve to increase our knowledge of the factors that tend to disturb the shape of the earth's crust. Many of the measurements made on the voyage of the *S-21* were over areas in which crustal movements have recently taken place. Final conclusions regarding these areas must, however, await more detailed knowledge of the topography before we can be assured that the recorded gravity anomalies do represent actual deviations from isostatic equilibrium.

The most complete investigation into these reductions and into the theory of isostasy is that initiated by J. F. HAYFORD⁴ at the United States Coast and Geodetic Survey and carried on in recent years by W. BOWIE. In HAYFORD's work the computation for attraction and compensation was extended for each gravity station to cover the entire surface of the world. For convenience the assumption was made that each feature is compensated locally by a corresponding deficiency in mass in the column directly underneath and extending down to a depth to be determined; the depths first determined by HAYFORD were 113.7 km.; later the value 122 km. was ascertained; still more recently BOWIE has found 96 km. to be a better depth on the basis of a larger number of stations. Local compensation assumes the crust to have no strength, just as a liquid has no strength. This assumption was made, as a simplification, to facilitate the mathematical computations.

Through the courtesy of the United States Coast and Geodetic Survey, the measurements obtained on the voyage of the *S-21* have all been reduced under the direction of Dr. W. BOWIE by this method of compensation. It is believed to be essential to proper interpretation that all gravity anomalies shall be computed by the same method and thus far the HAYFORD-BOWIE method is the one most widely

⁴ J. F. HAYFORD, Figure of the Earth and Isostasy from Measurements in the United States, U. S. C. and G. S., 1909; Supplementary Investigation of Figure of Earth and Isostasy, U. S. C. and G. S., 1909; J. F. HAYFORD and W. BOWIE, Effect of Topography and Isostatic Compensation upon the Intensity of Gravity, Special Publication No. 10, U. S. C. and G. S., 1912.

used in this connection. If it is desired later to introduce, instead of local compensation, regional compensation extending to a distance to be ascertained, this can easily be done on the basis of the tables and data already available for the HAYFORD-BOWIE method.*

CHAPTER 2

THE MEASUREMENT OF GRAVITY

The acceleration due to gravity was first measured by noting the distance a body falls in a second or unit of time. In 1590 GALILEO, by allowing bodies to fall from the Leaning Tower of Pisa, disproved the dictum of ARISTOTLE that heavy bodies fall with velocities proportional to their weights; in the absence of a good timepiece he was not able to measure the gravity acceleration accurately. Nine years earlier he had discovered the isochronism of the pendulum, but did not suggest, until 50 years later, that the principle be applied to the construction of an astronomical clock. The actual construction of a pendulum-regulated clock was first accomplished by HUYGENS in 1656. HUYGENS deduced also a formula for the pendulum period showing its relation to the gravity acceleration. Shortly thereafter, in 1666, NEWTON began his work on gravitation. Twenty years later he states in his Principia that "in the latitude of Paris a heavy body falling in a second of time describes 15 Paris feet, 1 inch, $1\frac{1}{2}$ line, that is, 2173 lines $\frac{1}{2}$. The weight of the body is diminished by the ambient air. Let us suppose the weight lost thereby to be $\frac{1}{1000}$ part of the whole weight; then that heavy body falling in vacuo will describe a height of 2174 lines in one second of time." From this single measurement, together with the measured length of an arc along a meridian and the known period of rotation of the earth, NEWTON computed the polar flattening of the earth considered as an oblate spheroid. He did this on the assumption that the earth is a liquid homogeneous mass. He found on computation that the ratio of the centrifugal acceleration of the earth at the equator to the gravity at Paris, corrected for centrifugal acceleration, is 1: 289; also that in a homogeneous globe whose equatorial axis is $\frac{1}{100}$ longer than the polar axis the ratio of the centrifugal acceleration at the Equator to the gravitational attraction is 4: 505; and therefore, by simple proportion, that the equatorial diameter of the earth is $\frac{1}{229}$ longer than its polar diameter, a value not greatly different from 1: 297, the present standard value.

NEWTON suggested in the Principia the use of a pendulum for measuring gravity and gives a table showing the variation in length of the seconds pendulum with latitude from the Equator to the pole. Astronomers had already noted that an astronomical pendulum clock that keeps time in one latitude fails to do so in another. Thus in 1672 M. RICHER found that on the island of Cayenne off French Guiana his clock, that had been adjusted for the latitude of Paris, lost 2 minutes 28 seconds a day. He found the length of a seconds pendulum at Cayenne (latitude 5°) to be shorter by $1\frac{1}{4}$ lines (approximately one-tenth inch) than the seconds pendulum at Paris (latitude 49°).

The pendulum still remains the best and most satisfactory method for gravity measurement. It is, however, an indirect method. The period of vibration of a pendulum is measured; and from this in turn the value of gravity is computed by the usual pendulum formula. But, if the length of the pendulum is not known,

* The isostatic reduction comprises the small BOWIE reduction which takes into account the slight shift of the sea level surface due to the elimination and compensation of the topography.

this value is only a relative value, because the period of vibration of the pendulum at one station is compared with the period of vibration at another station. The intensities of gravity at the two stations are inversely proportional to the square of the observed vibration periods.

For the absolute measurement of gravity the length of the pendulum must be ascertained and this is done by use of a reversible pendulum; but the measurement is a tedious process covering long periods of time. For this reason it is customary to establish a single base station for each country and to refer all gravity pendulum measurements to that station. By general agreement the base station with which all other base stations are compared is the gravity station at the Geodetic Institute at Potsdam, Germany. The base station for the United States is located at the United States Coast and Geodetic Survey in Washington. Comparisons between the Washington base station and the base stations of Canada, England, Germany, France, and Austria have been repeatedly made and the gravity measurements throughout the world thus rendered comparable. The outstanding uncertainties are small and within one hundredth of a dyne or one one-thousandth per cent of the total value of gravity. The Washington base station has been adopted as base station for the measurements of the United States submarine S-21; the station at Washington was occupied both before and after the voyage.

The single, free-swinging pendulum apparatus, to be satisfactory, in use, requires a steady platform of either cement or stone. A slight yielding or sway of the platform introduces horizontal accelerations in the swinging pendulum and affects its period. In order to measure the period with the required accuracy the pendulum is allowed to swing for many hours and the total number of swings during the total time is counted. This is done by a method of coincidences with the beats of chronometers whose rates are in turn ascertained by comparison with standard time signals sent out either by radio or by telegraph. In order to be as free as possible from changes in the rates of the chronometers the gravity pendulum is allowed to swing for practically the entire period between the standard time signals. To the vibration period thus obtained a number of corrections are made, such as correction for barometric pressure, for temperature, for arc of swing, and for flexure of support. The accuracy attainable by gravity measurements of this kind is about two parts in a million under ordinary conditions. Under special conditions in a local area, say, 100 miles from a base station, and using a free-swinging pendulum as the time keeper and radio time signals sent out at definite intervals, the accuracy can be pushed to several parts in 10,000,000. For most gravity work in geodesy such a high degree of accuracy is not needed and the accuracy commonly attained by the routine methods suffices.

In Holland the character of the ground is such that it is difficult to find a steady base on which to swing a pendulum. To overcome this difficulty, Dr. VENING MEINESZ devised a method, based on swinging two pendulums simultaneously, which answered the purpose satisfactorily. In 1923 this method was further elaborated and tested aboard a submerged submarine on which it was found to function well. A special apparatus was then constructed for the measurement of gravity at sea; this was better adapted to the purpose than the STÜCKRATH apparatus, which had previously been used.

The Vening Meinesz Apparatus.—The principle on which the new pendulum apparatus is based is not difficult to state.¹ If a single pendulum be set swinging and a horizontal acceleration or disturbance be then introduced, the character of the vibration is changed momentarily; it may be stopped altogether if the added acceleration is of the right magnitude and direction, or it may be greatly increased if the phase and amplitude relations are suitable. The resultant acceleration in each case is the algebraic sum of the original and added accelerations, and the motion of the pendulum is the expression of the action of the two accelerations. This response of the single pendulum to extraneous horizontal disturbances has prevented its use on board ship. If, however, instead of a single pendulum, a pair of free-swinging pendulums of very nearly the same vibration period be observed, both swinging in the same plane and suspended from the same frame, and the differences in angular elongation between the two pendulums be recorded by means of properly set mirrors, the result is free from the effects of added horizontal accelerations. These affect the angular elongations of both pendulums equally and hence disappear in the difference. This use of the pendulum pair to serve as a single fictitious pendulum is the essential feature of the new apparatus and enables it to be used successfully on a platform not perfectly steady, but subject to oscillations different in period from the pendulum period. The pendulum pair functions as a simple fictitious pendulum undisturbed by extraneous accelerations of not too great magnitude. On the submarine, rolls of 6° on a side are tolerated, provided the period of the roll is relatively long.

In the VENING MEINESZ apparatus three pendulums are actually employed. They are all suspended from a single frame and all swing in the same vertical plane. The first and second serve as one pair and the second and third as the second pair. In practice only the first and third are set swinging and then in opposite phase. The second or middle pendulum is stationary except in so far as it is set vibrating by accelerations due to the ship's motion. The pendulums are of the quarter meter, half-second type of Von STERNECK and STÜCKRATH. To the top of each pendulum a mirror is attached; this serves to reflect a beam of light from the one pendulum to the second and thus to indicate the resultant effect or phase of the pair. The arrangement of the mirrors in the actual apparatus is shown in Figures 1, 1A.

A beam of light from a pointolite source is reflected down through a narrow slit at the rear focus of a collimating lens into the pendulum case to the several pendulums and thence through a second lens up into the case containing the roll of photographic paper on which the movements are recorded.

In addition to the three pendulums, the pendulum case contains attached to it a dummy pendulum identical in shape and material with the swinging pendulums. A mercury thermometer reading to tenths of a degree centigrade is inserted into this dummy pendulum and indicates the temperature of the swinging pendulums. A hair hygrometer is mounted beside the dummy pendulum and indicates the humidity.

Two other pendulums are shown in Figure 1A; the first carries the prisms of *a* and *b* and the second, the prism *d*. These pendulums are not of the ordinary

¹ For a detailed treatment of the subject see "Theory and Practice of Pendulum Observations at Sea," by F. A. VENING MEINESZ. Netherlands Geodetic Commission. Delft, Holland. 1929.

View from above of interior of pendulum case of apparatus, showing the arrangement of prisms

FIGURE 1A

Arrangement of mirrors in pendulum case of apparatus. L_1 , L_2 are the entrance and exit lenses, respectively; m_1 , m_2 and m_3 are the three nearly isochronous, free-swinging pendulums mounted on a single supporting straight bar; a and d are two damped pendulums, a swinging in the same plane with m_1 , m_2 , m_3 , while d swings in a plane normal thereto. Prism c is vertically above d and prism n above o

The pendulum case with insulating cover removed. Two of the three pendulums are visible at the sides; the dummy pendulum with thermometer and the hair hygrometer can be seen in the central part. The damped pendulum *d* is shown at the upper left side. A reserve pendulum stands beside the apparatus. At the bottom the clutch and release levers are shown, and at the left side a spirit level. Photographed by James Stokley, Science Service

type; each one contains a self-damping device. Each pendulum actually consists of two pendulums one inside the other; the knife edge of the inner pendulum is in the same line with the knife edge of the outer pendulum. The inner pendulum has a vane extending down into an oil reservoir at the base of the outer pendulum. The moments of inertia and of mass of the inner pendulum are inversely proportional to the moments of inertia and of mass of the outer pendulum, with the result that the inner pendulum functions as an effective damping device for the outer pendulum, thus enabling it practically to maintain its position in space unchanged in spite of movements of the ship. One of these damped pendulums swings in the plane of the three free-swinging pendulums, and serves as a datum line to which movements of the middle pendulums are referred; a record of the motions of the middle pendulum is required for the computation of certain corrections that are applied to the two principal records made by the two pairs of pendulums. The prisms *a* and *b* of Figure 1A are attached to this damped pendulum. The second damped pendulum swings in a vertical plane perpendicular to that of the free-swinging pendulums and serves to record the pitching of the apparatus. Prism *d* of Figure 1A is mounted on this pendulum.

Prism *c* of Figure 1A reflects light from a bent bimetallic ribbon that indicates temperature fluctuations within the pendulum case. The pendulum case is enclosed by a lining of felt, 2 inches thick, to prevent rapid fluctuations in temperature.

In the lower part of the pendulum case is located the mechanism for clutching and releasing the pendulums. Means are provided for releasing any pendulum separately or in combination and in phase with, or in opposite phase to, the phase of any other pendulum. The usual practice is to set the two outer pendulums swinging in opposite phase and to leave the middle pendulum untouched. The release and clutching devices are provided with safeguards so that each pendulum can be lowered on the knife edge only after the pendulum bob has been released (fig. 2). The usual STÜCKRATH method for setting the pendulum in vibration with a given amplitude is used; a lever presses against the pendulum bob and pushes it out to the desired distance from its vertical position; on quick throwing of this lever the pendulum is free to swing with the given amplitude. An intermediate cog-gear system enables the two outer pendulums to be released at the same time and in opposite phase. On board ship care must be taken in this operation to release the pendulums when the apparatus is vertical in order that both pendulums may have the same initial amplitude of vibration.

The thermometer is read with the aid of a lens through a small glass window. The temperature should be read to twentieths of a degree. The hair hygrometer is read through the window at the same time. The readings of the hygrometer scale are checked at intervals by means of a psychrometer. For this purpose the hygrometer is taken out of the pendulum case and placed beside the psychrometer.

Mounted above the pendulum case is the recording apparatus (fig. 3A). This consists essentially of a roll of photographic paper that is slowly unwound by clockwork past the receiving slit on to a second drum. In addition, the illuminating apparatus and the lever signal arms for the chronometer signals are housed in this upper case. The light rays from the pendulum case pass into the recording

case through an opening at one corner and thence are reflected (fig. 3) through several prisms along one side of the box before they enter the cylindrical lens directly in front of the sensitive paper. This broken path inside the box serves to shut out extraneous light, to make the box light tight, and to increase the length of path of the rays.

At two points in the light path within the box, prisms can be inserted and enable the observer to see the movements of the light rays reflected from the several pendulums and to ascertain if the entire apparatus is functioning correctly. One of these prisms is placed beside the passing beam and, as it does not cut off the rays, it can be used while the instrument is recording, without disturbing the record. From time to time during the making of a record the observer may wish to assure himself that all is well and this he can do with the aid of this small prism. The second prism covers the entire beam and throws it on to a ground glass plate on which the entire recording light rays and their courses can be seen for any desired period of time. This arrangement is used only at the beginning of a measurement when it is important to know that the amplitudes of the two pendulum pairs are approximately equal and that the middle pendulum is not swinging violently.

The clock driving mechanism for winding the photographic paper on a drum can be run at two different speeds. At the beginning and end of a run the fast speed is used for five or six minutes in order to obtain a more open and easily read record on which to determine the coincidence and passage times for the two pendulum pairs. During the remainder of the exposure a speed one-third of the fast speed is used; photographic paper is thereby saved. Reproduction of part of an actual record is shown in Figure 4.

Time signals from the chronometers are marked on the record by means of lever arms operated through solenoids. At each make and break of the chronometer, the lever arm is made to pass in front of the slit thereby cutting off the light for an instant and leaving a blank spot on the record. The succession of these white interruptions on the record at half-second intervals are clearly shown on the record (fig. 4.) One lever arm is connected with a mean-time chronometer; the second with the sidereal time chronometer. It is through these time marks from the two chronometers whose rates are determined twice each day by comparison with radio time signals, that the time of vibration of each pendulum pair is evaluated from the record itself. For the time period during which the paper runs at slow speed, the length of stroke of one of the lever arms is reduced so that during the half second between the make and break of the chronometer, the slit is covered and the illumination is cut off.

In the design of the apparatus the effort has been made to reduce to a minimum the number of operations necessary to start the apparatus and to have these operations follow in a definite sequence, so that the procedure becomes one of routine. This is necessary because on a submarine the mind is not alert and mistakes are likely to be made if anything beyond routine and purely mechanical details is demanded. Proof that the present procedure is satisfactory is the fact that during the cruise of the *S-21* no station record was lost or was entirely unsatisfactory.

The pendulum case and recording chamber of apparatus. Examining apparatus are Hon. Curtis D. Wilbur, Secretary United States Navy; Capt. C. S. Freeman, United States Navy, Superintendent Naval Observatory; and Dr. F. A. Vening Meinesz, of the Holland Geodetic Commission. Photograph by James Stokley, Science Service

FIGURE 3A

Side view of pendulum case and recording chamber. Electrical connections to light source and to chronometers are shown

The entire apparatus, including pendulum case and recording apparatus, is mounted in a cradle supported on gimbal bearings. A rugged upright frame of angle irons is firmly attached to cross beams below the deck of the control room of the submarine. To this frame the gimbals are suspended on steel knife edges. Sufficient clearance is given to permit of an angle of roll of the boat of 13° on a side. Experience has shown that if the gimbal bearings work smoothly the ship may roll up to 6° on a side and yet not interfere seriously with the pendulum record. This means that measurements are possible on the surface of a fairly calm sea, if the vibrations from the engines are reduced below a certain limit.

A photograph of the gravity apparatus as mounted on the S-21 is reproduced in Figure 5. The levels near the base of the apparatus indicate the upright position. The apparatus is leveled roughly by means of a counterweight attached to an eccentric arm. The more exact leveling is done by moving a small mass over the top of the pendulum case. It is necessary to level the apparatus quite accurately in order that the pendulums swing from a horizontal support.

Adjustment of the apparatus.—The features of the instrument whose adjustment should be tested from time to time are the following:

A. On lowering the pendulums by the slow motion to the agate supporting planes the three knife edges should reach the planes at the same instant and each knife edge should be parallel with the agate plane.

B. Each one of the five different beams of light should be traced through the apparatus from the light source to the photographic paper to insure that the prisms and lenses are properly adjusted and that the beams impinge on the sensitive paper in the correct relative positions.

C. The lever arms actuated by the chronometers should be so adjusted that the solenoid control is positive; the tension of the opposing spring should not be so great as to cause the arm to rebound elastically on release by the solenoid.

D. The photographic paper should be maintained at uniform tension by adjustment of the pressure of the roller against the roll of paper as it is being unwound.

E. The speed of the photographic paper should be tested to insure easily legible records.

F. By levelling the apparatus on a fixed support on shore and noting the position of the record made by the second damped pendulum the normal position of the light beam can be determined.

Routine procedure in occupying a station at sea.—In all measurements with an apparatus on a ship at sea a definite routine must be prescribed and meticulously followed, otherwise failure may result because some operation has been overlooked. Experience with the new gravity apparatus has shown that if the following operations are performed in the order named, satisfactory results with a 30-minute record will be obtained.

1. Open the doors to the illuminating system of the recording case. Screw in the flexible cable release cords. Open the apertures at the base of the recording case and at the top of the pendulum case.

2. Wind the clock.

3. Connect the pointolite to its circuit. Plug in the chronometer circuits to the lever arms.

4. Release the cradle in the gimbals and level the apparatus.
5. Unclamp the pendulums.
6. Operate the levers that press against the outer pendulums and push each pendulum out to the desired amplitude.
7. Project the beams of light on the ground glass scale.
8. Lower the pendulums.
9. Set the pendulums swinging by quickly releasing the supporting levers.
10. Start the clockwork at the high speed; mark the photographic paper; and turn back the prism so that now the light rays impinge on the photographic paper instead of the ground glass scale. Record the time of day.
11. Read the thermometer, the hygrometer, the barometer, and the temperature outside the apparatus.
12. Five or six minutes after operation No. 10, change the speed of the clockwork from high to low speed and change the shutter movement from momentary to half-second interruptions.
13. Fifteen minutes after operation No. 10, reread the thermometer, hygrometer, the barometer, and the outside temperature.
14. Twenty-five minutes after No. 10, change the speed of the clockwork to high and the shutter to momentary interruptions.
15. Twenty-nine minutes after No. 10, read thermometer, hygrometer, barometer, and outside temperature.
16. Thirty minutes after No. 10, lift and clamp the pendulums, mark the paper, stop the clockwork, disconnect the pointolite and the chronometers, clamp the gimbals and cradle, screw off the flexible cords, close the doors.
17. Record the mean depth of the ship below the surface during the observation, its speed and course, the direction and velocity of the ocean current, the observed depths to ocean bottom, and the mean latitude and longitude during the interval.

EVALUATION OF GRAVITY FROM THE RECORDS

In the foregoing sections a brief description of the new pendulum apparatus is given and the method of making measurements with it at sea is explained. The task still remains to show how, from the data thus recorded, the value of gravity at a given station can be ascertained. Before undertaking the actual task of measuring and computing the results we have first to consider the principles on which the measurements are based and then to develop the formulas required in the computations.

THE PENDULUM EQUATION

In the pendulum apparatus the greatest amplitude of vibration permissible is less than 5 mm.; this corresponds to an angular deflection of $1^{\circ} 8'$ on each side. Although the pendulum swings in air at atmospheric pressure, the decrease of its amplitude during half an hour is not great, approximately 15 per cent. Thus if the amplitude at the beginning of a run is, in angular measure, $1^{\circ} 8'$ at the end, half an hour later it is still $0^{\circ} 57'$. These angles are so small that it is permissible, as a first approximation, to substitute the angle itself in radians for the sine or the tangent of the angle. This simplification greatly facilitates the mathematical

The gravity apparatus mounted in the U. S. submarine *S-21*. United States Navy official photograph

treatment of the problem and the computation of the effects of the action of certain disturbing factors on the period of the pendulum. In the derivation of the equations that arise in this connection it is the period of the pendulum and the disturbances to this period over a given time interval that are important; for this reason the ordinary pendulum equations expressing relations between moments or between accelerations, will be recast to express relations between velocities and to show the effect of external disturbances on the velocity of a certain vector, which may be called the pendulum vector; this is a rotating vector, the projection of which on a fixed line through the origin represents at each moment the angle of elongation of the pendulum; the mean time required for this vector to make a complete rotation is the period of the pendulum at that instant.

To derive the equation for the swinging pendulum let us suppose the whole mass of the pendulum to be concentrated in the bob at a point at the center of mass and the pendulum to be suspended from a fixed point by a weightless rod of length l .

FIGURE 6.—Diagram used in the derivation of the pendulum equation

In Figure 6 let s be the distance described by the particle, the point of reference being the lowest point C . At P the forces acting on the particle are its weight and the tension on the rod. The tension has no component in the direction in which the particle moves. The weight has a component, $-mg \sin \vartheta$, in which ϑ is the angle POC and $s = l\vartheta$. The equation of motion is therefore

$$m \frac{d^2 s}{dt^2} = -mg \sin \vartheta \quad \text{-----} (1)$$

Because of the small angle ϑ actually used, ϑ in radians can be substituted for $\sin \vartheta$ in this equation; also x in place of s if desired.

$$\frac{d^2 s}{dt^2} = -g\vartheta = -\frac{g}{l}s \quad \text{-----} (2a)$$

or

$$\frac{d^2 x}{dt^2} = -\frac{g}{l}x \quad \text{-----} (2b)$$

or

$$\frac{d^2 \vartheta}{dt^2} = -\frac{g}{l}\vartheta \quad \text{-----} (2c)$$

Thus the acceleration of the pendulum bob is proportional to its distance from C , and is directed toward C .

This is the equation of a simple, free swinging, undamped pendulum. Although in reality the pendulum mass will not be concentrated in P , a simple consideration shows that in this case the equation of motion remains the same, only that l means now the quotient of the moment of inertia over the moment of mass of the pendulum, both taken with regard to O . If now an external or disturbing effect, which is represented by S , is introduced into the equation of motion (2c), this equation takes the form

$$\frac{d^2\vartheta}{dt^2} + \frac{g}{l}\vartheta + S = 0 \quad (3a)$$

or

$$\frac{d^2x}{dt^2} + \frac{g}{l}x + Sl = 0 \quad (3b)$$

If the disturbing term is caused by a horizontal acceleration, $\frac{d^2x}{dt^2}$, of the pendulum knife-edge, we can prove that

$$S = \frac{1}{l} \frac{d^2x}{dt^2}$$

and the equation of motion reads

$$\frac{d^2\vartheta}{dt^2} + \frac{g}{l}\vartheta + \frac{1}{l} \frac{d^2x}{dt^2} = 0$$

If two pendulums be swung simultaneously from the same support, the disturbing force affects both pendulums and the two equations of motion are, if we write ϑ for $\frac{d^2\vartheta}{dt^2}$ and $\ddot{x}$ for $\frac{d^2x}{dt^2}$,

$$\vartheta_1 + \frac{g}{l_1}\vartheta_1 + \frac{1}{l_1}\ddot{x} = 0 \quad (4a)$$

$$\vartheta_2 + \frac{g}{l_2}\vartheta_2 + \frac{1}{l_2}\ddot{x} = 0 \quad (4b)$$

If the two pendulums are of the same length l , we find by subtracting (4b) from (4a)

$$(\vartheta_1 - \vartheta_2) + \frac{g}{l}(\vartheta_1 - \vartheta_2) = 0$$

an equation of the same form as that for a simple undisturbed pendulum of reduced length l and angle of elongation, $\vartheta_1 - \vartheta_2$. If therefore, we observe, by means of reflecting mirrors, the difference between the angles of elongation, the resulting angle, $\vartheta_1 - \vartheta_2$, may be considered the angle of elongation of a fictitious pendulum which is not disturbed by the horizontal accelerations of the apparatus and which is isochronous with the original pendulums.

This is the fundamental feature of the new gravity apparatus and the mathematical discussion that follows seeks to set up equations by means of which the effects of external disturbances on the period of this fictitious pendulum can be determined.

The presentation and interpretation of the subject is more direct if, instead of equation (3a), which expresses relations between accelerations, the equation be recast to express relations between velocities of the pendulum vector. If in equation

(3a) we write n^2 for $\frac{g}{l}$ we have

$$\ddot{\vartheta} + n^2\vartheta + S = 0 \quad (3c)$$

If now we introduce in place of ϑ two new variables a and φ defined by the equations

$$\vartheta = a \cos \varphi \quad \text{-----} (5a)$$

$$\frac{d\vartheta}{dt} = \dot{\vartheta} = -a n \sin \varphi \quad \text{-----} (5b)$$

and call a the amplitude and φ the phase of the pendulum vibration, we find that $\vartheta = OB$ is the horizontal projection (fig. 7) of the pendulum vector OA .

FIGURE 7.—Diagram illustrating the pendulum vector and the relations between φ , ϑ , and a

Differentiating (5a) and (5b), we find

$$\dot{\vartheta} = \dot{a} \cos \varphi - a \sin \varphi \dot{\varphi} \quad \text{-----} (6a)$$

$$\dot{\vartheta} = -\dot{a} n \sin \varphi - a n \cos \varphi \dot{\varphi} \quad \text{-----} (6b)$$

Combining (5b) and (6a) we have

$$(\varphi - n) a \sin \varphi - \dot{a} \cos \varphi = 0$$

and from (3c), (5a), and (6b)

$$(\varphi - n) a \cos \varphi + \dot{a} \sin \varphi - \frac{S}{n} = 0$$

From these two equations we obtain in turn

$$(\varphi - n) a = \frac{S}{n} \cos \varphi \quad \text{-----} (7a)$$

$$\dot{a} = \frac{S}{n} \sin \varphi \quad \text{-----} (7b)$$

If there is no disturbing factor, $S=0$, we have

$$\varphi = n \quad \text{-----} (7c)$$

$$\dot{a} = 0 \quad \text{-----} (7d)$$

In this case the amplitude a is constant, and the phase φ increases in a constant ratio n ; in other words, the pendulum vector has a constant length and rotates with a constant angular velocity $n = \sqrt{\frac{g}{l}}$. If we define the period of the pendulum T as half the time required for a complete rotation (2π) of the vector we have

$$T = \frac{\pi}{n} = \pi \sqrt{\frac{l}{g}} \quad \text{-----} (7e)$$

This is the period of the undisturbed pendulum, free swinging and undamped. The angular velocity of the pendulum vector is constant (fig. 8) and the horizontal

FIGURE 8.—Diagram illustrating positions of the pendulum vector in the case of a simple, free-swinging pendulum

projection OB of the vector OA is the angle ϑ which gives the angular position of the swinging pendulum with reference to the vertical.

Equations (7a) and (7b) express relations between velocities. In (7a) the angle φ refers to the angle between the pendulum vector and a fixed line; $\dot{\varphi} - n$ is the added angular velocity due to the disturbing acceleration S ; while $(\dot{\varphi} - n) \cdot a$ is the actual velocity along the circle, added to the velocity of the pendulum vector point A . Similarly in (7b), $\dot{a}$ is the velocity of the vector in the direction, OA , produced by the acceleration S . In equations (7a) and (7b) all terms represent velocities; therefore $\frac{S}{n} \cos \varphi$ represents a velocity normal to OA and $\frac{S}{n} \sin \varphi$, a velocity along

FIGURE 9.—Diagram showing the effect, on the pendulum vector, of a disturbing acceleration S

OA ; $\frac{S}{n}$ is therefore a velocity in a vertical direction, as shown in Figure 9; and the velocity of A , which for the undisturbed pendulum is an in a direction perpendicular to OA , is increased with a vertical component $\frac{S}{n}$.

If τ be the time required for the disturbed pendulum vector to describe half a rotation, or the angle π , we find by dividing (7a) by a and integrating over the time 2τ , ($\varphi = 2\pi$), for the left side of equation

$$2\pi - 2n\tau = 2n(T - \tau) = -2n \delta T$$

and therefore the disturbance to the period T caused by the disturbing acceleration S is

$$\delta T = -\frac{1}{2n^2} \int_0^{2\tau} \frac{S}{a} \cos \varphi dt \quad (8a)$$

The mean change in amplitude per unit of time during the same period is

$$\dot{a} = \frac{1}{2n\tau} \int_0^{2\tau} S \sin \varphi dt \quad (8b)$$

These equations are fundamental to the evaluation of the disturbing effects introduced by an acceleration S on the period T of the undisturbed pendulum. If φ_0 be the phase for $t=0$, φ may be written $\varphi = nt + \varphi_0$ and equation (8a) becomes

$$\delta T = -\frac{1}{2n^2} \int_0^{2\tau} \frac{S}{a} \cos (nt + \varphi_0) dt \quad (8a')$$

or

$$\delta T = -\frac{1}{2n^3} \int_0^{2\pi} \frac{S}{a} \cos \varphi d\varphi \quad (8a'')$$

Similarly equation (8b) may be written

$$\dot{a} = \frac{1}{2n\tau} \int_0^{2\tau} S \sin (nt + \varphi_0) dt \quad (8b')$$

or

$$\dot{a} = \frac{1}{2n^2\tau} \int_0^{2\pi} S \sin \varphi d\varphi \quad (8b'')$$

In the following paragraphs equations (8a) and (8b) will be found to answer the purpose, and the more easily understood equations (8a') to (8b'') will not be used.

The external disturbances which affect the period of vibration of the fictitious pendulum and for which mathematical expressions are to be found will now be considered in detail. In each case the expression for the function will be stated and evaluated on the basis of the assumption, actually met by swinging pendulums, that the angle of elongation, ϑ , is so small that the angle itself can be substituted for its sine.

Lack of isochronism between two pendulums.—It is physically impossible to produce two pendulums that have exactly the same length and vibration period. The pendulum vectors have consequently not exactly the same angular speed, and this slight difference affects the period of the fictitious pendulum. We have, therefore, to reconsider the formulas for the fictitious pendulum which were based on the assumption of absolute isochronism. Let the disturbing horizontal acceleration, of which we wish to eliminate the effect by means of the fictitious pendulum, be $\ddot{x}$. Then

$$S_1 = \frac{n_1^2}{g} \ddot{x} \quad S_2 = \frac{n_2^2}{g} \ddot{x}$$

wherein

$$n_1^2 = \frac{g}{l_1}, \quad n_2^2 = \frac{g}{l_2}$$

The velocity of the vector A_1 is thereby increased by a vertical disturbance $\frac{n_1}{g} \ddot{x}$, and

A_2 by a vertical velocity component $\frac{n_2}{g} \ddot{x}$. We eliminate these effects by taking the vectorial difference between OA_1 and a vector $\frac{n_1}{n_2} OA_2$; it is obvious that the vector

FIGURE 10.—Diagram illustrating the relations between the vectors of two pendulums when the difference between two angular elongations is observed, so that a fictitious pendulum vector is obtained

OA , which represents the difference, is free from the disturbance, because the two velocity disturbances are opposite and equal. The velocity of A , the extremity of the vector OA , depends only on the velocities, n_1 of OA_1 and n_2 of A_1A . The horizontal projection of this vector is the difference of the projections of OA_1 and AA_1 , namely,

$$\vartheta_1 - \frac{n_1}{n_2} \vartheta_2$$

The velocity of A can be separated into two parts.

(1) The velocity resulting from the rotation of the triangle OA_1A (fig. 10) with the angular velocity n_1 . This corresponds to that of a fictitious pendulum of period T_1 and angle of elongation

$$\vartheta = \vartheta_1 - \frac{n_1}{n_2} \vartheta_2$$

(2) An additional velocity resulting from the rotation of AA_1 (fig. 11) about A_1 with an angular velocity $n_2 - n_1$. This introduces the deviation into the

FIGURE 11.—Vectorial diagram to show the effect produced on the fictitious pendulum vector by a lack of isochronism between the two pendulums

period. In triangle A_1OA angle $A_1AC = \varphi_2 - \varphi$ and the velocity of A in the direction OA is

$$\dot{a} = (n_2 - n_1) \frac{n_1}{n_2} a_2 \sin (\varphi_2 - \varphi) \quad (9a)$$

and in the direction perpendicular to OA

$$a(\dot{\varphi} - n_1) = -(n_2 - n_1) \frac{n_1}{n_2} a_2 \cos (\varphi_2 - \varphi) \quad (9b)$$

From these equations we can deduce, as in equations (8a) and (8b), by integration over the double period $2T$, the equations

$$\delta T = \frac{n_2 - n_1}{2n_2} \int_0^{2T} \frac{a_2}{a} \cos (\varphi_2 - \varphi) dt \quad (9c)$$

wherein

$$T = T_1 + \delta T \quad (9d)$$

But a_2 , a , and $\cos (\varphi_2 - \varphi)$ are practically constant, and $\frac{n_2}{n_1} = 1$, very nearly. Also

$$n_2 - n_1 = -\frac{n^2}{\pi} (T_2 - T_1) = -\frac{\pi}{T^2} (T_2 - T_1)$$

wherein $T_1 = \frac{\pi}{n_1}$, $T_2 = \frac{\pi}{n_2}$, and $T = \frac{\pi}{n}$. Therefore to this degree of approximation we find

$$\vartheta = \vartheta_1 - \vartheta_2 \quad (10a)$$

$$T = T_1 + \delta T \quad \text{or} \quad T - T_1 = \delta T \quad (10b)$$

$$\delta T = -(T_2 - T_1) \frac{a_2}{a} \cos (\varphi_2 - \varphi) \quad (10c)$$

$$\dot{a} = -\frac{\pi}{T^2} (T_2 - T_1) a_2 \sin (\varphi_2 - \varphi) \quad (10d)$$

In the case of other disturbances causing slight changes δT_1 and δT_2 in the vibration periods of the two pendulums without affecting the ratio of the amplitudes, so that the shape of the triangle OA_1A of Figure 11 does not change because of variations in the sides, we find the value of the corresponding disturbance term for the vibration period of the fictitious pendulum from (10c), whereby

$$\delta T - \delta T_1 = -(\delta T_2 - \delta T_1) \frac{a_2}{a} \cos (\varphi_2 - \varphi)$$

or

$$\delta T = \delta T_1 - (\delta T_2 - \delta T_1) \frac{a_2}{a} \cos (\varphi_2 - \varphi) \quad (10e)$$

It should be noted that $a_2 \cos (\varphi_2 - \varphi)$ is the component of the second pendulum vector in the direction of the fictitious vector OA , while $a_2 \sin (\varphi_2 - \varphi)$ is the component normal thereto. In the computations the component $a_2 \cos (\varphi_2 - \varphi)$ is measured directly and is called b . Examples of this kind of disturbance are the effects of changes in temperature and in barometric pressure and humidity. They are considered in a later section.

In case there are additional disturbances of another kind, we determine the disturbance terms S_1' and S_2' in the equations of motion of the two pendulums, and we find the corresponding disturbance S in the equation of motion of the fictitious pendulum in the following way. The disturbance S_1' adds a vertical velocity

component $\frac{S_1'}{n_1}$ to A_1 and S_2' a vertical velocity component, $\frac{S_2'}{n_2}$; the effect on A by these two components is the vertical component,

$$\frac{S_1'}{n_1} - \frac{n_1}{n_2} \frac{S_2'}{n_1} = \frac{1}{n_1} \left(S_1' - \frac{n_1^2}{n_2^2} S_2' \right).$$

This corresponds to a disturbance factor

$$S = S_1 - S_2 \text{-----} (10f)$$

on the assumption adopted above that n_1 is practically equal to n_2 . From S the disturbance δT of the period can be deduced in the usual way.

Effect of an external vertical acceleration.—In the case of a vertical disturbing acceleration, $\ddot{z}$, this acts together with gravity and the equation of motion is

$$\ddot{\vartheta} + \frac{g + \ddot{z}}{l} \vartheta = 0$$

and S corresponds to the term

$$S = \frac{\ddot{z}}{l} \vartheta \text{-----} (11)$$

whereas in the case of a horizontal acceleration

$$S = \frac{\ddot{x}}{l}$$

In the first case the factor S is only 1 to 2 per cent of that in the second.

Substituting in equation (11) for ϑ its value, $\vartheta = a \cos \varphi$ from equation (5a), and introducing this value of S into equations (8a), (8b), we find

$$\delta T = -\frac{1}{2g} \int_0^{2\tau} \ddot{z} \cos^2 \varphi \, dt = -\frac{1}{4g} \int_0^{2\tau} \ddot{z} (1 + \cos 2\varphi) \, dt$$

or

$$\delta T = -\frac{\tau}{2g} [\ddot{z}] - \frac{1}{4g} \int_0^{2\tau} \ddot{z} \cos 2\varphi \, dt \text{-----} (12a)$$

in which $[\ddot{z}]$ indicates the mean value of $\ddot{z}$ during the time 2τ .

Equation (8b) gives

$$\dot{a} = \frac{\pi a}{4g\tau^2} \int_0^{2\tau} \ddot{z} \sin 2\varphi \, dt \text{-----} (12b)$$

If $\ddot{z}$ is the same for both pendulums we have from equation (10f)

$$S = S_1 - S_2 = \frac{\ddot{z}}{l} (\vartheta_1 - \vartheta_2) = \frac{\ddot{z}}{l} \vartheta$$

and therefore S is the same function for the fictitious pendulum.

The first term of equation (12a) is the variation of T corresponding to a change in g of $\ddot{z}$ in the formula

$$T^2 = \pi^2 \frac{l}{g}$$

On differentiation this formula gives

$$dT = -\frac{T dg}{2g}$$

This means that if we deduce gravity from the observed period of the pendulum we obtain not g , but $g + [\ddot{z}]$, where $[\ddot{z}]$ is the mean value of the vertical acceleration during the time of the observation.

If $\ddot{z}$ is not strongly variable during 2τ , the second term of (12a) is smaller than the first because $\cos 2\varphi$ is both positive and negative during the complete period 2τ . If $\ddot{z}$ is constant, the second term is zero. If the period of the pendulum is small as compared with that of the pitching ship, $\ddot{z}$ varies only slowly and the second member is negligible. The same statement holds for equation (12b), which represents the fluctuation in amplitude; this should be practically constant and the records show that it is.

If the mean value of $\ddot{z}$ is taken over the entire run, we have for

$$[\ddot{z}] = \frac{1}{t} \int_0^t \ddot{z} dt = \frac{1}{t} (\dot{z}_t - \dot{z}_0).$$

This can be made as small as desired by increasing t and by taking, at the beginning and at the end of the observation period, the mean of the positions of the fictitious pendulum for a great many seconds. Experience shows that if observations are extended over half an hour, this disturbance is negligible. The effect of the vertical accelerations appears on the record as a fluctuation in the curve of the chronometer time-break signals (phase-lag curve), but the amplitude remains rigorously constant.

Effects of angular movements of the apparatus.—In the foregoing discussion the assumption is made that an external horizontal or vertical acceleration is the same for each pendulum, thereby disregarding the movements of the line joining the two knife edges. But these movements disturb the period of the fictitious pendulum. They are, however, irregularly fluctuating quantities whose mean values over a long period of time are negligible, as was the case with the vertical accelerations. In other words the mean position of the line connecting the pendulums is horizontal as provided by the gimbal mountings. The pendulum system may, however, rotate; and if the rotation is about a horizontal axis in the plane of vibration, the period is affected. It is affected by the acceleration introduced by this rotation and by the rotation about the vertical axis in the plane of vibration. Rotation of the apparatus, through a small angle, about an axis normal to the plane of vibration has no effect on the period, because this rotation is not imparted to the pendulums.

If the plane of vibration is not strictly vertical, but includes a small angle β with the vertical, the effective value of gravity is not g but $g \cos \beta$. If we call the variation of gravity, due to the tilt $\ddot{z}$, it has the value

$$\ddot{z} = -g(1 - \cos \beta) = -\frac{1}{2}g\beta^2$$

On substituting this value for $\ddot{z}$ in equation (12a) and indicating the mean value of β^2 by $[\beta^2]$ we find

$$\delta T = \frac{1}{4} T [\beta^2]$$

providing we disregard, as heretofore, the second term of equation (12a). This is permissible if the variation of the amplitude is negligible, which, in turn, postulates that β does not contain a term of the same period as the pendulum.

The fact that β^2 is always positive signifies that its mean value is a positive quantity which is, in general, not negligible. It is essential, therefore, to make a continuous record of β and to take care that its value does not exceed 1° , because β^2 enters the formula and, with larger values, the computation of β^2 becomes laborious.

If the vibration planes of the two pendulums do not coincide exactly, the deviations β_1 and β_2 are not equal and a supplementary term is needed for the resulting disturbance of the fictitious pendulum. We have then for each pendulum

$$\delta T_1 = \frac{1}{4} T_1 [\beta_1^2] \text{ and } \delta T_2 = \frac{1}{4} T_2 [\beta_2^2].$$

For the supplementary disturbance we derive from equation (10e)

$$\delta T = \delta T_1 - (\delta T_2 - \delta T_1) \frac{a_2}{a} \cos(\varphi_2 - \varphi)$$

which reduces, if we neglect the difference $T_2 - T_1$, to the form

$$\delta T = \frac{1}{4} T_1 [\beta_1^2] + \frac{1}{4} T_1 [(\beta_1^2 - \beta_2^2)] \frac{a_2}{a} \cos(\varphi_2 - \varphi) \text{-----} (13a)$$

From equation (10d) we find to the same degree of accuracy

$$\dot{a} = \frac{\pi}{4T_1} [(\beta_1^2 - \beta_2^2) a_2 \sin(\varphi_2 - \varphi)] \text{-----} (13b)$$

The difference $(\beta_1^2 - \beta_2^2)$ is a fluctuating quantity. Therefore if the amplitude does not fluctuate, we may conclude, as in the foregoing sections, that the second term of equation (13a) is negligible.

FIGURE 12.—Pendulum curve of the damped pendulum *d* of Figure 1A, showing angular tilt of apparatus, β_c , and amplitude, α , of oscillating pendulum case in gimbals

If the apparatus is not accurately leveled, the term β_1 consists of a constant angle of tilt β_c and a fluctuating tilt β_f (fig. 12). If we assume the fluctuating tilt to be a simple pendulum or harmonic motion of period T_f , complete period $2 T_f$, and amplitude α we have

$$\beta_f = \alpha \cos \pi \frac{t}{T_f}$$

and

$$\beta_1 = \beta_c + \alpha \cos \pi \frac{t}{T_f} \text{-----} (14)$$

The mean value of β_1^2 is

$$[\beta_1^2] = \beta_c^2 + \frac{1}{2} \alpha^2$$

so that the disturbance to the fictitious pendulum is

$$\delta T = \frac{T}{4} \left(\beta_c^2 + \frac{1}{2} \alpha^2 \right) \text{-----} (15)$$

We have still to deduce the effect of the acceleration introduced by the rotation of the pendulum system about a horizontal axis and about a vertical axis in the plane of vibration. In Figure 13 let A be a point on the pendulum and let its polar coordinates be $\vartheta + \alpha$ and ρ . The angle of elongation is ϑ , as heretofore. Rotation about the horizontal axis gives a vertical acceleration

$$\rho(\dot{\beta})^2 \cos(\vartheta + \alpha) = \rho(\dot{\beta})^2$$

wherein β is the angular velocity about OX . Similarly rotation about OZ produces at A a horizontal acceleration

$$\rho(\dot{\gamma})^2 \sin(\vartheta + \alpha) = \rho(\dot{\gamma})^2(\vartheta + \alpha)$$

in which γ is the angular velocity about OZ . In both these expressions the angle $(\vartheta + \alpha)$ has been considered small, as it is in actual fact.

FIGURE 13.—Diagram used in derivation of formula for correcting term caused by the acceleration due to the rotation of the pendulum system about the horizontal and the vertical axes in the plane of vibration

These two accelerations in the plane of vibration, taken together, increase the momentum by the amount

$$\{(\dot{\beta})^2 - (\dot{\gamma})^2\}(\vartheta + \alpha)\rho^2 dm$$

in which dm is the element of mass of the pendulum at A .

If now we integrate over the whole pendulum and divide by the moment of inertia, we obtain for the disturbing term S

$$S = \{(\dot{\beta})^2 - (\dot{\gamma})^2\}\vartheta$$

This equation is of the form of equation (11), and we may write down directly the corresponding disturbances by substituting in (12a)

$$\ddot{z} = l\{(\dot{\beta})^2 - (\dot{\gamma})^2\} = g \frac{T^2}{\pi^2} \{(\dot{\beta})^2 - (\dot{\gamma})^2\}$$

and for the mean disturbance of the period, providing the amplitude does not show a disturbance and providing the second term of (12a) may be neglected,

$$\delta T = \frac{T^3}{2\pi^2} \{ -[(\dot{\beta})^2] + [(\dot{\gamma})^2] \} \dots \dots \dots (16)$$

in which, as heretofore $[]$ indicate mean values for the duration of the observation. The velocities β and γ are the same for all pendulums and therefore this formula is directly applicable to the fictitious pendulum if T be replaced by T_1 .

To find β differentiate equation (14) after t and find

$$\dot{\beta} = -\frac{\pi}{T_f} \alpha \sin \pi \frac{t}{T_f}$$

Therefore

$$[(\dot{\beta})^2] = \frac{1}{2} \frac{\pi^2}{T_f^2} \alpha^2$$

and

$$\delta T = -\frac{T_1^3}{4T_f^2} \alpha^2 \text{-----} (17)$$

This computation is based on the assumption that β is a simple harmonic function. In actual work this is only a first approximation, and the deviations that occur have, in general, the effect of increasing $[(\dot{\beta})^2]$.

If, in similar manner, we assume γ to be a simple periodic function, an equation similar to (17) is valid, if the ship be steered in a straight course. The period $2T_s$ is then that of the ship's movement, which is long compared with that of the pendulum, and, in general, renders this disturbance negligible.

Assuming the course of the ship to be straight, we find, on combining equations (15) and (17), that the total effect of the angular movements of the ship on the period of the fictitious pendulum is

$$\delta T = \frac{1}{4} T_1 [\beta_c^2 + C\alpha^2] \text{-----} (18)$$

in which

$$C = \frac{1}{2} \left(1 - 2 \frac{T_1^2}{T_f^2} \right) \text{-----} (18a)$$

provided the amplitude does not fluctuate.

Effects of changes in temperature and density of the air.—The deviations in the vibration period of the pendulum produced by these changes are practically proportional to the changes themselves or

$$\delta T_1 = c_1 t + d_1 D$$

$$\delta T_2 = c_2 t + d_2 D$$

in which t and D are the temperatures and density of the air, c_1 and c_2 , the temperature constants of the two pendulums, and d_1 and d_2 , the density constants.

Applying now equation (10e) to obtain the supplementary term for the period of the fictitious pendulum we find

$$\delta T = \delta T_1 - (\delta T_2 - \delta T_1) \frac{a_2}{a} \cos (\varphi_2 - \varphi) \text{-----} (10e)$$

or

$$\delta T = c_1 t + d_1 D - [(c_2 - c_1)t + (d_2 - d_1)D] \frac{a_2}{a} \cos (\varphi_2 - \varphi) \text{-----} (19a)$$

Swinging in air, both pendulums are damped; but inasmuch as the damping affects each pendulum equally, the ratio of the amplitudes remains the same, equation (10e) is valid, and no damping term is needed in equation (19a).

The amplitude is, however, affected by the damping and must be added to the equation deduced from (10d), which expresses the rate of decrease in amplitude

$$\dot{a} = -ka - \frac{\pi}{T^2} [(c_2 - c_1)t + (d_2 - d_1)D] a_2 \sin (\varphi_2 - \varphi) \text{-----} (19b)$$

wherein k is the damping coefficient, which is dependent on the density of the air.

Combining the second terms of equation (19a) and (19b) with the corrections for isochronism given in (10c) and (10d) we find

$$\delta T = -U_{21} \frac{a_2}{a} \cos(\varphi_2 - \varphi) \dots\dots\dots (20a)$$

$$\dot{a} = -\frac{\pi}{T_1^2} U_{21} a_2 \sin(\varphi_2 - \varphi) \dots\dots\dots (20b)$$

with

$$U_{21} = (T_2 - T_1) + (c_2 - c_1) t + (d_2 - d_1) D \dots\dots\dots (20c)$$

The quantity U_{21} represents the difference of the periods of the original pendulums 2 and 1 for the temperature and density of the air during the observation.

The first terms of equations (19a) and (19b) are identical with the corrections for a single pendulum

$$\delta T = c_1 t + d_1 D \dots\dots\dots (21a)$$

$$\dot{a} = -ka \dots\dots\dots (21b)$$

Correction for reduction to infinitely small amplitude.—The pendulums describe an arc not exceeding $1^\circ 8'$ on a side and therefore the assumption is justified, as a first approximation, that the angle itself may be used in place of the sine. However in the final accurate reduction of the time period we have to apply a small correction to reduce to vibrations of infinitely small amplitude. For the two pendulums this correction is found by integrating equation (1) and then expressing the integral in the form of an infinite, rapidly converging series. We thus find

$$T - T_1 = \delta T_1 = \frac{1}{16} T_1 a_1^2$$

$$T - T_2 = \delta T_2 = \frac{1}{16} T_2 a_2^2$$

and for the fictitious pendulum from equation (10e)

$$\delta T = \frac{1}{16} T_1 \left[a_1^2 - (a_2^2 - a_1^2) \frac{a_2}{a} \cos(\varphi_2 - \varphi) \right]$$

The value of a_1 is not recorded by the apparatus. To eliminate a_1 we substitute

$$a_1^2 = a^2 + a_2^2 + 2aa_2 \cos(\varphi_2 - \varphi)$$

deduced from Figure 14,

FIGURE 14.—Diagram used in derivation of formula for correction due to amplitude of vibration of the pendulums

and obtain

$$\delta T = \frac{1}{16} T_1 \left[a^2 + a_2^2 + 3aa_2 \cos(\varphi_2 - \varphi) + 2a_2^2 \cos^2(\varphi_2 - \varphi) \right] \dots\dots\dots (22a)$$

or

$$\delta T = \frac{1}{16} T_1 \left[\left\{ a + 1.5 a_2 \cos(\varphi_2 - \varphi) \right\}^2 + a_2^2 - \frac{1}{4} a_2^2 \cos^2(\varphi_2 - \varphi) \right] \dots\dots\dots (22b)$$

Similarly for the correction to the change in amplitude we find from (10d)

$$\dot{a} = \frac{\pi}{16T_1} a a_2 \sin(\varphi_2 - \varphi) \left[a + 2a_2 \cos(\varphi_2 - \varphi) \right] \quad (22c)$$

Correction for chronometer rate.—This affects only the time unit in which the vibration periods are measured. The corresponding correction to the period of the fictitious pendulum is therefore

$$\delta T = -\frac{r}{86400} T = -115.7 r T \cdot 10^{-7} \text{ sec} \quad (23)$$

in which r is the daily rate in seconds.

SUMMARY OF THE FORMULÆ FOR THE REDUCTION OF THE PERIOD OF THE FICTITIOUS PENDULUM

In the foregoing paragraphs the following corrections to the observed period of the pendulum have been deduced; they represent the values that have to be subtracted from the observed period to find the reduced period. The reduced period is the same as the period T_1 of the first original pendulum, $T = T_1$. For each factor the formula is given by use of which the desired correction δT can be computed.

- (1) Deviation of isochronism of the pendulums in each pair:

$$\delta T = -U_{21} \frac{a_2}{a} \cos(\varphi_2 - \varphi) \quad (20a)$$

$$U_{21} = (T_2 - T_1) + (c_2 - c_1) t + (d_2 - d_1) D \quad (20c)$$

- (2) Amplitude:

$$\delta T = \frac{1}{16} T_1 \left[a^2 + a_2^2 + 3a a_2 \cos(\varphi_2 - \varphi) + 2a_2^2 \cos^2(\varphi_2 - \varphi) \right] \quad (22a)$$

or

$$\delta T = \frac{1}{16} T_1 \left[\{a + 1.5a_2 \cos(\varphi_2 - \varphi)\}^2 + a_2^2 - \frac{1}{4} a_2^2 \cos^2(\varphi_2 - \varphi) \right] \quad (22b)$$

- (3) Angular movement about horizontal axis in plane of vibration:

$$\delta T = \frac{1}{4} T_1 \left[\beta_c^2 + C \alpha^2 \right] \quad (18)$$

$$C = \frac{1}{2} \left(1 - 2 \frac{T_1^2}{T_f^2} \right) \quad (18a)$$

- (4) Temperature:

$$\delta T = c_1 t \quad (21a)$$

- (5) Barometric pressure:

$$\delta T = d_1 D \quad (21a)$$

- (6) Rate of the chronometer:

$$\delta T = -115.7 r T \cdot 10^{-7} \text{ sec} \quad (23)$$

In these equations for the several corrections that have to be subtracted from the observed period to obtain the reduced period, the symbols

- a, a_2 = amplitudes of the fictitious and of the second pendulum,
 φ, φ_2 = phases of the fictitious and of the second pendulum,
 T_1, T_2 = periods of the two pendulums,
 c_1, c_2 = temperature constants of the two pendulums,
 d_1, d_2 = air-density constants of the two pendulums,
 t = temperature in Centigrade of the pendulums,
 D = density of air around pendulums,
 β_c = constant part of angle of deviation about the horizontal axis in the plane of vibration (angle of tilt of knife edges),
 α = amplitude of periodical part of this angle of deviation,
 T_f = period of this last movement.

FIGURE 15A

FIGURE 15B

FIGURE 15C

Reproduction in three slightly overlapping sections, of the photographic record for Station 20. Scale, approximately half the original size. The paper velocity for the middle part of the record (Fig. 15a) was one-third of that for the beginning and the end (Figs. 15a and 15c). In Figures 15a, 15c the interruptions of the light by the chronometer are momentary; in Figure 15b the interruptions extend over half a second. The arrows in Figure 15c indicate the points at which a_2 is measured on the lower a_1 pendulum curves. The plus or minus sign at each arrow is the sign of a_2 ; the plus sign indicates that the vector b is in phase with the a_1 vector at its maximum elongation; the negative sign, that it is in opposite phase.

On the record the amplitudes of the two fictitious pendulums should decrease gradually and uniformly from the beginning to the end of the observation period.

MEASUREMENTS OF A RECORD AND COMPUTATIONS NEEDED TO DETERMINE THE VALUE OF GRAVITY

After the photographic record has been taken and the necessary observations have been made of the temperature, barometric pressure, humidity, depth of submergence, ship's course and speed, ocean current, latitude and longitude of the station, and chronometer rates for the period of the exposure, the steps still required to ascertain the value of gravity for the station are certain measurements on the record itself and certain computations based on the data obtained. The derivation of the formulae for these computations is given in the preceding section.

On the photographic record (figs. 15 A, B, C) the instrument inscribes: (1) each oscillation of each pendulum pair during the entire period; (2) the oscillations of the middle pendulum referred to the damped pendulum swinging in its plane; inasmuch as the middle pendulum has no amplitude at the start, these indicate the effects due to the movements of the ship; (3) the oscillations of the pendulum case against the second damped pendulum swinging in the plane normal to the first; these disturbances are due to the pitching of the ship and swaying of the pendulum case; (4) each beat of the mean time chronometer; each beat of the sidereal time chronometer; (5) up and down movements of the ship due to ocean swell; (6) fluctuations of temperature within the pendulum case.

The object of the measurements on the photographic record is to determine the average vibration periods of the two pendulum pairs in terms of the time intervals registered by the chronometers. The vibration period is affected temporarily by extraneous accelerations due to movements of the ship and to other disturbances. Corrections must be added, for example, to reduce the value to that corresponding to standard conditions of temperature, barometric pressure, humidity, and for a stationary point at the given latitude and longitude.

FIGURE 16.—Sketch of pendulum curves to show the character of the time signal interruptions and the method for counting the time intervals

The shape of the curves inscribed by one of the pendulum pairs is sketched on an open scale in Figure 16. The breaks due to the interruptions caused by the shutter operated by one of the chronometers are indicated. In the actual record the difference in vibration period between the pendulum pair and the chronometer is slight,

and a great many vibrations occur between successive passages of these interruptions through the horizontal axis of the record. In other words the phase-lag for each vibration of the pendulum pair is only from 1° to 2° behind the sidereal time cycles and is of the order of $\frac{1}{2}^\circ$ for the mean time cycles or beats. In Figure 16 the phase-lag is drawn, for convenience, much greater, namely 36° , in order to show more clearly than appears on the actual record how the chronometer time intervals should be counted. If the vibration periods of chronometer and pendulum pair were exactly equal, the chronometer breaks would occur at the same relative position on the successive pendulum curves and the phase-lag sine curve passing through the breaks would be a horizontal straight line. But if the period of vibration of the pendulum pair is longer than the chronometer interval, the chronometer breaks occur at intervals less than a complete pendulum cycle. The result is that the chronometer breaks trace a sine curve which is the expression of the receding movement of the pendulum-vector whose angular velocity is the phase-lag of the pendulum pair. By determining this phase-lag we can easily compute the pendulum period, as will be shown below; the phase-lag may be found by measuring the interval of time between two passages of the phase-lag sine curve through the horizontal axis of the record.

Suppose in Figure 16 each fifth chronometer break were silent and therefore not marked. On the actual record (fig. 15A) each sixtieth break is silent and the curve for the sixtieth cycle is not interrupted on the record. In Figure 16 the interruptions both at the make of the electrical circuit and at the break of the circuit, half a second later, are shown as is the case in the photographic records (figs. 15 A, B, c). In ascertaining the elapsed time for a passage of the phase-lag curve, reference is made to the 60-second unmarked lines and the time is counted from these lines. Instead of counting the interruptions themselves, which are difficult to mark, being in the middle of the record, one may count the swings on the upper or the lower side of the record, but care must be taken to count correctly. In counting, it does not suffice to count simply the number of swings of the pendulum pair; thus in Figure 16, the number of swings between *A* and *B* on the lower lines is five, but between *B* and *C* it is four. In each case the number of circuit interruptions is the same.

The rule to follow in making these counts is briefly: Choose as reference curve that phase-lag curve that passes through the first break after the silent, 60-second line (b, fig. 16); this curve is marked by an arrow in Figure 16. Note the flank of the curve on which the 60-second blank occurs. If the counting is to be done on the lower side of the diagram, pass along this flank from *c* to *a* and mark *a* as a 60-second line; if the marking is to be done along the top of the diagram, pass from *c* to *d* and mark *d* as the 60-second line. If this rule is followed consistently, no trouble will be had in counting the time between passages.

If *N* is the angular phase lag of the pendulum pair for each complete cycle, then the number of vibrations required to complete the 360° cycle is $360/N$ and the period *T* is slightly longer than the half second of the chronometer in the proportion

$$T = \frac{360}{360-N} \times \frac{1}{2} = \frac{1}{1-\frac{N}{360}} \times \frac{1}{2} = 0.5 + \frac{0.25}{C-0.5} \dots\dots\dots (24)$$

Reproduction of part of a photographic gravity record, showing fluctuations in the time signal (phase-lag) curve, caused by vertical accelerations

wherein $2C = \frac{360}{N}$; C is the number of seconds between two consecutive passages of the phase-lag curve through the horizontal axis or reference line.

In Figures 15 A, B, c, the phase-lag curves for the two chronometers are clearly shown. The vibration period of the pendulum pairs is nearer that of the mean time chronometers than that of the sidereal time chronometer; therefore the phase-lag curve for the mean time is flatter and passes across the horizontal reference line of the pendulum curves less often than does the phase-lag curve for sidereal time. The horizontal axis, i. e., the line midway between the top and bottom of the vibration curves, is a convenient reference line and time is counted from the point of intersection of this line with the phase-lag curve. This point of intersection does not occur, in general, exactly on a vibration curve of the record, but between two such curves. Moreover, the phase-lag curve may fluctuate, because of vertical accelerations due to wave movement, even though the amplitudes remain constant, as in Figure 17.

The simplest and most direct method for finding the passages or points of intersection of the horizontal line of reference with a phase-lag curve is to draw a straight line with a sharp pencil through the chronometer breaks, or as nearly as possible, through the chronometer breaks immediately preceding and following the passage, as indicated in Figure 18. For most records this method suffices and is much

FIGURE 18.—Diagram to illustrate method of drawing a straight line through the fluctuating phase-lag curve for the purpose of finding its passage point

shorter than the method that requires the exact determination of the position of each chronometer break for many points before and after a passage. The exact point of intersection of the straight line thus drawn with the horizontal reference line, expressed in terms of the interval on the horizontal line between successive chronometer time breaks, determines the passage to 0.1 sec. in good records. The horizontal reference line is not actually drawn, but its position midway between the extremities of a vibration cycle is ascertained by a divided scale and the point of intersection is then determined. In the interest of accuracy passage points for the phase-lag curve should be determined both for the rising and falling slopes of the phase-lag curve, as indicated in Figures 15 A, B, c. A pair of intersections at the beginning and at the end of the run generally suffice; the passage times are marked in pencil upon the record below the middle and above the upper vibration curves. The time intervals between passages are compared; and if there is a sufficient number of passage intervals available their mean error can be computed. The mean error of the resulting pendulum period should be less than $5 \cdot 10^{-7}$ sec. It can be computed from the formula

$$m_T^2 = \frac{\mu^2}{8 \pi^2 C^2}$$

in which μ is the mean error of one passage time; p , the number of computed passages at the beginning and at the end; t , duration of the observation; C , number of seconds between two consecutive passages of the phase-lag curve through the horizontal axis.

Similar measurements of the passage time for the phase-lag curve of the mean-time chronometer can also be made. In this case, however, the curve is so much flatter that it is difficult to determine the passage point more accurately than 1 second. In Table 3 the actual records for Station 29 are listed as measured on the photographic record illustrated in Figures 15A, 15B, and 15C, which is a copy of the original record in three sections on a scale of approximately one-half the original. The width of the roll of photographic paper is 12 cm.

TABLE 3.—Station 29

Passages for sidereal time chronometer		
	Upper a_{32}	Middle a_{12}
	m s	m s
First (begin)-----	0 3.8	0 2.0
First (end)-----	22 54.0	22 52.0
Difference-----	22 50.2	22 50.0
Second (begin)-----	1 57.9	1 56.0
Second (end)-----	24 48.1	24 46.7
Difference-----	22 50.2	22 50.7
Mean Diff-----	22 50.2	22 50.35
Seconds (M. D.)-----	1,370 ^s .2	1,370 ^s .35
No. Passages-----	12	12
C -----	114 ^s .183	114 ^s .196
Sid. T -----	0 ^s .5021991	0 ^s .5021988
Passages for mean time chronometer		
	m s	m s
Begin-----	0 46	0 41
End-----	27 10	27 3
Difference-----	26 24	26 22
Seconds (M. D.)-----	1,584 ^s	1,582 ^s
No. Passages-----	5	5
C -----	316 ^s .80	316 ^s .40
Mean T -----	0 ^s .5007904	0 ^s .5007914
Mean T to Sid. T -----	0 ^s .5021615	0 ^s .5021625

From the values of C thus obtained for the sidereal and mean time chronometers, the periods of the two fictitious pendulums are ascertained by means of equation (24); these times are listed in the table under Sid. T . and Mean T . In order to make the two periods comparable the Mean T . is reduced to Sid. T . by adding to Mean T . a correction obtained by multiplying Mean T . by the factor 0.00273791.

These computations are made on the assumption that the two chronometers keep perfect time. This is not the case and corrections have to be made for the time rates of the chronometers. These rates were determined by comparison at noon and at 10 p. m. each day with the time signals sent out by the United States Naval Observatory by wireless from the stations at Arlington and at Annapolis.

The method for making the comparison has been long in use by astronomers; but it requires for its proper application much preliminary training on the part of the observer. The method consists essentially in noting (a) the time of coincidence of beats between the two chronometers, (b) the coincidence of beats of one of the chronometers with a third chronometer that gains, for instance, one second in about 65 seconds over this chronometer, (c) the coincidence of beats of the third chronometer with the wireless time signals which may be done by putting the contact of this third chronometer in series with the telephone of the wireless and noting the appearance and disappearance of the radio signals, (d) the coincidence of beats of the third chronometer with one of the other chronometers.

The actual time signal determinations made on November 6, 1928, on which Station 29 was occupied are given in Table 4.

The following chronometers were used: Sidereal time Nardin No. 212; mean time Nardin No. 2081; third chronometer Negus No. 1272.

TABLE 4

*Chronometer comparisons.**Observer, Vening Meinesz.*

Noon signals Nov. 6, 1928.					10 p. m. signals Nov. 6, 1928.										
	No. 212			No. 2081		No. 212			No. 2081						
	h	m	s	h	m s	Diff.	h	m	s	h	m s	Diff.			
	10	59	20.5	7	47	23	2.5	8	55	7.5	5	41	30	22.5	
	11	14	17.0	8	2	17	0.0	9	10	0	56	20	20		
								9	18	47.5	6	5	6	18.5	
Mean-----	11	6	48.75	7	54	50.00	1.25	9	7	58.333	5	54	18.667	20.333	
	Negus			No. 2081				Negus			No. 2081				
	10	23	22	7	42	40	18	8	32	50	5	42	51	1	
		24	28		43	45	17		33	59		43	59	0	
		25	31		44	47	16		35	2		45	1	-1	
		26	34		45	49	15		36	6		46	4	-2	
		27	41		46	55	14		37	14		47	11	-3	
									38	19		48	15	-4	
Mean-----	10	25	31.20	7	44	47.20	16.0	8	35	35	5	45	33.50	-1.5	
	Negus			Wireless time signals				Negus			Wireless time signals				
	10	33	7.5	11	56	52	44.5	8	40	37	9	55	8	31	
		33	56		57	40	44		40	48.5		55	19	30.5	
		34	8.5		57	52	43.5		41	43		56	13	30	
		35	1		58	44	43		41	54.5		56	24	29.5	
		35	14.5		58	57	42.5		42	49		57	18	29	
		36	7		59	49	42		43	1.5		57	30	28.5	
									43	55		58	23	28	
									44	6.5		58	34	27.5	
									45	1		59	28	27	
									45	11.5		59	38	26.5	
Mean-----	10	34	35.75	11	58	19	43.25	8	42	54.75	9	57	23.50	28.75	
	Negus			No. 2081				Negus			No. 2081				
	10	37	32	7	56	37	5	8	51	23	6	1	7	-16	
		38	35		57	39	4		52	32		2	15	-17	
		39	41		58	44	3		53	38		3	20	-18	
		40	47		59	49	2		54	45		4	26	-19	
		41	52		8	0	53	1		55	50		5	30	-20
Mean-----	10	39	41.40	7	58	44.40	3.0	8	53	37.60	6	3	19.60	-18	

By means of a series of coincidences of this kind it is possible from the averages to obtain coincidence values of small probable error.

For the noon period the time interval between the coincidences of the Negus No. 1272 and the Nardin No. 2081 was

Negus No. 1272	Nardin No. 2081
h m s 10 39 41.40 10 25 31.20	h m s 7 58 44.40 7 44 47.20
14 10.20 =850.20 sec.	13 57.20 =837.20 sec.

At Negus $10^h 34^m 35^s.75$ coincidence with the wireless signal was obtained. This was $9^h 4^s.55$, or 544.55 seconds, after the coincidence with No. 2081. Therefore, by proportion, No. 2081 was 536.223 seconds ahead of its time of coincidence with Negus; in other words, its time at wireless signal time, $11^h 58^m 19^s$, was $7^h 53^m 43^s.423$, and the difference in time is

h m s 11 58 19 7 53 43.42
4 4 35.58

A similar computation for the 10 o'clock signal period shows that Nardin No. 2081 at $5^h 52^m 46^s.55$ coincided with wireless signal time, $9^h 57^m 23^s.50$. The difference in time had become

h m s 9 57 23.50 5 52 46.55
4 4 36.95 4 4 35.58
0 0 1.37

The mean time chronometer, therefore, lost 1.37 seconds between $11^h 58^m 19^s$ and $9^h 57^m 23^s.50$, or in $9^h 59^m 4^s.50$; in 24 hours the difference would be in proportion or 3.29 seconds. The daily rate of the mean time chronometer is therefore +3.29 seconds.

The sidereal time chronometer registered $10^h 1^m 9^s.583$, while the mean time chronometer registered $9^h 59^m 28^s.667$. Reducing this mean time interval to sidereal time, we find that the sidereal chronometer has gained 2.437 seconds over the mean time chronometer during this time interval. In 24 hours it would gain 5.85 seconds. Therefore its daily rate of gain is -2.56 seconds, the algebraic sum of -5.85 and +3.29.

We may now correct the periods of each fictitious pendulum, after having ascertained the rates of both chronometers; we find both T 's in true sidereal time. If the chronometers are equally dependable, the mean of the two values should be taken.

A practical way for carrying this out, is to apply to the values of T given by the mean time chronometer and reduced to sidereal time (0.5021615 and 0.5021625 of Table 3) a correction corresponding to the difference in daily rate D of the two chronometers, given by (formula 23)

$$-115.7 D T \cdot 10^{-7} \text{ sec}$$

$$= -58.1 D \cdot 10^{-7} \text{ sec}$$

In the case of Station 29 D is 5.85 seconds, and this correction amounts to $340 \cdot 10^{-7}$ sec., thus yielding the values $T=0.5021955$ and $T=0.5021965$.

We may now compare these last values with those given by the sidereal time chronometer (0.5021991 and 0.5021988 of Table 3) and take their mean value if both chronometers are equally dependable.

Afterwards, together with the other corrections, these mean values are corrected for the rate of the sidereal chronometer.

The next step is the measurement of the amplitudes of the two fictitious pendulums and of the middle pendulum for the several passage periods for the purpose of computing the corrections for deviation of isochronism and for amplitude; for convenience the double amplitudes or total widths of the pendulum curves of the different pendulums are recorded in centimeters. These are listed in Table 5 under the headings $2a_{12}$, $2a_{32}$, $2a_2$.

TABLE 5.—Measurements in centimeters made on gravity record at Station 29

No.	$2a_2$	$2b$	Middle $2a_{12}$	$(2a_{12} + 1.32 \times 2b)^2$	Upper $2a_{32}$	$(2a_{32} - 1.32 \times 2b)^2$	$(2a_2)^2$	$(2b)^2$	2α	$(2\alpha)^2$
1.....	0.5	[0.0]	4.2	[17.6]	4.2	[17.6]	[0.3]	[0.0]	0.4	[0.2]
$\frac{1}{2} \times 1$		0.0		8.8		8.8	0.1	0.0		0.1
2.....	0.5	-0.3	4.2	14.4	4.1	20.3	0.3	0.1	0.7	0.5
3.....	0.7	-0.3	4.1	13.7	4.0	19.4	0.5	0.1	0.4	0.2
4.....	0.8	-0.2	4.1	14.7	4.0	18.1	0.6	0.0	0.5	0.3
5.....	0.8	+0.4	4.0	20.5	3.9	11.4	0.6	0.2	0.6	0.4
6.....	0.8	+0.8	4.0	25.6	3.8	7.5	0.6	0.6	0.4	0.2
7.....	0.8	+0.7	3.9	23.2	3.8	8.3	0.6	0.5	1.0	1.0
8.....	1.0	+0.0	3.9	15.2	3.7	13.7	1.0	0.0	0.8	0.6
9.....	1.3	-0.5	3.8	9.9	3.7	19.0	1.7	0.3	0.6	0.4
10.....	1.5	-1.0	3.8	6.2	3.6	24.2	2.3	1.0	0.4	0.2
11.....	1.2	-1.0	3.7	5.7	3.6	24.2	1.4	1.0	0.4	0.2
12.....	1.0	-0.9	3.7	6.3	3.5	22.0	1.0	0.8	0.9	0.8
$\frac{1}{2} \times 13$		0.0		6.5		6.1	0.2	0.0		0.1
13.....	0.6	[0.0]	3.6	[13.0]	3.5	[12.3]	[0.4]	[0.0]	0.4	[0.2]
Mean.....		-0.192	3.9	14.22	3.8	16.92	0.91	0.38		0.42

In Table 5 there are also two columns, headed $2b$ and 2α ; these are the measurements, in centimeters, of two additional quantities that appear on the photographic record. The quantity $b = a_2 \cos(\varphi_2 - \varphi)$ is the angle of elongation of pendulum a_2 at the moment that the fictitious pendulum has its maximum elongation. In the actual measurements the width of the vibration line of the lighter portion of the a_2 record is measured directly below the greatest elongation of the middle and upper curves on the photographic record. Attention must also be given to the signs of the measurements. If the phase of $2b$ coincides with that of a_{12} , the value $2b$ is positive; otherwise it is negative. These relations are clearly shown in Figure 15B

108069°—30—4

in which the points at which $2b$ is measured are indicated; the values with appropriate signs are also recorded in Table 5. In the computations the fact is taken into account that in the apparatus the phase of the fictitious pendulum a_{32} (upper record) is recorded as $(\vartheta_2 - \vartheta_3)$ and not $(\vartheta_3 - \vartheta_2)$ as it should be, while the phase $(\vartheta_1 - \vartheta_2)$ for a_{12} is correctly recorded. In the computations, $2b$ is therefore given, when used with a_{32} , a sign opposite to that when used with a_{12} . Used with a_{12} it has the sign of $a_2 \cos(\varphi_2 - \varphi)$ in equations (20) and (22).

The symbol α is the amplitude of the curve showing the oscillations of the apparatus about the horizontal axis in the vibration plane of the two fictitious pendulums. It is indicated by the curves of small amplitude superimposed on the upper, a_{32} , vibration curves. The amplitude 2α is measured in millimeters, or tenths of centimeters for each passage period. The curve registers in effect the oscillations of the pendulum case due to the pitching of the ship. The records show that the period, T_f of this oscillation which is the half period of the gimbal swinging is about 1.44 seconds.

In connection with the measurement of the amplitudes recorded on the photographic paper, it should be noted that the equivalent focal length of the lens used in the recording apparatus is 111 cm. and that the deviation x on the record is proportional to the angular deviation ω imparted by the apparatus

$$x = 111 \omega \text{ or } \omega = 0.00901 x \text{-----} (25)$$

wherein ω is expressed in radians. Also that the reflections at the pendulums take place under angles of 30° (fig. 1A), with the normals to the mirrors and hence the deflection of the beam is

$$\omega = 2(\vartheta_1 - \vartheta_2) \cos 30^\circ = 2\vartheta \cos 30^\circ \text{-----} (26)$$

in which ϑ is the angle of elongation of the fictitious pendulum. Combining the last two equations we find

$$x = 192.258 \vartheta$$

or

$$\vartheta = 0.0052 x \text{-----} (27)$$

in which x is in centimeters and ϑ in radians. In the case of a_2 , the prism arrangement is such that the reflected beams include an angle of 10° with the normals to the mirrors; therefore all measurements of amplitudes for a_2 and b have to be multiplied by

$$\frac{\cos 30^\circ}{\cos 10^\circ} = 0.88 \text{-----} (28)$$

to reduce them to the scale of the fictitious pendulums.

In Table 5 several additional columns are included and give the results of computation required to deduce the corrections resulting from the disturbing factors that are recorded. These corrections are computed by use of the equations derived in the preceding section.

Reduction to infinitely small amplitude.—This correction is found by using equation (22b), multiplied by the scale-conversion factors 0.0052^2 and 0.88 from equations (27) and (28). Supposing $T = 0.502$ sec., as a sufficiently close approximation, we find

$$\delta T = 2.12A \cdot 10^{-7} \text{ sec.} \text{-----} (29a)$$

in which

$$A = (2a + 1.32 \times 2b)^2 + 0.774 \left[(2a_2)^2 - \frac{1}{4} (2b)^2 \right] \text{-----} (29b)$$

The values of the terms in equation (29b), as obtained by measurements on the records, are listed in Table 5 for Station 29. Substituting these values in (29a) and (29b) we find

$$A_{32} = 16.92 + 0.774 (0.91 - 0.10) = 17.55$$

and

$$A_{12} = 14.22 + 0.774 (0.91 - 0.10) = 14.85$$

The correction for amplitude is accordingly (equation 29a),

$$\delta T_{32} = 37.2 \cdot 10^{-7} \text{ sec} \dots\dots\dots (29c)$$

$$\delta T_{12} = 31.5 \cdot 10^{-7} \text{ sec} \dots\dots\dots (29d)$$

Correction for isochronism.—This correction we find from equation (20a) to be

$$\delta T = -U_{21} \frac{a_2}{a} \cos (\varphi_2 - \varphi) \dots\dots\dots (20a)$$

which, multiplied with the factor 0.88 to reduce the expression, $b = a_2 \cos (\varphi_2 - \varphi)$, to the proper scale becomes

$$\delta T = -0.88 U_{21} \frac{2b}{2a} \cdot 10^{-7} \text{ sec} \dots\dots\dots (20d)$$

in which

$$U_{21} = (T_2 - T_1) + (c_2 - c_1)t + (d_2 - d_1)D \dots\dots\dots (20c)$$

To evaluate U_{21} and U_{32} , we must know certain constants of the pendulums; also the temperature at which the observations were taken, the barometric pressure, and the humidity.

The observations taken during the record for Station 29 are listed in Table 6.

TABLE 6.—Station 29, November 6, 1928

Eastern standard time, p. m.	Pendulum tempera- ture, C.	Humidity	Barometer	Barometer tempera- ture, C.
h m	°		mm.	°
3 53½	31. 77	78	771. 7	32. 1
58½				
4 8½	31. 78	78	773. 2	32. 9
18½				
4 23½	31. 80	78	773. 1	33. 1
Mean---	31. 78	78	772. 8	

Current, 1 knot west; ship's course, 98°; ship's speed, 3.85 knots.

Latitude, 20° 24'.0 N.; longitude, 71° 21'.5 W.

Depth of submergence, 42 feet; average depth (sonic soundings) to sea bottom, 2,257 fathoms.

In this table, under the heading time, the hour, 3^h 58½^m, was that at which the speed of the photographic paper was changed from fast to slow, and also the character of the chronometer signals was changed; at 4^h 18½^m both speed of paper and chronometer signals were restored to the initial conditions.

The observed pendulum temperatures are subject to a correction due to errors in the thermometer as shown in Table 7.

TABLE 7

Temperature, C.	Correction, C.
°	°
0	-0.16
10	-0.14
20	-0.16
30	-0.18
40	-0.22

The corrected mean pendulum temperature is therefore $t^\circ = 31^\circ.78 - 0^\circ.19 = 31^\circ.59$ C.

The scale of the hair hygrometer was checked against that of a psychrometer several times during the voyage; the correction for Stations 27 to 44 was found to be -9 . The corrected hygrometer reading for Station 29 is accordingly $78 - 9 = 69$.

The correction to the barometer reading was $+1.1$ mm. for the entire voyage. The corrected barometer reading for Station 29 is 772.8 mm. $+1.1$ mm. $= 773.9$ mm.

In the formula for U_{21} the difference $(d_2 - d_1)$ in the density constants is so small that the variation of $(d_2 - d_1) D$ may be neglected, because the observations are made at atmospheric pressure on the ship and this changes very little during an observation. For the two pendulum pairs a_{32} and a_{12} , the values of U_{ab} are

$$U_{32} = 28 - 0.16 t^\circ \cdot 10^{-7} \text{ sec.} \dots \dots \dots (30a)$$

$$U_{12} = -12 + 1.03 t^\circ \cdot 10^{-7} \text{ sec.} \dots \dots \dots (30b)$$

On the basis of these equations, of the corrected value for t° and of Table 5, we find the corrections for isochronism to be

$$\delta T_{32} = -0.88 (28 - 0.16 \times 31.59) \frac{+0.192}{3.8} \cdot 10^{-7} = -1.0 \cdot 10^{-7} \text{ sec.} \dots \dots \dots (31a)$$

$$\delta T_{12} = -0.88 (-12 + 1.03 \times 31.59) \frac{-0.192}{3.9} \cdot 10^{-7} = +0.9 \cdot 10^{-7} \text{ sec.} \dots \dots \dots (31b)$$

Correction for angular movement of the vibration plane.—If we assume the vibration period T_1 to be 0.502 sec., equation (18) for this correction becomes

$$\delta T = 1.02 (\beta_c^2 + C\alpha^2) \cdot 10^{-7} \text{ sec.} \dots \dots \dots (18b)$$

in which β_c and α are measured in millimeters; β_c is the constant part of the deviation, and α the amplitude of the fluctuating part.

The displacement β_c is found by measurement to be 0.6 mm. The half period T_f of the gimbal suspension is about 1.44 sec.; therefore from equation

$$C = 0.5 - \frac{0.25}{T_f^2} = 0.38 \dots \dots \dots (18a)$$

The value α^2 measured on the record (Table 5) is

$$\frac{1}{4} (2\alpha)^2 = \frac{42}{4} = 10.5$$

Accordingly the correction for tilt of the vibration plane is

$$\delta T = 1.02 (0.36 + 0.38 \times 10.5) \cdot 10^{-7} = 4.4 \cdot 10^{-7} \text{ sec.} \quad (18c)$$

Correction for rate of chronometer.—On the assumption that $T = 0.502 \text{ sec.}$, we find for equation (23)

$$\delta T = 58.1 r \cdot 10^{-7} \text{ sec.} \quad (23a)$$

in which r is the daily rate of the chronometer. For No. 212 the daily rate is -2.56 sec. and

$$\delta T = 148.7 \cdot 10^{-7} \text{ sec.} \quad (23b)$$

Correction for temperature coefficients of pendulums.—The temperature constant c for the fictitious pendulum, a_{32} (upper record) is $47.07 \cdot 10^{-7} \text{ sec.}$; for a_{12} , $45.88 \cdot 10^{-7} \text{ sec.}$

On the basis of these values we obtain from equation (21a)

$$\delta T = c_1 t \quad (21a)$$

and the corrections

$$\delta T_{32} = 47.07 \times 31.59 \cdot 10^{-7} = 1486.9 \cdot 10^{-7} \text{ sec.} \quad (21b)$$

$$\delta T_{12} = 45.88 \times 31.59 \cdot 10^{-7} = 1449.3 \cdot 10^{-7} \text{ sec.} \quad (21c)$$

This correction is applied to reduce the period to its value for 0° C. temperature.

Correction for air.—This correction is proportional to the density of the air surrounding the pendulums. It depends therefore on the barometric pressure and on the humidity because this increases the air density. The readings on the hygrometer indicate the percentage of saturation of water vapor present in the air at the temperature of the observations. To facilitate the computations the saturated vapor pressures in mm. of mercury at different temperatures from 0° C. to 39° C. are listed in Table 8; also the values $3/8 e$, where e is the vapor pressure of the saturated air.

TABLE 8.—Saturated vapor pressures, in mm. of mercury, at different temperatures

Temp. C.	e mm.	$3/8 e$ mm.	Temp. C.	e mm.	$3/8 e$ mm.	Temp. C.	e mm.	$3/8 e$ mm.	Temp. C.	e mm.	$3/8 e$ mm.
0	4.6	1.7	10	9.2	3.4	20	17.4	6.5	30	31.6	11.8
1	4.9	1.8	11	9.8	3.7	21	18.5	6.9	31	33.4	12.5
2	5.3	2.0	12	10.5	3.9	22	19.7	7.4	32	35.4	13.3
3	5.7	2.1	13	11.2	4.2	23	20.9	7.8	33	37.4	14.0
4	6.1	2.3	14	11.9	4.5	24	22.2	8.3	34	39.6	14.8
5	6.5	2.4	15	12.7	4.8	25	23.5	8.8	35	41.9	15.7
6	7.0	2.6	16	13.6	5.1	26	25.0	9.4	36	44.2	16.6
7	7.5	2.8	17	14.5	5.4	27	26.5	9.9	37	46.7	17.5
8	8.0	3.0	18	15.4	5.8	28	28.1	10.5	38	49.3	18.5
9	8.6	3.2	19	16.4	6.2	29	29.8	11.2	39	52.1	19.5

The correction to the air pressure due to the humidity is the hygrometer reading, h , in per cent multiplied by $3/8 e$ for the temperature at which the hygrometer was read. For Station 29 the corrected hygrometer reading is 0.69; the barometer correction is therefore

$$0.69 \times 13 = 9.0 \text{ mm.} \quad (32)$$

This is to be subtracted from the corrected barometer reading in order to get the pressure of dry air having the same density.

Formula for the computation of the air density.—This formula reads

$$D = B^{mm} \times \frac{1}{760} \times \frac{1}{1 + 0.00367 t} = B^{mm} \times d$$

wherein D is the density of the air (unity is the air density of dry air of 760 mm. and zero temperature), B the barometer reading in millimeters, t the temperature in centigrade, and $d = \frac{1}{760} \times \frac{1}{1 + 0.00367t}$. The values for d and $\log d$ are listed in Table 9 for temperatures from 0° to 39° C.

TABLE 9.—Temperature factors for computation of air density

Temp. C.	Log d (-10)	d (.001)	Temp. C.	Log d (-10)	d (.001)	Temp. C.	Log d (-10)	d (.001)	Temp. C.	Log d (-10)	d (.001)
0	7.1192	316	10	7.1035	269	20	7.0884	226	30	7.0738	185
1	.1176	311	11	.1020	265	21	.0869	222	31	.0724	181
2	.1160	306	12	.1005	260	22	.0855	218	32	.0710	178
3	.1144	301	13	.0989	256	23	.0840	213	33	.0695	174
4	.1129	297	14	.0974	251	24	.0825	209	34	.0681	170
5	.1113	292	15	.0959	247	25	.0811	205	35	.0667	166
6	.1097	287	16	.0944	243	26	.0796	201	36	.0653	162
7	.1082	283	17	.0929	239	27	.0782	197	37	.0639	159
8	.1066	278	18	.0914	234	28	.0767	193	38	.0625	155
9	7.1051	274	19	7.0899	230	29	7.0753	189	39	7.0611	151

For Station 29 the factor d for the temperature $31^\circ.59$ C. is 0.001179; and

$$D = B \times d = 764.9 \times 0.001179 = 0.902 \quad (33)$$

in which B is the barometer reading corrected for humidity, and is equal to $773.9 \text{ mm.} - 9.0 \text{ mm.} = 764.9 \text{ mm.}$

For each pendulum pair the density constants have been determined. The density constant for the pendulum a_{32} is $673.2 \cdot 10^{-7}$; and for a_{12} , $665.9 \cdot 10^{-7}$.

The correction factor for the pendulum periods is the product of the density and the density factor. For Station 29 the corrections for density of air are

$$\delta T_{32} = 673.2 \times 0.902 \cdot 10^{-7} = 607.2 \cdot 10^{-7} \text{ sec.} \quad (34a)$$

$$\delta T_{12} = 665.9 \times 0.902 \cdot 10^{-7} = 600.6 \cdot 10^{-7} \text{ sec.} \quad (34b)$$

Summary of corrections to pendulum period.—Assembling the several corrections we have for Station 29:

Station No. 29

	Upper pendulum pair a_{32}	Middle pendulum pair a_{12}	
Amplitude correction.....	$37.2 \cdot 10^{-7}$	$31.5 \cdot 10^{-7}$	(29c, d)
Isochronism correction.....	-1.0	0.9	(31a, b)
Angular vibration plane correction.....	4.4	4.4	(18c)
Chronometer rate correction.....	148.7	148.7	(23b)
Temperature correction.....	1486.9	1449.3	(21b, c)
Air density correction.....	607.2	600.6	(34a, b)
	<u>2283.4</u>	<u>2235.4</u>	
	$2283.4 \cdot 10^{-7} \text{ sec.}$	$2235.4 \cdot 10^{-7} \text{ sec.}$	
Sidereal Time.....	0.5021991	0.5021988	
Sid. T. corr.....	.5019708	.5019753	
Mean Time.....	.5021955	.5021965	
Mean T. corr.....	.5019672	.5019730	
Mean S. T.....	.5019730		
Mean M. T.....	.5019701		
Mean T_1	.5019715		(35a)
T (Washington).....	.5015438		(35b)
$V_T = T_1 - T$	0.0004277		(36)

In this list, mean T_1 refers to the mean vibration period of the pendulum pairs for the station occupied; T refers to the mean vibration period of the same pendulums at Washington, the base gravity station for the United States. V_T is the difference in vibration periods for Station 29 and Washington, for which station the value of gravity is known to be $g=980.112$ cm. per sec² or dynes.

Computation of the value of gravity from the observed pendulum period.—If V_T is the difference between the mean of the two fictitious pendulum periods at the station and at the base station, and if V_g represents the difference in gravity between the two stations, we deduce from the gravity pendulum equation (7e)

$$T^2 = \pi^2 \frac{l}{g} \quad T_1^2 = \pi^2 \frac{l}{g_1} \quad \text{-----} \quad (7e)$$

$$g_1 = g \frac{T^2}{T_1^2} \text{ and } g_1 - g = g_1 \left(1 - \frac{T_1^2}{T^2}\right) \quad \text{-----} \quad (37a)$$

or

$$V_g = -CV_T \quad \text{-----} \quad (37b)$$

in which

$$C = \frac{g_1(T_1 + T)}{T^2} = \frac{g(T_1 + T)}{T_1^2} \quad \text{-----} \quad (37c)$$

wherein g is the value of gravity at the base station, g_1 the value of gravity at the observation station, T the mean of the vibration periods at the base station, T_1 the mean of the vibration periods at the observation station, $V_g = g_1 - g$ the difference in gravity values at the two stations, and $V_T = T_1 - T$ the difference in their vibration periods. An approximate but sufficiently accurate formula for C is obtained from (37c) by noting that

$$T_1 + T = 2T_1 - (T_1 - T) = 2T_1 \left(1 - \frac{V_T}{2T_1}\right)$$

Therefore

$$\frac{T_1^2}{T_1 + T} = \frac{T_1^2}{2T_1 \left(1 - \frac{V_T}{2T_1}\right)} = \frac{T_1}{2} \left(1 - \frac{V_T}{2T_1}\right)^{-1} = \frac{T_1}{2} \left(1 + \frac{V_T}{2T_1} + \dots\right)$$

and

$$C = \frac{g(T_1 + T)}{T_1^2} = \frac{g}{\frac{T_1^2}{T_1 + T}} = \frac{2g}{T_1 + \frac{1}{2}V_T} \quad \text{-----} \quad (37d)$$

if we neglect powers of V_T higher than the first.

For the Washington station

$$g = 980.112 \text{ cm/sec}^2 \quad \text{-----} \quad (38)$$

Substituting this value in equation (37d) and also the values, T_1 and V_T , from equations (35a) and (36) we obtain

$$C = 3903.4$$

and

$$V_g = -1.669 \text{ cm/sec}^2 \quad \text{-----} \quad (37b)$$

Correction to gravity for East-West component of the ship's velocity.—This effect of the ship's velocity on the observed value of gravity was first studied by von Eötvös. If the ship is traveling east, its velocity coincides with the direction of the earth's rotation, and hence increases the centrifugal acceleration and to that extent reduces the gravity value. If the ship is traveling west it is reducing the apparent rotation of the earth slightly and hence is causing gravity to increase.

This change in the centrifugal acceleration, which acts in a plane normal to the earth's axis, is

$$\frac{(V+v)^2}{r} - \frac{v^2}{r}$$

As a first approximation this expression reduces to

$$\frac{2V}{r} v \dots\dots\dots (39)$$

in which r is the radius of the parallel; V , the East-West velocity, caused by the earth's rotation; v , the East-West component of the ship's velocity. To get the component in the direction of gravity we multiply this disturbance by the cosine of the latitude (φ). The correction to be applied to the observed value of gravity is therefore

$$dg = \frac{2V}{r} \cos \varphi \times v = \frac{4\pi \cdot 10^5}{86400 \times 3600} \cos \varphi \times v = 0.0040 \cos \varphi \times v \cdot \text{cm/sec}^2 \dots\dots\dots (39a)$$

in which v , the East-West component of the ship's velocity, is expressed in kilometers per hour. If v is expressed in nautical miles per hour, or knots

$$dg = 0.0075 \cos \varphi \times v \cdot \text{cm/sec}^2 \dots\dots\dots (39b)$$

If the ship is going Eastward, the value dg is to be added to the observed gravity.

To obtain the East-West component of the ship's velocity multiply the speed of the vessel by the sine of the course. In this case the speed is 3.85 knots and the course 98° ; the East-West component is

$$3.85 \times \sin 98^\circ = 3.81 \text{ knots} \dots\dots\dots (39c)$$

To this must be applied the velocity of the current, positive if East and negative if West. The current is given 1 knot West. Therefore the effective East component of the ship's velocity is

$$v = +3.81 - 1.00 = 2.81 \text{ knots} \dots\dots\dots (39d)$$

The total Eötvös correction is accordingly

$$dg = 0.0075 \times \cos 20^\circ 24' \times 2.81 = +0.020 \text{ cm/sec}^2 \dots\dots\dots (39e)$$

Correction for depth of submergence.—Two corrections are needed to refer the value of gravity, measured in a submarine below the surface, to the actual surface. When submerged the submarine is nearer the center of the earth than when it is at the surface; the first correction is one for negative elevation, and the second for the attraction of the layer of water between the ship and the surface, which is reversed when bringing the station to the surface; we have therefore to correct for double this attraction.

If the depth of the submarine is d (in meters) and the earth's radius a , the first correction is

$$\delta g = -\frac{2g}{a} d = -0.308 d \cdot 10^{-3} \text{ cm/sec}^2 \dots\dots\dots (40a)$$

which must be applied to g .

If the depth d is expressed in feet, the equation reads

$$\delta g = -0.0939 d \cdot 10^{-3} \text{ cm/sec}^2 \dots\dots\dots (40b)$$

In Table 6 the depth of submergence for Station 29 is 42 feet. The gravity apparatus was mounted, however, 15 feet above the keel. The correct depth for the gravity apparatus is therefore 27 feet. From (40b) we find for this station

$$\delta g = -2.5 \cdot 10^{-3} \text{ cm/sec}^2 \dots\dots\dots (40c)$$

To compute the second correction we assume that the layer of water is a horizontal cake indefinitely extended. The attraction of such a layer on a point at its surface is

$$2\pi k \rho' d$$

in which k is the gravitation constant, ρ' , the density of sea water. The attraction of the earth is

$$g = \frac{4}{3} \pi k \rho a; \text{ or } k = \frac{3}{4} \frac{g}{\pi a \rho}$$

in which ρ is the mean density of the earth and a its radius. Substituting in the first formula the value of k from the second we have for the attraction the expression

$$\frac{3}{2} \frac{\rho'}{\rho} \frac{d}{a} g$$

and for the correction to gravity, twice this amount or

$$\delta'g = +3 \frac{\rho'}{\rho} \frac{d}{a} g \quad (41)$$

This correction must be added to g .

By substituting $\rho' = 1.03$, $\rho = 5.52$, $g = 980$ cm., $a = 6.370 \cdot 10^8$ cm. we obtain

$$\delta'g = +0.086 d \cdot 10^{-3} \text{ cm/sec}^2 \quad (41a)$$

wherein d is given in meters. If the depth be stated in feet the equation reads

$$\delta'g = +0.026 d \cdot 10^{-3} \text{ cm/sec}^2 \quad (41b)$$

For Station 29 the correct depth is 27 feet and the correction is from (41a)

$$\delta'g = +0.7 \cdot 10^{-3} \text{ cm/sec}^2 \quad (41c)$$

Summary of corrections to the gravity value deduced from the vibration period of the pendulum.—Assembling the several gravity corrections we have for Station 29

E-W component of ship's velocity and of ocean current in knots.....	$dg = +0.020 \text{ cm/sec}^2$	(39e)
---	--------------------------------------	-------

Depth of submergence (position).....	$\delta g = -0.0025 \text{ cm/sec}^2$	(40c)
--------------------------------------	---	-------

Depth of submergence (water layer).....	$\delta'g = +0.0007 \text{ cm/sec}^2$	(41c)
---	---	-------

Total correction to gravity.....	$\Delta g = +0.018 \text{ cm/sec}^2$	(42)
----------------------------------	--	------

If this correction be added to V_g equation (37b), the result, combined with the value of gravity at the base station, Washington, is the gravity value for Station 29

Gravity at Washington.....	$g = 980.112 \text{ cm/sec}^2$	(38)
----------------------------	--------------------------------------	------

Pendulum corrections.....	$Vg = -1.669 \text{ cm/sec}^2$	(37b)
---------------------------	--------------------------------------	-------

Gravity corrections.....	$\Delta g = +0.018 \text{ cm/sec}^2$	(42)
--------------------------	--	------

Gravity at Station 29.....	$g = 978.461 \text{ cm/sec}^2$
----------------------------	--------------------------------

For the actual measurements and computations to deduce the value of gravity from the station records, it is advisable to formulate a definite procedure and to use it for the entire series of stations. By this method the measurements and computations are made step by step as a routine matter and the results can be readily tested and compared. It is convenient also to assemble all tables and instrumental constants on one sheet for easy reference.

A form of procedure for this purpose has been developed by Miss Eleanor A. Lamson, of the United States Naval Observatory, and used by her in computing the results obtained on the trip of the U. S. submarine S-21. This form is presented in abstract in the Appendix to this report.

CHAPTER 3

RESULTS OBTAINED ON THE CRUISE OF THE U. S. SUBMARINE S-21

The cruise of the *S-21* was planned as a preliminary or reconnaissance trip to obtain data on the different geological features of the Gulf of Mexico and the Caribbean Sea. This region is one of the most interesting areas of the earth's surface, especially from the viewpoint of the geological structure and geological history.

The islands of the West Indies are commonly grouped under two heads, (a) the Greater Antilles, including Cuba, Jamaica, Haiti, Porto Rico, and the Virgin Islands; (b) the Lesser Antilles, comprising the arc of islands, chiefly volcanic, which extends, as a curved line convex to the Atlantic, from the east end of the Greater Antilles to South America and bounds the Caribbean Sea on the east.

The geological formations, exposed in the West Indies and also bordering the Gulf of Mexico and the Caribbean Sea, were formed largely during Cretaceous and Cenozoic times and are geologically young. They are underlain by Paleozoic rocks, chiefly Permian and Carboniferous in age. These in turn rest on Proterozoic metamorphic and intrusive rocks that we find exposed in Honduras, Guatemala, and southern Mexico. This ancient protaxis of Pre-Cambrian rocks extends in an easterly direction in a line slightly convex to the south, through Honduras and probably across the Caribbean into Jamaica and perhaps beyond to Haiti and Porto Rico. To the north of this protaxis is a folded foreland, an area of intense movement, and now represented in part by the Bartlett Deep. Still farther to the north is another trough, due to orogenic movement, represented by the Nares Deep north of Porto Rico and its extension to the WNW., north of Haiti and Cuba. This feature is not well shown on the chart, but the gravity measurements made on this expedition clearly indicate its existence.

To the north and west of this depressed and folded zone occurs the region called by SUESS the "Gulf of Mexico Plate" of nearly horizontal Cretaceous and Cenozoic beds that cover the land areas bordering the Gulf of Mexico, and extend apparently to the Bahama Banks.

The bottom of the Gulf of Mexico is fairly flat and undisturbed by folds and faults. In this respect it is quite different from the Caribbean Sea, whose bottom is less regular and shows evidence of normal faulting. In the eastern part of the Gulf of Mexico occur the up-faulted blocks of Yucatan and Campeche Bank, tilting to the west; they are the single large disturbing feature in the Gulf of Mexico area. A down-faulted block forms the Yucatan channel and separates Cuba from the mainland.

The Caribbean basin is different in its features from the Gulf of Mexico; it resembles the Mediterranean area and was considered by SUESS to be the western extension of his Tethyan Mediterranean. The western part of the Caribbean Sea is a much folded and faulted area with major structural features trending northeast. The eastern part of the Caribbean Sea bottom is less disturbed. It is bordered by the arc of volcanic islands of the Lesser Antilles. These volcanoes occur on a ridge that appears to have developed in late Mesozoic time. The volcanic islands were probably formed during Cretaceous and Tertiary time. Volcanic activity has

continued on some of the islands down to the present. Nearly all the islands of the Greater and Lesser Antilles began as volcanic islands. Of these, Cuba is the oldest and was in existence in Paleozoic time.

There is evidence that orogenic movements took place in this region in the Pre-Cambrian and then again in the late Paleozoic. The most important cycle of movement began in the late Cretaceous. It followed the general lines of the Paleozoic deformation and appears to have ended before Middle Eocene time. The major anticlinorium of this disturbance appears to have extended, as an arc convex to the north, from British Honduras across the Antillean Sea to southeastern Cuba, Haiti, Porto Rico, and the Virgin Islands to the east. A second anticlinorium reached from southern Guatemala and northern Honduras across the Honduran Banks possibly to Jamaica.

A still later diastrophic cycle began in the late Miocene and attained its maximum effect during the Pliocene. At this time the valley between the two anticlinoria noted above was depressed and finally faulted to the present Bartlett Deep. As a result of the orogenic movements of this period of diastrophism, the Central American and Antillean lands were raised and the Caribbean Sea was depressed. At this time or possibly still later the sea bottom of the Nares Deep and its westerly extension was depressed with accompanying faulting and movement in Porto Rico and possibly northern Haiti and Cuba.

This generalized and incomplete sketch of certain phases of the geological history of this region serves to indicate that in the geologically recent past the area has been subjected to movement on a large scale. Some of these movements are still active and give rise to earthquakes in certain parts of the field.¹

Wherever orogenic movements occur, they indicate the presence of stresses of great magnitude in the rock masses affected; there the isostatic equilibrium must be disturbed because other forces besides gravity are acting on the crust. The crustal movements are the direct expression of this unbalanced condition and of the isostatic tendency at work to restore the ordinary balance or equilibrium. Gravity measurements over the disturbed areas should indicate where these departures from equilibrium exist.

The gravity anomalies thus obtained are the expression of these departures from the normal state or balanced condition of the earth's crust and may serve as a rough measure of the stresses present in the rock masses in any given area. This is true, providing proper regard is taken of local inhomogeneities in density near the station that affect the gravity value and with it the gravity anomaly. In the case of gravity measurements at sea, this factor of the disturbing effect of local departures from normal density in the rock masses is much less serious than on land. The depths at sea average several miles, as a rule; furthermore the average gravity attraction over a course several miles in length is taken during the half hour period of observation. These two facts tend to integrate the effects of local inhomogeneities, of excess density here and deficiency of density there, and to yield a value of gravity that

¹ An excellent summary of the geological history of the Antillean region has recently been given by CHARLES SCHUCHERT, *Science*, vol. 69, pp. 139-145, 1929, Feb. 8.

expresses the resultant of an area of appreciable extent, wherein the gravity anomalies refer actually to questions of load rather than to local disturbances due to masses of abnormal density in the near vicinity of the gravity station.

From a geological viewpoint, therefore, gravity measurements, especially at sea, afford a powerful tool for detecting unstable parts of the earth's crust where movements, such as faulting and folding, are likely to occur and to be centers of earthquake disturbance.

These figures illustrate the effect of distance on decreasing the gravitational attraction of a mass on a given point

FIGURE 19a

FIGURE 19b

FIGURE 19c

The effect of distance on decreasing the gravitational attraction of a mass on a given point is well shown in Figures 19, a, b, c, in which the gravitational attraction of a cylindrical mass of unit density ($\rho=1$), radius 1 cm. and height as indicated, on a point in the produced axis of the cylinder. In Figure 19a each cylinder exerts a gravitational attraction of $4 \cdot 10^{-8}$ dynes on the point directly below the numeral 4. In Figure 19b the given point is located in each case on the upper surface of the cylinder; the gravitational attractions for the different cylinders are, respectively, 4, 8, 16, 24, 32, 40 dynes, multiplied by 10^{-8} . In Figure 19c the cylinders are all of the same dimensions, but are located at different distances from the given point; in each case the gravitational attraction in terms of 10^{-8} dyne is given. This figure shows how rapidly with depth of burial the attraction of a local inhomogeneity decreases. The formula by which these attractions are computed is the standard equation

$$g = 2\pi \cdot k \cdot \rho [\sqrt{c^2 + h^2} - \sqrt{c^2 + (h+t)^2} + t] \dots\dots\dots (45a)$$

in which g is the gravitational attraction in dynes or cm./sec.²; k , the gravitation constant $= 6.673 \cdot 10^{-8}$; ρ , the density of the attracting cylinder; c , its radius; t , its length; and h , the distance of the attracted point to the top of the cylinder.

Equation (45a) may be expressed in another form better adapted to practical use

$$g = 2\pi \cdot k \cdot \rho \cdot t \left[\sqrt{\left(\frac{c}{t}\right)^2 + \left(\frac{h}{t}\right)^2} - \sqrt{\left(\frac{c}{t}\right)^2 + \left(1 + \frac{h}{t}\right)^2} + 1 \right] \dots\dots\dots (45b)$$

or

$$g = 2\pi \cdot k \cdot \rho \cdot c \left[\sqrt{1 + \left(\frac{h}{c}\right)^2} - \sqrt{1 + \left(\frac{h+t}{c}\right)^2} + \frac{t}{c} \right] \dots\dots\dots (45c)$$

If, in equation (45c), c is very large as compared with h or t , we find for g the value

$$g = 2\pi \cdot k \cdot \rho \cdot c \dots\dots\dots (46)$$

The attraction, therefore, of a horizontal plate indefinitely extended varies directly as its thickness and is not affected by its distance from the point. Thus the attraction of such a plate of density $\rho=2.67$ and thickness 10 meters is 1.12 millidynes; this means roughly 1 millidyne for a plate 30 feet thick.

Figure 19c shows in a particular case how rapidly with increasing distance the gravitational attraction falls off. An example illustrating the relations may aid in further visualizing them. Consider the gravitational attraction of a cylindrical mass, of radius 1 kilometer, thickness 1 kilometer, and density one unit higher than that of the adjacent rocks ($\rho=1$) on a point of unit mass, located on the produced axis of the cylinder, at a distance h in kilometers from the top of the cylinder. The value of the gravitational attraction can be computed directly by use of equation (45c); the results are given in Table 10.

TABLE 10.—*Gravitational attraction of a cylindrical mass of density=1, radius=1 km., thickness 1 km., on a point of unit mass located on the produced axis of the cylinder*

h km.	Gravita- tional attraction dynes	h km.	Gravita- tional attraction dynes	h km.	Gravita- tional attraction dynes
0. 0	0. 0246	0. 6	0. 0117	2. 0	0. 0031
. 1	. 0217	. 7	. 0104	3. 0	. 0016
. 2	. 0192	. 8	. 0093	4. 0	. 0010
. 3	. 0169	. 9	. 0083	5. 0	. 0007
. 4	. 0150	1. 0	. 0075	7. 5	. 0004
. 5	. 0132	1. 5	. 0046	10. 0	. 0002

This table shows clearly how at a depth of 2,000 fathoms (3.66 km.) the gravitational attraction of a local feature, 2 km. wide and 1 km. thick and excess density $\rho=1$, has decreased from its value of 25 millidynes at the surface to 1 millidyne; at 4,000 fathoms it is only 0.4 millidyne.

In the discussion of the meaning of gravity anomalies and the excesses or deficiencies of load they represent, the following table (No. 11), computed by means of equation (45c), has been found useful. It gives the gravitational attraction of a cylinder of density, $\rho=1$, and radius, $c=1$, cm. on a point of unit mass located on the produced axis of the cylinder and at a distance h in centimeters from the top of the cylinder. In equation (45c) the ratios $\frac{h}{c}$ and $\frac{t}{c}$ are considered to be the variables; the quantities ρ and c appear before the parenthesis and in the first power only. Table 11 is computed on the assumption that $\rho=1$ and $c=1$ cm. But in case it is desired to ascertain the gravitational attraction of a cylinder of another density ρ_1 , or another radius c_1 , this can be obtained directly from the table by multiplying the values there given by the ratios ρ_1/ρ , or c_1/c . The values of the gravitational attraction are expressed in units of a microdyne or one millionth of a dyne. In this connection, a set of curves plotted on logarithm paper has been prepared from which the gravitational attraction of a cylindrical mass of any given size and density can be ascertained. A greatly reduced illustration of this graphical plot is shown in Figure 20.

In the figure the curves give the graphical solution of equation (45c) for the gravitational attraction in dynes of a cylinder, of unit density and of radius, 1 centimeter, on a point of unit mass located on the produced axis of the cylinder in which the ratio of the height of the cylinder to its radius, $\frac{h}{c}$, is represented by the several curves; the ratio of the distance of the top of the cylinder, from the given point, to the radius of the cylinder $\frac{t}{c}$, is given by the ordinates; and the gravitational attraction in dynes of the cylinder on the given point is given by the abscissae. Both the ordinates and the abscissae are plotted on a logarithmic scale. The data used in the construction of this diagram are listed in Table 11.

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Curves giving the graphical solution of equation 45c

TABLE 11.—*Gravitational attraction in microdynes (dynes $\cdot 10^{-6}$) on a point of unit mass located in the produced axis of a cylinder of unit density, radius $c=1$ cm., length t , and at a distance h in centimeters from the top of the cylinder*

$\frac{h}{c}$ \ $\frac{t}{c}$	0.001	0.01	0.1	0.5	1	5	10
0	0.0004191	0.0041718	0.039836	0.160149	0.245607	0.377760	0.398365
0.1	.0003773	.0037550	.035716	.142048	.217345	.338723	.358735
.2	.0003368	.0033508	.031770	.125426	.191926	.303776	.323221
.3	.0003028	.0029697	.028091	.110440	.169350	.272746	.291650
.4	.0002638	.0026189	.024737	.097134	.149501	.245370	.263753
.5	.0002316	.0023028	.021737	.085437	.132180	.221321	.239207
.6	.0002034	.0020225	.019091	.075295	.117143	.200249	.217657
.7	.0001787	.0017769	.016784	.066500	.104127	.181799	.198749
.8	.0001573	.0015637	.014785	.058910	.092869	.165635	.182146
.9	.0001387	.0013794	.013060	.052366	.083130	.151450	.167538
1.0	.0001228	.0012207	.011575	.046723	.074693	.138970	.154652
1.5	.0000704	.0007006	.006700	.027969	.046202	.094884	.108752
2	.0000442	.0004408	.004246	.018233	.030938	.069181	.081538
3	.0000216	.0002145	.002088	.0093176	.016424	.041936	.051937
4	.0000125	.0001249	.001223	.005591	.009871	.028394	.036660
5	.0000082	.0000813	.000799	.003711	.006816	.020605	.027557
10	.0000021	.0000208	.000206	.000992	.001893	.006952	.010437
15	.0000009	.0000093	.000092	.000449	.000870	.003485	.005578
20	.0000006	.0000052	.000052	.000255	.000498	.002093	.003489
30	.0000002	.0000023	.000023	.000114	.000225	.000997	.001746
40	.0000001	.0000013	.000013	.000065	.000128	.000582	.001048
50	.0000001	.0000008	.000008	.000042	.000082	.000381	.000699
100	.0000001	.0000002	.000002	.000010	.000021	.000100	.000190

As noted above, it is possible by the use of Table 11 or of the graphical plot (fig. 20) to deduce the attraction of a mass of irregular shape by passing through it concentric cylinders of the proper thickness and of different radii; if the space between two such cylinders be called a zone, and the sector or compartment of this zone be taken that most nearly fits the given mass or part of it, it is possible by simple proportion to deduce the gravitational attraction of the compartment from that of the zone itself. In many cases, as for example an ocean deep, it might seem to be simpler to compute the attraction of a rectangular block that approximates the shape of the ocean deep; but this is unfortunately not true because of the intricate integrations required.

In geological literature emphasis is placed on the fact that extremes in topography, such as lofty mountain ranges and great ocean deeps or troughs are geologically young features and essentially evanescent in character. They are the expression of the culminating centralized effects of the action of mountain-making or orogenic forces; there we may expect to find departures from normal gravity equilibrium or balance. In view of the fact that the magnitude of orogenic forces is quite beyond direct study in the laboratory, it is necessary, if we would evaluate them, to study their effects in the field where they are now active. We know from a study of the rocks themselves what changes they produce and how large are the masses they can move. But the mechanical relations are so complex and the quantities involved so prodigious that we can not, by any known direct method, measure the order of magnitude of the orogenic forces themselves. Gravity measurements afford the only available approach to this problem which is fundamental to geological theory.

The geology of the Antillean region is known only imperfectly and much remains to be done before its geological history can be stated in detail. Special

geological field studies with the aid of good topographic maps will be needed; also more detailed and accurate maps and charts of the sea bottom, especially of the Caribbean region. As an aid and guide in this investigation a detailed gravity survey will be of great assistance in locating critically important regions where careful geological field work should be done.

It was with this viewpoint in mind that the *S-21* expedition was planned. Other factors that entered into consideration were chiefly those that might contribute to the theory of isostasy, such as the rate at which isostatic compensation is attained in the case of the depression of a great sea bottom, like the Caribbean, or in the case of an immense load that is in the course of being added locally, such as the Delta of the Mississippi. One might expect, in a recently subsided ocean bottom, to find a deficiency of gravity attraction, a negative gravity anomaly; while over a recently added load, comparable in size to the Mississippi Delta, there might be a corresponding excess of gravity attraction or a positive anomaly. As a result of the *S-21* expedition, tentative answers to these and other questions were obtained. More definite answers will be possible only after more detailed work has been done; the reconnaissance cruise of the *S-21* has shown, however, where such detailed work should be done and the kind of work that is needed.

Details of the itinerary of the *S-21* cruise are given in chapter 1. In planning the trip the effort was made to include as many diverse features as possible and yet to have the voyages between ports not to exceed five days or about 1,000 miles in length. On the trip from Washington to Key West gravity was measured first at Norfolk, Va., and then at three stations on the coastal shelf. These measurements were preliminary in character and intended primarily to give the crew practice in handling the ship, maintaining it on an even keel, at constant depth, and along a constant course. From Key West to Galveston measurements were made across the profile of the continental shelf (Stations 5, 6, 7) which drops off rapidly from 100 fathoms to 1,800 fathoms, thence along a course nearly due west along the 25° parallel over the deep flat portion of the Gulf of Mexico (Stations 8, 9, 10) and then in a northerly direction to Galveston over the gradually rising continental shelf (Station 11).

From Galveston to Key West the interest centered chiefly in the Mississippi Delta. Is it an extra load on the earth's crust and not yet isostatically compensated? Two profiles were obtained as indicated on Chart 1. (Stations 13, 14, and 15, and 15, 16, and 17.) Station 12 was occupied midway between Galveston and Station 13 and near the 100-fathom line of the continental shelf. Station 18 is near the eastern edge of the deep flat bottom of the Gulf of Mexico.

On the trip from Key West to Guantanamo Bay, Stations 19 and 20 were occupied north of Cuba near its east end, and definite evidence was obtained of the extension of the Nares Deep downwarping to this distance. After rounding Cape Maysi into the Windward Passage two *N-S* profiles were taken across the Bartlett Deep. (Stations 21, 22, 23, and Stations 24, 25, 26.) Unfortunately time was not available for a study of Bartlett Deep, farther to the west, nor of any of the other interesting features of the western portion of the Caribbean Sea. From Guantanamo

Bay further study of the Nares Deep downwarping was made along the north coast of Haiti. (Stations 28, 29, 30.) Two *N-S* profiles across the Nares Deep proper were then taken. (Stations 31, 32, 33, and 34, 35, 36, 37.)

From St. Thomas a profile was made across the small Virgin Island Deep between St. Thomas and St. Croix and extending westward. (Stations 38, 39, 40.) Station 41 is at the southern end of the Nares Deep profile, which includes not only Stations 33, 32, 31, 41, but also Stations 27 and 28 occupied by Dr. VENING MEINESZ on his voyage in 1926-27 from Holland to Java via the Panama Canal. Station 27 is in Mona Passage between Porto Rico and Haiti. Stations 42 and 43 are over the deep Caribbean Sea south of Haiti. Station 44 is the south end of the western *N-S* profile over the Bartlett Deep.

On leaving Guantanamo Bay, Station 45 was occupied near the southeastern shore of Cuba not far from Cape Maysi. Thence the course led through Crooked Island Passage and from there in a northerly direction over the deep Atlantic toward Cape Hatteras. Stations 46 and 47 were taken on a profile at the margin of the continental shelf. Stations 48 and 49 are over the deep Atlantic, with depths averaging 2,500 fathoms over vast areas. The attempt was made to occupy a station off Cape Hatteras, but was unsuccessful because of the heavy ocean swell, caused by a severe storm that caused the ship, even at 80 feet depth, to roll 10° on a side.

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TABLE 12.—*The assembled results of the gravity measurements*

No. of Station	Depth, fm.	Latitude ϕ	Longitude λ	Observed Gravity at Sta. (g_o)	Corr. for Topog. and Compens.	Bowie Formula				Helmert Formula		
						Theoretical Gravity (γ)	Computed Gravity (g_c)	Iso- static Anom- aly ($g_o - g_c$)	Free Air Anom- aly ($g_o - \gamma$)	Theoretical Gravity (γ)	Computed Gravity (g_c)	Anom- aly ($g_o - g_c$)
1, Hampton Roads		+36 56.7	76 19.9	979.874	+0.020	979.903	979.923	-0.049	-0.029	979.897	979.917	-0.043
2	15	32 3.6	79 58.0	979.511	+0.018	979.492	979.510	+0.001	+0.019	979.486	979.504	+0.007
3	26	28 20.5	80 5.5	979.235	+0.029	979.201	979.230	+0.005	+0.034	979.194	979.223	+0.012
4	37	24 53.5	80 29.6	978.980	+0.031	978.952	978.983	-0.003	+0.028	978.945	978.976	+0.004
5	210	24 52.0	84 7.0	979.022	+0.057	978.950	979.007	+0.015	+0.072	978.943	979.000	+0.022
6	1867	24 47.0	84 27.0	978.867	-0.079	978.945	978.866	+0.001	-0.078	978.937	978.858	+0.009
7	1823	24 45.0	86 5.0	978.970	-0.042	978.942	978.900	+0.070	+0.028	978.935	978.893	+0.077
8	1900	25 6.7	89 22.0	978.948	-0.036	978.967	978.931	+0.017	-0.019	978.960	978.924	+0.024
9	1925	24 59.7	90 36.2	978.988	-0.034	978.960	978.926	+0.062	+0.028	978.952	978.918	+0.070
10	1993	25 1.3	93 22.5	979.000	-0.041	978.961	978.920	+0.080	+0.039	978.954	978.913	+0.087
11	907	26 24.0	93 45.2	979.102	+0.004	979.058	979.062	+0.040	+0.044	979.051	979.055	+0.047
12	73	28 4.5	92 5.5	979.228	+0.025	979.181	979.206	+0.022	+0.047	979.174	979.199	+0.029
13	1021	27 21.7	89 4.8	979.156	-0.002	979.128	979.126	+0.030	+0.028	979.121	979.119	+0.037
14	535	28 15.0	89 11.0	979.216	+0.007	979.194	979.201	+0.015	+0.022	979.187	979.194	+0.022
15	530	28 46.3	88 44.5	979.247	-0.003	979.234	979.231	+0.016	+0.013	979.226	979.223	+0.024
16	1220	28 20.0	87 55.5	979.153	-0.034	979.200	979.166	-0.013	-0.047	979.193	979.159	-0.006
17	1727	26 52.0	86 17.5	979.112	-0.042	979.092	979.050	+0.062	+0.020	979.084	979.042	+0.070
18	1815	26 8.5	85 29.0	979.018	-0.054	979.040	978.986	+0.032	-0.022	979.032	978.978	+0.040
Key West				978.964	+0.035							
Galveston				979.274	+0.007							
19	1332	21 33.5	76 16.0	978.627	-0.063	978.735	978.672	-0.045	-0.108	978.727	978.664	-0.037
20	1470	21 8.3	74 59.3	978.630	-0.044	978.709	978.665	-0.035	-0.079	978.701	978.657	-0.027
21	1582	19 12.5	75 52.5	978.564	-0.028	978.596	978.568	-0.004	-0.032	978.588	978.560	+0.004
22	2867	19 30.5	75 53.0	978.421	-0.136	978.614	978.478	-0.057	-0.193	978.606	978.470	-0.049
23, Moro Castle		19 57.4	75 52.5	978.742	+0.087	978.639	978.726	+0.016	+0.103	978.631	978.718	+0.024
24	2530	19 18.0	76 45.5	978.522	-0.109	978.602	978.493	+0.029	-0.080	978.594	978.485	+0.037
25	3770	19 36.5	76 51.0	978.432	-0.208	978.620	978.412	+0.020	-0.188	978.612	978.404	+0.028
26	942	19 53.3	76 51.0	978.680	-0.010	978.635	978.625	+0.055	+0.045	978.627	978.617	+0.063
27, Guantanamo	5	19 54.5	75 8.9	978.742	+0.085	978.636	978.721	+0.021	+0.106	978.628	978.713	+0.029
28	2205	20 27.0	72 26.7	978.458	-0.096	978.668	978.572	-0.114	-0.210	978.660	978.564	-0.106
29	2257	20 24.0	71 21.5	978.461	-0.095	978.665	978.570	-0.109	-0.204	978.657	978.562	-0.101
30	2520	19 32.5	68 35.0	978.382	-0.069	978.616	978.547	-0.165	-0.234	978.608	978.539	-0.157
31	4300	19 38.0	67 45.8	978.342	-0.176	978.621	978.445	-0.103	-0.279	978.613	978.437	-0.095
32	2540	20 13.0	67 46.0	978.670	-0.012	978.655	978.643	+0.027	+0.015	978.646	978.634	+0.036
33	2800	22 9.5	67 35.0	978.783	-0.004	978.772	978.768	+0.015	+0.011	978.764	978.760	+0.023
34	3045	21 48.5	66 30.3	978.736	-0.017	978.750	978.733	+0.003	-0.014	978.742	978.725	+0.011
35	2920	20 35.0	66 25.5	978.701	-0.010	978.676	978.666	+0.035	+0.025	978.668	978.658	+0.043
36	4210	19 46.5	66 14.0	978.369	-0.139	978.629	978.490	-0.121	-0.260	978.621	978.482	-0.113
37, San Juan	5	18 27.8	66 6.8	978.689	+0.148	978.556	978.704	-0.015	+0.133	978.548	978.696	-0.007
38, St. Thomas	5	18 20.0	64 56.0	978.687	+0.141	978.549	978.690	-0.003	+0.138	978.541	978.682	+0.005
39	2367	17 55.5	65 6.0	978.398	-0.066	978.527	978.461	-0.063	-0.129	978.519	978.453	-0.055
40	970	17 37.5	65 13.3	978.557	+0.058	978.512	978.570	-0.013	+0.045	978.503	978.561	-0.004
41	2517	15 47.0	67 43.0	978.436	-0.029	978.421	978.392	+0.044	+0.015	978.412	978.383	+0.053
42	1652	16 18.0	71 48.7	978.473	+0.006	978.444	978.450	+0.023	+0.029	978.436	978.442	+0.031
43	2296	16 50.0	73 4.8	978.471	-0.052	978.471	978.419	+0.052	0.000	978.463	978.411	+0.060
44	1737	18 45.8	76 26.0	978.554	-0.051	978.572	978.521	+0.033	-0.018	978.564	978.513	+0.041
45	1100	19 57.0	74 31.0	978.618	-0.010	978.639	978.629	-0.011	-0.021	978.631	978.621	-0.003
46	952	22 56.5	74 26.8	978.818	+0.029	978.822	978.851	-0.033	-0.004	978.814	978.843	-0.025
47	2510	23 31.8	73 59.5	978.804	-0.056	978.861	978.805	-0.001	-0.057	978.853	978.797	+0.007
48	2512	26 16.5	74 0.0	979.054	-0.010	979.050	979.040	+0.014	+0.004	979.042	979.032	+0.022
49	2562	+32 27.0	74 12.0	979.482	-0.031	979.524	979.493	-0.011	-0.042	979.517	979.486	-0.004

The results of the gravity measurements are assembled in Table 12. In column 1 the number of each station is given; in column 2, the depth to sea bottom in fathoms (1 fathom=6 feet=1.8288 meters); in columns 3 and 4, the latitude and longitude of the station.

In column 5 the observed value of gravity deduced from the photographic record is given for each station. These values were first computed by VENING MEINESZ, WRIGHT, and COLLINS, during the trip while the ship was in port. Later the measurements and computations were all repeated by Miss ELEANOR A. LAMSON at the United States Naval Observatory, after certain corrections had been made, as a result of restandardization of the barometers and thermometers and other measuring apparatus used. Because of these corrections, the values deduced by Miss LAMSON average, if Washington is used as base, 0.003 dynes higher than the tentative values first obtained.

In column 6 the isostatic correction for topography and compensation is listed for each station. These values were computed at the United States Coast and Geodetic Survey under the direction of Dr. W. BOWIE and Mr. C. H. SWICK, according to the method devised by J. F. HAYFORD and based on the idea of perfect local compensation. The writers are greatly indebted to the United States Coast and Geodetic Survey for its courtesy in this matter. The computation for each station requires several days' labor and the task of computing the values for 49 stations within the short time available was not easy. The writers wish also to express their appreciation of the assistance rendered by Mr. C. H. SWICK of the United States Coast and Geodetic Survey in connection with the two series of gravity measurements at the Survey gravity base station.

In column 7 the theoretical gravity value for the latitude of each station is given, as deduced from the BOWIE formula of 1917,

$$\gamma = 978.039 (1 + 0.005294 \sin^2 \varphi - 0.000007 \sin^2 2\varphi) \dots\dots\dots (43)$$

in which γ is the value of gravity in dynes or cm/sec.² and φ is the latitude.

In column 8 the "computed" gravity value is given for each station. Each value is the algebraic sum of the theoretical gravity (column 7) and the correction for topography and compensation (column 6).

In column 9 the isostatic anomaly for each station is listed. This is the difference between the observed value g_o (column 5) and the "computed value" g_c (column 8) or $(g_o - g_c)$.

In column 10 the "free air" anomaly is given for each station. This is the difference $(g_o - \gamma)$ between the observed value and the theoretical value for the given latitude as computed from equation (43).

For the sake of comparison the theoretical value of gravity computed by use of the HELMERT equation of 1915, namely,

$$\gamma = 978.052 [1 + 0.005285 \sin^2 \varphi - 0.000007 \sin^2 2\varphi + 0.000018 \cos^2 \varphi \cos 2(\lambda + 17^\circ)] \dots\dots\dots (44)$$

is given, in column 11, for each station; in this equation φ is the latitude, λ , the longitude from Greenwich, East being positive. In column 12 the computed gravity, g_c is listed for each station. Each value is the algebraic sum of the

HELMERT theoretical gravity (column 11) and the correction for topography and compensation (column 6).

In column 13 the HELMERT anomaly is given; it is the difference between the observed gravity value (column 5) and the HELMERT computed value (column 12).

The mean of the isostatic anomalies without regard to sign is 0.037; the mean with regard to sign is -0.002 . The mean without regard to sign is relatively high as compared with other areas of this extent. There are several factors that tend to produce this result. The ocean deeps give high negative values while the floor of the deeper parts of the Gulf of Mexico and of the Caribbean Sea exhibit abnormally high positive values. The mean of the free air anomalies without regard to sign is 0.070; the mean with regard to sign is -0.026 .

The results of the computations, by the United States Coast and Geodetic Survey, of the corrections for topography and for compensation on the assumption of a depth of 113.7 km. for the depth of compensation are listed in Table 13 for each zone.

TABLE 13

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 1				Sea Station No. 2			
A.....	-15	-1	0	-1	-55	-1	0	-1
B.....	-7	-2	0	-2	-55	-12	0	-12
C.....	-13	0	0	0	-55	0	0	0
D.....	-9	0	0	0	-55	0	0	0
E.....	-6	0	0	0	-55	0	0	0
F.....	-4	0	0	0	-55	0	0	0
G.....	-7	0	0	0	-55	0	0	0
H.....	-7	0	0	0	-55	0	0	0
I.....	-2	0	0	0	-55	0	0	0
J.....	+4	0	0	0	-55	-1	+1	0
K.....	+7	0	0	0	-51	-1	+1	0
L.....	+10	0	0	0	-53	-1	+1	0
M.....	+18	+1	-1	0	-61	-2	+4	+2
N.....	+31	+2	-2	0	-177	-1	+9	+8
O.....	+30	+1	-2	-1	-340	-3	+17	+14
18.....				+3				+3
17.....				+6				+4
16.....				+8				+5
15.....				+11				+4
14.....				+15				+7
13.....				+32				+22
12.....				+25				+20
11.....				+24				+22
10.....				+18				+20
9.....				+13				+13
8.....				+14				+12
7.....				+5				+5
6.....				+6				+7
5.....				+7				+9
4.....				+7				+7
3.....				+6				+6
2.....				+3				+3
1.....				+1				+1
Total.....				+200				+181

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 3				Sea Station No. 4			
A-----	-96	-1	0	-1	-137	-1	0	-1
B-----	-96	-20	0	-20	-137	-24	0	-24
C-----	-92	-4	0	-4	-137	-12	0	-12
D-----	-92	-6	0	-6	-137	-6	0	-6
E-----	-92	0	0	0	-136	-3	0	-3
F-----	-101	0	0	0	-125	0	0	0
G-----	-101	0	0	0	-118	0	0	0
H-----	-101	0	0	0	-115	-1	+1	0
I-----	-108	-1	+1	0	-124	-1	+1	0
J-----	-120	-1	+1	0	-137	-2	+2	0
K-----	-152	-1	+2	+1	-174	+1	+3	+4
L-----	-230	0	+6	+6	-215	+4	+5	+9
M-----	-440	-1	+26	+25	-422	-1	+25	+24
N-----	-575	+2	+29	+31	-742	+2	+38	+40
O-----	-633	-7	+32	+25	-673	-8	+34	+26
18-----				+7				+6
17-----				+7				+5
16-----				+7				+4
15-----				+8				+5
14-----				+7				+6
13-----				+39				+35
12-----				+33				+42
11-----				+32				+49
10-----				+25				+28
9-----				+14				+15
8-----				+14				+17
7-----				+5				+5
6-----				+6				+5
5-----				+10				+10
4-----				+8				+8
3-----				+6				+6
2-----				+2				+2
1-----				+1				+1
Total-----				+288				+306
	Sea Station No. 5				Sea Station No. 6			
	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
A-----	-775	-1	0	-1	-6,889	-1	0	-1
B-----	-775	-40	0	-40	-6,889	-41	+1	-40
C-----	-775	-70	0	-70	-6,919	-107	+3	-104
D-----	-756	-70	+1	-69	-6,919	-212	+8	-204
E-----	-796	-39	+2	-37	-6,919	-345	+17	-328
F-----	-747	-16	+2	-14	-6,919	-371	+21	-350
G-----	-784	-6	+3	-3	-6,919	-301	+25	-276
H-----	-778	-5	+4	-1	-6,919	-257	+33	-224
I-----	-780	-6	+6	0	-6,863	-247	+55	-192
J-----	-922	0	+10	+10	-6,527	-134	+73	-61
K-----	-1,227	-7	+20	+13	-5,978	-89	+96	+7
L-----	-1,694	-13	+41	+28	-5,220	-49	+125	+76
M-----	-2,775	-20	+163	+143	-4,507	-43	+265	+222
N-----	-2,949	-9	+151	+142	-3,979	-12	+204	+192
O-----	-3,305	-9	+167	+158	-3,790	-8	+191	+183

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 5—Continued				Sea Station No. 6—Continued			
18	-----	-----	-----	+33	-----	-----	-----	+36
17	-----	-----	-----	+32	-----	-----	-----	+33
16	-----	-----	-----	+29	-----	-----	-----	+29
15	-----	-----	-----	+21	-----	-----	-----	+23
14	-----	-----	-----	+18	-----	-----	-----	+20
13	-----	-----	-----	+39	-----	-----	-----	+39
12	-----	-----	-----	+28	-----	-----	-----	+27
11	-----	-----	-----	+29	-----	-----	-----	+27
10	-----	-----	-----	+16	-----	-----	-----	+15
9	-----	-----	-----	+11	-----	-----	-----	+11
8	-----	-----	-----	+14	-----	-----	-----	+14
7	-----	-----	-----	+6	-----	-----	-----	+6
6	-----	-----	-----	+7	-----	-----	-----	+7
5	-----	-----	-----	+10	-----	-----	-----	+10
4	-----	-----	-----	+8	-----	-----	-----	+8
3	-----	-----	-----	+6	-----	-----	-----	+6
2	-----	-----	-----	+2	-----	-----	-----	+2
1	-----	-----	-----	+1	-----	-----	-----	+1
Total	-----	-----	-----	+569	-----	-----	-----	-786
	Sea Station No. 7				Sea Station No. 8			
A	-6,727	-1	0	-1	-7,011	-1	0	-1
B	-6,727	-41	+1	-40	-7,011	-41	+1	-40
C	-6,734	-107	+3	-104	-7,011	-108	+4	-104
D	-6,734	-212	+8	-204	-7,011	-215	+8	-207
E	-6,734	-344	+16	-328	-7,011	-349	+17	-332
F	-6,734	-363	+20	-343	-7,011	-376	+21	-355
G	-6,734	-292	+24	-268	-7,011	-307	+25	-282
H	-6,734	-240	+32	-208	-7,011	-258	+34	-224
I	-6,734	-234	+54	-180	-7,011	-256	+56	-200
J	-6,734	-139	+75	-64	-7,011	-151	+79	-72
K	-6,785	-109	+109	0	-7,011	-112	+112	0
L	-6,788	-67	+163	+96	-7,041	-73	+169	+96
M	-6,761	-72	+398	+326	-7,064	-76	+415	+339
N	-5,990	-19	+307	+288	-7,080	-21	+362	+341
O	-5,706	-8	+288	+280	-6,365	-13	+321	+308
18	-----	-----	-----	+45	-----	-----	-----	+51
17	-----	-----	-----	+39	-----	-----	-----	+50
16	-----	-----	-----	+37	-----	-----	-----	+47
15	-----	-----	-----	+32	-----	-----	-----	+42
14	-----	-----	-----	+28	-----	-----	-----	+41
13	-----	-----	-----	+37	-----	-----	-----	+52
12	-----	-----	-----	+21	-----	-----	-----	+21
11	-----	-----	-----	+20	-----	-----	-----	+14
10	-----	-----	-----	+8	-----	-----	-----	-3
9	-----	-----	-----	+9	-----	-----	-----	+4
8	-----	-----	-----	+13	-----	-----	-----	+11
7	-----	-----	-----	+6	-----	-----	-----	+7
6	-----	-----	-----	+7	-----	-----	-----	+8
5	-----	-----	-----	+10	-----	-----	-----	+10
4	-----	-----	-----	+8	-----	-----	-----	+8
3	-----	-----	-----	+6	-----	-----	-----	+6
2	-----	-----	-----	+2	-----	-----	-----	+2
1	-----	-----	-----	+1	-----	-----	-----	+1
Total	-----	-----	-----	-421	-----	-----	-----	-361

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 9				Sea Station No. 10			
A-----	-7, 103	-1	0	-1	-7, 354	-1	0	-1
B-----	-7, 103	-41	+1	-40	-7, 354	-41	+1	-40
C-----	-7, 103	-108	+4	-104	-7, 380	-108	+4	-104
D-----	-7, 103	-219	+9	-210	-7, 380	-219	+9	-210
E-----	-7, 103	-353	+17	-336	-7, 380	-354	+18	-336
F-----	-7, 103	-381	+21	-360	-7, 380	-392	+22	-370
G-----	-7, 103	-314	+26	-288	-7, 380	-327	+27	-300
H-----	-7, 103	-258	+34	-224	-7, 380	-275	+35	-240
I-----	-7, 103	-257	+57	-200	-7, 380	-279	+59	-220
J-----	-7, 103	-160	+80	-80	-7, 380	-163	+83	-80
K-----	-7, 103	-114	+114	0	-7, 380	-118	+118	0
L-----	-7, 103	-74	+170	+96	-7, 334	-80	+176	+96
M-----	-7, 215	-82	+424	+342	-7, 387	-85	+434	+349
N-----	-7, 176	-24	+367	+343	-7, 317	-25	+375	+350
O-----	-7, 004	-21	+353	+332	-6, 823	-22	+344	+322
18-----				+65				+63
17-----				+59				+60
16-----				+50				+55
15-----				+43				+52
14-----				+37				+46
13-----				+52				+41
12-----				+18				+10
11-----				+8				-6
10-----				-4				-5
9-----				+4				+4
8-----				+11				+10
7-----				+7				+8
6-----				+8				+9
5-----				+10				+10
4-----				+8				+8
3-----				+6				+6
2-----				+2				+2
1-----				+1				+1
Total-----				-345				-410
	Sea Station No. 11				Sea Station No. 12			
	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
A-----	-3, 347	-1	0	-1	-269	-1	0	-1
B-----	-3, 347	-41	+1	-40	-269	-32	0	-32
C-----	-3, 321	-102	+2	-100	-258	-32	0	-32
D-----	-3, 321	-187	+4	-183	-258	-12	0	-12
E-----	-3, 321	-240	+8	-232	-258	-9	+1	-8
F-----	-3, 321	-185	+10	-175	-258	-1	+1	0
G-----	-3, 429	-118	+12	-106	-258	-1	+1	0
H-----	-3, 413	-84	+16	-68	-221	-1	+1	0
I-----	-3, 519	-64	+28	-36	-202	-2	+2	0
J-----	-3, 563	-40	+40	0	-196	-2	+2	0
K-----	-3, 616	-38	+58	+20	-199	-3	+3	0
L-----	-3, 775	-34	+91	+57	-223	-5	+5	0
M-----	-4, 138	-31	+243	+212	-498	0	+29	+29
N-----	-4, 359	-8	+223	+215	-844	-2	+43	+41
O-----	-4, 352	-13	+219	+206	-1, 148	-6	+58	+52

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 11—Continued					Sea Station No. 12—Continued			
18				+41				+14
17				+39				+15
16				+37				+17
15				+34				+23
14				+32				+27
13				+30				+46
12				+8				+23
11				-4				+5
10				-6				-7
9				+3				-7
8				+9				+10
7				+8				+7
6				+9				+9
5				+10				+10
4				+8				+8
3				+6				+6
2				+2				+2
1				+1				+1
Total				+36				+246
Sea Station No. 13					Sea Station No. 14			
A	-3,767	-1	0	-1	-1,974	-1	0	-1
B	-3,767	-41	+1	-40	-1,974	-40	0	-40
C	-3,764	-102	+2	-100	-1,974	-93	+1	-92
D	-3,764	-191	+5	-186	-1,974	-158	+2	-156
E	-3,782	-261	+9	-252	-1,974	-149	+5	-144
F	-3,782	-221	+11	-210	-1,974	-86	+6	-80
G	-3,782	-134	+14	-120	-1,937	-46	+7	-39
H	-3,828	-102	+18	-84	-1,937	-25	+9	-16
I	-3,916	-72	+31	-41	-1,956	-29	+16	-13
J	-3,938	-46	+44	-2	-1,983	-6	+22	+16
K	-3,976	-44	+64	+20	-2,048	-13	+33	+20
L	-4,013	-36	+96	+60	-2,037	-9	+49	+40
M	-4,052	-24	+238	+214	-1,983	-9	+117	+108
N	-4,140	-6	+212	+206	-2,038	-2	+104	+102
O	-4,178	-9	+211	+202	-2,396	-7	+121	+114
18				+41				+26
17				+40				+26
16				+37				+26
15				+37				+26
14				+35				+27
13				+39				+37
12				+17				+17
11				+9				+8
10				+3				+4
9				+3				+1
8				+9				+9
7				+7				+7
6				+9				+9
5				+10				+10
4				+8				+8
3				+6				+6
2				+3				+3
1				+1				+1
Total				-20				+70

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 15				Sea Station No. 16			
A-----	-1,956	-1	0	-1	-4,502	-1	0	-1
B-----	-1,956	-40	0	-40	-4,502	-41	+1	-40
C-----	-1,956	-93	+1	-92	-4,502	-102	+2	-100
D-----	-1,937	-155	+2	-153	-4,502	-203	+5	-198
E-----	-1,937	-149	+5	-144	-4,502	-291	+11	-280
F-----	-1,919	-84	+6	-78	-4,502	-264	+14	-250
G-----	-1,922	-46	+7	-39	-4,502	-172	+16	-156
H-----	-1,915	-25	+9	-16	-4,502	-134	+22	-112
I-----	-1,886	-26	+15	-11	-4,553	-104	+36	-68
J-----	-1,810	-4	+20	+16	-4,578	-70	+51	-19
K-----	-1,697	-7	+27	+20	-4,599	-54	+74	+20
L-----	-1,622	-4	+39	+35	-4,612	-31	+111	+80
M-----	-1,601	-7	+94	+87	-4,461	-27	+262	+235
N-----	-1,695	-3	+87	+84	-3,609	-5	+185	+180
O-----	-1,852	-5	+93	+88	-2,665	-10	+134	+124
18-----				+18				+25
17-----				+19				+25
16-----				+20				+24
15-----				+21				+23
14-----				+22				+23
13-----				+36				+33
12-----				+16				+15
11-----				+6				+9
10-----				+6				+8
9-----				+3				+4
8-----				+7				+8
7-----				+7				+7
6-----				+9				+9
5-----				+10				+10
4-----				+8				+8
3-----				+6				+6
2-----				+3				+3
1-----				+1				+1
Total-----				-26				-344
	Sea Station No. 17				Sea Station No. 18			
	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
A-----	-6,373	-1	0	-1	-6,697	-1	0	-1
B-----	-6,373	-41	+1	-40	-6,697	-41	+1	-40
C-----	-6,365	-107	+3	-104	-6,697	-107	+3	-104
D-----	-6,365	-212	+8	-204	-6,697	-212	+8	-204
E-----	-6,365	-343	+15	-328	-6,697	-344	+16	-328
F-----	-6,365	-349	+19	-330	-6,697	-360	+20	-340
G-----	-6,365	-275	+23	-252	-6,697	-288	+24	-264
H-----	-6,365	-223	+31	-192	-6,697	-240	+32	-208
I-----	-6,365	-211	+51	-160	-6,688	-234	+54	-180
J-----	-6,383	-121	+71	-50	-6,569	-136	+74	-62
K-----	-6,365	-102	+102	0	-6,582	-105	+105	0
L-----	-6,358	-57	+153	+96	-6,546	-61	+157	+96
M-----	-6,332	-68	+372	+304	-6,385	-63	+375	+312
N-----	-6,273	-17	+321	+304	-5,374	-14	+275	+261
O-----	-4,853	-7	+245	+238	-4,356	-7	+220	+213

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 17—Continued					Sea Station No. 18—Continued			
18.....				+40				+40
17.....				+36				+39
16.....				+33				+36
15.....				+31				+27
14.....				+24				+22
13.....				+27				+31
12.....				+13				+19
11.....				+16				+21
10.....				+15				+15
9.....				+7				+8
8.....				+11				+12
7.....				+6				+6
6.....				+8				+8
5.....				+10				+10
4.....				+8				+8
3.....				+6				+6
2.....				+2				+2
1.....				+1				+1
Total.....				-425				-538
Sea Station No. 19					Sea Station No. 20			
A.....	-4,915	-1	0	-1	-5,424	-1	0	-1
B.....	-4,915	-41	+1	-40	-5,424	-41	+1	-40
C.....	-4,915	-102	+2	-100	-5,424	-103	+3	-100
D.....	-4,915	-210	+6	-204	-5,424	-211	+7	-204
E.....	-4,915	-308	+12	-296	-5,424	-325	+13	-312
F.....	-4,915	-285	+15	-270	-5,424	-311	+16	-295
G.....	-4,915	-198	+18	-180	-5,397	-223	+19	-204
H.....	-4,982	-160	+24	-136	-5,356	-170	+26	-144
I.....	-5,037	-135	+40	-95	-5,304	-150	+42	-108
J.....	-5,005	-86	+56	-30	-5,131	-85	+57	-28
K.....	-4,723	-56	+76	+20	-5,000	-60	+80	+20
L.....	-4,074	-33	+98	+65	-4,716	-34	+113	+79
M.....	-2,558	-15	+150	+135	-3,858	-26	+227	+201
N.....	-1,089	-4	+56	+52	-2,826	-4	+145	+141
O.....	-996	-4	+50	+46	-2,136	-10	+108	+98
18.....				+10				+36
17.....				+17				+34
16.....				+28				+33
15.....				+37				+36
14.....				+42				+34
13.....				+67				+65
12.....				+51				+58
11.....				+47				+53
10.....				+37				+37
9.....				+18				+17
8.....				+15				+15
7.....				+7				+7
6.....				+7				+7
5.....				+9				+9
4.....				+7				+7
3.....				+5				+5
2.....				+2				+2
1.....				+1				+1
Total.....				-627				-441

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TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 21				Sea Station No. 22			
A-----	-5,838	-1	0	-1	-10,579	-1	0	-1
B-----	-5,838	-41	+1	-40	-10,579	-46	+2	-44
C-----	-5,838	-103	+3	-100	-10,590	-109	+5	-104
D-----	-5,812	-211	+7	-204	-10,609	-235	+13	-222
E-----	-5,812	-334	+14	-320	-10,627	-402	+26	-376
F-----	-5,821	-333	+17	-316	-10,664	-482	+32	-450
G-----	-5,812	-245	+21	-224	-10,670	-457	+38	-419
H-----	-5,743	-188	+28	-160	-10,632	-425	+51	-374
I-----	-5,553	-165	+44	-121	-10,194	-435	+82	-353
J-----	-5,604	-101	+63	-38	-9,340	-251	+105	-146
K-----	-5,858	-82	+94	+12	-8,782	-170	+141	-29
L-----	-6,458	-59	+155	+96	-8,010	-99	+192	+93
M-----	-7,446	-93	+438	+345	-6,764	-83	+398	+315
N-----	-5,469	-18	+280	+262	-4,193	-17	+215	+198
O-----	-2,716	-8	+137	+129	-2,835	-11	+143	+132
18-----				+17				+23
17-----				+23				+28
16-----				+31				+32
15-----				+32				+30
14-----				+32				+32
13-----				+62				+62
12-----				+48				+55
11-----				+47				+52
10-----				+35				+35
9-----				+15				+15
8-----				+15				+15
7-----				+7				+7
6-----				+7				+7
5-----				+9				+9
4-----				+8				+8
3-----				+5				+5
2-----				+2				+2
1-----				+1				+1
Total				-284				-1,362
	Sea Station No. 23				Sea Station No. 24			
	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
A-----	-18	-1	0	-1	-9,336	-1	0	-1
B-----	-14	-2	0	-2	-9,336	-46	+2	-44
C-----	+4	-1	0	-1	-9,336	-109	+5	-104
D-----	+24	0	0	0	-9,336	-227	+11	-216
E-----	-5	-2	0	-2	-9,363	-385	+22	-363
F-----	-205	-5	+1	-4	-9,391	-452	+28	-424
G-----	-382	-4	+1	-3	-9,456	-413	+34	-379
H-----	-187	-1	+1	0	-9,271	-370	+45	-325
I-----	-307	-2	+2	0	-9,391	-385	+75	-310
J-----	-1,289	-17	+14	-3	-9,456	-256	+106	-150
K-----	-1,416	-23	+23	0	-9,668	-198	+155	-43
L-----	-1,653	-20	+40	+20	-9,963	-148	+239	+91
M-----	-2,993	-48	+176	+128	-9,436	-143	+555	+412
N-----	-3,299	-16	+169	+153	-5,414	-19	+280	+261
O-----	-3,434	-8	+173	+165	-2,549	-11	+128	+117

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 23—Continued				Sea Station No. 24—Continued			
18				+39				+20
17				+33				+20
16				+27				+28
15				+24				+32
14				+30				+35
13				+62				+64
12				+50				+43
11				+53				+46
10				+34				+31
9				+16				+14
8				+15				+15
7				+7				+7
6				+7				+7
5				+9				+9
4				+8				+8
3				+5				+5
2				+2				+2
1				+1				+1
Total				+872				-1,091
	Sea Station No. 25				Sea Station No. 26			
A	-13,911	-1	0	-1	-3,476	-1	0	-1
B	-13,911	-47	+3	-44	-3,476	-41	+1	-40
C	-13,911	-115	+7	-108	-3,487	-102	+2	-100
D	-13,653	-244	+16	-228	-3,475	-188	+4	-184
E	-13,653	-441	+33	-408	-3,436	-245	+8	-237
F	-13,653	-546	+41	-505	-3,432	-190	+10	-180
G	-13,653	-547	+49	-498	-3,014	-103	+11	-92
H	-13,561	-541	+65	-476	-3,110	-87	+15	-72
I	-13,413	-634	+107	-527	-2,922	-102	+23	-79
J	-13,099	-443	+147	-296	-2,589	-73	+29	-44
K	-11,826	-298	+189	-109	-3,322	-86	+53	-33
L	-9,902	-155	+238	+83	-4,117	-86	+99	+13
M	-6,613	-106	+389	+283	-4,527	-79	+266	+187
N	-4,826	-19	+247	+228	-4,529	-23	+232	+209
O	-3,013	-8	+152	+144	-3,214	-8	+162	+154
18				+20				+27
17				+24				+24
16				+30				+30
15				+32				+32
14				+32				+32
13				+58				+57
12				+44				+52
11				+47				+49
10				+29				+28
9				+14				+14
8				+15				+15
7				+7				+7
6				+7				+7
5				+9				+9
4				+8				+8
3				+5				+5
2				+2				+2
1				+1				+1
Total				-2,078				-100

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TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 27					Sea Station No. 28			
A	-18	-1	0	-1	-8, 136	-1	0	-1
B	-15	-3	0	-3	-8, 136	-46	+2	-44
C	-9	0	0	0	-8, 136	-108	+4	-104
D	-8	0	0	0	-8, 118	-220	+10	-210
E	+46	0	0	0	-8, 118	-363	+19	-344
F	+81	0	0	0	-8, 118	-414	+24	-390
G	+31	0	0	0	-8, 187	-353	+29	-324
H	-310	-3	+1	-2	-8, 251	-323	+40	-283
I	-1, 117	-20	+9	-11	-8, 326	-318	+67	-251
J	-1, 476	-21	+17	-4	-8, 406	-201	+94	-107
K	-1, 973	-26	+32	+6	-8, 095	-145	+130	-15
L	-2, 379	-21	+57	+36	-7, 526	-85	+181	+96
M	-2, 591	-37	+152	+115	-5, 904	-64	+347	+283
N	-2, 463	-10	+126	+116	-3, 298	-10	+169	+159
O	-3, 670	-8	+185	+177	-1, 487	-7	+75	+68
18				+33				+20
17				+35				+25
16				+29				+30
15				+25				+36
14				+30				+50
13				+58				+103
12				+58				+72
11				+51				+58
10				+35				+38
9				+17				+19
8				+15				+16
7				+7				+7
6				+7				+7
5				+9				+9
4				+8				+7
3				+5				+5
2				+2				+2
1				+1				+1
Total				+854				-962
Sea Station No. 29					Sea Station No. 30			
A	-8, 328	-1	0	-1	-9, 299	-1	0	-1
B	-8, 328	-46	+2	-44	-9, 299	-46	+2	-44
C	-8, 321	-108	+4	-104	-9, 225	-109	+5	-104
D	-8, 321	-220	+10	-210	-9, 225	-227	+11	-216
E	-8, 321	-364	+20	-344	-9, 225	-382	+22	-360
F	-8, 321	-420	+25	-395	-9, 225	-448	+28	-420
G	-8, 322	-366	+30	-336	-9, 163	-403	+33	-370
H	-8, 312	-328	+40	-288	-9, 271	-373	+45	-328
I	-8, 302	-315	+66	-249	-9, 557	-394	+76	-318
J	-8, 233	-193	+92	-101	-10, 032	-281	+112	-169
K	-7, 841	-137	+125	-12	-10, 387	-234	+166	-68
L	-6, 734	-69	+162	+93	-10, 424	-171	+250	+79
M	-4, 731	-45	+278	+233	-8, 961	-146	+527	+381
N	-2, 887	-11	+148	+137	-6, 066	-26	+311	+285
O	-1, 736	-6	+87	+81	-5, 016	-13	+253	+240

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 29—Continued					Sea Station No. 30—Continued			
18				+21				+57
17				+27				+58
16				+45				+68
15				+59				+79
14				+70				+75
13				+126				+128
12				+74				+86
11				+57				+57
10				+39				+39
9				+19				+21
8				+16				+17
7				+7				+8
6				+7				+8
5				+9				+8
4				+7				+6
3				+5				+4
2				+2				+2
1				+1				0
Total				-949				-692
Sea Station No. 31					Sea Station No. 32'			
A	-15,867	-1	0	-1	-9,373	-1	0	-1
B	-15,867	-47	+3	-44	-9,373	-46	+2	-44
C	-15,867	-116	+8	-108	-9,373	-109	+5	-104
D	-15,867	-247	+19	-228	-9,373	-227	+11	-216
E	-15,867	-462	+38	-424	-9,410	-387	+23	-364
F	-15,996	-588	+48	-540	-9,502	-459	+29	-430
G	-16,052	-598	+58	-540	-9,594	-419	+35	-384
H	-16,224	-652	+78	-574	-9,871	-395	+47	-348
I	-16,301	-814	+130	-684	-10,055	-425	+80	-345
J	-16,155	-609	+181	-428	-10,482	-300	+117	-183
K	-15,941	-517	+255	-262	-10,683	-239	+171	-68
L	-15,321	-364	+368	+4	-10,893	-177	+261	+84
M	-13,693	-283	+805	+522	-11,966	-218	+704	+486
N	-11,024	-55	+564	+509	-12,500	-67	+640	+573
O	-6,986	-14	+352	+338	-9,752	-19	+492	+473
18				+59				+75
17				+58				+71
16				+64				+67
15				+70				+67
14				+77				+75
13				+123				+130
12				+78				+84
11				+60				+64
10				+41				+42
9				+21				+22
8				+17				+17
7				+8				+8
6				+7				+7
5				+8				+8
4				+6				+6
3				+4				+4
2				+2				+2
1				0				0
Total				-1,757				-122

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 33				Sea Station No. 34			
A-----	-10, 332	-1	0	-1	-11, 236	-1	0	-1
B-----	-10, 332	-46	+2	-44	-11, 236	-46	+2	-44
C-----	-10, 332	-109	+5	-104	-11, 236	-110	+6	-104
D-----	-10, 332	-234	+12	-222	-11, 236	-235	+13	-222
E-----	-10, 332	-401	+25	-376	-11, 236	-411	+27	-384
F-----	-10, 332	-471	+31	-440	-11, 236	-499	+34	-465
G-----	-10, 332	-445	+37	-408	-11, 236	-478	+40	-438
H-----	-10, 332	-418	+50	-368	-11, 162	-438	+54	-384
I-----	-10, 332	-443	+83	-360	-11, 070	-489	+89	-400
J-----	-10, 424	-297	+117	-180	-11, 070	-332	+124	-208
K-----	-10, 424	-227	+167	-60	-11, 070	-257	+177	-80
L-----	-10, 486	-159	+252	+93	-11, 070	-194	+266	+72
M-----	-10, 635	-172	+625	+453	-10, 978	-184	+646	+462
N-----	-10, 920	-53	+559	+506	-10, 897	-52	+558	+506
O-----	-11, 044	-24	+557	+533	-11, 050	-20	+557	+537
18-----				+110				+114
17-----				+111				+119
16-----				+111				+122
15-----				+107				+121
14-----				+103				+104
13-----				+127				+124
12-----				+85				+83
11-----				+66				+70
10-----				+45				+46
9-----				+23				+24
8-----				+19				+20
7-----				+8				+8
6-----				+7				+7
5-----				+8				+8
4-----				+6				+6
3-----				+4				+4
2-----				+2				+2
1-----				0				0
Total-----				-36				-171
	Sea Station No. 35				Sea Station No. 36			
	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
A-----	-10, 775	-1	0	-1	-15, 535	-1	0	-1
B-----	-10, 775	-46	+2	-44	-15, 535	-47	+3	-44
C-----	-10, 775	-109	+5	-104	-15, 535	-116	+8	-108
D-----	-10, 775	-235	+13	-222	-15, 535	-247	+19	-228
E-----	-10, 775	-410	+26	-384	-15, 535	-461	+37	-424
F-----	-10, 793	-482	+32	-450	-15, 572	-577	+47	-530
G-----	-10, 793	-459	+39	-420	-15, 652	-586	+56	-530
H-----	-10, 793	-436	+52	-384	-15, 821	-634	+76	-558
I-----	-10, 793	-466	+86	-380	-15, 922	-790	+127	-663
J-----	-10, 886	-322	+122	-200	-15, 982	-600	+179	-421
K-----	-10, 978	-256	+176	-80	-15, 932	-518	+255	-263
L-----	-11, 016	-192	+264	+72	-15, 522	-373	+373	0
M-----	-11, 399	-196	+670	+474	-14, 641	-323	+861	+538
N-----	-12, 431	-64	+636	+572	-12, 823	-70	+657	+587
O-----	-12, 236	-24	+617	+593	-9, 640	-17	+486	+469

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 35—Continued					Sea Station No. 36—Continued			
18				+113				+77
17				+106				+75
16				+90				+81
15				+75				+82
14				+76				+82
13				+127				+125
12				+82				+80
11				+71				+67
10				+45				+45
9				+23				+23
8				+18				+18
7				+8				+8
6				+7				+7
5				+8				+8
4				+6				+6
3				+4				+4
2				+2				+2
1				0				0
Total				-97				-1,386
Sea Station No. 37, San Juan, P. R.					Sea Station No. 38, St. Thomas			
A	-18	-1	0	-1	-18	-1	0	-1
B	-18	-4	0	-4	-11	-2	0	-2
C	-18	0	0	0	+17	0	0	0
D	-2	0	0	0	+34	0	0	0
E	+1	0	0	0	+1	0	0	0
F	-17	0	0	0	+124	0	0	0
G	-48	0	0	0	+274	+1	-1	0
H	-109	-1	+1	0	+224	+1	-1	0
I	-230	-2	+2	0	+29	0	0	0
J	-422	+1	+5	+6	-49	-1	+1	0
K	-588	-1	+9	+8	-107	-2	+2	0
L	-754	-6	+18	+12	-734	-6	+18	+12
M	-1,292	-6	+76	+70	-2,455	-18	+144	+126
N	-4,221	-14	+216	+202	-3,215	-6	+165	+159
O	-8,110	-18	+409	+391	-6,646	-12	+335	+323
18				+84				+78
17				+88				+81
16				+91				+80
15				+85				+85
14				+80				+84
13				+120				+129
12				+76				+83
11				+57				+57
10				+40				+37
9				+22				+23
8				+18				+20
7				+9				+9
6				+8				+8
5				+8				+8
4				+6				+6
3				+4				+4
2				+2				+2
1				0				0
Total				+1,482				+1,411

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 39					Sea Station No. 40			
A-----	-8,734	-1	0	-1	-3,579	-1	0	-1
B-----	-8,734	-46	+2	-44	-3,579	-41	+1	-40
C-----	-8,734	-108	+4	-104	-3,579	-102	+2	-100
D-----	-8,856	-227	+11	-216	-3,579	-190	+4	-186
E-----	-8,856	-373	+21	-352	-3,579	-249	+9	-240
F-----	-8,801	-434	+26	-408	-3,568	-201	+11	-190
G-----	-8,548	-375	+31	-344	-3,573	-132	+13	-119
H-----	-8,291	-325	+40	-285	-3,593	-97	+17	-80
I-----	-8,072	-310	+65	-245	-3,603	-69	+29	-40
J-----	-7,242	-158	+81	-77	-3,722	-42	+42	0
K-----	-5,886	-88	+94	+6	-3,981	-44	+64	+20
L-----	-4,267	-38	+102	+64	-5,151	-43	+124	+81
M-----	-2,769	-17	+163	+146	-5,654	-57	+332	+275
N-----	-3,491	-8	+179	+171	-4,640	-17	+238	+221
O-----	-5,338	-12	+269	+257	-4,848	-14	+244	+230
18-----				+72				+60
17-----				+77				+76
16-----				+84				+79
15-----				+85				+85
14-----				+82				+85
13-----				+127				+123
12-----				+79				+76
11-----				+52				+49
10-----				+36				+35
9-----				+22				+22
8-----				+20				+20
7-----				+9				+9
6-----				+8				+8
5-----				+8				+8
4-----				+6				+6
3-----				+4				+4
2-----				+2				+2
1-----				0				0
Total				-659				+578
Sea Station No. 41					Sea Station No. 42			
A-----	-9,288	-1	0	-1	-6,096	-1	0	-1
B-----	-9,288	-46	+2	-44	-6,096	-41	+1	-40
C-----	-9,299	-109	+5	-104	-6,088	-103	+3	-100
D-----	-9,336	-227	+11	-216	-6,088	-211	+7	-204
E-----	-9,373	-382	+22	-360	-6,088	-335	+15	-320
F-----	-9,409	-453	+28	-425	-6,088	-343	+18	-325
G-----	-9,409	-412	+34	-378	-6,088	-262	+22	-240
H-----	-9,409	-381	+45	-336	-6,088	-205	+29	-176
I-----	-9,409	-385	+75	-310	-6,070	-198	+49	-149
J-----	-9,409	-257	+105	-152	-6,204	-115	+69	-46
K-----	-9,409	-191	+151	-40	-6,273	-91	+100	+9
L-----	-9,409	-130	+226	+96	-6,519	-60	+156	+96
M-----	-9,423	-139	+554	+415	-6,853	-80	+403	+323
N-----	-9,479	-41	+485	+444	-7,311	-26	+374	+348
O-----	-9,469	-13	+477	+464	-7,256	-20	+366	+346

108069°—30—6

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
	Sea Station No. 41—Continued				Sea Station No. 42—Continued			
18				+93				+70
17				+88				+66
16				+80				+58
15				+71				+52
14				+61				+46
13				+82				+77
12				+40				+46
11				+39				+33
10				+29				+25
9				+20				+15
8				+18				+16
7				+10				+9
6				+8				+8
5				+8				+8
4				+7				+7
3				+3				+4
2				+2				+2
1				0				0
Total				-288				+63
	Sea Station No. 43				Sea Station No. 44			
A	-8,472	-1	0	-1	-6,410	-1	0	-1
B	-8,472	-46	+2	-44	-6,410	-41	+1	-40
C	-8,469	-108	+4	-104	-6,421	-107	+3	-104
D	-8,462	-222	+10	-212	-6,421	-212	+8	-204
E	-8,459	-367	+20	-347	-6,458	-343	+15	-328
F	-8,456	-425	+25	-400	-6,458	-354	+19	-335
G	-8,449	-370	+30	-340	-6,458	-275	+23	-252
H	-8,469	-337	+41	-296	-6,458	-223	+31	-192
I	-8,473	-328	+68	-260	-6,458	-222	+52	-170
J	-8,475	-207	+95	-112	-6,400	-126	+72	-54
K	-8,487	-156	+136	-20	-6,338	-96	+101	+5
L	-8,502	-108	+204	+96	-6,242	-54	+150	+96
M	-8,540	-122	+502	+380	-5,640	-50	+332	+282
N	-7,726	-29	+396	+367	-4,836	-19	+248	+229
O	-6,135	-16	+309	+293	-3,541	-13	+178	+165
18				+46				+27
17				+43				+28
16				+46				+25
15				+47				+32
14				+46				+40
13				+78				+65
12				+39				+37
11				+38				+43
10				+26				+30
9				+15				+14
8				+16				+15
7				+8				+7
6				+8				+7
5				+8				+9
4				+7				+8
3				+4				+5
2				+2				+2
1				0				+1
Total				-523				-508

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 45					Sea Station No. 46			
A-----	-4,059	-1	0	-1	-3,513	-1	0	-1
B-----	-4,059	-41	+1	-40	-3,513	-41	+1	-40
C-----	-4,059	-102	+2	-100	-3,506	-102	+2	-100
D-----	-4,059	-197	+5	-192	-3,475	-189	+4	-185
E-----	-3,967	-270	+10	-260	-3,402	-243	+8	-235
F-----	-3,874	-224	+12	-212	-3,386	-189	+10	-179
G-----	-3,613	-130	+13	-117	-3,659	-133	+13	-120
H-----	-3,367	-82	+16	-66	-3,967	-111	+19	-92
I-----	-3,007	-54	+24	-30	-4,160	-95	+33	-62
J-----	-2,664	-28	+30	+2	-4,174	-61	+47	-14
K-----	-2,251	-24	+36	+12	-4,262	-49	+68	+19
L-----	-2,215	-20	+53	+33	-4,042	-27	+97	+70
M-----	-2,557	-23	+150	+127	-3,843	-29	+226	+197
N-----	-3,081	-4	+158	+154	-4,355	-8	+223	+215
O-----	-3,295	-10	+166	+156	-5,094	-12	+257	+245
18-----				+26				+58
17-----				+29				+59
16-----				+34				+52
15-----				+32				+50
14-----				+26				+47
13-----				+64				+85
12-----				+59				+54
11-----				+53				+53
10-----				+36				+38
9-----				+17				+19
8-----				+15				+16
7-----				+7				+7
6-----				+7				+7
5-----				+9				+9
4-----				+8				+7
3-----				+5				+5
2-----				+2				+2
1-----				+1				0
Total				-104				+286
Sea Station No. 47					Sea Station No. 48			
A-----	-9,269	-1	0	-1	-9,269	-1	0	-1
B-----	-9,269	-46	+2	-44	-9,269	-46	+2	-44
C-----	-9,269	-109	+5	-104	-9,225	-109	+5	-104
D-----	-9,269	-227	+11	-216	-9,225	-227	+11	-216
E-----	-9,269	-382	+22	-360	-9,225	-382	+22	-360
F-----	-9,225	-448	+28	-420	-9,225	-448	+28	-420
G-----	-9,225	-405	+33	-372	-9,225	-405	+33	-372
H-----	-9,225	-372	+44	-328	-9,363	-378	+45	-333
I-----	-9,253	-376	+74	-302	-9,454	-389	+76	-313
J-----	-9,248	-249	+104	-145	-9,513	-262	+107	-155
K-----	-9,299	-182	+149	-33	-9,640	-195	+154	-41
L-----	-9,202	-125	+221	+96	-9,686	-136	+232	+96
M-----	-8,141	-108	+479	+371	-9,726	-148	+572	+424
N-----	-6,725	-22	+344	+322	-9,721	-45	+498	+453
O-----	-6,939	-13	+350	+337	-10,062	-13	+507	+494

TABLE 13—Continued

Zone	Elevation in feet	Topography	Compensation	Topography and compensation	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 47—Continued					Sea Station No. 48—Continued			
18				+71				+105
17				+70				+105
16				+65				+101
15				+64				+87
14				+63				+81
13				+88				+105
12				+54				+62
11				+52				+44
10				+39				+34
9				+19				+18
8				+17				+16
7				+7				+7
6				+7				+7
5				+9				+9
4				+7				+7
3				+5				+5
2				+2				+2
1				0				0
Total				-560				-97

Zone	Elevation in feet	Topography	Compensation	Topography and compensation
Sea Station No. 49				
A	-9,454	-1	0	-1
B	-9,454	-46	+2	-44
C	-9,446	-109	+5	-104
D	-9,446	-227	+11	-216
E	-9,410	-387	+23	-364
F	-9,410	-453	+28	-425
G	-9,410	-412	+34	-378
H	-9,410	-381	+45	-336
I	-9,410	-385	+75	-310
J	-9,410	-257	+105	-152
K	-9,382	-189	+150	-39
L	-9,294	-127	+223	+96
M	-9,462	-143	+556	+413
N	-9,479	-42	+485	+443
O	-8,922	-18	+450	+432
18				+86
17				+84
16				+81
15				+76
14				+70
13				+101
12				+56
11				+35
10				+22
9				+16
8				+15
7				+6
6				+6
5				+8
4				+7
3				+6
2				+3
1				+1
Total				-306

The isostatic correction for topography and compensation is the algebraic sum of the isostatic corrections for the several zones. This correction is listed in Table 13 and is to be added to the value of the theoretical gravity to obtain the computed gravity.

The Gulf of Mexico.—Passing now to the results obtained by the expedition we shall consider first the relations in the Gulf of Mexico and in particular over the "Gulf of Mexico Plate." As indicated on Chart No. 1, the depths are here fairly uniform over a great expanse and average about 2,000 fathoms or a little over 2 miles. There is little evidence of folding and faulting over the central area. The slopes leading from the deeper parts of the Gulf to land are gentle and the formations bordering the Gulf are flat lying or nearly so and are relatively young in age. Abrupt changes in gravity are not to be expected in this area and the measurements show this to be the case, but they show systematically positive anomalies.

Several of the profiles taken with the aid of the sonic depth finder between certain stations are shown in Figures 21 and 22, a, b, c. In these profiles and in those to follow the vertical scale is greatly exaggerated; the ratio of the vertical scale to the horizontal scale is 25 to 1. In the profiles each 1,000-fathom line is indicated by a horizontal line.

FIGURE 21.—Profile of ocean bottom between Stations 5 and 6. The interval between the successive horizontal lines is 1,000 fathoms. The vertical scale is exaggerated 24.7 fold. The gravity anomaly in dynes or cm/sec^2 for each station is given directly above the station

The profile between Stations 5 and 6 shows the edge of the continental shelf nearly 200 miles off the southwest coast of Florida. The slope of the shelf from 200 fathoms to 2,000 fathoms is about 1:6 or a little over $9^\circ.5$. The value of the gravity anomaly decreases from +15 millidynes at Station 5 to +1 millidyne at Station 6. Farther out to sea the anomalies rise and commonly exceed the values found in shallow water. This rise is clearly indicated by the gravity anomalies, +70, +62, +80 millidynes, obtained for Stations 7, 9, and 10. The value for Station 8 is lower, +17 millidynes, and may be due to the nearness of the Campeche bank.

In an earlier cruise similar large positive isostatic anomalies have been found over the broad deeps of the Atlantic and smaller positive values over the Pacific; in the Atlantic the mean isostatic anomaly for 23 stations (Stations 1 to 4, 6 to 10, 12 to 25) was 38 millidynes without regard to sign and +38 millidynes with regard to sign; for the same stations the "free air" anomaly was 23 without, and +20 with regard to sign. For the Pacific the mean isostatic anomaly for 28 stations

(Stations 59 to 70, 75 to 90) was 20 without regard, and +18 with regard to sign; the mean free air anomaly was 25 without regard, and +18 with regard to sign. For 18 stations, occupied along the Pacific seaboard between Balboa and San Francisco (Stations 38 to 47, 49 to 56), the mean isostatic anomalies were 43 without, and +37 with regard to sign; the mean free-air anomalies for the same stations were 36 without, and -7 with regard to sign. These large positive anomalies are difficult to explain satisfactorily (both isostatic and free air), and the fact that the free-air anomalies are smaller for the Atlantic and of the same order of magnitude as the isostatic anomalies for the Pacific is worth mentioning. Furthermore there seems to be a tendency for the isostatic anomalies to fall as the depth increases over many of the ocean bottom slopes. Just what the significance of these relations is, can not at present be stated.

More data of measurement and more detailed computations will be necessary before final conclusions can be drawn. The general theory of isostasy has been so well established that it is not easy to understand actual excess loads on the Earth's crust over areas of such vast extent. If stresses, active in the crust, are responsible for them, the engineering difficulty arises of explaining their maintainance over such great areas. An explanation based on a down-warping of the geoid surface over the oceans is not in harmony with potential theory and with the theorem of STOKES. On the land it is possible, from measurements of the deflection of the vertical and also from measurements, with the Eötvös torsion balance, of the gravity gradient and of the difference of the reciprocals of the radii of curvature (differential curvature) of the geoid surface, to determine also along this line the actual shape of the geoid surface, and this generally agrees well with the shape given by the gravity theorem of STOKES. This subject is one which requires thorough investigation and for the purpose additional profiles across the oceans will be essential.

The fact that these pronounced positive anomalies are found at the foot of the continental slopes bordering the Pacific Ocean might seem to point toward a westward drift of the American Continent in accord with the theory of WEGENER, because they might be explained—in connection with the time element required to reestablish equilibrium—by the effect of the pressure exerted by the continent on its front side; but in this case one would expect a negative anomaly on the lee side, i. e., the east coast. The gravity stations thus far occupied are not sufficient in number to warrant positive assertions; but the evidence on the east side, so far as it goes, is not in favor of this hypothesis.

Profiles across the Mississippi Delta.—The Mississippi River deposits each year about one-twentieth of a cubic mile of sediment over its delta. This is spread over a large area and the actual increase of load at any particular point is not great. In the course of thousands of years, however, the load would become appreciable and, if the disturbance to the isostatic balance were not compensated, but the load supported as an extra burden on a strong, competent crust, the gravity measurements would indicate this fact by the appearance of pronounced positive anomalies. It was to test the Mississippi Delta gravimetrically from this viewpoint that the trip over this part of the Gulf of Mexico was planned.

The results of the measurements are reproduced in the profiles of Figures 22 a, b, c.

The slopes on the Delta are gentle; the slope between Stations 14 and 13 is 1:107 or about $0^{\circ} 32'$; from Station 15 to 16 the slope is about 1:80 or $0^{\circ} 45'$.

The gravity anomalies from Stations 15 to 13 increase only slightly, from +16 to +30 millidynes, in common with the general increase in gravity toward deep parts of a large body of ocean water. Station 16 with its gravity anomaly -13 millidynes is an exception to this general relation. Station 11 with the anomaly +40 millidynes is in accord with the rule; also Stations 17 and 18 with the gravity anomalies, +62 and +32 millidynes.

FIGURE 22a

FIGURE 22b

FIGURE 22c

FIGURES 22a, b, c.—Profiles across the Mississippi Delta between Stations 12 and 13, 13 and 15, 15 and 18. Ratio of vertical scale to horizontal scale, 25.47

The general conclusion that can be drawn from the few measurements available is that the Mississippi Delta does not show evidence of the excess positive anomalies that might be expected if the earth's crust were of great strength and capable of supporting the extra load. The evidence, so far as it goes, is that the Mississippi Delta is a compensated load and that compensation is proceeding concomitantly with the deposition of the load.

The Nares Deep and its extension in a northwesterly direction north of Haiti and Cuba.—The Nares Deep, north of Porto Rico, is the deepest area thus far measured in the Atlantic Ocean. As outlined on the bathymetric chart its deepest portion extends in an East-West direction along the 20° N. parallel of latitude from longitude 65° to 68° west of Greenwich, a distance of 200 miles with an average width of 50

miles. The disturbance, however, of which the Nares Deep is the most pronounced feature, extends from the passage between the Bahama Banks and Cuba and Haiti in an ESE. to easterly direction through 12° to 15° of longitude or a distance of roughly 1,000 miles, from longitude 78° W. to 65° or 60° W.

The gravity measurements taken north of Cuba and of Haiti all show pronounced negative anomalies as depicted on profiles 23 a, b, c, d and indicate clearly that the Nares Deep structural disturbance does extend far to the west even though the depths are there no longer excessive. Thus north of Cuba the gravity anomalies for Stations 19 and 20 (fig. 23a) are -45 and -35 millidynes respectively over depths of 1,332 fathoms at No. 19 to 1,470 fathoms at No. 20.

Similarly north of Haiti the profiles between Stations 28 to 31 (fig. 23b) show the following gravity anomalies, -114 at 28, -109 at 29, -165 at 30, and -103 millidynes at 31 at the depths in fathoms 2,205, 2,257, 2,520, and 4,300 respectively. These profiles are in general along the trend of the trough. This proves that the negative anomalies found above the deep, are not caused by the deep itself, but by the tectonical disturbance, which gave rise to the formation of the deep.

The profiles across the Nares Deep itself are reproduced in Figures 23c and 23d. Station 31, over a depth of 4,300 fathoms, shows an anomaly -103 millidynes (fig. 23c), whereas Nos. 32 and 33 at depths of 2,540 and 2,800 fathoms north of the Nares depression show the positive anomalies $+27$ and $+15$ millidynes. The continuation of this profile to the south through the Mona Passage and into the Caribbean Sea is possible by virtue of earlier measurements made in July, 1926; at Station No. 27 (1926) latitude $18^{\circ} 23'.7$ N. and longitude $67^{\circ} 42'.3$ W., the depth was 159 fathoms and the gravity anomaly -131 millidynes; at Station No. 28 (1926), latitude $16^{\circ} 54'$ N., longitude $67^{\circ} 42'.3$ W., depth 2,679 fathoms, the gravity anomaly is $+36$ millidynes; at Station No. 41 of the present cruise at latitude $15^{\circ} 47'$ N. longitude $67^{\circ} 43'$ W., and depth 2,517 fathoms, the gravity anomaly is $+44$ millidynes. The gravity anomalies for the last two stations indicate that the Nares Deep disturbance does not extend south of Porto Rico.

The second profile across the Nares Deep is given in Figure 23d, and presents relations similar to those of the first profile. At Station No. 34, several hundred miles north of the Nares Deep, the gravity anomaly is $+3$ millidynes and the depth 3,045 fathoms; at Station No. 35, which is located on the top of the Nares Deep north slope, the gravity anomaly is $+35$ millidynes and the depth 2,920 fathoms. In the deep itself at No. 36 the depth is 4,210 fathoms and the gravity anomaly -121 millidynes; at San Juan at the surface the gravity anomaly is -15 millidynes.

The slopes leading into the Nares Deep are from Stations 31 to 32, 1:14 or $4^{\circ} 5'$; from Stations 35 to 36, 1:13 or $4^{\circ} 22'$; from a point beyond Station 36 to 37, it is 1:11 or $5^{\circ} 12'$. These are not excessive slopes, but the gravity anomalies do show an area of considerable size that is not isostatically compensated. A gravity anomaly of -165 millidynes at Station 30 indicates a deficiency of load equivalent to a layer of rock nearly a mile thick and of density 2.67.

This structural element, over 1,000 miles in length, is one of the outstanding features of the region. It is clearly marked and easy of access to submarines operating from Guantanamo Bay as a base; its detailed study, both with reference to its

FIGURES 23a, b, c, d.—Profiles along and across the Nares Deep and its westerly extension between Stations 19 and 20, 28 and 31, 31 and 33, 34 and 37. Ratio vertical to horizontal scale, 24.7 for Stations 19 to 20; 25.0 for 28 to 31, 31 to 33, and 34 to 37.

exact configuration, determined by measurements with the sonic depth finder, and to its gravity anomaly relations, is greatly to be desired. It is a critical area whose elucidation would add much to the understanding of the development of the Caribbean region, and to the measurement of the orogenic forces there active in the earth's crust. There are relatively few areas on the earth's surface that show structural relations so clearly as this area and exceptional opportunity is here offered to make contributions of fundamental importance to the science of geology.

The Virgin Island Deep.—This deep extends as a narrow depression from a point between St. Thomas and St. Croix in a westerly direction to a point SE. of Porto Rico. The results of the measurements of depths and of gravity anomalies are shown in Figure 24. At Station 38 in the harbor of St. Thomas the gravity anomaly is -3 millidynes; at Station 39, 17 miles to the south, the depth is 2,367 fathoms and the gravity anomaly, -63 millidynes; at Station 40, 30 miles south of St. Thomas, the depth is 970 fathoms and the gravity anomaly -13 millidynes; still farther to the southwest at Station 41 approaching the center of the Caribbean

FIGURE 24.—Profiles across Virgin Island Deep and into the Caribbean Sea between Stations 38, 39, 40, and 41. Ratio vertical to horizontal scales, 25.47

Sea basin the depth is 2,517 fathoms and the gravity anomaly $+44$ millidynes. The north slope of this deep shows a gradient 5.4:1 or $10^{\circ} 30'$; the south slope, a gradient of 1:8 or $7^{\circ} 13'$.

This small deep shows relations similar to those observed for the Nares Deep. The gravity anomalies are smaller, but they indicate a deficiency of load equivalent to a bed of rock nearly half a mile thick and of density 2.67. This means that the deep is a downwarped or depressed structural feature that is not completely compensated isostatically; in other words, it is an unstable portion of the earth's crust where movements in the earth's crust are likely to occur and to give rise to earthquakes.

The Caribbean Sea.—Only three measurements, at Stations 41, 42, and 43, were made over the deeper portions of the eastern part of the Caribbean Sea. At these stations the depths and gravity anomalies are, respectively, 2,517, 1,652, and 2,296 fathoms, and $+44$, $+23$, and $+52$ millidynes. These relatively high positive anomalies are in accord with anomalies found over the Gulf of Mexico Basin. A complete explanation of positive anomalies of this magnitude over deep-sea areas has still to be worked out.

The Bartlett Deep.—This deep, between Cuba and Jamaica and extending far to the west in a direction somewhat south of west, is one of the major structural features of the Antillean region. It is nearly 1,000 miles long and merits detailed study both geologically and gravimetrically. Unfortunately time permitted us to make only two profiles across the eastern end of Bartlett Deep. The first profile (fig. 25a) extends from the entrance of Santiago Bay, Station 23, in a southerly direction to Station 21, a distance of 52 miles. At Stations 23, 22, and 21, the following depth and gravity anomaly values were obtained: 5, 2,867 and 1,582 fathoms and +16, -57, and -4 millidynes. These relations are similar in character to those noted for the Nares and the Virgin Island Deeps. For the profile to the west, Stations 26, 25, 24, and 44 (fig. 25b) the corresponding measurements for the depths were 942, 3,770, 2,530, and 1,737 fathoms and the gravity anomalies +55, +20, +29, and +33 millidynes. This series of measurements does not show a large negative anomaly for the station located directly above the deepest part of the deep; the anomaly

Profiles across the Bartlett Deep, between Stations 21, 22, and 23, and 44, 24, 25, and 26. Ratio vertical to horizontal scales, 26.0

FIGURE 25a

FIGURE 25b

is less than that for the stations at the top of the slopes; but it is not greatly different. The soundings indicate a ridge between Stations 24 and 44; but this does not explain the difference in behavior of this section of the deep. The values indicate that the Bartlett Deep at this point is isostatically nearly compensated. It may be an older part of the depressed area, at which the isostatic equilibrium has been more nearly restored than is the case along the eastern profile.

The differences in behavior of the two profiles are interesting, because they indicate more complete compensation for the western profile than for the eastern. The values of the gravity anomalies for the Bartlett Deep are lower than those obtained for the Nares Deep and its westerly extension; this might indicate that the Bartlett Deep is an older and more nearly compensated structural feature than the Nares Deep. A complete gravimetric survey of these two deeps is essential, if the exact significance of the anomaly values thus far obtained is to be determined. The detailed study of this region with the aid of topographic maps of the land and ocean-bottom areas and of a sufficient number of gravity stations, properly placed on the

land and over the sea area, will contribute much toward solving the intricate geological problems here presented and will furnish quantitative data on the nature and order of magnitude of the forces and stresses involved in mountain-making processes.

The slopes on these two profiles are, from Station 23 to a point near Station 22 as 1:4.5 or $12^{\circ} 32'$; from Station 22 to a point near Station 21 as 1:8.9 or $6^{\circ} 24'$; from Turquino Peak, north of Station 26, to a point near Station 25 the slope gradient is 1:3.3 or $17^{\circ} 22'$; the difference in elevation between Turquino Peak and the bottom of the deep is 29,120 feet and is attained over a horizontal distance of 18.5 miles. This is certainly one of the longest steep slopes on the earth's surface; and yet the gravity anomalies show that it is nearly in isostatic equilibrium. A gravity station at the top of Turquino Peak would be of much interest in this connection. It will have to wait, however, until after topographic maps of this region have become available, because, without them, the isostatic reductions can not be satisfactorily made.

Section from Guantanamo Bay to Cape Hatteras.—The measurements made on this part of the voyage are the least satisfactory of the entire group, chiefly because of the heavy storm and seas encountered during the whole trip. The ocean swell was at all times heavy and gave rise to unusually large vertical accelerations, even at depths of 80 feet and over. The only station occupied in relatively still water was Station 45 off the southeast end of Cuba. The gravity anomaly of -11 millidynes was found here over a depth of 1,100 fathoms.

At Crooked Island Passage a profile was taken from Station 46, depth 952 fathoms and gravity anomaly -33 millidynes to Station 47, depth 2,510 fathoms and gravity anomaly -1 millidyne. Stations 48 and 49 are over the deep part of the Atlantic, the depths being 2,512 and 2,562 fathoms, respectively, and the gravity anomalies $+14$ and -11 millidynes. The gravity anomaly for Station 49 is lower than usual; the record is poor and not much emphasis should be placed on the result. The attempt was made to occupy a station 100 miles east of Cape Hatteras, but was unsuccessful because of the excessive roll of the ship; this caused the pendulum case to strike against the supporting frame.

As a result of this experience the gimbal frame and its support are being redesigned so that rolls up to 15° on a side will be permitted before the pendulum case strikes against the framework. A damping arrangement, consisting of a vane attached to the pendulum case and projecting into a basin or pail of water covered with a layer of thick oil, aids in preventing violent oscillations of the pendulum case. Accelerations of small magnitude are adequately cared for on the record; but violent or rapid changes, especially of high frequency, such as are produced by vibrations from the ship's engines, or of a frequency approximating in period that of the pendulums or of the pendulum case, are inimical to satisfactory records. Vertical accelerations up to one or two feet per second per second are permissible. Their effect is to cause a fluctuation in the phase-lag curve of the chronometer signal breaks, and this can be taken into account in the computations and measurements from the records.

SUMMARY

The gravity measurements made during the cruise of the U. S. submarine S-21 prove that over those parts of the Gulf of Mexico and the Antillean regions, where

movements of the earth's crust have taken place in geologically recent times and given rise to unusual depressions of the ocean floor, such as the Nares Deep, the Virgin Island Deep, and the Bartlett Deep, negative gravity anomalies of appreciable magnitude exist and are the expression of a lack of isostatic balance. The remarkable fact that these negative anomalies continue far to the westward of the Nares Deep, north of east Cuba, proves that they are not caused by under compensation of the Deep itself but that they must be connected in some other way with the tectonic disturbance. The gravity anomalies over the Nares Deep are larger than those over the Bartlett Deep and may indicate that the Nares Deep is a younger feature, even now in process of development, in which the departure from isostatic equilibrium is greater than in the Bartlett Deep.

Measurements over the deeper portions of the Gulf of Mexico and the Caribbean Sea yielded large positive anomalies, in accord with the values obtained on an earlier trip across the Atlantic Ocean. An entirely satisfactory explanation for these positive anomalies over vast areas of the ocean is still outstanding.

Measurements over the Mississippi Delta indicate that it is in isostatic equilibrium and is not an extra load supported by a mechanically strong competent crust. The number of observations available is not sufficient to determine how rapidly the isostatic balance is restored in the case of a disturbance. For a small load deposited slowly, such as that of the Mississippi Delta, it appears that deposition and isostatic compensation progress hand in hand, or that at least the time-lag is small.

In the case of a large structural feature, such as the Nares or Bartlett Deep, the large negative anomalies which are obtained indicate the existence of stresses of great magnitude in the earth's crust which are disturbing its equilibrium. Possibly these stresses were recently active and are now subsiding; in this case there has been an appreciable lag in the reestablishment of isostatic balance, which postulates a crust of some residual strength. For a more exact study of the orogenic stresses involved in the deformation of the crust, additional detailed measurements should be made over these deeps; they are critical regions for studies of this kind and are better adapted for the purpose, than are mountain ranges and basins, because sea stations are in general more distant than land stations from disturbing local inhomogeneities in the crust, which tend to confuse the general aspect of the gravity field.

Gravity measurements made across the continental shelf commonly show an increase in the gravity anomaly as the land recedes until deep water is reached; at this point the anomaly may surpass the value obtained at the top of the shelf in shallow water. There are exceptions to this sequence, however, and a detailed study of the geological relationships together with a greater number of cross sections will be required to ascertain the significance of the variations.

Further investigations, both theoretical and instrumental, into the meaning of the positive anomalies obtained over the broad deep ocean areas are much needed. With the elucidation of these features and the spread of gravity stations at sea over the surface of the earth a more exact determination of the figure of the earth can be made. This in turn will stimulate a critical discussion of the factors that enter into the formation of the earth's surface and underlie the theory of isostasy.

ADDITIONAL EVIDENCE ON THE MEANING OF GRAVITY ANOMALIES

Since the writing of this report new data have become available which throw light on the problem of the meaning of the gravity anomalies. First, the United States Coast and Geodetic Survey, thanks to the interest taken in the results of the U. S. submarine *S-21* expedition by Dr. W. BOWIE, has organized a gravity expedition in the West Indies in order to supplement these results by values on the islands. Four stations have been occupied on the island of Porto Rico, five on Haiti, and four on Cuba. Second, the results have been published of 33 gravity stations occupied in Mexico by the geographical and climatological service of that country. Third, an extensive and detailed maritime gravity survey has been made in the Netherland East Indies, where gravity has now been determined at 290 stations; the results show remarkable analogies with those of the *S-21* expedition and therefore help to an understanding of these results.

The Netherland East Indies are tectonically, like the West Indies, one of the most active areas of the earth's crust, as is shown by the great number of earthquakes and active volcanoes. The results of the gravity survey show great deviations from isostasy and these deviations show a remarkable independence of the topography. The principal feature is a narrow strip of strong negative anomalies which could be followed over a distance of more than 5,000 miles and which is bordered on both sides by fields of positive anomalies. This strip has a breadth of about 100 miles, and the difference of the anomalies in the negative strip and in the positive fields ranges from 130 to 430 millidynes.

The strip shows a general parallelism with the tectonic folding axes, but its location is more or less independent of these features. This strip coincides, for instance, with the deepest part of the Philippine Deep, but towards the south it gradually deviates and follows a submarine ridge. While the continuation of the deep runs more to the eastward, south of Java, in the Indian Ocean, the strip coincides with a ridge which separates the principal Java Deep from a less pronounced secondary deep.

Nearly all the earthquake centers which are known are located above or in the immediate vicinity of the strip. The distribution of volcanoes shows a curious correlation to the location of the strip; nearly all of them are situated in rows at some distance on the concave side of a curved part of this strip and on a line parallel to it.

Tentatively, the following explanation may be suggested. The strip is probably a concentration of light surface material in the crust, which is caused by the tectonic disturbance which is going on. If it is accepted that this disturbance is a crustfolding caused by strong lateral compression, the supposition seems reasonable that the tendency towards isostatic equilibrium brings it about that only a thin surface layer folds upwards and that the principal crustlayer folds downwards. Likewise it seems acceptable that this downward fold has a less local character than the surface fold of this thin upper layer, so that only one downward fold accompanies several parallel surface folds. This would explain why the strip of negative anomalies, which represents the effect of the downward fold, often coincides with a ridge, though it does not do so always.

The presence of a row of volcanoes on the concave side of a curved part of the strip might find its explanation in the tensile stresses which must develop in the earth's crust in a sense parallel to the fold if the crust moves from the inside, i. e., the concave side, toward the curved fold. In this way fissures may come into being, or at least a decrease of pressure in the crust and this may lead to the formation of volcanoes.

The results of the S-21 expedition in combination with those which have been found on the islands and in Mexico show remarkable analogies with these East Indian results and throw new light on the matter.

As in the East Indies, we find extensive fields of positive anomalies: The Gulf of Mexico and the Caribbean. The meaning of these fields is not yet clearly explained but if the folding supposition is accepted, it appears reasonable to see whether an explanation can be found in elastic compression and therefore increased density of the crust and in the vertical resultant of the compressive forces caused by the earth's curvature. This must be studied before an opinion can be given.

The Caribbean is surrounded by a recent folding arc and along that part of the arc which has been investigated, a strip of strong negative anomalies has been found over the Nares Deep and north of the islands of Haiti and east Cuba. This is similar to what has been found in the East Indies and especially the continuation of the strip to the westward of the Nares Deep, where the topography does not show any special feature. This proves that the negative anomalies are not caused by the Deep itself by means of under compensation, but that we have to look for the cause in the nature of the tectonic process. It seems plausible to suppose that here also it is caused by a concentration of light surface material and a downward folding of the main part of the crust.

The negative anomaly of 131 millidynes in the Mona Passage to the west of Porto Rico seems to indicate that this downward fold is not limited to a strip on the northern side of the islands but that it is also present below the northern part of Porto Rico and the same conclusion may be derived from the results found on this island; three of the four stations show negative anomalies, although much less pronounced. That they are smaller, may perhaps be explained by the possibility that, if the folding hypothesis is right, the surface folds are not locally compensated, so that the islands are excess weights, of which the attraction locally diminishes the negative anomaly caused by the fold.

The negative anomaly obtained in the Virgin Island Deep points probably in the same direction, viz, that the strip of concentrated light matter is also present below the island row—i. e., between the islands of St. Thomas and St. Croix—and that the attraction of the island ridges brings the gravity near the islands back to the normal value.

A remarkable point is the location of volcanoes, which corresponds to the situation in the East Indies; here also the curved part of the tectonic disturbance line, i. e. the smaller Antillean Islands, is strongly volcanic. It is of the highest importance to investigate this eastern part of the Archipelago in order to see whether the strip of negative anomalies continues from the Nares Deep along the island row towards the east and the south and whether, as in the East Indies, it is situated on the outside of the row of volcanoes.

The present material does not allow a clear insight in the continuation of the strip toward the westward. For the first part, west of the Nares Deep this is not doubtful: The values of -114 and -109 millidynes found on the Stations 28 and 29 and the value found by the United States Coast and Geodetic Survey on the north coast of Haiti in Puerto Plata, -87 millidynes, clearly indicate that the strip continues here; the values of $+5$ and $+14$ millidynes found in Bonao and Santo Domingo show that it is not present below the southern part of the island.

More to the west, however, things become more doubtful. The anomalies at the Stations 19 (-45 millidynes), 20 (-35 millidynes) and 46 (-33 millidynes) suggest the possibility of the continuation toward the northwest; possibly greater negative values will be found if measurements are made between the Stations 20 and 46.

On Cuba the United States Coast and Geodetic Survey has found -48 millidynes at Chaparra, which corresponds well with Station 19 (-45 millidynes) in the immediate vicinity, and $+65$ and $+73$ millidynes at Alto Cedro and Cayo Mambi somewhat more to the south. This makes it probable that, as on the other islands, the strip is not present below the southern part of east Cuba. The fact that, with one exception, no negative values have been found over the eastern part of the Bartlett Deep, seems to point in the same direction. The presence of the Deep, however, and the strong seismicity of this part of the crust apparently contradict this view and no decisive opinion is yet possible. A continuation of the gravity research to the east and to the west of the investigated part of the Bartlett Deep will probably clear up the problem.

It will also be important to pursue this continuation over the whole region up to the coast of Honduras, because this would give the connecting link with the remarkable results found in Mexico by the geographical and climatological service. These results have not yet been reduced by the isostatic method and so no well founded opinion can be given. A provisional isostatic reduction, however, makes it probable that here also a strip of strong negative anomalies is found, running east-west and somewhat to the south of the line of volcanoes Orizaba—Popocatepetl and bending toward the northwest along the west coast.

This would again present the phenomenon, which was found in the East Indies: A row of volcanoes on the concave side of a curved part of a strip of negative anomalies.

According to these results it appears possible that the positive anomalies found in 1926 over the foot of the continental slope of the west coast would simply be the positive field bordering the negative strip toward the southwest side and that it would not have any general meaning as a continental coastal effect.

From what has been said, it may be concluded that the West Indies still present a great many problems of the highest importance of which the solution would decisively contribute to the clearing up of the great geophysical problems concerning tectonic activity, mountain formation, ocean deeps, volcanoes, seismic activity, etc. The expedition of the U. S. submarine *S-21* has given us an important hold on these problems: new facts and probabilities have been discovered and clear indications have been obtained regarding promising possibilities for further research.

APPENDIX

REDUCTION OF OBSERVATIONS MADE AT SEA WITH THE VENING MEINESZ APPARATUS
FOR THE DETERMINATION OF THE VALUE OF GRAVITY

In order that the computations for the determination of the value of gravity at sea may be performed with facility the formulæ have been assembled and an example has been computed. The observation made at the sea station listed as No. 28 (November 6, 1928) off the northern coast of the island of Haiti, has been used for this purpose, since it is typical of the series made during the voyage of the U. S. submarine *S-21*.

The data taken from the observing book are given, followed immediately by the measures from the photographic record and the reductions. The tables of corrections and constants of the instruments used are also listed.

Measures from the photographic record.—To determine the value of the corrections to the observed period of the pendulum for amplitude, for deviation from isochronism, and for angular movement of swinging plane, where adopted $T=0^s.502$.

$$\text{Corr. for amplitude} = +2.12 A \cdot 10^{-7} \text{ sec.}$$

$$\text{Corr. for deviation from isochronism} = -0.88 \frac{2b}{2a} U \cdot 10^{-7} \text{ sec.}$$

where

$$A = (2a \mp 1.32 \times 2b)^2 + 0.774 [(2a_2)^2 - \frac{1}{4} (2b)^2]$$

and

$2a$ = total breadth of records of fictitious pendulums (upper and middle pendulum pairs)

$2a_2$ = total breadth of record (lower pendulum)

$b = x$ component of chronometer mark in the lower record when the chronometer mark of the fictitious pendulum record has its greatest elongation. If the fictitious pendulums are in opposite phase the values of b are equal with opposite signs. Note that the phase of the upper record must be reversed to make comparable with others.

The above measures are made in cm. The temperature constants are,

$$U_{12} = -12 + 1.03 t^\circ \text{ (middle pendulum pair).}$$

$$U_{32} = 28 - 0.16 t^\circ \text{ (upper pendulum pair).}$$

Correction for angular movement of swinging plane measured in mm. is

$$(\beta_c)^2 + 0.095 (2\alpha)^2 \cdot 10^{-7} \text{ sec.}$$

where

β_c = deviation of tilt axis from normal position.

2α = total fluctuation.

These quantities are measured from the curve of the damped pendulum.

To determine the period of the pendulums from the photographic record.—The general equation for the period T is used,

$$T = 0.5 + \frac{0.25}{C - 0.5}$$

Two readings are made when the pendulum vector passes through the Y axis at the beginning of the record (one up, one down) and two near the end if possible. To obtain C take the difference of each group and divide by the number of coincidence intervals.

In case a mean time chronometer is used as well as a sidereal time chronometer reduce the mean time to the sidereal time by the following formula:

$$\text{Mean Time to Sidereal Time} = \text{M. T.} + 0.00273791 \times \text{M. T.}$$

To correct for difference of the daily rates of the chronometers

$$\begin{aligned}\delta T &= \frac{T}{86400} \Delta \cdot 10^{-7} \text{ sec.} \\ &= 58.1 \times \Delta \cdot 10^{-7} \text{ sec.}\end{aligned}$$

where Δ = difference of daily rates of the two chronometers.

The correction to the observed period of the pendulum for rate of the sidereal chronometer

$$\begin{aligned}\delta T &= 58.1 \times r \cdot 10^{-7} \text{ sec.} \\ r &= \text{daily rate of chronometer.}\end{aligned}$$

where

Complete reduction of the observed period for each pair of pendulums.—

- $t^\circ \times$ temperature constant.
- $D \times$ density constant.
- Amplitude correction.
- Deviation from isochronism correction.
- Swing correction.
- Chronometer rate correction.

The sum of the above corrections is to be *subtracted* from the periods of the pendulum pairs as obtained from the photographic record. The mean period of the several pendulum pairs is taken as the observed period at the Observation Station = T_1 .

The results for gravity.—

$$V_T = T_1 - T_b$$

where

T_1 = period at observation station.

and

T_b = period at base station (in this case, Washington).

$$V_g = -CV_T$$

where

$$C = \frac{2g_b}{T_1 + \frac{1}{2}V_T}$$

and

g_b = 980.112 (in this case, Washington).

To correct for the (E - W) velocity, V , caused by earth's rotation and the (E - W) component of the velocity, v , of the vessel.

$$v = \text{recorded velocity} \times \sin \text{course} \pm \text{current (in miles)}$$

Gravity formulæ assembled.—

$$dg = 7.5 \cos \varphi \times v \cdot 10^{-3} \text{ cm.}$$

$$\delta g = -0.094d \cdot 10^{-3} \text{ cm.}$$

$$\delta^1 g = 0.026d \cdot 10^{-3} \text{ cm.}$$

where

d = depth of observing pier below surface (in feet).

Then

$$\text{Observed gravity} = g_b + V_g + dg + \delta g + \delta^1 g$$

Tables of Corrections and Constants used in the Reductions

Table I		Table II		Table III
Pendulum Thermometer C.	Corr.	Hair hygrometer (Station)	Corr.	Barometer
°	°			Constant Corr. for voyage = +1.1 mm.
0	-0.16	1-20	0	
10	-0.14	20-27	-4	
20	-0.16	27-44	-9	
30	-0.18	44-49	-10	
40	-0.22			

TABLE IV.—*Correction to the Barometer Readings for Humidity where e = Vapor Pressure of Saturated Air*

Corr. t° C.	$\frac{3}{8}e$	Corr. t° C.	$\frac{3}{8}e$	Corr. t° C.	$\frac{3}{8}e$	Corr. t° C.	$\frac{3}{8}e$
°	mm.	°	mm.	°	mm.	°	mm.
0	1.7	+10	3.4	+20	6.5	+30	11.8
+1	1.8	11	3.7	21	6.9	31	12.5
2	2.0	12	3.9	22	7.4	32	13.3
3	2.1	13	4.2	23	7.8	33	14.0
4	2.3	14	4.5	24	8.3	34	14.8
5	2.4	15	4.8	25	8.8	35	15.7
6	2.6	16	5.1	26	9.4	36	16.6
7	2.8	17	5.4	27	9.9	37	17.5
8	3.0	18	5.8	28	10.5	38	18.5
9	3.2	19	6.2	29	11.2	39	19.5
+10	3.4	+20	6.5	+30	11.8	+40	20.6

TABLE V.—*Correction for Density of Atmosphere*

$$D = B^{mm} \cdot d = B^{mm} \cdot \frac{1}{760} \times \frac{1}{1 + 0.00367t^{\circ}}$$

Corr. t° C.	d	Corr. t° C.	d	Corr. t° C.	d	Corr. t° C.	d
°		°		°		°	
0	0.00132	+10	0.00127	+20	0.00123	+30	0.00119
+1	131	11	126	21	122	31	118
2	131	12	126	22	122	32	118
3	130	13	126	23	121	33	117
4	130	14	125	24	121	34	117
5	129	15	125	25	121	35	117
6	129	16	124	26	120	36	116
7	128	17	124	27	120	37	116
8	128	18	123	28	119	38	115
9	127	19	123	29	119	39	115
+10	0.00127	+20	0.00123	+30	0.00119	+40	0.00115

Constants

Temperature constant: First pair pendulums (upper) = $47.07 \cdot 10^{-7}$ sec.; second pair pendulums (middle) = $45.88 \cdot 10^{-7}$ sec.

Density constant: First pair pendulums (upper) = $673.2 \cdot 10^{-7}$ sec.; second pair pendulums (middle) = $665.9 \cdot 10^{-7}$ sec.

Axis of tilt curve normally 9.5 mm. below axis of upper curve; half period T_f of the gimbal suspension = 1.44 .

Mean value of $U_{32} = (28 - 0.16t^\circ)10^{-7}$ sec.; mean value of $U_{12} = (-12 + 1.03t^\circ)10^{-7}$ sec.

Depth of ship given in feet, to be diminished by 15 feet, for position of gravity apparatus.

Speed of ship given in knots. The usual speed of 4 knots corrected to 3.85 knots.

g_b at base station (Washington) = 980.112.

T_b at base station (Washington) = 0.5015438

THE EXAMPLE

Reduction of an Observation for the Determination of the Value of Gravity at Sea

Data from Observing Book

[November 6, 1928. Sea Station No. 28. U. S. S-21]

75th Meridian Time	Pendulum Thermometer C.	Hair Hygrometer	Barometer	Depth of submergence	Room Thermometer C.
<i>h. m.</i>	<i>°</i>		<i>mm.</i>	<i>ft.</i>	<i>°</i>
9 12 a. m.	31. 71	77	773. 1	60	32. 0
9 17 a. m.					
9 27 a. m.	31. 68	77	774. 1		32. 5
9 37 a. m.					
9 42 a. m.	31. 65	77	774. 8		33. 0

Current = 1.2 knots West; ship's speed = 4 knots; ship's course = 98° ; sounding = 2,205 fathoms; latitude = $20^\circ 27' 0''$ N.; longitude = $72^\circ 26' 40''$ W.

Daily Rate of Chronometers (Arlington Time Signals from Naval Observatory): No. 2081, Mean Time = $+3.49$; No. 212, Sid. Time = -2.34 .

Measures from Photographic Record

[Amplitude, isochronism, swinging plane]

Lower		Middle		Upper		$(2a_2)^2$	$(2b)^2$	2α	$(2\alpha)^2$	β_c	$(\beta_c)^2$
$2a_2$	$2b^*$	$2a_{12}$	$(2a_{12} \pm 1.32 \times 2b)^2$	$2a_{32}$	$(2a_{32} - 1.32 \times 2b)^2$						
<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>cm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>
0.6	0.0 $\frac{1}{2}wt$	4.4	19.4 $\frac{1}{2}wt$	3.7	13.7 $\frac{1}{2}wt$	0.4 $\frac{1}{2}wt$	0.0 $\frac{1}{2}wt$	2.5	6.2 $\frac{1}{2}wt$	1.0	1.0 $\frac{1}{2}wt$
0.8	-0.5	4.4	13.7	3.7	19.4	0.6	0.2	3.0	9.0	0.0	0.0
1.1	-0.7	4.3	11.6	3.6	20.2	1.2	0.5	2.0	4.0	0.0	0.0
1.1	+0.3	4.2	21.2	3.6	10.2	1.2	0.1	2.5	6.2	0.2	0.0
0.7	+0.4	4.2	22.1	3.5	9.0	0.5	0.2	1.0	1.0	0.5	0.2
1.0	+0.8	4.1	27.0	3.5	5.8	1.0	0.6	3.0	9.0	0.5	0.2
1.0	+0.7	4.1	25.0	3.4	6.2	1.0	0.5	2.5	6.2	1.0	1.0
0.8	+0.4	4.0	20.2	3.3	7.8	0.6	0.2	2.0	4.0	0.0	0.0
0.8	+0.2	4.0	18.5	3.3	9.0	0.6	0.0	1.5	2.2	0.0	0.0
1.0	0.0	3.9	15.2	3.3	10.9	1.0	0.0	1.0	1.0	0.5	0.2
1.0	-0.7	3.9	9.0	3.2	16.8	1.0	0.5	1.5	2.2	1.0	1.0
0.7	-0.5	3.8	9.6	3.2	15.2	0.5	0.2	2.0	4.0	0.5	0.2
0.6	0.0 $\frac{1}{2}wt$	3.7	13.7 $\frac{1}{2}wt$	3.1	9.6 $\frac{1}{2}wt$	0.4 $\frac{1}{2}wt$	0.0 $\frac{1}{2}wt$	2.0	4.0 $\frac{1}{2}wt$	0.5	0.2 $\frac{1}{2}wt$
Means	+0.033	4.1	17.47	3.4	11.84	0.80	0.25	-----	4.5	-----	0.28

* Note signs.

Period of the Pendulums from Photographic Record

	Sid. Time Chronometer No. 212		Mean Time Chronometer No. 2081	
	Upper ₃₂	Middle ₁₂	Upper ₃₂	Middle ₁₂
Readings (begin)-----	<i>m.</i> 0 <i>s.</i> 25.8	<i>m.</i> 0 <i>s.</i> 30.3	<i>m.</i> 0 <i>s.</i> 15	<i>m.</i> 0 <i>s.</i> 25
Readings (end)-----	23 18.2	23 22.0	26 41	26 51
Diff-----	22 52.4	22 51.7	26 26	26 26
Readings (begin)-----	2 20.2	2 24.4	-----	-----
Readings (end)-----	25 13.0	25 16.3	-----	-----
Diff-----	22 52.8	22 51.9	-----	-----
Mean-----	22 52.6	22 51.8	-----	-----
No. Passages-----	1,372 ^s .6	1,371 ^s .8	1,586 ^s .0	1,586 ^s .0
<i>C</i> -----	12	-----	5	-----
$T = 0.5 + \frac{0.25}{C-0.5}$	114 ^s .383	114 ^s .317	317 ^s .20	317 ^s .20
(<i>C</i> -0.5)-----	113 ^s .883	113 ^s .817	316 ^s .70	316 ^s .70
Log 0.25-----	9.397940	9.397940	9.397940	-----
Log (<i>C</i> -0.5)-----	2.056459	2.056207	2.500648	-----
T -----	7.341481	7.341733	6.897292	-----
M. T. red. to S. T. = $+ .00273791 \times M. T.$	0 ^s .0021952	0 ^s .0021965	0 ^s .0007894	-----
Corr. for Diff. Daily Rate-----	-----	-----	0.5021605	-----
Sid. T-----	0.5021952	0.5021965	339	0.5021944

General Reductions

Thermometer (Centigrade):		Barometer:	
Reading	31. 68	Reading	774. 0
Corr. (Table I)	-. 19	Corr. (Table III)	+1. 1
t°	31. 49	Corr. for Hyg. (Table IV)	-8. 8
Hair Hygrometer:		B^{mm}	766. 3
Reading	77	Corr. for Density:	
Corr. (Table II)	-9	d (Table V)	.00118
Hyg	68	$B^{mm} \times d = D$	0. 904
-1% Hyg. $\times \frac{3}{8}e$ (Table IV) ---			
	$mm.$		
	$68 \times 12. 9 = -8. 8$		

	Pendulums	
	Upper pair	Middle pair
<i>Corr. for Amplitude</i>		
$\delta T = +2.12 A \cdot 10^{-7}$ sec.		
$A = (2a \mp 1.32 \times 2b)^2 + .774[(2a_2)^2 - \frac{1}{4}(2b)^2]$		
From Photographic Record:		
$2a$	3. 4	4. 1
$(2b)^2$	0. 25	
(1) $(2a \mp 1.32 \times 2b)^2$	11. 84	17. 47
$(2a_2)^2$	0. 80	}
$-\frac{1}{4}(2b)^2$	-0. 06	
$[(2a_2)^2 - \frac{1}{4}(2b)^2]$	0. 74	
(2) $.774 [(2a_2)^2 - \frac{1}{4}(2b)^2]$	0. 57	0. 57
$A = (1) + (2)$	12. 41	18. 04
Corr. = +2. 12A	26. 3	38. 2
<i>Corr. for Deviation from Isochronism</i>		
$\delta T = -0.88 \frac{2b}{2a} U \cdot 10^{-7}$ sec.		
Upper pair $U_{32} = 28 - 0.16t^{\circ}$		
Middle pair $U_{12} = -12 + 1.03t^{\circ}$		
$U_{32} \dots U_{12}$	23. 0	20. 4
From Photographic Record:		
Log $2b$	8. 5185 n^*	8. 5185
Log $2a$	0. 5315	0. 6128
Log -0.88	7. 9870 n	7. 9057
	9. 9445 n	9. 9445 n
Log U	7. 9315	7. 8502 n
	1. 3617	1. 3096
Corr.	9. 2932	9. 1598 n
	+0. 2	-0. 1
<i>Corr. for Angular Movement or Swing.</i>		
$\delta T = (\beta_0)^2 + 0.095 (2a)^2 \cdot 10^{-7}$ sec.		
From Photographic Record:		
$(\beta_0)^2$	0. 28	}
$.095 (2a)^2 = .095 (4.5)$	0. 43	
Corr.	0. 7	

* Note change of sign.

Corr. for Rate of Sid. Chron. $\delta T = 58.1 \times r \cdot 10^{-7} \text{ sec.}$ $r = \text{Daily Rate} = -2.34$ Corr. $-2.34 \times 58.1 = -136.0$	Corr. for Diff. of Daily Rate of Chrons. $\text{Diff. Daily Rates } \delta T = 58.1 \times \Delta \cdot 10^{-7} \text{ sec.}$ Daily Rate, M. T. $+3.49$ Daily Rate, S. T. -2.34 $\Delta = +5.83$ Corr. $+5.83 \times 58.1 = 338.7$
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Complete Reduction of Observed Period

	Pendulums	
	Upper pair	Middle pair
Temperature constant.....	47.07 $\cdot 10^{-7} \text{ sec.}$	45.88 $\cdot 10^{-7} \text{ sec.}$
Density constant.....	673.2 $\cdot 10^{-7} \text{ sec.}$	665.9 $\cdot 10^{-7} \text{ sec.}$
$t^\circ \times \text{Temp. Const.}$	1482.2	1444.8
$D \times \text{Density Const.}$	608.6	602.0
Ampl. Corr.....	26.3	38.2
Isoch. Corr.....	0.2	-0.1
Swing Corr.....	0.7	0.7
Rate Corr.....	136.0	136.0
—Sum of Corr.....	-2254.0	-2222.0
T (Sid. Time Chron.).....	0.5021952	0.5021965
T (Mean Time Chron.).....	0.5021944	0.5021944
Corr. T (Sid. T.).....	0.5019698	0.5019743
Corr. T (Mean T.).....	0.5019690	0.5019722
Mean (Sid. T.).....	0.5019720	(45)
Mean (M. T.).....	0.5019706	(32)
Mean T_1 (Obs. Station).....	0.5019713	
T_b (Washington).....	0.5015438	
$V_T = (T_1 - T_b)$	0.0004275	
$\frac{1}{2} V_T$	0.0002138	

Results for Gravity

Velocity of Vessel (E-W) = $v \sin$ course:	
From Obs. Book:	
Course.....	98°
Speed = v	4 knots; corrected v (=3.85).
Log 3.85.....	0. 5855
Log sin course.....	9. 9958
	<hr/> 0. 5813
Velocity of Vessel.....	3. 81
Current.....	-1. 2 West.
True v	<hr/> 2. 61

$V_o = -CV_T$		$\delta g = -0.0939d \cdot 10^{-3} \text{ cm.}$	
$g_b = 980.112$		d (feet).....	60
$C = \frac{2g_b}{T_1 + \frac{1}{2}V_T}$		Corr. for pier (feet).....	-15
T_1	0. 5019713	Corr. d = Σ	45
$\frac{1}{2}V_T$	. 0002138	δg	-4. 2
$\left(T_1 + \frac{1}{2}V_T\right)$	0. 5021851	$\delta^1 g = 0.026d \cdot 10^{-3} \text{ cm.}$	1. 2
Log $2g_b$	3. 292306	Base Station (Washington):	
Log $\left(T_1 + \frac{1}{2}V_T\right)$	9. 700864	g_b	980. 112
Log C	3. 591442	V_o	-1. 669
Log V_T	6. 630936	g	978. 443
Log CV_T	0. 222378	dg	18
V_o	-1. 669	δg	-4
$dg = 7.5 \cos \varphi \times v \cdot 10^{-3} \text{ cm.}$		$\delta^1 g$	1
$\varphi = \text{latitude}$	+20° 27' 0''	Gravity at Observation Station...	978. 458
Log 7.5.....	0. 8751		
Log cos φ	9. 9717		
Log true v	0. 4166		
Σ	1. 2634		
dg	18. 3		

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