LICK OBSERVATORY BULLETIN

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SOME FORMULAS AND TABLES RELATING TO ORBIT COMPUTATION
AND NUMERIC INTEGRATION

By

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1. FORMULAS RELATING CIVIL AND SIDEREAL TIME,
WITH APPLICATION TO TOPOCENTRIC COORDINATES

The determination of the local sidereal time from the civil time is very expeditious with a computator and is independent of the annual ephemerides, if the day is adopted as the unit.

Let θ^M = mean equinoctial time, that is, the hour angle of the mean equinox of date

θ^T = so-called sidereal time, referred to the "true" equinox—or correctly, mean equinoctial time plus long period terms in the nutation

θ^F = sidereal time (properly so-called), referred to a fixed equinox

L = east longitude from Greenwich

t = GCT for which θ is wanted, expressed in JCT

t_0 = an epoch for which θ is given

and let subscripts "1900" and "date" designate the equinox to which θ is referred, and subscript "0" the Greenwich θ at epoch t_0 .

$$\text{Then } \theta^M_{date} = (\theta^M_{date})_0 + L + 1.0027\ 3790\ 9265(t - t_0) \quad (1A)$$

$$\theta^T_{date} = (\theta^M_{date})_0 + L + 1.0027\ 3790\ 9265(t - t_0) + m(A - \tau) \quad (1B)$$

$$\theta^F_{1900} = (\theta^F_{1900})_0 + L + 1.0027\ 3780\ 3094(t - t_0) \quad (1C)$$

where the numeric factors convert mean equinoctial and sidereal intervals to mean time intervals. The last term in (1B) is the nutation, where A includes only long period terms¹ and τ is the fraction of the year from the beginning of the Besselian fictitious year.

¹ Cf. *American Ephemeris*, p. 200; *British Nautical Almanac* 1931, p. 807-8.

t_0 (JCT)	$(\theta^M_{date})_0$	$(\theta^F_{1900})_0$	GCT	
239 0000	27730 607	27757 172	1831 Jul 1	
240 0000	.1521 534	.1537 481	1858 Nov 16	
241 0000	.5312 460	.5317 791	1886 Apr 3	(1D)
242 0000	.9103 387	.9098 100	1913 Aug 20	
243 0000	.2894 313	.2878 410	1941 Jan 5	

Many computators have only 8-place capacity, and hence the constants are given for intervals of 10000 days. The data are based on Newcomb's constants at 1900, and are accurate to .01 in 300 years, if terms in $(t - t_0)^2$ are disregarded.

For $t_0 = 242\ 0000$, (1A) becomes

$$\theta^M_{date} = 291034 + L + 1.0027\ 3791(t - 242\ 0000) \quad (1E)$$

which with a 9-place computator gives θ at once to .0001, sufficiently accurate for refraction, parallax, etc.

A very convenient method of dealing with parallax in the preliminary determination of an orbit is by the use of topocentric² coordinates,³ applied as corrections to the solar coordinates.

$$\text{Let } \theta_0 = 291034 + L = \text{local } \theta \text{ at } 242\ 0000. \quad (1F)$$

$$\theta = \theta^M_{date} = \theta_0 + 1.0027\ 3791(t - 242\ 0000) \quad (1G)$$

It scarcely needs mention that the decimal fraction in the result is the θ desired.

$$\text{Let } A = -.0000\ 4266\ 4\ \rho_E \cos \varphi', \quad \Delta X = A \cos \theta \quad (1H)$$

$$\Delta Z = -.0000\ 4266\ 4\ \rho_E \sin \varphi', \quad \Delta Y = A \sin \theta$$

where the numeric factor converts to astronomic units. θ_0 , A , ΔZ are constant for an observatory, and are tabulated for various observatories in Table I. An auxiliary table is appended for finding A and ΔZ for observatories

² B. J. 1916, [17].

³ Charlier, *Arkiv för Matematik, Astronomi och Fysik* 7:10, 12-13, 1911; Moulton, *A.J.*, 23, 108, 1914.

not in the main table. Table II gives four-place values of cosine and sine with argument in decimal of a day.

Example.—Find the topocentric coordinates for Washington, 1921 Nov 21.9764 GCT (*A.J.* **34**, 199). Quantities in () are not recorded on the computing sheet.

θ_0	$^{\circ} 6963$	Table I		
t	242 3015. (9764)	Use any JD table as if argu-		
t_0	(242 0000)	ments were civil time		
$t-t_0$	(3015. 9764)			
θ	(3024).9302			
$\cos\theta$	+ .905	Table II	ΔX	-300
$\sin\theta$	- .425		ΔY	+141
A	- .332		ΔZ	-267
		Table I		Table I

In 50 years the precession in ΔX , ΔY , ΔZ can amount to 6 units in the 7th place, but (1C) includes the larger part of the precession, so that it refers these quantities to 1900 with a maximum error of 2 units for the interval 1850–1950. Rigid formulas⁴ will rarely be required for this.

This article was written while the author was at the U. S. Naval Observatory, Washington, D. C., 1925 January.

2. CORRECTION OF SOLAR COORDINATES FOR ERROR IN SUN'S LONGITUDE

An error in Earth's computed place, that is, in the solar coordinates, used to represent the position of an object, produces a corresponding, fictitious error of observation. Let errors in Earth's latitude and radius vector be neglected and let

ΔL = error in Sun's (or Earth's) mean longitude (observed-computed)

ρ = geocentric distance of an object at opposition

The present accepted value of $\Delta L = +1''.47$ corresponds to 70 units in the 7th place of the solar coordinates and causes an apparent error of observation of $\frac{\Delta L}{\rho}$, more than $15''$ for objects like Comets 1927 *c* (Pons-Winnecke) and 1930 *d* (Schwassmann-Wachmann),⁵ and about $8''.5$ for *Eros* at the approaching 1931 opposition.⁶

ΔL may be easily offset by interpolating the solar coordinates with a time increased by Δt , the interval during which Earth moves through the angle ΔL . Since Earth's mean daily motion is $3548''$

$$\Delta t \text{ (in days)} = \frac{\Delta L''}{3548''} \quad (2A)$$

⁴ See p. 36; and *A.N.* **236**, 266, 1929.

⁵ *Lick Obs. Bull.* **15**, 182, 1930.

⁶ Professor Leuschner suggests that the departure of *Eros* from its prediction during the recent opposition is in part attributable to the neglect of ΔL in deriving the elements and ephemeris.

Jones⁷ has reduced the ΔL 's observed at various observatories to the basis of Greenwich before the use of the "impersonal" micrometer in 1915, which evidently caused a sudden increase in ΔL of $.03$ or $.04$.⁸ On the assumption that the results of the impersonal micrometer are more reliable, $+.45$ is added to Jones' adopted values. ΔL is assumed constant after 1923, which accords with the value used in eclipse predictions.⁹ The corresponding values of Δt in $^{\circ}00001$ are

Date	Δt	Date	Δt	Date	Δt	Date	Δt
1899.7		1904.5	$^{+18}$	1906.5	$^{+26}$	1909.2	
1900.2	$^{+10}$	1905.0	$^{+18}$	1906.8	$^{+26}$	1910.3	$^{+34}$
1900.7	11	1905.2	19	1907.0	27	1911.4	35
1901.3	12	1905.4	20	1907.3	28	1913.2	36
1902.1	13	1905.6	21	1907.6	29	1915.0	37
1902.8	14	1905.8	22	1907.9	30	1917.3	38 (2B)
1903.4	15	1906.0	23	1908.3	31	1919.0	39
1904.0	16	1906.3	24	1908.7	32	1920.8	40
1904.5	$^{+17}$	1906.5	$^{+25}$	1909.2	$^{+33}$	date	$^{+41}$

Alternatively, corrections may be applied directly to the solar coordinates.

Let L , M = Sun's mean longitude, and mean anomaly

λ , V = Sun's true longitude, and true anomaly

π = Sun's longitude of perihelion

$$L = \pi + M, \quad \Delta L = \Delta M \quad (2C)$$

$$\lambda = \pi + V = \pi + (M + 2e \sin M + \dots) \quad (2D)$$

$$\Delta \lambda = (1 + 2e \cos M) \Delta M = (1 + 2e \cos M) \Delta L \quad (2E)$$

Regarding λ as variable, differentiate

$$\begin{aligned} X &= R \cos \lambda \\ Y &= R \sin \lambda \cos \epsilon \\ Z &= R \sin \lambda \sin \epsilon \end{aligned} \quad (2F)$$

$$\begin{aligned} \Delta X &= -R \sin \lambda \Delta \lambda = [-Y \sec \epsilon (1 + 2e \cos M)] \Delta L \\ \Delta Y &= R \cos \lambda \cos \epsilon \Delta \lambda = [X \cos \epsilon (1 + 2e \cos M)] \Delta L \\ \Delta Z &= R \cos \lambda \sin \epsilon \Delta \lambda = [X \sin \epsilon (1 + 2e \cos M)] \Delta L \end{aligned} \quad (2G)$$

ΔX , ΔY , ΔZ corresponding to $\Delta L = +1''.47$ are given below. For values corresponding to a new ΔL , multiply by

$$\frac{\text{new } \Delta L''}{+1''.47} \quad (2H)$$

Conversely, it may be possible to determine ΔL from *Eros*, 719 *Albert*, etc., or a comet with stellar nucleus that approaches close to Earth like 1927 *c* (Pons-Winnecke).¹⁰ The geocentric distance of *Eros* at opposition ranges from 0.15 to 0.7, closer, with an occasional

⁷ *Mon. Not.* **87**, 13, 1926.

⁸ *A.J.* **41**, 177, 1932; *Mon. Not.* **85**, 1003, 1925.

⁹ *A.J.* **35**, 92, 1923; Robertson, Total Phase of the Solar Eclipse of April 28, 1930.

¹⁰ The recent Delporté and Reinmuth objects afford additional possibilities.

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Page 41, Tashkent θ_0 , for 0.9028 read 0.1028.

Page 38, col. 1, for entire last paragraph substitute the following:

Example.—To find the auxiliaries when neither date is 1900, e.g. for transferring from $t_1=1960.0$ to $t_2=1950.0$.

Accurately: from (4B), $\tau=+.06$, $T=-.01$; then from (4D)

$X_x +.9999\ 9703$	$Y_x -.0022\ 3501$	$Z_x -.0009\ 7167$
$X_y +.0022\ 3501$	$Y_y +.9999\ 9750$	$Z_y -.0000\ 0109$
$X_z +.0009\ 7167$	$Y_z -.0000\ 0109$	$Z_z +.9999\ 9953$

Approximately: (a) use the auxiliaries in Table IV for $1900+(t_1-t_2)$; (b) same as (a), except for X_y , X_z , Y_x , Z_x , use the differences of their values as tabulated for t_1 and t_2 , taken in the sense (t_1-t_2) :

$X_x +.9999\ 9703$	$Y_x -.0022\ 3481$	$Z_x -.0009\ 7158$
$X_y +.0022\ 3481$	$Y_y +.9999\ 9750$	$Z_y -.0000\ 0109$
$X_z +.0009\ 7158$	$Y_z -.0000\ 0109$	$Z_z +.9999\ 9953$

The difference, accurate—approximate, is the error made. For solar coordinates, the maximum errors in the 8th place for (a) and (b) are, respectively, ($Z=.4Y$):

$$\begin{array}{lll} \Delta X = +68Y - 21Z \leq 60, & +20Y + 9Z \leq 24 \\ \Delta Y = -68X \leq 68, & -20X \leq 20 \\ \Delta Z = +21X \leq 21, & -9X \leq 9 \end{array}$$

Thus even for relatively long intervals far from 1900, the approximate methods may suffice.

exception, than any other asteroid in perihelion. These close approaches, with an accurate theory of the orbit, can be utilized to investigate how much of ΔL is attributable to error in theory of Earth's motion and how much to error in time caused by non-uniformity of Earth's rotation.

CORRECTIONS TO X, Y, Z CORRESPONDING TO ERROR IN SUN'S MEAN LONGITUDE, $\Delta L = +1''.47$ (UNIT = .0000 001)

Date	ΔX	ΔY	ΔZ	Date	ΔX	ΔY	ΔZ
Jan 0	+71	+10	+4	Jul 4	-69	-13	-5
5	70	16	6	9	67	18	8
10	68	22	9	14	66	23	10
15	66	27	12	19	63	28	12
20	63	32	14	24	60	33	14
25	60	37	16	29	57	37	16
30	56	42	18	Aug 3	-54	-41	-18
Feb 4	+52	+46	+20	8	50	45	20
9	47	50	22	13	46	49	21
14	42	54	23	18	41	52	23
19	36	57	25	23	36	56	24
24	31	60	26	28	31	58	25
Mar 1	+25	+62	+27	Sep 2	-26	-60	-26
6	19	64	28	7	20	62	27
11	13	65	28	12	14	64	28
16	+7	66	28	17	8	65	28
21	0	66	28	22	-2	65	28
26	-6	65	28	27	+4	65	28
31	12	65	28	Oct 2	+10	-65	-28
Apr 5	-18	+63	+27	7	16	64	28
10	24	62	27	12	22	62	27
15	29	59	26	17	28	60	26
20	35	57	25	22	34	58	25
25	40	54	23	27	39	55	24
30	44	51	22	Nov 1	+44	-52	-23
May 5	-49	+47	+20	6	49	48	21
10	53	43	19	11	53	44	19
15	57	39	17	16	58	40	17
20	60	34	15	21	61	35	15
25	63	29	13	26	64	30	13
30	65	24	10	Dec 1	+67	-25	-11
Jun 4	-67	+19	+8	6	69	19	8
9	68	14	6	11	71	14	6
14	70	9	4	16	72	8	3
19	70	+3	+1	21	72	-2	-1
24	70	-2	-1	26	72	+4	+2
29	70	7	3	31	+71	+10	+4
Jul 4	-69	-13	-5				

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3. WILLIS'S FORMULA FOR INTERPOLATING SOLAR COORDINATES

Willis's method of interpolating solar coordinates uses no differences and requires fewer operations, with positive numbers in general. The formula used is justified here and varied slightly from that originally given.¹¹

¹¹ Willis, *A.J.* **36**, 118, 1926.

Let the schedule of differences be

$$\begin{array}{c|ccc} \text{Arg.} & \text{Func.} & \text{Differences} & \\ t_m & X_m & b_m & d_m \dots\dots \\ & & a & c \dots\dots \\ t_n & X_n & b_n & d_n \dots\dots \end{array} \quad (3A)$$

Let t = time between t_m and t_n for which X is wanted. By Everett's interpolation formula¹²

$$n = \frac{t - t_m}{w}, \quad m = 1 - n \quad (3B)$$

$$X = mX_m + nX_n + B_m b_m + B_n b_n \text{ (similarly } Y, Z) \quad (3C)$$

The effect of third differences is included. Fourth differences for $w=1$ are negligible, and hence if only Sun were acting, a second difference would be proportional to the second derivative. Actually

$$X'' = -\frac{(wk)^2}{R^3} X - \frac{(wk)^2 m}{r^3} x \quad (3D)$$

where the second term is Moon's action. If this term is neglected, (3D) substituted for the second differences in (3C) gives

$$X = \left[m - B_m \frac{(wk)^2}{R_m^3} \right] X_m + \left[n - B_n \frac{(wk)^2}{R_n^3} \right] X_n \quad (3E)$$

The error for $w=1$ thus introduced is

$$|\epsilon_1| \leq (B_m + B_n) \frac{(wk)^2 m}{r^2} \leq \frac{1}{8} \frac{.000296}{27100000} \left(\frac{93000000}{221000} \right)^2 = .0000 0024 \quad (3F)$$

If $\Delta R = R_n - R_m$, the error for $w=1$ of setting $R_m = R_n = R$ in (3E) is

$$|\epsilon_2| \leq B(wk)^2 \left[\frac{1}{(R + \Delta R)^2} - \frac{1}{R^2} \right] X = B(wk)^2 (2R^{-3} \Delta R + \dots) X \leq (.06)(.0003)(2)(1)(.0003)(1) = .0000 0001 \quad (3G)$$

Further, substitute $\frac{B_m + B_n}{2}$ for each of B_m and B_n in (3E)

$$X = \left[m - \frac{B_m + B_n}{2} \frac{(wk)^2}{R^3} \right] X_m + \left[n - \frac{B_m + B_n}{2} \frac{(wk)^2}{R^3} \right] X_n \quad (3H)$$

The error for $w=1$ thus arising is

$$|\epsilon_3| = \frac{(wk)^2}{R^3} \left[\left(B_m - \frac{B_m + B_n}{2} \right) X_m + \left(B_n - \frac{B_m + B_n}{2} \right) X_n \right] = \frac{(wk)^2}{R^3} \frac{B_n - B_m}{2} (X_n - X_m) \leq (.00030)(.008)(.017) = .0000 0004 \quad (3I)$$

¹² Everett, *British Association Report for 1900*, p. 648; Thompson, *Tracts for Computers* 5, iii, 1921; Whittaker and Robinson, *Short Course on Interpolation*, p. 40, 1923; and *Calculus of Observations*, p. 40, 1924.

If these errors could occur simultaneously their total would be

$$|\epsilon| \leq |\epsilon_1| + |\epsilon_2| + |\epsilon_3| = .0000\ 0029 \quad (3J)$$

That is, (3H) gives the coordinates from day intervals with a maximum error of 3 units in the 7th place. If this is too large to neglect, the *American Ephemeris* or *Berliner Jahrbuch* which tabulate for half-day intervals is to be used. The maximum error then becomes $\frac{1}{4}\epsilon$, or 7 units in the 8th place.

Let

$$p = -\frac{B_m + B_n}{2} \frac{(wk)^2}{R^3} = -\frac{1}{2} \left[\frac{m(m^2-1)}{3!} + \frac{n(n^2-1)}{3!} \right] \frac{(wk)^2}{R^3} \\ = \frac{mn}{4} \frac{(wk)^2}{R^3} \quad (3K)$$

Substitution of this into (3H) gives the working formula

$$X = (m+p)X_m + (n+p)X_n \quad (\text{similarly } Y, Z) \quad (3L)$$

The advantages are that m, n, p are always positive, p being merely the end figures of $(m+p)$ and $(n+p)$; the products are in general formed positively, the sign being attached afterward; all three coordinates are formed by six multiplications, requiring four keyboard settings; practically all writing except the result is eliminated; and since the differences are not used, they need not be published.

$$p = (23042''.531 + 139''.726\tau + 0''.060\tau^2)T + (+30''.226 - 0''.270\tau)T^2 + (+17''.995)T^3 \\ m-p = (23042.531 + 139.726\tau + 0.060\tau^2)T + (+109.500 + 0.390\tau)T^2 + (+18.325)T^3 \\ n = (20046.854 - 85.331\tau - 0.370\tau^2)T + (-42.666 - 0.370\tau)T^2 + (-41.800)T^3 \quad (4A)$$

Table V gives p for $w = \frac{1}{2}$ with arguments m or n and time of year. Interpolation is not to be made between vertical columns, because the differences do not exceed .0000 0002. The table is based on data for a year intermediate between leap years.

Similarly, this idea is applicable to all rectangular coordinates in Newtonian motion and all other functions, such as cosine and sine, in Willis's very expeditious solution of Kepler's equation, etc., where second differences are proportional to the function because fourth differences are negligible.

The greater part of this article was written while the author was at the U. S. Naval Observatory, Washington, D. C., 1926 April.

Example.—Find solar coordinates for 1931 Feb 28.75350 GCT (same example as in *British Nautical*

$$X_x = 1 + (-296\ 8268 - 2\ 6250\tau)T^2 + (-1\ 3125 - 96\tau)T^3 + (+1\ 4660)T^4 \\ X_y = (-2234\ 2669 - 13\ 5482\tau - 58\tau^2)T + (-6\ 7741 - 58\tau)T^2 + (+22\ 1042 + 3252\tau)T^3 + (+1535)T^4 \\ X_z = (-971\ 8989 + 4\ 1370\tau + 179\tau^2)T + (+2\ 0685 + 179\tau)T^2 + (+9\ 6212 + 282\tau)T^3 + (+350)T^4 \\ Y_x = (+2234\ 2669 + 13\ 5482\tau + 58\tau^2)T + (+6\ 7741 + 58\tau)T^2 + (-22\ 1042 - 3252\tau)T^3 + (-1717)T^4 \\ Y_y = 1 + (-249\ 5974 - 3\ 0270\tau)T^2 + (-1\ 5135 - 105\tau)T^3 + (+1\ 2320)T^4 \\ Y_z = (-108\ 5741 - 1962\tau)T^2 + (-2849 + 38\tau)T^3 + (+5379)T^4 \\ Z_x = (+971\ 8989 - 4\ 1370\tau - 179\tau^2)T + (-2\ 0685 - 179\tau)T^2 + (-9\ 6212 - 282\tau)T^3 + (+68)T^4 \\ Z_y = (-108\ 5741 - 1962\tau)T^2 + (+886 + 53\tau)T^3 + (+5387)T^4 \\ Z_z = 1 + (-47\ 2294 + 4021\tau)T^2 + (+2010 + 9\tau)T^3 + (+2339)T^4 \quad (4D)$$

¹³ Republished from *A.N.* 236, 266, 1929, with examples. ¹⁴ Andoyer, *B.A.* 23, 75, 1911. ¹⁵ Cf. Oppolzer, *Bahnbestimmung* 1, p. 236.

Almanac 1931, 800). The data in () are from the *American Ephemeris* 1931, 19 or *Berliner Jahrbuch* 1931, 22, and are not to be entered on the computing sheet.

$$\begin{array}{rcl} m & .49300 & \\ n & .50700 & 476 \end{array} \quad (3B) \text{ and Table V}$$

Arg.	Date	X	Y	Z
(t_m)	Feb 28.5	+ .9245 920	-.3264 899	-.1416 064
(t_n)	Mar 1.0	+ .9277 845	-.3190 867	-.1383 954
t		+ .9262 194	-.3227 396	-.1399 798

(The beginning computer will note that if $(m+p)$ is set in the keyboard for the first product, only the first five figures are changed to set $(n+p)$ for the second product, and that the two products in X are accumulated without recording the first product; then because $(n+p)$ is already set in the keyboard, it is best to form the second product in Y first, etc.)

4. PRECESSION IN EQUATORIAL RECTANGULAR COORDINATES, WITH TABLE FOR TRANSFER TO 1900¹³

The following analytic expressions of Newcomb's precessional motions between two equinoxes t_1 and t_2 are derived from Andoyer's expressions for 1850.¹⁴

$$\text{where } \tau = \frac{t_1 - 1900}{1000}, \quad T = \frac{t_2 - t_1}{1000} \quad (4B)$$

The nine direction-cosines¹⁵ which transfer equatorial rectangular coordinates from t_1 to t_2 are

$$\begin{aligned} X_x &= +\cos n \cos(m-p) \csc p - \sin(m-p) \sin p \\ X_y &= -\cos n \cos(m-p) \sin p - \sin(m-p) \csc p \\ X_z &= -\sin n \cos(m-p) \\ Y_x &= +\cos n \sin(m-p) \csc p + \cos(m-p) \sin p \\ Y_y &= -\cos n \sin(m-p) \sin p + \cos(m-p) \csc p \\ Y_z &= -\sin n \sin(m-p) \\ Z_x &= +\sin n \csc p \\ Z_y &= -\sin n \sin p \\ Z_z &= +\cos n \end{aligned} \quad (4C)$$

General series expressions are obtained by expanding these and substituting (4A) therein:

where the unit of coefficients is .0000 0001. For about 50 years, $Y_z = -X_y$, $Z_x = -X_z$, $Z_y = Y_z$.

The general formulas for transferring from equinox t_1 to equinox t_2 are

$$\begin{aligned}x_2 &= X_{x_1} \cdot x_1 + X_{y_1} \cdot y_1 + X_{z_1} \cdot z_1 \\y_2 &= Y_{x_1} \cdot x_1 + Y_{y_1} \cdot y_1 + Y_{z_1} \cdot z_1 \\z_2 &= Z_{x_1} \cdot x_1 + Z_{y_1} \cdot y_1 + Z_{z_1} \cdot z_1\end{aligned}\quad (4E)$$

Any equatorial rectangular quantities, as Δx , Δy , Δz ; vector elements; etc.; velocities in the coordinates, as x' , y' , z' ; etc., may be similarly transferred. Though not so conveniently as by a usual method, polar coordinates (a, δ) may be rigorously treated by letting $x = \cos \delta \cos a$, $y = \cos \delta \sin a$, $z = \sin \delta$, transferring, and re-forming a, δ .

Table IV gives the auxiliaries that transfer to 1900. Let quantities without subscript "1900" refer to equinox t . Then

$$\begin{aligned}x_{1900} &= X_x \cdot x + X_y \cdot y + X_z \cdot z \\y_{1900} &= Y_x \cdot x + Y_y \cdot y + Y_z \cdot z \\z_{1900} &= Z_x \cdot x + Z_y \cdot y + Z_z \cdot z\end{aligned}\quad (4F)$$

and inversely

$$\begin{aligned}x &= +X_x \cdot x_{1900} - X_y \cdot y_{1900} - X_z \cdot z_{1900} \\y &= -Y_x \cdot x_{1900} + Y_y \cdot y_{1900} + Y_z \cdot z_{1900} \\z &= -Z_x \cdot x_{1900} + Z_y \cdot y_{1900} + Z_z \cdot z_{1900}\end{aligned}\quad (4G)$$

The formulas above and the accompanying table were developed while the author was at the U. S. Naval Observatory, Washington, D. C., 1923.

Example.—To refer the vector elements of *Pluto* (*Lick Obs. Bull.* 15, 40) from 1930.0 to 1900.0.

$$\left. \begin{array}{ll} aX_c - 27.66610 & bX_s + 25.24472 \\ aY_c - 28.32223 & bY_s - 24.34350 \\ aZ_c - 0.58200 & bZ_s - 15.39646 \end{array} \right\} 1930.0$$

The following auxiliaries are to be used directly from Table IV, and not recorded:

$$\left. \begin{array}{lll} X_x + .9999\ 733 & Y_x - .0067\ 034 & Z_x - .0029\ 155 \\ X_y + .0067\ 034 & Y_y + .9999\ 775 & Z_y - .0000\ 098 \\ X_z + .0029\ 155 & Y_z - .0000\ 098 & Z_z + .9999\ 958 \end{array} \right\} \begin{array}{l} 1930.0 \\ \text{to} \\ 1900.0 \end{array}$$

By formula (4F)

$$\left. \begin{array}{ll} aX_c - 27.85691 & bX_s + 25.03597 \\ aY_c - 28.13613 & bY_s - 24.51203 \\ aZ_c - 0.50106 & bZ_s - 15.46976 \end{array} \right\} 1900.0$$

When transferring for any interval of a few years, if neither date is 1900, Table IV may give directly the auxiliaries with the accuracy required, instead of computing them from (4D). For example, in the rather extreme case of transferring solar coordinates from 1960 to 1950, the auxiliaries from (4D), with $\tau = +.06$, $T = -.01$, are

$$\begin{array}{rrr} +.9999\ 9703 & +.0022\ 3501 & +.0009\ 7167 \\ -.0022\ 3501 & +.9999\ 9750 & -.0000\ 0109 \\ -.0009\ 7167 & -.0000\ 0109 & +.9999\ 9953 \end{array}$$

If the values for 1910 to 1900 from Table IV be used instead, the differences in the 8th place from those above will indicate the maximum error made ($Z = .4Y$):

$$\begin{array}{rrr} \Delta X = & 0X + 68Y - 21Z = & 6 \\ \Delta Y = & -68X + 0Y + 0Z = & 7 \\ \Delta Z = & +21X + 0Y + 0Z = & 2 \end{array}$$

Error in 7th place

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5. FORMULA FOR TIME OF PERIHELION IN A PARABOLA

The time of perihelion in a parabolic orbit may be expressed directly in terms of quantities already available in an orbit computation, so that Barker's table may be avoided. Using the well known formulas

$$r = \frac{p}{1 + \cos V} = \frac{2q}{2 \cos^2 \frac{V}{2}} = q \sec^2 \frac{V}{2} = q \left(1 + \tan^2 \frac{V}{2} \right) \quad (5A)$$

differentiate first and fourth members with respect to

$$\begin{aligned}\tau &= k\sqrt{M+m}t \\ r' &= 2q \sec^2 \frac{V}{2} \tan \frac{V}{2} \cdot \frac{V'}{2} = r \tan \frac{V}{2} \cdot V'\end{aligned}\quad (5B)$$

Solve for $\tan \frac{V}{2}$, and substitute for V' from $r^2 V' = \sqrt{p}$

$$\tan \frac{V}{2} = \frac{r'}{rV'} = \frac{r'}{r} \cdot \frac{r^2}{\sqrt{p}} = \frac{rr'}{\sqrt{2q}} \quad (5C)$$

From first and last members of (5A)

$$\tan^2 \frac{V}{2} = \frac{r}{q} - 1 = \frac{r-q}{q} \quad (5D)$$

Substitute (5C) and (5D) into the well known integral

$$\begin{aligned}t - T &= \frac{\sqrt{2} q^{3/2}}{k\sqrt{M+m}} \tan \frac{V}{2} \left(1 + \frac{1}{3} \tan^2 \frac{V}{2} \right) \\ &= \frac{\sqrt{2} q^{3/2}}{k\sqrt{M+m}} \frac{rr'}{\sqrt{2q}} \left(1 + \frac{r-q}{3q} \right) = \frac{q rr'}{k\sqrt{M+m}} \frac{2q+r}{3q}\end{aligned}\quad (5E)$$

whence, if $M=1$ and m is neglected

$$T = t - \frac{rr'(2q+r)}{3k} = t - 19.377480 rr'(2q+r) \quad (5F)$$

NOTE.—This formula has been adopted by Crawford, *Determination of Orbits of Comets and Asteroids*, p. 65, formula (234a).

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6. DERIVATION OF CLOSED EXPRESSIONS FOR df AND dg

In correcting an orbit by Leuschner's method,¹⁶ when the intervals are long compared to the heliocentric distance, expressions for df and dg , closed with respect to the time, are needed. These arise in differentiating

$$x = fx_0 + gx_0' \quad (6A)$$

$$\text{that is, } dx = f dx_0 + g dx_0' + x_0 df + x_0' dg \quad (6B)$$

(similarly y, z)

Neither Kühnert¹⁷ nor Leuschner¹⁸ give the general derivation of the closed expressions for df and dg .¹⁹

$$\begin{aligned} \text{Let } \Delta E &= E - E_0 \\ F &= a(1 - \cos \Delta E) \\ G &= a^{1/2} \sin \Delta E \end{aligned} \quad (6C)$$

Then Kühnert's finite expressions for f and g are^{17, 20}

$$\begin{aligned} f &= 1 - \frac{a}{r_0}(1 - \cos \Delta E) = 1 - \frac{F}{r_0} \\ g &= \tau - a^{3/2}(\Delta E - \sin \Delta E), \end{aligned} \quad (6D)$$

and also

$$g = a(rr')_0(1 - \cos \Delta E) + a^{1/2}r_0 \sin \Delta E = F(rr')_0 + Gr_0$$

Differentiate these:

$$\begin{aligned} df &= \frac{a}{r_0^2}(1 - \cos \Delta E)dr_0 - \frac{1}{r_0}(1 - \cos \Delta E)da - \frac{a}{r_0} \sin \Delta E d\Delta E \\ dg &= -\frac{3}{2}a^{1/2}(\Delta E - \sin \Delta E)da - a^{3/2}(1 - \cos \Delta E)d\Delta E \\ dg &= a^{1/2} \sin \Delta E dr_0 + \left[(rr')_0(1 - \cos \Delta E) + \frac{r_0}{2a^{1/2}} \sin \Delta E \right] da \\ &\quad + a(1 - \cos \Delta E)d(rr')_0 + [a^{1/2}r_0 \cos \Delta E + a(rr')_0 \sin \Delta E]d\Delta E \end{aligned} \quad (6E)$$

and substitute second and third of (6C) and second of (6D) therein:

$$\begin{aligned} df &= \frac{F}{r_0^2}dr_0 - \frac{F}{ar_0}da - \frac{G}{r_0}a^{1/2}d\Delta E \\ dg &= -\frac{3(\tau - g)}{2a}da - F a^{1/2}d\Delta E \\ dg &= G dr_0 + \left[\frac{F(rr')_0}{a} + \frac{Gr_0}{2a} \right] da + F d(rr')_0 \\ &\quad + \left[r_0 - \frac{Fr_0}{a} + G(rr')_0 \right] a^{1/2}d\Delta E \end{aligned} \quad (6F)$$

Equate the last two and collect coefficients

¹⁶ *Lick Publ.* 7, 255, 1913.

¹⁷ *A.N.* 95, 145, 1879.

¹⁸ *Lick Publ.* 7, 262, 1913.

¹⁹ Leuschner's derivation is given by Crawford, *Determination of Orbits of Comets and Asteroids*, pp. 86-88, 1930.

²⁰ *Lick Publ.* 7, 248, 1913.

$$\begin{aligned} \left[F + r_0 - \frac{Fr_0}{a} + G(rr')_0 \right] a^{1/2}d\Delta E = \\ -Gdr_0 - \frac{1}{2a} \left[3(\tau - g) + 2F(rr')_0 + Gr_0 \right] da - Fd(rr')_0 \end{aligned} \quad (6G)$$

$$\begin{aligned} \text{Now } r &= a(1 - e \cos E) = a - ae \cos(E_0 + \Delta E) \\ &= a - e \cos E_0 \cdot a \cos \Delta E + e \sin E_0 \cdot a \sin \Delta E \end{aligned}$$

$$\text{Substitute } e \cos E_0 = 1 - \frac{r_0}{a}, \quad e \sin E_0 = \frac{(rr')_0}{a^{1/2}},$$

$$\begin{aligned} a \cos \Delta E &= a - F, \quad a \sin \Delta E = Ga^{1/2} \\ r &= a - \left(1 - \frac{r_0}{a} \right) (a - F) + \frac{(rr')_0}{a^{1/2}} Ga^{1/2} = F + r_0 - \frac{Fr_0}{a} + G(rr')_0 \end{aligned} \quad (6H)$$

Introduce an auxiliary L , and subtract and add $2g$ as given in third of (6D). Let

$$\begin{aligned} L &= \frac{3\tau - g - Gr_0}{r} = \frac{3\tau - g - Gr_0 - 2g + 2[F(rr')_0 + Gr_0]}{r} \\ &= \frac{3(\tau - g) + 2F(rr')_0 + Gr_0}{r} \end{aligned} \quad (6I)$$

Substitute (6H) and (6I) into (6G), and divide by r ,

$$a^{1/2}d\Delta E = -\frac{G}{r}dr_0 - \frac{L}{2a}da - \frac{F}{r}d(rr')_0 \quad (6J)$$

Substitute this into first and second of (6F)

$$\begin{aligned} df &= \left(\frac{F}{r_0^2} + \frac{G^2}{rr_0} \right) dr_0 + \frac{1}{2ar_0}(-2F + GL)da + \frac{FG}{rr_0}d(rr')_0 \\ dg &= \frac{FG}{r}dr_0 + \frac{1}{2a}(3g - 3\tau + FL)da + \frac{F^2}{r}d(rr')_0 \end{aligned} \quad (6K)$$

$$\text{Let } 2M = \frac{a}{r_0}(GL - 2F), \quad 2N = a(3g - 3\tau + FL) \quad (6L)$$

The factor "2" is introduced merely that M and N may have the same significance given them by other writers.

$$\begin{aligned} df &= \left(\frac{F}{r_0^2} + \frac{G^2}{rr_0} \right) dr_0 + M \frac{da}{a^2} + \frac{FG}{rr_0}d(rr')_0 \\ dg &= \frac{FG}{r}dr_0 + N \frac{da}{a^2} + \frac{F^2}{r}d(rr')_0 \end{aligned} \quad (6M)$$

$$\text{Now } \frac{1}{a} = \frac{2}{r_0} - v^2, \quad \text{whence } \frac{da}{a^2} = \frac{2}{r_0^2}dr_0 + d(v^2) \quad (6N)$$

Substitution in (6M) gives the closed expressions:^{2, 3}

$$\begin{aligned} df &= \left(\frac{F}{r_0^2} + \frac{G^2}{rr_0} + \frac{2M}{r_0^2} \right) dr_0 + Md(v^2) + \frac{FG}{rr_0}d(rr')_0 \\ dg &= \left(\frac{FG}{r} + \frac{2N}{r_0^2} \right) dr_0 + Nd(v^2) + \frac{F^2}{r}d(rr')_0 \end{aligned} \quad (6O)$$

where F , G , and v^2 are γ^2 , $\sqrt{2} \gamma \gamma_0$, and G_0^2 , respectively, of Leuschner's notation.

These formulas were derived while the author was at Perkins Observatory, Delaware, Ohio, 1927 May.

7. ON INTERPOLATING DIRECTLY FROM AN INTEGRATION TABLE

In numeric integration it is usual, besides the integration table, to form another table, here called function table, with its differences, the purpose being to interpolate non-tabular values therefrom. However, the functions may be interpolated directly from the integration table, by using the proper interpolation formula, with no more labor than an ordinary interpolation. It will be noted that an error in the integration table is best detected there, because in the interpolation for the function table it will tend to be smoothed.

Let the schedule of differences of a table of double integration be

2nd sum	1st sum	Function to be summed	1st diff.	2nd diff.	
F_{-2}		b_{-2}		d_{-2}	
F_{-1}	$a_{-3/2}$	b_{-1}	$c_{-3/2}$	d_{-1}	
F_0	$a_{-1/2}$	b_0	$c_{-1/2}$	d_0	
F_1	$a_{1/2}$	b_1	$c_{1/2}$	d_1	

(7A)

let $a_0 = \frac{1}{2}(a_{-1/2} + a_{1/2})$, $c_0 = \frac{1}{2}(c_{-1/2} + c_{1/2})$, etc.; and let the corresponding quantities of the function table be denoted the same but in (). The tabular values of the function table are usually found from

$$(F_0) = F_0 + \frac{1}{12}b_0 - \frac{1}{240}d_0 + \frac{31}{60480}f_0 - \frac{289}{3628800}h_0 + \dots \quad (7B)$$

and differenced. The desired quantity F is then interpolated, perhaps with Stirling's formula:

$$F = (F_0) + n(a_0) + \frac{n^2}{2}(b_0) + \frac{n(n^2-1)}{6}(c_0) + \dots \quad (7C)$$

Now (a_0) , (b_0) , etc., may be expressed in terms of quantities in (7A). Evidently they will be of the same form as (7B), that is,

$$(a_0) = a_0 + \frac{1}{12}c_0 - \frac{1}{240}e_0 + \frac{31}{60480}g_0 - \dots$$

$$(b_0) = b_0 + \frac{1}{12}d_0 - \frac{1}{240}f_0 + \frac{31}{60480}h_0 - \dots \quad (7D)$$

Substitution of (7B) and (7D) into (7C) gives a formula for interpolating directly from a table of double integration:

$$F = F_0 + na_0 + \left(\frac{n^2}{2} + \frac{1}{12}\right)b_0 + \left(\frac{n^3}{6} - \frac{n}{12}\right)c_0 + \left(\frac{n^4}{24} - \frac{1}{240}\right)d_0$$

$$+ \left(\frac{n^5}{120} - \frac{n^3}{36} + \frac{11n}{720}\right)e_0 + \left(\frac{n^6}{720} - \frac{n^4}{288} + \frac{31}{60480}\right)f_0$$

$$+ \left(\frac{n^7}{5040} - \frac{n^5}{480} - \frac{n^3}{180} - \frac{191n}{60480}\right)g_0 \quad (7E)$$

$$+ \left(\frac{n^8}{40320} - \frac{n^6}{4320} + \frac{n^4}{2160} - \frac{289}{3628800}\right)h_0 + \dots$$

The coefficients are given in Table III. Most are already available in connection with other interpolation formulas.²¹ It is intended to extend this table.

²¹ Cf. Oppolzer, *Bahnbestimmung* II, p. 565 *et seq.*; Rice, *Theory and Practice of Interpolation*, p. 226.

Other formulas are readily derived. The odd differences above in terms of other quantities are

$$a_0 = F_1 - F_0 - \frac{b_0}{2}, \quad c_0 = b_1 - b_0 - \frac{d_0}{2}, \quad \text{etc.} \quad (7F)$$

Substituting these in (7E), and introducing $m = 1 - n$, gives another formula for direct interpolation:

$$F = mF_0 + \left(\frac{m^3}{6} - \frac{m}{12}\right)b_0 + \left(\frac{m^5}{120} - \frac{m^3}{36} + \frac{11m}{720}\right)d_0$$

$$+ \left(\frac{m^7}{5040} - \frac{m^5}{480} - \frac{m^3}{180} - \frac{191m}{60480}\right)f_0 + \dots$$

$$+ nF_1 + \left(\frac{n^3}{6} - \frac{n}{12}\right)b_1 + \left(\frac{n^5}{120} - \frac{n^3}{36} + \frac{11n}{720}\right)d_1$$

$$+ \left(\frac{n^7}{5040} - \frac{n^5}{480} + \frac{n^3}{180} - \frac{191n}{60480}\right)f_1 + \dots \quad (7G)$$

It is intended to give tables of the coefficients of this and other interpolation formulas; and also the corresponding data for a table of single integration.

These formulas and table were developed while the author was at the U.S. Naval Observatory, 1920 October.

8. AN EXTRAPOLATION FORMULA IN NUMERIC INTEGRATION

In extending a table of double integration for disturbed coordinates (or perturbations), the coordinate for the next date may be expressed as a function of quantities already tabulated. Let the schedule of differences be as in (7A). The usual procedure is to build by estimation from the run of the differences the quantities needed to find the coordinate (F_0) from (7B). Now

$$b_0 = b_{-1} + c_{-3/2} + d_{-2} + e_{-5/2} + f_{-3} + g_{-7/2} + h_{-4} + \dots$$

$$d_0 = d_{-2} + 2e_{-5/2} + 3f_{-3} + 4g_{-7/2} + 5h_{-4} + \dots$$

$$f_0 = f_{-3} + 3g_{-7/2} + 6h_{-4} + \dots$$

$$h_0 = h_{-4} + \dots$$

where the coefficients follow the diagonals of Pascal's triangle. Substitution into (7B) gives a formula for forward extrapolation:

$$(F_0) = F_0 + .0833 \ 3333 \ b_{-1} + .0833 \ 3333 \ c_{-3/2}$$

$$+ .0791 \ 667 \ d_{-2} + .075 \ e_{-5/2} + .07135 \ f_{-3} \quad (8A)$$

$$+ .0682 \ g_{-7/2} + .065 \ h_{-4} + .06 \ i_{-9/2} + \dots$$

similarly the formula for backward extrapolation is:

$$(F_0) = F_0 + .0833 \ 3333 \ b_1 - .0833 \ 3333 \ c_{3/2}$$

$$+ .0791 \ 667 \ d_2 - .075 \ e_{5/2} + .07135 \ f_3 \quad (8B)$$

$$- .0682 \ g_{7/2} + .065 \ h_4 - .06 \ i_{9/2} + \dots$$

Mr. J. E. Willis suggests introducing the factor 12 throughout the process of integration to facilitate the numeric working.

These formulas were derived while the author was at Perkins Observatory, Delaware, Ohio, 1927 June.

TABLE I. TOPOCENTRIC COORDINATES (See pages 34-35)

Observatory	θ_0	A	ΔZ	φ	Observatory	θ_0	A	ΔZ	φ	Observatory	θ_0	A	ΔZ	φ
Aarhus	0.9387	-238	-353	+56.1	Jena	0.9425	-269	-330	+50.9	Philadelphia	0.7012	-327	-273	+40.0
Albany	.7054	-314	-288	+42.7	Johannesburg	.9883	-383	+187	-26.2	Pola	.9488	-303	-299	+44.9
Alger	.9188	-342	-254	+36.8	Juvisy	.9169	-282	-319	+48.7	Potsdam	.9466	-261	-336	+52.4
Ann Arbor	.6778	-316	-286	+42.3	Kasan	.0459	-240	-351	+55.8	Poughkeepsie	.7051	-319	-282	+41.7
Arcetri	.9416	-309	-294	+43.8	Kiel	.9385	-249	-345	+54.3	Poznan	.9572	-261	-336	+52.4
Arequipa	0.7116	-410	+120	-16.4	Kiev	0.9951	-272	-327	+50.5	Prag	0.9504	-274	-326	+50.1
Athens	.9762	-337	-261	+38.0	Kioto	.2875	-350	-243	+35.0	Princeton	.7029	-326	-275	+40.3
Babelsberg	.9467	-261	-336	+52.4	Königsberg	.9673	-247	-347	+54.7	Pulkovo	.9946	-215	-367	+59.8
Bamberg	.9406	-275	-325	+49.9	Königstuhl	.9346	-278	-322	+49.4	Rome	.9450	-318	-283	+41.9
Barcelona	.9162	-320	-281	+41.4	Kopenhagen	.9453	-241	-351	+55.7	Sanfernando	.8931	-344	-252	+36.5
Bergedorf	0.9388	-254	-341	+53.5	Krakau	0.9658	-274	-326	+50.1	Santiago	0.7140	-356	+235	-33.6
Berkeley	.5707	-337	-260	+37.9	Kremsmünster	.9496	-286	-316	+48.1	Simeis	.0048	-305	-297	+44.4
Berlin	.9475	-260	-337	+52.5	Lapata	.7494	-350	+243	-34.9	Sonneberg	.9414	-273	-327	+50.4
Besançon	.9270	-290	-312	+47.2	Leiden	.9228	-262	-335	+52.2	Staradala	.9609	-287	-315	+47.9
Bloemfontein	.9832	-373	+206	-29.1	Leipzig	.9448	-267	-332	+51.3	Stockholm	.9605	-218	-365	+59.3
Bonn	0.9301	-271	-329	+50.7	Lemberg	0.9771	-276	-324	+49.8	Strasbourg	0.9319	-283	-318	+48.6
Bordeaux	.9089	-303	-299	+44.8	Leningrad	.9945	-214	-368	+59.9	Swarthmore	.7010	-328	-272	+39.9
Bothkamp	.9385	-250	-344	+54.2	Lisbon	.8848	-333	-265	+38.7	Sydney	.3304	-355	+236	-33.9
Breslau	.9578	-268	-331	+51.1	Liverpool	.9018	-255	-341	+53.4	Tacubaya	.6348	-403	-141	+19.4
Cambridge	.9106	-262	-336	+52.2	Lund	.9470	-241	-351	+55.7	Tartu	.9846	-224	-362	+58.4
Cape	0.9617	-354	+237	-33.9	Lyon	0.9236	-299	-304	+45.7	Tashkent	0.9028	-321	-280	+41.3
Catania	.9522	-339	-258	+37.5	Madrid	.9001	-325	-275	+40.4	Taunton	.7129	-318	-283	+41.9
Charlottesville	.6922	-336	-261	+38.0	Marseille	.9253	-311	-291	+43.3	Teramo	.9485	-314	-288	+42.7
Cincinnati	.6758	-331	-268	+39.1	Melbourne	.3130	-337	+260	-37.8	Tokio	.2980	-347	-247	+35.7
Columbia	.6539	-332	-267	+38.9	Middletown	.7085	-320	-282	+41.6	Torino	.9319	-302	-300	+45.0
Columbus	0.6798	-327	-273	+40.0	Milano	0.9359	-300	-303	+45.5	Toulouse	0.9144	-309	-293	+43.6
Cordova	.7320	-364	+221	-31.4	Minneapolis	.6513	-302	-300	+45.0	Triest	.9486	-299	-304	+45.6
Denver	.6188	-329	-271	+39.7	Moskau	.0147	-241	-351	+55.8	Uccle	.9224	-270	-329	+50.8
Des Moines	.6502	-320	-282	+41.6	Mt. Hamilton	.5724	-340	-257	+37.3	Upsala	.9593	-215	-367	+59.9
Dresden	.9485	-269	-330	+51.0	Mt. Wilson	.5824	-353	-239	+34.2	Utrecht	.9246	-263	-335	+52.1
Dublin	0.8927	-255	-341	+53.4	München	0.9426	-285	-316	+48.1	Victoria	0.5675	-283	-318	+48.5
Düsseldorf	.9291	-268	-331	+51.2	Nantucket	.7156	-321	-280	+41.3	Warsaw	.9688	-262	-336	+52.2
Edinburgh	.9015	-240	-352	+55.9	Napoli	.9499	-323	-278	+40.9	Washington	.6963	-332	-267	+38.9
Evanston	.6668	-317	-284	+42.1	Neuchâtel	.9297	-292	-310	+47.0	Wellesley	.7123	-316	-286	+42.3
Flagstaff	.6001	-349	-245	+35.2	Nice	.9306	-309	-293	+43.7	Wien	.9557	-285	-317	+48.2
Frankfurt	0.9344	-274	-326	+50.1	Nikolaiev	0.9992	-292	-310	+47.0	Williams Bay	0.6644	-315	-287	+42.6
Geneva	.9274	-296	-306	+46.2	Northampton	.7086	-316	-286	+42.3	Winchester	.7127	-315	-286	+42.5
Göttingen	.9380	-266	-332	+51.5	Northfield	.6516	-305	-297	+44.5	Windsor	.3293	-356	+235	-33.6
Greenwich	.9103	-266	-332	+51.5	Oslo	.9401	-214	-368	+59.9					
Hamburg	.9380	-254	-342	+53.6	Ottawa	.7000	-300	-302	+45.4					
Harvard	0.7128	-316	-286	+42.4	Oxford	0.9069	-265	-334	+51.8					
Heidelberg	.9345	-278	-322	+49.4	Padova	.9433	-300	-302	+45.4					
Helsingfors	.9797	-213	-369	+60.2	Palermo	.9474	-336	-262	+38.1					
Helwan	.9974	-370	-211	+29.9	Paris	.9168	-281	-320	+48.8					
Hyderabad	.1283	-407	-127	+17.4	Perth	.2321	-362	+224	-32.0					

$\theta_0 = 0.910339 + \text{east longitude}$
 $\theta = \theta_0 + 1.00273791(t - 2420000)$

A is always negative

ΔZ has the sign opposite to φ

$\Delta X = A \cos \theta$

$\Delta Y = A \sin \theta$

Unit of A and $\Delta Z = .0000001$

φ	A	ΔZ	φ	A	ΔZ	φ	A	ΔZ	φ	A	ΔZ	φ	A	ΔZ
0°	-427	0	10°	-420	74	20°	-401	145	30°	-370	212	40°	-327	273
1	427	7	11	419	81	21	398	152	31	366	218	41	322	278
2	426	15	12	417	88	22	396	159	32	362	225	42	318	284
3	426	22	13	416	95	23	393	166	33	358	231	43	313	289
4	426	30	14	414	103	24	390	172	34	354	237	44	307	295
5	-425	37	15	-412	110	25	-387	179	35	-350	243	45	-302	300
6	424	44	16	410	117	26	384	186	36	346	249	46	297	305
7	423	52	17	408	124	27	380	193	37	341	255	47	291	310
8	423	59	18	406	131	28	377	199	38	337	261	48	286	316
9	421	66	19	404	138	29	373	206	39	332	267	49	280	320
10	-420	74	20	-401	145	30	-370	212	40	-327	273	50	-275	325

TABLE II. FOUR-PLACE TRIGONOMETRIC TABLE. (See page 35)

	cos	sin		sin	cos	
.000	1.0000	.0000	.250	.9251	.3798	.188
.001	1.0000	.0063	.249	.9227	.3856	.187
.002	.9999	.0126	.248	.9202	.3914	.186
.003	.9998	.0188	.247	.9178	.3971	.185
.004	.9997	.0251	.246	.9152	.4029	.184
.005	.9995	.0314	.245	.9127	.4086	.183
.006	.9993	.0377	.244	.9101	.4144	.182
.007	.9990	.0440	.243	.9075	.4201	.181
.008	.9987	.0502	.242	.9048	.4258	.180
.009	.9984	.0565	.241	.9021	.4315	.179
.010	.9980	.0628	.240	.8994	.4371	.178
.011	.9976	.0691	.239	.8966	.4428	.177
.012	.9972	.0753	.238	.8938	.4484	.176
.013	.9967	.0816	.237	.8910	.4540	.175
.014	.9961	.0879	.236	.8881	.4596	.174
.015	.9956	.0941	.235	.8852	.4652	.173
.016	.9950	.1004	.234	.8823	.4707	.172
.017	.9943	.1066	.233	.8793	.4762	.171
.018	.9936	.1129	.232	.8763	.4818	.170
.019	.9929	.1191	.231	.8733	.4873	.169
.020	.9921	.1253	.230	.8702	.4927	.168
.021	.9913	.1316	.229	.8671	.4982	.167
.022	.9905	.1378	.228	.8639	.5036	.166
.023	.9896	.1440	.227	.8607	.5090	.165
.024	.9887	.1502	.226	.8575	.5144	.164
.025	.9877	.1564	.225	.8543	.5198	.163
.026	.9867	.1626	.224	.8510	.5252	.162
.027	.9856	.1688	.223	.8477	.5305	.161
.028	.9846	.1750	.222	.8443	.5358	.160
.029	.9834	.1812	.221	.8409	.5411	.159
.030	.9823	.1874	.220	.8375	.5464	.158
.031	.9811	.1935	.219	.8341	.5516	.157
.032	.9799	.1997	.218	.8306	.5569	.156
.033	.9786	.2059	.217	.8271	.5622	.155
.034	.9773	.2120	.216	.8235	.5673	.154
.035	.9759	.2181	.215	.8200	.5724	.153
.036	.9745	.2243	.214	.8163	.5775	.152
.037	.9731	.2304	.213	.8127	.5827	.151
.038	.9716	.2365	.212	.8090	.5878	.150
.039	.9701	.2426	.211	.8053	.5929	.149
.040	.9686	.2487	.210	.8016	.5979	.148
.041	.9670	.2548	.209	.7978	.6029	.147
.042	.9654	.2608	.208	.7940	.6079	.146
.043	.9637	.2669	.207	.7902	.6129	.145
.044	.9620	.2730	.206	.7863	.6179	.144
.045	.9603	.2790	.205	.7824	.6228	.143
.046	.9585	.2850	.204	.7785	.6277	.142
.047	.9567	.2910	.203	.7745	.6326	.141
.048	.9549	.2970	.202	.7705	.6374	.140
.049	.9530	.3030	.201	.7665	.6423	.139
.050	.9511	.3090	.200	.7624	.6471	.138
.051	.9491	.3150	.199	.7584	.6518	.137
.052	.9471	.3209	.198	.7543	.6566	.136
.053	.9451	.3269	.197	.7501	.6613	.135
.054	.9430	.3328	.196	.7459	.6660	.134
.055	.9409	.3387	.195	.7417	.6707	.133
.056	.9387	.3446	.194	.7375	.6753	.132
.057	.9365	.3505	.193	.7333	.6800	.131
.058	.9343	.3564	.192	.7290	.6846	.130
.059	.9321	.3623	.191	.7247	.6892	.129
.060	.9298	.3681	.190	.7203	.6937	.128
.061	.9274	.3740	.189	.7159	.6982	.127
.062	.9251	.3798	.188	.7115	.7026	.126
.063	.9227	.3856	.187	.7071	.7071	.125

TABLE III
COEFFICIENTS FOR INTERPOLATING A FUNCTION
DIRECTLY FROM A TABLE OF DOUBLE
INTEGRATION

(See page 40)

n	B	C	D	E
$\pm .00$	+.083333	$\mp .00000$	-.0042	$\pm .000$
.01	.083383	.00083	.42	0
.02	.083533	.00167	.42	0
.03	.083783	.00250	.42	0
.04	.084133	.00332	.42	1
$\pm .05$	+.084583	$\mp .00413$	-.0042	$\pm .001$
.06	.085133	.00496	.42	1
.07	.085783	.00578	.42	1
.08	.086533	.00658	.42	1
.09	.087383	.00738	.42	1
$\pm .10$	+.088333	$\mp .00817$	-.0042	$\pm .002$
.11	.089383	.00894	.42	2
.12	.090533	.00971	.42	2
.13	.091783	.01047	.42	2
.14	.093133	.01121	.42	2
$\pm .15$	+.094583	$\mp .01194$	-.0041	$\pm .002$
.16	.096133	.01265	.41	2
.17	.097783	.01335	.41	2
.18	.099533	.01403	.41	3
.19	.101383	.01469	.41	3
$\pm .20$	+.103333	$\mp .01533$	-.0041	$\pm .003$
.21	.105383	.01596	.41	3
.22	.107533	.01656	.41	3
.23	.109783	.01714	.41	3
.24	.112133	.01770	.40	3
$\pm .25$	+.114583	$\mp .01823$	-.0040	$\pm .003$
.26	.117133	.01874	.40	3
.27	.119783	.01922	.39	4
.28	.122533	.01967	.39	4
.29	.125383	.02010	.39	4
$\pm .30$	+.128333	$\mp .02050$	-.0038	$\pm .004$
.31	.131383	.02087	.38	4
.32	.134533	.02121	.37	4
.33	.137783	.02151	.37	4
.34	.141133	.02178	.36	4
$\pm .35$	+.144583	$\mp .02202$	-.0035	$\pm .004$
.36	.148133	.02222	.35	4
.37	.151783	.02239	.34	4
.38	.155533	.02252	.33	4
.39	.159383	.02261	.32	4
$\pm .40$	+.163333	$\mp .02267$	-.0031	$\pm .004$
.41	.167383	.02268	.30	4
.42	.171533	.02265	.29	4
.43	.175783	.02258	.27	4
.44	.180133	.02247	.26	4
$\pm .45$	+.184583	$\mp .02231$	-.0025	$\pm .004$
.46	.189133	.02211	.23	4
.47	.193783	.02186	.21	4
.48	.198533	.02157	.20	4
.49	.203383	.02123	.18	4
$\pm .50$	+.208333	$\mp .02083$	-.0016	$\pm .004$

$$F = F_0 + na_0 + Bb_0 + Cc_0 + Dd_0 + Ee_0 + \dots$$

TABLE IV
TO TRANSFER EQUATORIAL RECTANGULAR COORDINATES TO 1900.0 (See pages 36-37)

t	X_x	X_y	X_z	Y_x	Y_y	Y_z	Z_x	Z_y	Z_z
1840	+ .9998 9317	-.0134 0268	-.0058 3193	+ .0134 0268	+ .9999 1018	-.0000 3908	+ .0058 3193	-.0000 3909	+ .9999 8299
41	.9998 9670	131 7936	57 3473	131 7936	1315	3779	57 3473	3780	8356
42	.9999 0018	129 5604	56 3752	129 5604	1607	3652	56 3752	3653	8411
43	0359	127 3271	55 4032	127 3271	1894	3527	55 4032	3528	8465
44	0694	125 0938	54 4311	125 0938	2175	3404	54 4311	3405	8518
1845	+ .9999 1023	-.0122 8605	-.0053 4591	+ .0122 8605	+ .9999 2452	-.0000 3284	+ .0053 4591	-.0000 3284	+ .9999 8571
46	1347	120 6272	52 4871	120 6272	2724	3166	52 4871	3166	8622
47	1664	118 3938	51 5150	118 3938	2991	3049	51 5150	3050	8673
48	1976	116 1604	50 5430	116 1604	3253	2935	50 5430	2936	8723
49	2281	113 9271	49 5710	113 9271	3510	2824	49 5710	2824	8771
1850	+ .9999 2581	-.0111 6936	-.0048 5989	+ .0111 6936	+ .9999 3762	-.0000 2714	+ .0048 5989	-.0000 2714	+ .9999 8819
51	2875	109 4602	47 6269	109 4602	4009	2607	47 6269	2607	8866
52	3163	107 2268	46 6548	107 2268	4251	2501	46 6548	2502	8912
53	3444	104 9933	45 6828	104 9933	4488	2398	45 6828	2398	8956
54	3720	102 7598	44 7108	102 7598	4720	2297	44 7108	2297	9000
1855	+ .9999 3990	-.0100 5263	-.0043 7388	+ .0100 5263	+ .9999 4947	-.0000 2198	+ .0043 7388	-.0000 2199	+ .9999 9043
56	4255	98 2927	42 7667	98 2927	5169	2102	42 7667	2102	9086
57	4513	96 0592	41 7947	96 0592	5386	2007	41 7947	2008	9127
58	4765	93 8256	40 8227	93 8256	5598	1915	40 8227	1915	9167
59	5011	91 5920	39 8507	91 5920	5805	1825	39 8507	1825	9206
1860	+ .9999 5252	-.0089 3584	-.0038 8786	+ .0089 3584	+ .9999 6007	-.0000 1737	+ .0038 8786	-.0000 1737	+ .9999 9244
61	5486	87 1248	37 9066	87 1248	6204	1651	37 9066	1651	9282
62	5714	84 8912	36 9346	84 8912	6397	1568	36 9346	1568	9318
63	5937	82 6575	35 9626	82 6575	6584	1486	35 9626	1486	9353
64	6154	80 4238	34 9906	80 4238	6766	1407	34 9906	1407	9388
1865	+ .9999 6364	-.0078 1901	-.0034 0186	+ .0078 1901	+ .9999 6943	-.0000 1330	+ .0034 0186	-.0000 1330	+ .9999 9421
66	6569	75 9564	33 0466	75 9564	7115	1255	33 0466	1255	9454
67	6768	73 7226	32 0746	73 7226	7282	1182	32 0746	1182	9486
68	6961	71 4889	31 1026	71 4889	7445	1112	31 1026	1112	9516
69	7148	69 2551	30 1306	69 2551	7602	1043	30 1306	1043	9546
1870	+ .9999 7329	-.0067 0213	-.0029 1586	+ .0067 0213	+ .9999 7754	-.0000 0977	+ .0029 1586	-.0000 0977	+ .9999 9575
71	7504	64 7875	28 1866	64 7875	7901	0913	28 1866	0913	9603
72	7673	62 5537	27 2146	62 5537	8044	0851	27 2146	0851	9630
73	7836	60 3198	26 2426	60 3198	8181	0791	26 2426	0792	9656
74	7994	58 0860	25 2706	58 0860	8313	0734	25 2706	0734	9681
1875	+ .9999 8145	-.0055 8521	-.0024 2986	+ .0055 8521	+ .9999 8440	-.0000 0679	+ .0024 2986	-.0000 0679	+ .9999 9705
76	8290	53 6182	23 3266	53 6182	8562	0625	23 3266	0625	9728
77	8430	51 3843	22 3547	51 3843	8680	0574	22 3547	0574	9750
78	8564	49 1504	21 3827	49 1504	8792	0525	21 3827	0526	9771
79	8691	46 9164	20 4107	46 9164	8899	0479	20 4107	0479	9792
1880	+ .9999 8813	-.0044 6824	-.0019 4387	+ .0044 6824	+ .9999 9002	-.0000 0434	+ .0019 4387	-.0000 0434	+ .9999 9811
81	8929	42 4485	18 4668	42 4485	9099	0392	18 4668	0392	9830
82	9038	40 2145	17 4948	40 2145	9191	0352	17 4948	0352	9847
83	9142	37 9805	16 5228	37 9805	9279	0314	16 5228	0314	9864
84	9240	35 7464	15 5509	35 7464	9361	0278	15 5509	0278	9879
1885	+ .9999 9332	-.0033 5124	-.0014 5789	+ .0033 5124	+ .9999 9438	-.0000 0244	+ .0014 5789	-.0000 0244	+ .9999 9894
86	9418	31 2784	13 6070	31 2784	9511	0213	13 6070	0213	9907
87	9498	29 0443	12 6350	29 0443	9578	0184	12 6350	0184	9920
88	9573	26 8102	11 6631	26 8102	9641	0156	11 6631	0156	9932
89	9641	24 5761	10 6911	24 5761	9698	0131	10 6911	0131	9943
1890	+ .9999 9703	-.0022 3420	-.0009 7192	+ .0022 3420	+ .9999 9750	-.0000 0109	+ .0009 7192	-.0000 0109	+ .9999 9953
91	9760	20 1078	08 7472	20 1078	9798	0088	08 7472	0088	9962
92	9810	17 8737	07 7753	17 8737	9840	0070	07 7753	0070	9970
93	9854	15 6395	06 8034	15 6395	9878	0053	06 8034	0053	9977
94	9893	13 4054	05 8315	13 4054	9910	0039	05 8315	0039	9983
1895	+ .9999 9926	-.0011 1712	-.0004 8596	+ .0011 1712	+ .9999 9938	-.0000 0027	+ .0004 8596	-.0000 0027	+ .9999 9988
96	9952	08 9370	03 8876	08 9370	9960	0017	03 8876	0017	9992
97	9973	06 7027	02 9157	06 7027	9978	0010	02 9157	0010	9996
98	9988	04 4685	01 9438	04 4685	9990	0004	01 9438	0004	9998
99	.9999 9997	02 2343	00 9719	02 2343	.9999 9998	0001	00 9719	0001	.9999 9999
1900	+1.0000 0000	-.0000 0000	-.0000 0000	+ .0000 0000	+1.0000 0000	-.0000 0000	+ .0000 0000	-.0000 0000	+1.0000 0000

$$x_{1900} = X_x \cdot x + X_y \cdot y + X_z \cdot z \quad y_{1900} = Y_x \cdot x + Y_y \cdot y + Y_z \cdot z \quad z_{1900} = Z_x \cdot x + Z_y \cdot y + Z_z \cdot z$$

TABLE IV—(Continued)

<i>t</i>	X_x	X_y	X_z	Y_x	Y_y	Y_z	Z_x	Z_y	Z_z
1900	+1.0000 0000	+ .0000 0000	+ .0000 0000	-.0000 0000	+1.0000 0000	-.0000 0000	-.0000 0000	-.0000 0000	+1.0000 0000
01	.9999 9997	02 2343	00 9719	02 2343	.9999 9998	0001	00 9719	0001	0000
02	9998	04 4686	01 9438	04 4686	9990	0004	01 9438	0004	.9999 9998
03	9973	06 7029	02 9157	06 7029	9978	0010	02 9157	0010	9996
04	9952	08 9372	03 8876	08 9372	9960	0017	03 8876	0017	9992
1905	+ .9999 9926	+ .0011 1715	+ .0004 8594	-.0011 1715	+ .9999 9938	-.0000 0027	-.0004 8594	-.0000 0027	+ .9999 9988
06	9893	13 4058	05 8313	13 4058	9910	0039	05 8313	0039	9983
07	9854	15 6402	06 8032	15 6402	9878	0053	06 8032	0053	9977
08	9810	17 8746	07 7751	17 8746	9840	0070	07 7751	0070	9970
09	9760	20 1089	08 7469	20 1089	9798	0088	08 7469	0088	9962
1910	+ .9999 9703	+ .0022 3433	+ .0009 7188	-.0022 3433	+ .9999 9750	-.0000 0109	-.0009 7188	-.0000 0109	+ .9999 9953
11	9641	24 5777	10 6906	24 5777	9698	0131	10 6906	0131	9943
12	9573	26 8121	11 6625	26 8121	9641	0156	11 6625	0156	9932
13	9498	29 0466	12 6343	29 0466	9578	0184	12 6343	0184	9920
14	9418	31 2810	13 6062	31 2810	9511	0213	13 6062	0213	9907
1915	+ .9999 9332	+ .0033 5154	+ .0014 5780	-.0033 5154	+ .9999 9438	-.0000 0244	-.0014 5780	-.0000 0244	+ .9999 9894
16	9240	35 7499	15 5498	35 7499	9361	0278	15 5498	0278	9879
17	9142	37 9844	16 5216	37 9844	9279	0314	16 5216	0314	9864
18	9038	40 2189	17 4935	40 2189	9191	0352	17 4935	0352	9847
19	8928	42 4534	18 4653	42 4534	9099	0392	18 4653	0392	9830
1920	+ .9999 8813	+ .0044 6879	+ .0019 4371	-.0044 6879	+ .9999 9002	-.0000 0434	-.0019 4371	-.0000 0434	+ .9999 9811
21	8691	46 9224	20 4089	46 9224	8899	0479	20 4089	0479	9792
22	8563	49 1569	21 3807	49 1569	8792	0526	21 3807	0526	9771
23	8430	51 3914	22 3525	51 3914	8679	0574	22 3525	0574	9750
24	8290	53 6260	23 3242	53 6260	8562	0625	23 3242	0625	9728
1925	+ .9999 8145	+ .0055 8606	+ .0024 2960	-.0055 8606	+ .9999 8440	-.0000 0679	-.0024 2960	-.0000 0679	+ .9999 9705
26	7993	58 0951	25 2678	58 0951	8312	0734	25 2678	0734	9681
27	7836	60 3297	26 2396	60 3297	8180	0792	26 2396	0791	9656
28	7673	62 5643	27 2113	62 5643	8043	0851	27 2113	0851	9630
29	7503	64 7989	28 1831	64 7989	7900	0913	28 1831	0913	9603
1930	+ .9999 7328	+ .0067 0335	+ .0029 1548	-.0067 0335	+ .9999 7753	-.0000 0977	-.0029 1548	-.0000 0977	+ .9999 9575
31	7147	69 2681	30 1266	69 2681	7601	1043	30 1266	1043	9546
32	6960	71 5028	31 0983	71 5028	7444	1112	31 0983	1112	9516
33	6767	73 7374	32 0701	73 7374	7281	1182	32 0701	1182	9486
34	6568	75 9720	33 0418	75 9720	7114	1255	33 0418	1255	9454
1935	+ .9999 6363	+ .0078 2067	+ .0034 0135	-.0078 2067	+ .9999 6942	-.0000 1330	-.0034 0135	-.0000 1330	+ .9999 9422
36	6152	80 4414	34 9852	80 4414	6764	1407	34 9852	1407	9388
37	5936	82 6760	35 9569	82 6760	6582	1486	35 9569	1486	9354
38	5713	84 9107	36 9286	84 9107	6395	1568	36 9286	1568	9318
39	5484	87 1454	37 9003	87 1454	6203	1652	37 9003	1651	9282
1940	+ .9999 5250	+ .0089 3801	+ .0038 8720	-.0089 3801	+ .9999 6005	-.0000 1737	-.0038 8720	-.0000 1737	+ .9999 9244
41	5009	91 6148	39 8437	91 6148	5803	1825	39 8437	1825	9206
42	4763	93 8495	40 8154	93 8495	5596	1915	40 8154	1915	9167
43	4511	96 0842	41 7871	96 0842	5384	2008	41 7871	2008	9127
44	4252	98 3190	42 7587	98 3190	5166	2102	42 7587	2102	9086
1945	+ .9999 3988	+ .0100 5537	+ .0043 7304	-.0100 5537	+ .9999 4944	-.0000 2199	-.0043 7304	-.0000 2199	+ .9999 9044
46	3718	102 7885	44 7020	102 7885	4717	2298	44 7020	2297	9001
47	3442	105 0232	45 6737	105 0232	4485	2399	45 6737	2398	8957
48	3160	107 2580	46 6453	107 2580	4248	2502	46 6453	2501	8912
49	2872	109 4928	47 6170	109 4928	4005	2607	47 6170	2607	8866
1950	+ .9999 2578	+ .0111 7275	+ .0048 5886	-.0111 7275	+ .9999 3758	-.0000 2715	-.0048 5886	-.0000 2714	+ .9999 8820
51	2278	113 9623	49 5602	113 9623	3506	2824	49 5602	2824	8772
52	1972	116 1971	50 5318	116 1971	3249	2936	50 5318	2936	8723
53	1660	118 4319	51 5034	118 4319	2987	3050	51 5034	3050	8674
54	1343	120 6667	52 4750	120 6667	2719	3166	52 4750	3166	8623
1955	+ .9999 1019	+ .0122 9015	+ .0053 4466	-.0122 9015	+ .9999 2447	-.0000 3285	-.0053 4466	-.0000 3284	+ .9999 8572
56	0689	125 1363	54 4182	125 1363	2170	3405	54 4182	3405	8519
57	0354	127 3711	55 3897	127 3711	1888	3528	55 3897	3527	8466
58	.9999 0012	129 6060	56 3613	129 6060	1601	3653	56 3613	3652	8412
59	.9998 9665	131 8408	57 3329	131 8408	1308	3780	57 3329	3779	8356
1960	+ .9998 9312	+ .0134 0756	+ .0058 3044	-.0134 0756	+ .9999 1011	-.0000 3909	-.0058 3044	-.0000 3908	+ .9999 8300

$$x = +X_x \cdot x_{1900} - X_y \cdot y_{1900} - X_z \cdot z_{1900}$$

$$y = -Y_x \cdot x_{1900} + Y_y \cdot y_{1900} + Y_z \cdot z_{1900}$$

$$z = -Z_x \cdot x_{1900} + Z_y \cdot y_{1900} + Z_z \cdot z_{1900}$$

TABLE V
FOR INTERPOLATING SOLAR COORDINATES (See pages 35-36)

$$p = \frac{mn (wk)^2}{4 R^3} \quad w = \frac{1}{2} \text{ day}$$

<i>m</i> or <i>n</i>	Jan 1 31	Jan 22 15	Feb 2 4	Feb 10 25	Feb 17 Nov 18	Feb 24 Nov 12	Mar 1 Nov 6	Mar 7 Nov 1	Mar 12 Oct 26	Mar 17 Oct 21	Mar 22 Oct 16	Mar 28 Oct 11	Apr 1 Oct 6	Apr 6 Oct 1	Apr 11 Sep 26	Apr 17 Sep 21	Apr 22 Sep 15	Apr 28 Sep 10	May 3 Sep 4	May 10 Aug 29	May 17 Aug 22	May 24 Aug 14	Jun 2 Aug 5	Jun 16 Jul 23	Jun 30 Jul 1	<i>m</i> or <i>n</i>
.00	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	.00
.01	019	019	019	019	019	019	019	019	019	019	018	018	018	018	018	018	018	018	018	018	018	018	018	018	017	.01
.02	038	038	038	038	037	037	037	037	037	037	037	036	036	036	036	036	036	035	035	035	035	035	035	035	034	.02
.03	057	056	056	056	056	055	055	055	055	054	054	054	054	054	053	053	053	052	052	052	052	052	051	051	051	.03
.04	075	074	074	074	073	073	073	073	072	072	072	071	071	071	070	070	070	069	069	069	069	068	068	068	068	.04
.05	092	092	092	091	091	090	090	090	089	089	089	088	088	087	087	087	086	086	086	085	085	085	084	084	084	.05
.06	110	109	109	108	108	107	107	106	106	106	105	105	104	104	103	103	102	102	102	101	101	100	100	099	099	.06
.07	127	126	126	125	124	124	123	123	122	122	121	121	120	120	119	119	118	118	117	117	116	116	115	115	115	.07
.08	143	142	142	141	141	140	140	139	138	138	137	137	136	135	135	134	134	133	132	132	131	131	130	130	130	.08
.09	159	159	158	157	157	156	155	155	154	153	153	152	151	151	150	149	149	148	147	147	146	145	145	144	144	.09
.10	175	174	174	173	172	171	171	170	169	168	168	167	166	166	165	164	163	163	162	161	161	160	159	158	158	.10
.11	190	190	189	188	187	186	186	185	184	183	182	182	181	180	179	179	178	177	176	175	175	174	173	172	172	.11
.12	205	204	204	203	202	201	200	199	199	198	197	196	195	194	193	193	192	191	190	189	188	188	187	186	186	.12
.13	220	219	218	217	216	215	214	214	213	212	211	210	209	208	207	206	205	204	203	202	201	200	199	199	199	.13
.14	234	233	232	231	230	229	228	227	226	225	224	223	222	222	221	220	219	218	217	216	215	214	213	212	212	.14
.15	248	247	246	245	244	243	242	241	240	239	238	237	236	235	234	233	232	231	230	228	227	226	225	224	224	.15
.16	261	260	259	258	257	256	255	254	253	252	251	249	248	247	246	245	244	243	242	241	240	239	238	237	237	.16
.17	274	273	272	271	270	269	268	266	265	264	263	262	261	260	258	257	256	255	254	253	252	251	249	248	248	.17
.18	287	286	285	283	282	281	280	279	277	276	275	274	273	272	270	269	268	267	266	264	263	262	261	260	260	.18
.19	299	298	297	295	294	293	292	291	289	288	287	286	284	283	282	281	279	278	277	276	275	273	272	271	271	.19
.20	311	310	308	307	306	305	303	302	301	300	298	297	296	294	293	292	291	289	288	287	285	284	283	282	282	.20
.21	323	321	320	319	317	316	315	313	312	311	309	308	307	305	304	303	301	300	299	297	296	295	293	292	292	.21
.22	334	332	331	329	328	327	325	324	323	321	320	318	317	316	314	313	312	310	309	308	306	305	303	302	302	.22
.23	344	343	341	340	339	337	336	334	333	332	330	329	327	326	324	323	322	320	319	317	316	315	313	312	312	.23
.24	355	353	352	350	349	347	346	344	343	341	340	339	337	336	334	333	331	330	328	327	325	324	322	321	321	.24
.25	364	363	362	360	358	357	356	354	352	351	350	348	346	345	344	342	340	339	338	336	334	333	332	330	330	.25
.26	374	372	371	369	368	366	365	363	362	360	359	357	356	354	352	351	349	348	346	345	343	342	340	339	339	.26
.27	383	382	380	378	377	375	374	372	371	369	367	366	364	363	361	360	358	356	355	353	352	350	348	347	347	.27
.28	392	390	389	387	385	384	382	381	379	377	376	374	373	371	369	368	366	364	363	361	360	358	356	355	355	.28
.29	400	399	397	395	394	392	390	389	387	385	384	382	381	379	377	376	374	372	371	369	367	366	364	362	362	.29
.30	408	407	405	403	402	400	398	396	395	393	391	390	388	386	385	383	381	380	378	376	375	373	371	370	370	.30
.31	416	414	412	411	409	407	406	404	402	400	399	397	395	394	392	390	388	387	385	383	382	380	378	376	376	.31
.32	423	421	420	418	416	414	413	411	409	407	406	404	402	400	399	397	395	393	392	390	388	386	385	383	383	.32
.33	430	428	426	425	423	421	419	417	416	414	412	410	409	407	405	403	402	400	398	396	394	393	391	389	389	.33
.34	436	434	433	431	429	427	425	424	422	420	418	416	415	413	411	409	408	406	404	402	400	399	397	395	395	.34
.35	442	440	439	437	435	433	431	430	428	426	424	422	420	419	417	415	413	411	410	408	406	404	402	400	400	.35
.36	448	446	444	442	441	439	437	435	433	431	429	428	426	424	422	420	418	417	415	413	411	409	407	406	406	.36
.37	453	451	449	448	446	444	442	440	438	436	434	433	431	429	427	425	423	421	420	418	416	414	412	410	410	.37
.38	458	456	454	452	450	449	447	445	443	441	439	437	435	434	432	430	428	426	424	422	420	418	417	415	415	.38
.39	462	461	459	457	455	453	451	449	447	445	443	442	440	438	436	434	432	430	428	426	424	423	421	419	419	.39
.40	467	465	463	461	459	457	455	453	451	449	447	445	444	442	440	438	436	434	432	430	428	426	424	422	422	.40
.41	470	468	466	464	463	461	459	457	455	453	451	449	447	445	443	441	439	437	435	433	432	430	428	426	426	.41
.42	474	472	470	468	466	464	462	460	458	456	454	452	450	448	446	444	442	440	438	437	435	433	431	429	429	.42
.43	476	475	473	471	469	467	465	463	461	459	457	455	453	451	449	447	445	443	441	439	437	435	433	431	431	.43
.44	479	477	475	473	471	469	467	465	463	461	459	457	455	453	451	449	447	445	444	442	440	438	436	434	434	.44
.45	481	479	477	475	473	471	469	467	465	463	461	459	457	455	453	451	449	447	446	444	442	440	438	436	436	.45
.46	483	481	479	477	475	473	471	469	467	465	463	461	459	457	455	453	451	449	447	445	443	441	439	437	437	.46
.47	484	482	480	478	476	474	472	470	468	466	464	462	460	458	456	454	452	450	448	446	444	442	440	438	438	.47
.48	485	483	481	479	477	475	473	471	469	467	465	463	461	459	457	455	453	451	449	447	445	443	441	439	439	.48
.49	486	484	482	480	478	476	474	472	470	468	466	464	462	460	458	456	454	452	450	448	446	444	442	440	440	.49
.50	486	484	482	480	478	476	474	472	470	468	466	464	462	460	458	456	454	452	450	448	446	444	442	440	440	.50

Interpolate vertically only

unit = .0000 0001

$$n = 2(t - t_m)$$

$$m = 1 - n$$

$$X = (m+p)X_m + (n+p)X_n$$

(similarly Y, Z)

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