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A PHOTOMETRIC STUDY OF STELLAR SPECTRA*

BY

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INTRODUCTION

The significance of the application of photographic spectrophotometry to the solution of many problems of astrophysical importance has long been recognized. For some years, this problem has engaged the attention of a number of astrophysicists. However, on account of numerous difficulties encountered in this field, especially those inherent in the photographic process yet imperfectly understood, progress has not been rapid, and it was not until very recently that the method of stellar spectrophotometry was put on a fairly satisfactory working basis, the latest contribution being the method developed at Harvard by Shapley, Miss Payne¹ and their collaborators.

A few years ago, the writer made a similar study of this problem at the Lick Observatory.² Although the method followed is subject to improvement it was under the circumstance the best one practicable. The study has yielded some interesting results. All of my photographs were measured visually with the Hartmann microphotometer, which, while well adapted to the special problem in view, was nevertheless a slow instrument. Moreover it has not the manifold advantages possessed by a self-registering instrument, such as the Moll microphotometer. The latter, besides producing more faithful and more permanent records, from which, in general, results of greater reliability are obtainable, is far more powerful for the analysis

of details. To do justice to the Hartmann instrument, however, I may say here that I found that at the dense parts of a spectrogram the visual measures obtained with this instrument are more accurate than the records secured with the Moll photometer at the same density, for it is obvious that the galvanometer deflection is not very sensitive at heavy parts of the spectrogram. On the other hand, in lightly exposed portions the Moll is very sensitive, and here the Hartmann can not compete in detecting slight variations of the photographic blackening.

In 1926 a Moll self-registering microphotometer became available at the Lick Observatory. Accordingly many of my old spectrograms and some additional new ones were examined with this instrument. Some of the results thus obtained have already appeared.³ This paper deals with the others derived from the same analysis.

OBSERVATIONS

As just noted, besides using my former observations, I secured a number of new plates. These were taken in exactly the same manner as before, that is, with the two-prism quartz spectrograph attached to the Crossley reflector, the photometric scale on each plate being likewise determined by a set of six standard exposures impressed simultaneously on the plate through an ultra-violet color filter.⁴ The new material

¹ *Harvard Reprints* 28, 1926; *H. C. O. Cir.* 301, 1927; and subsequent papers.

² *Lick Obs. Bull.*, 12, 104, 1926. This will be referred to again as Paper A.

³ *Publ. A. S. P.*, 39, 112, 1927. This will be referred to again as Paper D.

⁴ For a detailed account of the photometric method followed see Paper A.

* Read before the Fourth Pacific Science Congress held at Bandoeng, Java, May 16-25, 1929.

includes some re-observations of the old stars, but is mainly concerned with the brighter Class B stars, especially the Be type, a few additional Class A stars, and four brighter Cepheid variables. As many plates as possible were obtained of the Cepheids to cover the different phases of their light periods. A complete series of new observations of ζ *Geminorum* was also secured with the slit spectrograph.

It was originally intended to observe the fainter Class A stars of known trigonometric parallaxes and more "c" stars of the same spectral class in order to test further the intensity-magnitude-temperature relationship⁵ which formed the basis of my proposed method for the determination of the spectroscopic parallaxes of Class A stars. On account of the unfavorable weather prevailing at Mount Hamilton during the year 1925-26, the demands of other investigations upon the Crossley reflector, and finally on account of my call to the University of Amoy, I was unable to carry out this part of the program.

Not all of my earlier spectrograms could be utilized to advantage for analysis with the Moll microphotometer because many of them, while of correct density for visual reduction, were found to be too dense for the self-recording process. This point has already been referred to. It may be recalled, however, that the old observations consist of two sets, one taken with the slit, and the other with the slitless spectrograph. Of the latter set there are three exposures of varying intensity on each plate. Thus on almost every plate the faintest exposure, which was not measured with the Hartmann on account of its comparative faintness, turned out to be of the right density for the Moll. In many instances two or even three of the exposures were analysed. Moll tracings were also obtained from slit plates of suitable density. These are, however, not numerous and for the sake of homogeneity will not be included in the main discussion given below, which is based, therefore, entirely on the slitless observations. Suffice it to say that the results from the slit tracings are found to be in general accord with those presented in this paper. The more recent observations were also made with the slitless spectrograph, except, as already noted, those of ζ *Geminorum*, which are not included in the present discussion.

REDUCTIONS

The Moll Microphotometer.—The working of the Moll microphotometer as here employed may be briefly described as follows: A condensing lens throws on the spectrogram to be analysed an image of the straight short filament of a 25-watt auto-lamp which

⁵ *Lick Obs. Bull.*, 12, 155, 1926. This will be referred to again as Paper B.

draws the current from a 6-volt storage battery. By means of this beam an objective produces an image of the spectrogram, magnified about thirteen times, on the slit of the thermo-couple casing. There are two thermo-couples behind the slit, one above the other, but actually only the upper one was used. The thermo-couple is connected with a very sensitive galvanometer whose mirror throws a spot of light on the photographic paper. The galvanometer deflection is proportional to the amount of radiation of the analysing beam incident on the thermo-couple, the amount being dependent on the photographic blackening of the spectrogram. The spotlight, registering as it does the galvanometer deflection, therefore traces a curve on the print which reproduces the intensity variation of the photographic spectrum, as the plate and the drum on which the print is mounted are moved by a small electric motor. Two magnifications of the trace are available, one of about 7 and the other of 40 times. For my purpose it was found sufficient, in several respects even better, to use the smaller magnification.

In general, other things being equal, the galvanometer deflection is a function of the following factors: (1) voltage of the lamp, (2) temperature of the room, (3) atmospheric pressure in the room, and (4) width of the thermo-couple slit. It is clear that these factors should remain constant during a run. Factor (1) was kept constant by having the battery fully charged the night before the day on which a series of runs was to be made. The voltage was noted before and after each run. In no case was the drop more than .05 volt. Moreover this constancy was amply checked by the identity of the zero mark (darkness) and the maximum deflection mark (clear film) at the beginning and end, respectively, of the tracings, the registration of which will be described later. The room temperature was fairly constant, at least during the short interval (about 15 minutes) of a run. Factor (3) may ordinarily be neglected. But in the course of the work it was found that a sudden opening of the door of the dark-room in which the Moll is housed affected the sensitivity of the thermo-couple in such a way as to produce a deflection of the spotlight of 1 or 2 mm. Special precaution was therefore taken not to let the door be violently disturbed when a tracing was in progress. The regularity and smoothness of the tracings obtained attest the freedom from this pressure effect.

The slit of the thermo-couple was of course of constant width during a run. Its constancy, too, for all observations is theoretically desirable. For otherwise the difference in the integrating effect arising from the changes of the slit width would render the intercomparison of line depths from different plates meaningless. However, in order to preserve the max-

imum deflection of the galvanometer, which for obvious reasons was kept the same for all tracings, it was found necessary occasionally to vary the slit width by a small amount. In practice the slit width used varied from .10 mm. to .19 mm. It is important therefore to inquire whether such a variation exerts any practical influence on the depth of the lines here encountered. Constancy of the slit width, it will be noticed, is necessary only for lines whose widths are narrower than the effective width of the slit. Now the lines chiefly concerned in this discussion are the Balmer lines of hydrogen and the *H* and *K* lines. The widths of these lines are considerably greater than the effective widths of the slit employed, .15 mm. being equivalent to 0.8 Å. We are therefore safe in concluding that the employment of different slit widths to accommodate the uniformity of maximum deflection is legitimate. To test the effect experimentally, an identical section of a spectrogram was analysed five times with the slit width set at .12, .14, .16, .18 and .20 mm. respectively. Figure 1 represents

Fig. 1. Moll tracings of an identical section of a spectrogram made at different slit widths of the thermo-couple.

such an analysis. It is evident that, as far as can be detected, the depths of the lines, including the narrow *K* line as well, remain practically the same in all the tracings. This further proves that errors of line depths arising from this source are negligible. Incidentally, the exact reproduction of the tracings even to minor details, as shown by Figure 1, speaks well for the accuracy of the Moll microphotometer.

One more important factor remains to be considered. It was noted that the line depth depends to a marked degree also on the speed of the run. The effect is clearly shown in Figure 2, where the tracings of a section of spectrogram identical with that used in Figure 1 were made with different lengths of time, as indicated. It is to be noticed that although the continuous background remains unaltered the lines are shallower at the greater speed, as evidenced by the slope of the dotted line in the figure. This is to be expected since the response of the thermo-couple to incident radiation is not instantaneous. It is clear

then that the true strength⁶ of a line will be approached asymptotically with decrease of speed to zero. Of course to run the tracing at a very slow speed would be impracticable. A compromise therefore had to be made between attaining the truer strength of the lines and the employment of not too slow a speed. Finally the middle speed of Figure 2 was adopted throughout

Fig. 2. Moll tracings of an identical section of a spectrogram made at different lengths of time.

the analysis. Fortunately, the deviation from the true line strength does not vitiate the relative values from different tracings, therefore the intercomparison of line strengths is still valid, provided of course that the speed of all the runs was the same. The accomplishment of maintaining equality of speed would have been a simple matter had the motor run true to form. Actually the one used varied in speed a good deal from day to day and sometimes even changed speed during a run, because of some sudden fluctuation of load on the main power line. Therefore checking of the speed at frequent intervals by noting the speed of the crank handle with the aid of a watch had to be resorted to. Any change thus detected could be brought back to normal by means of a rheostat. It was only through such checkings that constancy of speed throughout the investigation was assured. In consequence of this speed effect it is to be remembered that the line strengths presented in this paper are not absolute. However, it is believed that they are not far from their true values.

The Moll Tracings.—The procedure of securing a Moll tracing will now be described. After the plate was set in place and adjusted for parallelism, focus, etc., it was cranked out to one side so that the analysing beam fell on a clear section near the red end of the spectrogram where the tracing was to begin. The beam was then cut off by the insertion of a diaphragm

⁶ By this we mean of course the strength as registered by a spectrogram obtained with the spectrograph of our dispersion and resolution, for it is well known that line strength, other things being equal, depends on the dispersion and resolution employed. It should be remarked here that errors affecting the line depth arising from unsteady seeing, imperfect guiding and poor focus have been demonstrated by the Harvard observers to be of negligible importance, especially in the case of wide lines such as those here considered.

Fig. 4. Microphotometer Tracings of Typical Stellar Spectra.

Fig. 4 (Continued)—Microphotometer Tracings of Typical Stellar Spectra.

in its path. The galvanometer spotlight accordingly registered the zero point, or darkness, the position of which was adjusted. The diaphragm was then removed. The spotlight registering the clear film could now be put at a standard maximum deflection mark by regulating the thermo-couple slit. In practice, however, the slit was left undisturbed as much as possible. It was found that more deflection could also be obtained conveniently by throwing the image of the analysing beam on the thermo-couple slit with the condensing lens a little out of focus. This may seem strange at first, but remembering that it is the thermal image and not the visual image which produces the most effect on the thermo-couple, this is what is to be expected. In case the maximum deflection could not yet be secured with this thermal focusing, the slit was widened until the maximum mark was reached. For too much deflection the beam was put out of focus in the other direction. In all the tracings the distance from the zero point to the maximum deflection mark was kept at 10 cm.

Everything being adjusted for a run, the motor was started and the beam covered up for a while to enable the spotlight to trace the initial zero point on the print. The diaphragm was then again removed. The beam now still falling on the clear plate caused the galvanometer light to register the clear film mark. Then the spectrogram approached, to have its curve traced on the print. During the run the speed of the motor was constantly checked as has already been noted. At the end the darkness and clear film marks were once more impressed on the print, and, in addition, the six standard exposures were also registered. These eight markings thus appearing at the left margin constitute the photometric scale of the tracing. They correspond to arbitrary intensity steps of 0, 1, 2, 4, 8, 16, 32 and ∞ . The average time of securing a tracing in this manner amounts to about 15 minutes. One such print is shown in Figure 3. Altogether 119 tracings were obtained from the slitless plates, and 49 from the slit ones, for 90 stars, not counting the

Fig. 3. Microphotometer Tracings of a Spectrum of β Ursae Majoris.

Cepheids; 27 tracings from slitless observations for RT *Aurigae*, η *Aquilae*, S *Sagittae* and T *Vulpeculae*; and a total of 49 tracings from slit plates comprising a complete series for ζ *Geminorum*. (As mentioned before this series will be deferred for discussion in a later paper.)

Measurement of the Tracings.—With the aid of the intensity scale, the continuous background spectrum was measured at ten different wave-lengths. In this discussion, however, only the values (J'_λ) at $\lambda 4000$ and $\lambda 4800$ will be utilized for color study. These J'_λ 's reduced to outside the atmosphere give a ratio $\frac{J'_{\lambda 4000}}{J'_{\lambda 4800}}$

which will be denoted by C . This is in effect a color equivalent. The line and band intensities were reduced as follows. The intensity at the center of a line or band was noted. Then that of the corresponding continuous background, had the line or band not existed, was estimated and measured. This is quite an easy procedure as it is always possible to draw a smooth curve over the line or band joining the adjacent continuous backgrounds. The background for the continuous hydrogen absorption is, however, more difficult to estimate and its location must be considered largely a matter of judgment. The difference in intensity between the background and the line or band gives the depth or strength of these members and when this is expressed in terms of the background it measures the percentage loss of light. This method of expressing the strength of lines, etc., has the well known advantage that the variation of plate intensity with wave-length will not enter in, and therefore lines at different wave-lengths can be intercompared and averaged for statistical studies. In the early type stars four hydrogen lines were usually measured, $H\beta$, $H\gamma$, $H\delta$ and the line at the point where the continuous absorption begins. This line is usually, though not always, $H\kappa$. Other measurements include the intensity of the continuous hydrogen absorption whenever present, the H and K lines, the G band, and the cyanogen band whose head lies at $\lambda 3883$. All will be discussed in the following pages.

RESULTS AND DISCUSSION

For illustration, a set of tracings of typical stars (mostly giants) of the spectral sequence from Oe5 to K5 was chosen and is given in Figure 4. The enlarged photographs of the spectra of these stars are also shown in Plate I. In Plate II we have similar photographs of the spectra of three Class Be stars, Moll tracings of two of which have already been published,⁷ and commented upon. The familiar variations of spectral characteristics from class to class can clearly be

⁷ Paper D.

TABLE I
GENERAL LIST OF STARS

Bos3	Name	Cl.	Ph.m.	C	H β	H γ	H δ	H ϵ	H ζ	R	N	G	H	K	λ 3883	n
913	ξ Per	Oe5	4.05	1.36	.19	.18	.20	.22	.20		10					2
1357	λ Ori	Oe5	3.66	1.37	.23	.18	.40	.28	.27		10					1
1366	ι Ori	Oe5	2.87	2.06	.17	.25	.15	.21	.20		10					1
1706	S Mon	Oe5	4.68	2.26	.15	.22	.32	.21	.22		10					1
5073	9 Sge	Oe5	6.29	1.67	.15	.15	.22	.06	.14		11					1
	44° 3639	Oe5	6.01	1.87	.17	.24	.25	.20	.22		10					1
5844	10 Lac	Oe5	4.91	1.86	.12	.20	.34	.29	.24		11					1
103	κ Cas	B0	4.00	1.20	.11	.20	.14	.12	.14		12					2
1339	δ Ori	B0	2.24	1.88	.16	.17	.18	.12	.16		10					2
1370	ϵ Ori	B0	1.51	1.50	.06	.20	.20	.19	.16		13.5					2
1389	σ Ori	B0	3.54	1.83	.14	.24	.24	.21	.21		10					2
1435	κ Ori	B0	1.96	1.84	.14	.18	.20	.18	.18		10					2
4225	ζ Oph	B0	2.46	1.84	.14	.20	.20	.17	.18		10					2
844	$\circ$ Per	B1	3.72	1.48	.17	.23	.25	.30	.24	.10	12					1
894	ζ Per	B1	2.69	1.12	.10	.27	.17	.14	.17	.08	13					1
910	ϵ Per	B1	2.74	1.98	.23	.26	.26	.22	.24	.08	12.5					2
1301	η Ori	B1	3.22	2.22	.14	.24	.24	.26	.22	.09	13					2
1609	β CMa	B1	1.77	1.86	.16	.26	.26	.25	.18	.11	13.3					3
5532	β Cep	B1	3.10	1.50	.22	.28	.32	.27	.27	.11	13					2
1079	ν Eri	B2	3.93	1.51	.12	.29	.30	.24	.24	.11	13					1
1303	γ Ori	B2	1.51	1.54	.18	.30	.24	.32	.26	.12	11.5					2
3476	α Vir	B2	1.02	1.33	.15	.27	.22	.27	.23	.10	11					1
122	ζ Cas	B3	3.55	1.60	.27	.25	.13	.30	.24	.20	12					1
419	ϵ Cas	B3	3.27	1.69	.25	.50	.50	.35	.40	.11	14					1
1147	π^4 Ori	B3	3.61	1.71	.20	.29	.33	.23	.26	.13	13					1
1159	π^5 Ori	B3	3.72	1.74	.20	.33	.25	.25	.24	.12	13					1
1204	η Aur	B3	3.11	1.84	.25	.44	.42	.36	.37	.23	13					1
4479	ι Her	B3	3.62	1.59	.31	.40	.47	.40	.40	.25	13					1
947	c Per	B3p	3.86	1.42	.08	.27	.30	.33	.24	.27	13					1
1375	ζ Tau	B3p	2.83	1.29	.09	.26	.25	.20	.20	.12	13					1
838	δ Per	B5	2.98	1.43	.30	.33	.37	.46	.36	.34	12					1
860	20 Tau	B5	3.90	1.39	.26	.42	.34	.38	.35	.37	14					1
1262	τ Ori	B5	3.56	1.42	.32	.50	.52	.46	.45	.36	13.5					2
1819	γ CMa	B5	3.95	1.62	.25	.40	.41	.38	.36	.33	13					1
4368	ζ Dra	B5	3.10	1.84	.27	.58	.64	.53	.50	.42	13					1
852	17 Tau	B5p	3.69	1.87	.24	.43	.42	.43	.38	.31	13.5					2
869	η Tau	B5p	2.84	1.61	.28	.40	.46	.44	.40	.40	14					1
3281	κ Dra	B5p	3.76	1.88	.13	.40	.47	.44	.36	.36	12					1
4162	τ Her	B5p	3.79	1.73	.33	.53	.56	.50	.48	.33	12					1
4548	67 Oph	B5p	3.80	1.58	.21	.26	.43	.39	.32	.27	13					1
784	ξ Tau	B8	3.70	1.35	.29	.50	.50	.40	.42	.31	13					1
1304	β Tau	B8	1.73	1.66	.23	.40	.44	.38	.36	.38	14					1
1944	β CMi	B8	3.04	1.70	.33	.51	.58	.56	.50	.35	13.5					2
2698	α Leo	B8	1.29	1.66	.47	.46	.50	.50	.48	.50	12					1
3890	β Lib	B8	2.69	1.70	.38	.46	.58	.50	.46	.47	13					1
5310	α Del	B8	3.81	1.83	.40	.62	.64	.50	.54	.42	13					1
1250	β Ori	B8p	0.29	1.52	.23	.33	.37	.33	.32	.25	14					2
4859	λ Aql	B9	3.53	1.51	.39	.61	.59	.50	.52	.41	12					1
932	ν Tau	A0	3.94	1.12	.48	.60	.58	.48	.54	.44	14			.10		2
1690	γ Gem	A0	1.93	1.67	.41	.51	.60	.58	.52	.57	14			.22		2
1732	α CMa	A0	1.58	1.65	.54	.60	.61	.49	.56	.47	12					1
	c Hya	A0	3.95	1.33	.56	.65	.64	.49	.58	.44	13			.10		1
2479	θ Hya	A0	3.84	1.23	.45	.61	.63	.55	.56	.43	14			.08		1
2930	β UMa	A0	2.44	1.88	.50	.69	.70	.64	.64	.60	16			.25		1
3117	γ UMa	A0	2.54	1.57	.54	.67	.67	.57	.61	.53	13			.15		1
4203	λ Oph	A0	3.85	1.61	.53	.65	.68	.50	.59	.50	14			.30		1
4328	ϵ Her	A0	3.92	1.77	.46	.55	.63	.55	.55	.55	13			.17		1
4500	γ Oph	A0	3.74	1.62	.50	.55	.62	.55	.56	.48	11			.06		1
5048	δ Cyg	A0	2.97	1.60	.44	.60	.69	.65	.60	.53	14			.20		1
5813	7 Lac	A0	3.85	1.61	.43	.61	.65	.55	.56	.50	14			.20		1
1478	β Aur	A0	2.07	1.26	.53	.70	.62	.50	.59	.40	13			.18		1
1482	θ Aur	A0p	2.71	1.63	.41	.54	.57	.50	.50	.50	14.5					2

TABLE I.—GENERAL LIST OF STARS (*Continued*)

Boss	Name	Cl.	Ph.m.	C	H β	H γ	H δ	H ϵ	H ζ	R	N	G	H	K	λ 3883	n
2694	η Leo	A0p	3.58	1.50	.17	.36	.40	.36	.32	.33	15			.17		1
3363	ϵ UMa	A0p	1.68	1.70	.42	.59	.63	.59	.56	.50	16					1
3371	α CVn	A0p	2.90	1.83	.42	.61	.62	.58	.56	.42	14					1
4824	γ Lyr	A0p	3.30	1.72	.31	.45	.53	.50	.45	.45	14					1
3626	α Dra	A0p	3.64	1.79	.54	.72	.74	.62	.66	.55	14			.15		1
1763	θ Gem	A2	3.70	1.43	.46	.58	.62	.56	.56	.50	13			.38		1
1886	λ Gem	A2	3.71	1.53	.47	.63	.66	.51	.57	.45	13			.42		1
3190	δ UMa	A2	3.50	1.71	.54	.61	.67	.50	.58	.45	13			.22		1
3928	γ UMi	A2	3.20	1.60	.31	.50	.56	.55	.48	.51	15			.45		1
4026	ϵ Ser	A2	3.81	1.74	.46	.56	.70	.50	.56	.45	14			.25		1
4988	ι Cyg	A2	4.00	1.32	.50	.59	.70	.55	.58	.46	13			.44		1
5320	α Cyg	A2p	1.39	1.12	.16	.19	.27	.25	.22	.40	17.5			.14		2
4581	η Oph	A3	3.81	1.61	.38	.52	.62	.42	.49	.45	13			.30		1
5062	α Aql	A5	1.03	1.45	.38	.55	.50	.44	.47	.44	13			.36		1
3722	γ Boo	F0	3.28	1.33	.57	.62	.57	.47	.56	.20	13			.68		1
2008	α CMi	F5	0.90	1.04	.20	.35	.36	.27	.30		12		.50	.50		1
3758	μ Vir	F5	4.37	1.05	.25	.40	.44	.48	.39		13		.71	.62		1
3105	β Vir	F8	4.30	0.90		.25							.58	.64		1
1246	α Aur	G0	0.77	0.89	.17	.15						.30	.55	.60	.27	1
3596	η Boo	G0	3.36	0.78	.30	.18						.24	.72	.78	.37	1
4192	η Dra	G5	3.67	0.68								.22	.78	.79	.59	1
3662	α Boo	K0	1.24	0.53								.47	.80	.82	.55	1
4541	γ Dra	K5	3.60	0.33								.38	.80	.87	.66	1

seen both in Figure 4 and in Plate I. In the former, however, the effects are more vivid, particularly in respect to the details, which are not so well brought out in the latter. The changes in the ultra-violet region are especially noteworthy.

The chief features of the hydrogen spectrum, both the line and the continuous absorption, which were studied⁸ earlier by the writer, again reveal themselves here, but with greater beauty and force, as is to be expected from the more refined method of analysis. Special attention may here be directed to the intensity phenomenon of the continuous hydrogen absorption as exhibited in Plate I. Its variation in strength from class to class makes the ultra-violet ends of these spectra trace out a reverse S figure.

It will be seen from Figure 4 that the continuous hydrogen absorption makes its first appearance as early as B1, and not at B3 as was concluded in the paper just cited. In one or two instances a trace of it can be detected even in B0 stars. However, the majority of these stars do not show the absorption.

The various points just mentioned will now be taken up in more detail. All the measures from the slitless observations reduced in the manner indicated in the foregoing section are collected for discussion in Table I. The table is arranged in the order of spectral class beginning with Oe5. The first four columns give successively the Boss number, name of star, the Harvard Class and the photographic magnitude. Column 5 contains the color equivalent C, which is, as already

noted, the ratio $\frac{J'\lambda 4000}{J'\lambda 4800}$. Both of the J' 's are corrected

⁸ Paper A.

TABLE II
CLASS BE STARS

Boss	Name	Sp. Cl.	Ph. m.	C	H β	H γ	H δ	R	n
199	γ Cas	B0e	2.01	1.54	1.45	1.10		1.10	1
384	ϕ Per	B0e	3.95	1.62	1.41	.14		1.12	2
5410	f' Cyg	B0e	4.62	1.54	1.07	.24	.24	1.06	2
5208	P Cyg	B1e	4.66	0.95	1.78	1.42	1.33		3
4183	χ Oph	B3e	4.68	1.56	1.44		.17	1.22	3

for atmospheric absorption. In the next four columns we have the strength of four hydrogen lines expressed in terms of their respective backgrounds. The line at the beginning of the continuous absorption is designated H δ_b . The following column, H ϵ_m , represents the mean strength of these lines. Column 11, headed by R, gives the intensity of the continuous hydrogen absorption measured at the wave-length where it has the greatest value. N denotes the number of hydrogen lines visible. Where a fractional part of a line occurs, it stands for the average of two or three observations. The remaining columns, except the last one, contain the intensities of the lines and bands indicated. The last column, n, gives the number of observations.

Table II contains similar data for a few Class Be stars. The total radiation of the bright hydrogen lines and of the continuous emission at the head of the Balmer series is here again expressed in terms of the estimated background. These intensities are therefore greater than unity.

(1) *Relation between Color Equivalent and Spectral Class.*—It must be emphasized here that in Table I are embodied materials of observations taken over a period of several years, with different silver surfaces

Typical Stellar Spectra in the Ultra-Violet

(a) Spectra of 3 Be Stars

(b) Spectra of RT Aurigae

(c) Spectra of η Aquilae

(d) Spectra of S Sagittae

(e) Spectra of T Vulpeculae

of the Crossley mirror. For this reason the material may not be considered strictly homogeneous, since our color equivalent (C) is liable to be affected by a possible variation in the reflectivity of the different surfaces, and therefore too much significance should not be attached to the individual values of C. However, errors from this source are only differential. It is believed that if present they will not affect the general results derived from a statistical study in which the errors will tend to average out. Fortunately from the way the intensity is expressed, the other quantities of Table I are free from this difficulty.

A plot of the color equivalent against spectral class taken from Tables I and II is shown in Figure 5. The

Fig. 5. Relation between Color Equivalent and Spectral Class.

general increase of color with the advance of spectral class is well indicated. It will be seen that among the early type stars of which the observations are most numerous there is considerable dispersion. Part of this could be attributed to errors of observation. The evidence seems strong, however, that this dispersion, or, at least a greater part of it, is real. The heavy line in the figure represents the mean of the observations. The bright line B-type stars (indicated by 0) were not used in taking the mean, since it was found⁹ that these stars exhibit lower color temperatures than normal stars of the class. It is interesting to note that in spite of this precaution, the mean color curve from B0 to B5 is still lower than those that were to be expected from our common knowledge of its position. In other words the mean color equivalent seems to have reached a maximum at B8. Above B8 it drops a little or remains constant from B5 to B0. Then it increases again toward Oe5. Several of the Oe5 stars also show exceptional color equivalents. It will be noticed from the figure that all the bright line stars

⁹ Paper D.

except one fall below the average color curve, a point in substantial agreement with their lower color temperatures just mentioned. *P Cygni* and ζ *Persei* too, as indicated, show exceptional color equivalents. The extraordinarily low figure of the former was noted and discussed by the writer,¹⁰ while that of the latter was found and called attention to by Davidson and Greaves.¹¹

Another interesting point brought out by the figure is that the five stars with "c" characteristics (indicated by x), namely, ζ *Tauri*, 67 *Ophiuchi*, β *Orionis*, η *Leonis*, and α *Cygni*, are all found to possess much lower color temperatures than those common to normal stars of their classes. This again is to be expected¹² from the ionization theory in consequence of the great luminosity peculiar to stars of this character.

(2) *Relation between Mean Strength of Hydrogen Lines and Spectral Class.*—Figure 6, which shows this

Fig. 6. Relation between Mean Strength of Hydrogen Lines and Spectral Class.

relation, was obtained by plotting column H_m of Table I against spectral class. The full line again represents the mean values. It is evident that the strength of the hydrogen lines attains maximum at A0 and gradually falls off toward both the earlier and later classes. We note also that the "c" stars as a special group once more refuse to fall in line with the rest. The hydrogen lines of these stars appear to be unusually weak. This may be in part explained by their extraordinary sharpness, as a result of which their line depths are liable to suffer diminution through observational errors like poor seeing, imperfect guiding, etc. There seems to be no doubt however that

¹⁰ Paper D. Gerasimovic (*Harv. Bull.*, 857, 15, 1928) has since confirmed the low color temperature of this star and obtained a figure of 7700°.

¹¹ *M. N.*, 87, 352, 1927. *Ibid*, 88, 703, 1928.

¹² Cf. Russell-Dugan-Stewart "Astronomy," p. 873.

the general character is real. It may be of interest to note that this weakness of lines, together with the lower color equivalents of these stars noted above, is in good accord, qualitatively at least, with the intensity-magnitude-temperature correlation given by the writer elsewhere.¹³

The *Pleiades* stars of which three are available in this discussion, viz. η *Tauri*, 17 *Tauri*, and 20 *Tauri*, (denoted by $\oplus$ in Figure 6) also exhibit shallower lines, a fact in harmony with the tendency of these stars to produce emission lines as recently discussed by Miss Payne.¹⁴

(3) *Relation between the Strength of the Continuous Absorption and the Mean Strength of the Hydrogen Lines.*—It was stated previously¹⁵ that the intensity of the continuous absorption follows closely the strength of the Balmer series lines. This statement is fully borne out by a plot of R against H_m from Table I, as appears in Figure 7. The straight line relation is

Fig. 7. Relation between the Strength of the Continuous Absorption and the Mean Strength of the Hydrogen Lines.

good, considering the difficulty in estimating the background for the continuous absorption. The approximately 45° inclination of this line also indicates that the rate of increase and decrease of the two quantities is about the same. Of course the line should not be expected to pass through the origin, for at B0, where the continuous absorption does not exist, the hydrogen lines though weak are still of appreciable intensity.

¹³ Paper B.

¹⁴ *H. C. O. Cir.*, 303, 1927.

¹⁵ Paper A.

(4) *Relation between the Intensity of K, of $\lambda 3883$ Cyanogen Band, and the Spectral Class.*—In Figure 8

Fig. 8. Relation between the Intensity of K, of $\lambda 3883$ Cyanogen Band, and the Spectral Class.

first appearance near B8, and rapidly increases in intensity until the maximum is attained at about K0, these two quantities taken from Table I are plotted against spectral class. The observations here are few and scattered. Nevertheless, the familiar variation of K and $\lambda 3883$ is clearly shown. The K line makes its where the line strength is about 0.9 of the continuous background. The band $\lambda 3883$ first becomes visible at F8 and is strengthened down the sequence.

(5) *Relation between Luminosity and the Number of Hydrogen Lines Visible.*—According to the Bohr theory of the hydrogen atom, the higher quantum orbits give rise to the higher members of the Balmer series lines. It is consequently to be expected that stars of higher luminosity, and, on the assumption of equal mass, therefore of lower pressure which permits the existence of higher quantum orbits, should exhibit more Balmer lines. This correlation has been the subject of a statistical study at Harvard by Payne and Howe.¹⁶ By plotting the number of hydrogen lines observable against the reduced proper motions of over 200 Class A stars they obtained a diagram in which the existence of such a relation is indicated though the dispersion is considerable, due, as pointed out, to the dispersion of the proper motion among Class A stars.

It is of interest to inquire whether a similar relationship is shown by the data here available. Among the list of Class A stars (B5 to F5) of Table I, there are 39 whose trigonometric parallaxes are more or less accurately known. Hence their absolute magnitudes can be derived. This number, though small, affords

¹⁶ *H. C. O. Cir.*, 287, 1925.

some material for such a study. A plot of the absolute magnitude against the number of hydrogen lines visible, counting from $H\beta$ (column N of Table I), is shown in Figure 9. The expected correlation was not

Fig. 9. Relation between Luminosity and the Number of Hydrogen Lines Visible.

evident until stars of each spectral class were grouped together and averaged, as shown by the three full lines in the figure. This figure is essentially the same as that given by the Harvard investigators. It is difficult to see, from the general consideration mentioned above, why there should be a correlation curve for each class of stars. However, the fact that these curves fall in regular order in the diagram suggests that it may be an effect of mass. For if the mass is taken into account according to the mass-luminosity law, this discrepancy comes out of the right order and in the right direction.

THE CEPHEID VARIABLES

The results of the measurements from the Cepheid observations are embodied in Table III. The first five columns give successively the Boss number, the name

of star, the range of visual magnitude, the period and the phase at which the observations were taken. The rest of the table has the same significance as the corresponding part of Table I. All these observations were secured within a short time. Their homogeneity is thus assured. It will be noted that for *RT Aurigae* observations are more numerous and well scattered throughout the period. For each of the others observations covering only three phases are available. Fortunately, there is always one near the maximum and one near the minimum light phase of these stars. Therefore the variation of spectral characteristics from phase to phase can in general be safely traced. A plot of the relevant quantities in Table III against phase for each of the stars listed is provided in Figures 10, 11, 12, and 13. The approximate light curve is added in the upper part of each figure to facilitate comparison. The characteristic spectral changes brought out by these figures are so similar that it is of advantage to discuss all four stars together.

(1) *Changes of Color*.—Figures 10–13 show that the color equivalent C of these four Cepheid variables changes in a clear fashion with phase. The color corresponding to the highest temperature coincides in every case with the brightest phase, and the color corresponding to the lowest one coincides invariably with the minimum light. This variation of temperature with phase is a familiar phenomenon. That for ζ *Geminorum* has already been discussed by the writer¹⁷ in a preliminary paper.

(2) *Behavior of lines*.—Unlike the peculiar variation of the $H\gamma$ line in the spectrum of ζ *Geminorum*,¹⁸ the behavior of the hydrogen lines in all four of the stars under discussion confirms the characteristic changes of their intensity, as first noted by Shapley,¹⁹ that is, the strength of the hydrogen lines in Cepheids decreases

¹⁷ *Publ. A. S. P.*, **38**, 357, 1926. This will be referred to again as Paper C.

¹⁸ Paper C.

¹⁹ Shapley, *Ap. J.*, **44**, 273, 1916.

TABLE III

THE CEPHEID VARIABLES															
Boss	Name	Vis. Mag.	Period	Phase	C	$H\beta$	$H\gamma$	H δ	H ϵ	G	H	K	$\lambda 4217$	$\lambda 3883$	n
1629	RT Aur	5.0–5.9	3.473	0.63	0.68	.35	.45	.42	.40	.11	.70	.75	.08	.00	1
				1.09	0.65	.22	.48	.26	.32	.20	.78	.80	.11	.00	1
				1.63	0.58	.16	.38	.32	.28	.36	.77	.81	.12	.15	1
				2.09	0.55	.22	.30	.19	.24	.36	.68	.71	.14	.24	2
				3.07	0.62	.23	.36	.28	.29	.36	.64	.65	.25	.19	2
5071	η Aql	3.7–4.3	7.18	0.42	0.58	.17	.42	.35	.31	.19	.79	.81	.12	.00	3
				1.41	0.54	.14	.38	.36	.29	.28	.75	.78	.24	.00	2
				5.25	0.37	.26	.30	.22	.26	.37	.70	.72	.26	.36	2
5098	S Sge	5.4–6.1	8.38	4.70	0.32	.20	.29	.24	.24	.37	.82	.90	.24	.40	2
				5.68	0.32	.17	.16	.14	.16	.38	.78	.85	.26	.46	2
				8.30	0.52	.36	.26	.26	.29	.14	.71	.73	.12	.00	2
5370	T Vul	5.5–6.4	4.43	2.00	0.50	.14	.30	.16	.20	.22	.71	.71	.11	.21	2
				3.31	0.50	.25	.25	.22	.24	.36	.72	.78	.20	.28	2
				4.29	0.72	.22	.34	.33	.30	.12	.67	.73	.10	.00	2

Fig. 10. Spectrum Intensity Curves of RT Aurigae.

Fig. 12.—Spectrum Intensity Curves of S Sagittae.

Fig. 11. Spectrum Intensity Curves of η Aquilae.

Fig. 13. Spectrum Intensity Curves of T. Vulpeculae.

Fig. 14. Microphotometer Tracings of Spectra of 4 Cepheid Variables.

from maximum to minimum light. This variation in η *Aquilae* is slight but definite. The effect is most pronounced in RT *Aurigae*.

The behavior of the H and K lines does not appear so consistent. For RT *Aurigae* these lines seem to reach a maximum strength at $\frac{1}{4}$ period following maximum phase and to fall to minimum at $\frac{1}{4}$ period following minimum light. This peculiar phenomenon is in partial agreement with what was observed in case of ζ *Geminorum*. In S *Sagittae* and in T *Vulpeculae* the maximum strength of the H and K lines falls exactly at minimum light, and just the opposite effect is shown in the case of η *Aquilae*. However, in view of the scantiness of observations for the latter stars, these apparently conflicting behaviors of the H and K lines may have no significance.

(3) *Behavior of Bands*.—Both the G and the $\lambda 4217$ bands are found to increase in intensity from maximum to minimum light, as is to be expected from temperature change considerations.

The appearance and strengthening of the $\lambda 3883$ band from maximum to minimum phase which were first noted in ζ *Geminorum*²⁰ again occur in all the stars here considered. Some idea of the variation of intensity of this band can be gained by referring again to Figures 10–13. To show this effect as well as others noted above, two prints, one made from an observation near maximum light and the other made from an observation near the minimum, were chosen for each star and are reproduced in Plate II, b–e. The behavior of the $\lambda 3883$ band at once stands out clearly. This can also be seen in the microphotometer tracings from these observations shown in Figure 14.

(4) *Changes in Spectral Type*.—Changes of spectral type of these Cepheid variables have been estimated by Shapley.²¹ By comparing the microphotometer tracings of these stars with those (Fig. 4), of the typical stars of the spectral sequence, the change of spectral type from maximum to minimum was estimated. These estimates appear in the second column of Table IV. The spectral classes can also be obtained quanti-

tatively by comparing the mean strength of the hydrogen lines (H_m) of these stars with Figure 6, in which the average curve of the H_m for all the stars in the spectral sequence is shown. The types thus derived are listed in the third column of the table. For comparison those estimated by Shapley are also entered. It will be seen that the agreement from these three estimates is quite satisfactory.

SUMMARY

One hundred and forty-six slitless spectrograms secured with the two-prism quartz spectrograph attached to the Crossley Reflector for 90 stars, mostly of Classes B and A, and four Cepheid variables, were analysed with the Moll microphotometer. The plates were standardized photometrically by means of a set of standard exposures. While the method employed has limitations, it is believed that for our purpose the general results obtained are fairly trustworthy.

The working of the Moll microphotometer as used here is described and some of the instrumental effects are discussed. It was found that the line depth is independent of the thermo-couple slit width within the range used, whereas it is strongly affected by the speed of a run.

Spectral intensity measurements were made of: (a) the continuous background spectrum at ten different wave-lengths, (b) four hydrogen lines, (c) the continuous hydrogen absorption whenever present, (d) the H and K lines, (e) the G band, and (f) the 3883A cyanogen band.

A study of the foregoing quantities yielded the following results:

(1) The stellar temperature increases from the lower spectral sequence up to B8 where it seems to reach a maximum. It then decreases a little from B5 to B0 before increasing again toward the upper sequence Oe5. This drop of temperature among the Class B stars was unexpected. However, owing to the limited observational material the reality of this phenomenon must await confirmation.

(2) The strength of the hydrogen lines attains a maximum at A0 and gradually falls off toward both the earlier and later classes. The "c" stars and the *Pleiades* stars exhibit shallower lines.

(3) The intensity of the continuous hydrogen absorption follows closely the strength of the Balmer series lines, a result in agreement with that previously obtained.

(4) The K line makes its first appearance at about B8, rapidly increases in intensity until the maximum is reached at about K0, at which the line strength is 0.9 of the continuous background.

TABLE IV

CHANGES IN SPECTRAL TYPE OF THE CEPHEID VARIABLES

Star	Spectral class		Shapley
	This paper		
	From general spectral characteristics	From strength of hydrogen lines	
RT Aurigae	F8-G0	A8-G0	A8-G0
η Aquilae	F5-G5	F6-F2	A8-G5
S Sagittae	F5-G5	F6-F2	F4-G3
T Vulpeculae	F5-G0	F5-G0	A9-G1

²⁰ Paper C.

²¹ Shapley. *Loc. cit.*

(5) The 3883 cyanogen band first becomes visible at F8 and is strengthened through the sequence.

(6) A correlation was found between the luminosity and the number of hydrogen lines visible as is to be expected from the modern theory of the hydrogen atom.

(7) The variation of the spectral characteristics with phase of the four Cepheid variables here investigated may be summarized as follows: (a) The color of these stars corresponding to the highest temperature coincides in every case with the brightest phase, and the color corresponding to the lowest temperature coincides invariably with minimum light. (b) The strength

of the hydrogen lines decreases from maximum to minimum light. (c) All spectral bands are found to increase in intensity from maximum to minimum light. (d) The 3883 cyanogen band disappears during maximum phases in all cases.

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