LICK OBSERVATORY BULLETIN

NUMBER 418

 β
 A SPECTROGRAPHIC STUDY OF β CEPHEI

BY

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INTRODUCTION

β Cephei ($\alpha = 21^{\text{h}} 27^{\text{m}} 4^{\text{s}}$; $\delta = +70^{\circ} 07'$; 1900.0) is described in the *Henry Draper Catalogue*¹ as of visual magnitude 3.32 and spectral class B_1 . It was found to have a variable radial velocity by Frost and Adams,² in 1902, from observations secured with the Bruce spectrograph of the Yerkes observatory. From eight plates obtained in May, 1906, Frost³ found the period to be about four hours and thirty minutes. A velocity curve³ was obtained by him from plates taken on July 6 and August 27 of 1906 which led him to conclude that the eccentricity of the orbit must be small and that the two bodies must be revolving nearly in contact with each other.

A study of β Cephei was made by C. C. Crump⁴ at the University of Michigan in 1912, using a single prism spectrograph that gave a linear dispersion of 44 Å/mm. He determined the period, by means of Frost's observations, together with his own, to be accurately $4^{\text{h}} 34^{\text{m}} 17^{\text{s}} \cdot 4$, on the average, for the six year interval preceding 1912. The other orbital elements were determined by Crump from his own velocity curve by a careful least squares solution. He also determined definitive elements for an anomalous velocity curve of Oct. 1, 1912, and concluded that the change in amplitude of the curve at that epoch must be real. The velocity of the center of mass of the system was shown to be changing by a comparison of his value with the two previously obtained by

Frost. In addition to the elements for an elliptical orbit, Crump found it necessary to postulate a secondary oscillation in the velocity in order to account satisfactorily for the observed velocity curve.

But few years have passed since 1912 during which β Cephei has not been made the subject of a special investigation at some one of the observatories equipped for radial velocity work. Velocity curves and, in most cases, orbital elements were determined at Poulkovo in 1914, 1916, and 1917 by Gerasimovic⁵ and Belopolsky⁶, at Ottawa in 1919, 1920, 1922, and 1923 by Boothroyd and Van Arnam,⁷ Henderson, and Henroteau,^{8, 9} and at the Lick Observatory in 1918, 1921, 1926, 1927, and 1928 by Henroteau, Miss Hobe, and the writer.

The discovery of the variation in light of β Cephei was made by Guthnick¹⁰ at Berlin-Babelsberg from photo-electric measures made in August and September, 1913, by Guthnick and Prager. The observations were continued in January and February, 1914,¹¹ and in January and February, 1917.¹² The range of variation in light was found to be about 0.05 magnitude in a period equal to that for the velocity variation. The light curves obtained in February, 1917, indicated that the variation might not be identical for different epochs.

⁵ *Bull. de l'Obs. Central de Russie*, Poulkovo, 7 No. 80.

⁶ *Bull. de l'Academie des Sciences de Russie*, 1918.

⁷ *Pop. Ast.* 36, 347, 1928.

⁸ *Pub. Dom. Obs.* 5, 77, 1922.

⁹ *Pub. Dom. Obs.* 9, 36, 1925.

¹⁰ *A. N.* 4701, 1913-14.

¹¹ *Berlin-Babelsberg*, 1, 23, 1914.

¹² *Berlin-Babelsberg*, 2, 78, 1918.

¹ *H. A.* Vol. 99.

² *Ap. J.* 15, 340, 1902.

³ *Ap. J.* 24, 260, 1906.

⁴ *Pub. Detroit Obs.* 2, 144, 1916.

A photometric study of β Cephei was made by the photo-electric method by Miss E. E. Cummings¹³ in 1921 at the Lick Observatory. She determined the mean light curve for the summer of 1921 and compared it with those previously obtained by Guthnick and Prager. She also made four comparisons of photometric with spectrographic data, which showed that the light maximum occurs (in the mean) about 0.033 day before the maximum velocity of approach, or about 0.015 day after the time of maximum contraction on the pulsation hypothesis. The mean curve, with a secondary maximum on the ascending branch, is found to be very similar in shape even when derived at intervals of several years. The disparity between individual curves observed at separate epochs, noted by Guthnick and Prager, was again pointed out by Miss Cummings.

The variation in light of β Cephei was studied by Miss M. Gussow¹⁴ in 1925 at Berlin-Babelsberg. The mean curve found by her shows a secondary maximum on the descending branch instead of the ascending branch as found by previous observers, proving that the mean light curve is not constant in form.

β Cephei has been taken by Ludendorff¹⁵ as the typical star of a small group of variables of early spectral type with small light changes, to form a subdivision of the δ Cephei type variables because of the similar relation of the phase of the light curve to that of the velocity curve. The underlying cause of this variability and phase relation is probably very similar for these stars.

Approximately a thousand spectrograms of β Cephei have been obtained since 1900 but as yet its behavior has not been completely determined nor satisfactorily interpreted and explained. Previous investigators have drawn the following conclusions: (1) The radial velocity varies with a constant period of 0.1904795 day. (2) The amplitude of the velocity variation ranges from 20 to 40 km./sec. with (3) a period of the order of a month. (4) A secondary oscillation exists in the radial velocity curve relative to simple elliptical motion having one-third of the period and one-tenth of the amplitude of the principal variation. (5) The velocity of the system, or of the center of mass, varies from one revolution to the next and from year to year. (6) A periodic variation in luminosity exists, ranging from 0.04 to 0.07 magnitude, with a phase

lag for luminosity relative to positive radial velocity ranging from approximately one-fourth to one-half of the entire period.

The present investigation comprises an examination of existing data and conclusions concerning β Cephei and a collection and discussion of new material. Three general topics will be considered, namely, the variation of the orbital elements as derived on the binary hypothesis, the effect of stellar atmospheric level to be expected on the pulsation hypothesis, and a test of a proposed spectrophotometric method for determining the light variation at any point in the spectrum, including the ultraviolet region.

RADIAL VELOCITY OBSERVATIONS

Lick Observatory observations.—The majority of the observations of β Cephei obtained by the writer, and all earlier observations of this star secured at the Lick Observatory, were made with the new Mills spectrograph attached to the 36-inch refractor. This spectrograph contains three dense flint prisms set for minimum deviation at $\lambda 4500$ Å. With the camera of 21 inches focal length that was used, the linear dispersion at $\lambda 4500$ Å is 10.9 Å/mm. The instrument is furnished with a curved slit and a set of small lenses and reflecting prisms for introducing the comparison spectra. A full description of the new Mills spectrograph is given in *Publ. Lick Obs.*, Vol. 16, page ix. A titanium spark was used for the comparison spectra, a part of the spectrum being introduced at the beginning of the exposure, a part at the middle, and a part at the end, in order to eliminate the effects of any possible instrumental changes occurring during the exposure. The slit was set at a width of 0.0010 inch, which necessitated an exposure of from 25 to 40 minutes on Eastman 33 plates for this star.

The data for the 119 Lick observations obtained with the new Mills spectrograph in the years 1903 to 1928 are collected in Table 1. Columns one and two contain respectively the Greenwich Civil Time, and the corresponding Julian date of the observation. The fourth column gives the radial velocity of the star relative to the Sun. The number of lines upon which the determination of the velocity depends is quoted in column five, and the initials given in column six refer to the one who measured the spectrogram. They have the following significance: Cs, H. D. Curtis; Ht, F. Henroteau; H, Miss A. M. Hobe; and Md, H. S. Mendenhall.

¹³ *L. O. B.* 11, 115, 1923–24.

¹⁴ *A. N.* 5484, 201, 1925.

¹⁵ *Handbuch der Astrophysik*, 7, 222.

TABLE I
LICK OBSERVATIONS OF β Cephei WITH THE NEW MILLS III-PRISM SPECTROGRAPH

Date G. C. T. 1903	Jul. Day 241+	Phase	V	No. of lines	Mens. by	Date G. C. T. 1926	Jul. Day 242+	Phase	V	No. of lines	Mens. by
Sept. 16	6374.875		- 6.1	7	Cs	July 1	.949	.155	+ 6.3	14	Md†
			- 5.3	13	H		.989	.005	- 4.6	18	Md†
			- 8.7	15	Md*		4699.011	.027	- 7.8	14	Md†
20	6378.962		- 4.4	6	Cs	6	4704.006	.070	-19.0	14	Md†
			- 3.2	13	H	12	4709.785	.134	+ 6.0	14	Md†
			- 3.7	14	Md*		.808	.157	+ 1.8	12	Md†
1918	242+						.833	.182	+ 0.1	17	Md†
May 15	1729.851		+11.7	?	Ht		.856	.015	-13.8	16	Md†
			+12.2	11	H		.880	.039	-17.1	16	Md†
			+ 8.6	14	Md		.903	.062	-23.7	14	Md†
	.884		+ 6.2	?	Ht		.926	.085	-14.4	16	Md†
			+ 8.0	12	H		.956	.115	- 2.9	16	Md†
			+ 3.3	17	Md		.982	.141	+10.5	11	Md†
	.908		- 3.5	?	Ht		4710.008	.167	+ 9.9	14	Md†
			- 2.7	13	H	Aug. 8	4736.728	.029	-11.5	20	Md
			- 5.8	13	Md		.756	.057	-19.0	17	Md
	.931		-14.5	?	Ht		.790	.091	-14.6	11	Md
			-13.5	11	H		.859	.160	+ 7.3	20	Md
			-17.5	13	Md		.895	.006	+ 1.7	20	Md
	.954		-21.4	?	Ht		.923	.034	- 9.9	19	Md
			-22.4	12	H		.949	.060	-16.9	19	Md
			-26.6	14	Md	Aug. 11	4739.744	.187	- 1.8	14	Md
	.985		-19.1	?	Ht		.775	.028	- 8.2	16	Md
			-21.5	13	H		.804	.057	-20.1	18	Md
			-24.7	18	Md		.834	.087	-17.5	13	Md
1921							.874	.127	+ 4.7	16	Md
Nov. 6	3000.573		+ 9.2	11	H		.901	.154	+ 9.2	16	Md
	.586		+11.6	11	H*		.927	.180	+ 7.1	15	Md
	.596		+10.2	11	H*	12	4740.826	.127	+ 4.7	15	Md
	.610		+ 2.9	12	H*	21	4750.023	.181	+ 5.3	13	Md
	.631		- 6.9	14	H	24	4753.018	.128	+ 4.9	15	Md
	.648		-14.5	12	H	Sept. 12	4771.890	.142	+ 7.5	15	Md
	.666		-17.9	12	H*		.922	.174	+ 6.8	16	Md
	.684		-14.3	12	H*		.955	.017	- 3.5	14	Md
	.702		-13.1	13	H*		.989	.051	-16.8	14	Md
	.720		- 5.6	12	H		4772.024	.086	-13.8	16	Md
	.738		+ 2.5	13	H	14	4773.838	.186	+ 5.2	19	Md
	.760		+ 9.9	13	H*		.858	.016	- 5.1	14	Md
	.778		+12.3	13	H		.961	.119	- 0.4	10	Md
	.794		+ 9.1	11	H		.982	.140	+12.0	11	Md
	.811		+ 1.1	14	H	Oct. 7	4796.723	.023	- 7.7	12	Md
	.827		- 1.8	13	H		.743	.043	-17.6	14	Md
1926							.765	.065	-22.6	15	Md
June 15	4682.912	.119	+ 3.2	17	Md		.791	.091	-18.0	18	Md
	.932	.139	+ 5.2	11	Md		.813	.113	- 6.9	18	Md
19	4686.916	.123	+ 1.4	16	Md		.835	.135	+ 2.7	15	Md*
23	4690.799	.006	- 0.4	11	Md		.855	.155	+12.8	16	Md
	.817	.024	-10.0	14	Md		.876	.176	+10.9	14	Md
	.844	.051	-19.4	15	Md		.898	.008	+ 3.3	11	Md
	.861	.068	-21.6	15	Md		.919	.029	- 7.9	10	Md
	.878	.085	-17.0	17	Md	Oct. 24	4813.602	.139	+11.1	12	Md
	.894	.101	-12.3	15	Md		.621	.158	+12.7	11	Md
	.910	.117	- 7.0	14	Md		.640	.177	+ 4.4	10	Md
	.927	.134	+ 4.0	14	Md		.665	.012	- 2.5	17	Md
	.944	.151	+ 7.4	14	Md*		.685	.032	-13.7	17	Md
	.963	.170	+ 7.1	15	Md		.704	.051	-21.5	15	Md
	.983	.000	- 2.0	15	Md		.723	.070	-20.0	15	Md
	4691.011	.028	- 9.6	19	Md		.743	.090	- 9.8	13	Md
July 1	4698.880	.086	-14.0	11	Md†		.762	.109	- 4.0	19	Md
	.904	.110	-10.4	18	Md†		.782	.129	+ 5.1	16	Md
	.927	.133	+ 0.9	15	Md†		.801	.148	+11.0	11	Md

TABLE I—(Concluded)

Date G. C. T. 1927	Jul. Day 242+	Phase	V	No. of lines	Meas. by
Jan. 6	4887.756	.007	+ 1.3	14	Md
	.849	.100	−13.5	18	Md
7	4888.589	.077	−14.7	18	Md
	.614	.102	− 4.2	16	Md
	.642	.130	+ 5.8	19	Md
	.675	.163	+ 3.9	20	Md
	.712	.010	− 4.5	15	Md
	.751	.049	−17.8	21	Md
1928					
Sept. 10	5500.800	.088	−16.8	14	Md
	.824	.112	− 4.5	16	Md
	.851	.139	+ 9.8	18	Md
	.878	.166	+11.3	13	Md
	.904	.001	+ 0.8	16	Md
	.938	.035	−16.0	17	Md
	.965	.062	−21.5	16	Md
	.992	.089	−13.1	15	Md

* Measured twice. † Eastman 40 plate. ‡ Poor; ½ weight.

A three-prism spectrograph was designed in 1925 by T. S. Jacobsen to give spectra extending farther into the violet than those obtained with the Mills spectrograph. The prisms are mounted in a wooden box without temperature control, but spectra of excellent definition, with a linear dispersion of 21 Å/mm. at 4200Å, have been obtained. A more complete description of this instrument has been previously given by the designer.¹⁶

Seventeen violet spectrograms of β Cephei were obtained in 1926 on Eastman 33 plates with an exposure of 20 minutes each. An iron spark spectrum was used for comparison. The data for these observations are given in Table II.

TABLE II

LICK OBSERVATIONS OF β Cephei WITH THE III-PRISM VIOLET SPECTROGRAPH

Date G. C. T. 1926	Jul. Day 242+	Phase	V	No. of lines	Meas. by
July 19	4717.011	.121	− 0.9	45	Md
28	4725.682	.031	−18.5	48	Md
	.698	.047	−21.3	49	Md
	.713	.062	−20.6	37	Md*
	.812	.161	+ 7.0	51	Md
	.829	.178	+ 1.7	47	Md
	.846	.005	− 7.8	47	Md
	.870	.029	−17.1	49	Md
Aug. 5	4734.010	.168	+ 3.7	48	Md
14	4742.832	.037	−15.7	51	Md
	.849	.054	−20.7	51	Md
	.866	.071	−15.5	50	Md
	.887	.092	− 9.9	50	Md
	.952	.157	+ 4.5	50	Md
	.970	.175	+ 6.2	48	Md
	.990	.005	− 4.5	49	Md
	4743.009	.024	− 8.4	50	Md

* Underexposed.

¹⁶ L. O. B. 12, 138, 1926.

Method of measurement and reduction.—The Mills plates have all been measured on a standard measuring engine having a screw-pitch of ½ mm. One or two settings were made on each comparison line and two or three on each star line. Each plate was measured in both the direct and reverse position in order to eliminate as far as possible the effect of personal equation. From 10 to 20 star lines and from 10 to 20 comparison lines were measured on each plate.

The wave-lengths adopted for the stellar and comparison lines are given in the third column of Table III, and are based upon recent laboratory determinations by the investigators noted in the final column. Identification of the absorbing atom for each of the stellar lines is quoted in column one, the absence of such notation indicating that the wave-length refers to a titanium line of the comparison spectrum. The measures of the spectrograms were reduced in the usual manner by means of a standard dispersion table computed from the Hartmann-Cornu dispersion formula. To the value of the radial velocity thus derived a correction was applied for the observer's velocity due to the orbital and rotational motions of the Earth. A further correction has been applied to each observed velocity for the effect of flexure in the tube of the 36-inch telescope. Data for its derivation were taken from the investigation by Jacobsen.¹⁷ This correction for β Cephei although small, ranging from +0.20 to −0.20 km./sec. for different positions of the instrument, has been used in obtaining the values of the radial velocities for the years 1926 to 1928, given in Table I. Measures of the individual lines on each spectrogram, after correction for the radial velocity obtained from the mean of all lines on the plate, were utilized for a determination of the wave-lengths of the lines in the spectrum of β Cephei. The wave-lengths thus derived are given in the second column of Table III.

TABLE III

Absorbing atoms	λ , I. A., in β Cephei	Assumed λ , I. A.	Authority
		<i>4386.858</i>	H. Crew ¹⁸
He I	4387.922	87.928	P. W. Merrill ¹⁹
		<i>95.036</i>	F. L. Brown ²⁰
O II	4395.950	95.950	A. Fowler ²¹
		<i>4411.080</i>	H. Crew
O II	4414.899	14.888	J. S. Clark ²²
O II	4416.968	16.974	J. S. Clark
		<i>17.718</i>	H. Crew
		<i>27.098</i>	F. L. Brown
He I	4437.543	37.548	P. W. Merrill
		<i>43.799</i>	F. L. Brown

¹⁷ L. O. B. 12, 138, 1926.

¹⁸ Ap. J. 56, 53, 1922.

¹⁹ Bull. Bureau Standards 14, 162, 1917.

²⁰ Ap. J. 60, 108, 1924.

²¹ Proc. Roy. Soc. 110, 476, 1926.

²² Ap. J. 40, 332, 1914.

TABLE III—(Concluded)

Absorbing atoms	λ , I. A., in β Cephei	Assumed λ , I. A.	Authority
N II, O II	4446.984	47.040	J. S. Clark
O II	4452.402	52.380	A. Fowler
		57.427	F. L. Brown
		68.493	F. L. Brown
*O II(?)	4469.642	69.320	A. Fowler
He I	4471.527	71.477	P. W. Merrill
Mg II, Mg II	4481.221	81.230	A. Fowler
		81.259	F. L. Brown
		4501.269	F. L. Brown
		12.734	F. L. Brown
		22.797	F. L. Brown
		27.305	F. L. Brown
N II	4530.493	30.421	J. S. Clark
		49.624	H. Crew
Si III, N II	4552.602	52.630	A. Fowler
		63.757	F. L. Brown
Si III	4567.844	67.872	A. Fowler
		71.970	F. L. Brown
Si III	4574.753	74.777	A. Fowler
		89.961	H. Crew
O II	4590.983	90.983	J. S. Clark
O II	4596.148	96.189	J. S. Clark
N II	4601.637	4601.490	J. S. Clark
N II	4607.174	07.167	J. S. Clark
O II, O II	4609.542	09.610	A. Fowler
N II	4613.690	13.884	J. S. Clark
		17.270	F. L. Brown
		23.098	F. L. Brown

* Probably line for which Struve gives $\lambda = 4470.046$ (Rowland's Scale). *Ap. J.* 62, 198, 1925.

Two Mills plates of β Cephei taken in 1903 and measured by both H. D. Curtis and Miss A. M. Hobe, and six obtained in 1918 that both Miss Hobe and F. Henroteau had measured, were remeasured by the writer with the following results:

TABLE IV

COMPARISON OF MEASURES OF MILLS PLATES OF β Cephei

Plate taken	Curtis and Henroteau	Miss Hobe	H. S. Mendenhall	Mean of Cs, Ht, and Hobe	Mendenhall minus Mean	Residuals, $\Delta + 3.2$
1903	- 6.1	- 5.3	- 8.7	- 5.7	-3.0	+0.2
1903	- 4.4	- 3.2	- 3.7	- 3.8	+0.1	+3.3
1918	+11.7	+12.2	+ 8.6	+12.0	-3.4	-0.2
1918	+ 6.2	+ 8.0	+ 3.3	+ 7.1	-3.8	-0.6
1918	- 3.5	- 2.7	- 5.8	- 3.1	-2.7	+0.5
1918	-14.5	-13.5	-17.5	-14.0	-3.5	-0.3
1918	-21.4	-22.4	-26.6	-21.9	-4.7	-1.5
1918	-19.1	-21.5	-24.7	-20.3	-4.4	-1.2

The next to last column gives the difference between the velocity derived by the writer and the mean of previous results. The average value of this difference is -3.2 km./sec. Adding 3.2 to each value in the difference column furnishes the residuals quoted in the final column. This limited material indicates that the mean square error for a single measurement of a plate is about ± 1.5 km./sec. The "probable error" would be about $2/3$ of this or ± 1 km./sec. A comparison of the wave-lengths used by Miss Hobe (Albrecht's provisional table) with those in Table III gives a mean difference corresponding to 3.6 km./sec., weights having been assigned to the lines according to the frequency of their appearance in the reductions. This difference is of the right sign and order to account for the observed systematic difference in the measures. It is of interest to note that this result is in harmony with Albrecht's prediction²³ that the large K term in the B type stars could be accounted for largely by the wave-lengths used in the reductions.

TABLE V

Assumed λ I. A.	Absorbing atoms	Est. int.	Assumed λ I. A.	Absorbing atoms	Est. int.	Assumed λ I. A.	Absorbing atoms	Est. int.
3927.921			4227.437			18.549		
30.298			41.787	N	2	19.222	O II	3
		3	53.79	O II, S	3	20.92	He I, O II	4
45.050	O II	2	67.16	C II	3	32.82	O II	2
54.368	O II	3	75.80	O II	1	43.759	He I	5
56.59			94.128			4325.765		
64.727	He I	3	4303.82	O II	2	36.86	O II	
		2	17.160	O II	2	40.466	H I (γ)	15
69.260			19.647	O II	2	45.570	O II	1
70.075	H I (ϵ)	20	25.77	O II	1	47.429	O II	2
73.266	O II	3	4035.05	N	2	49.434	O II	3
82.725	O II	1	41.34	N	1	51.275	O II	3
94.995	N II	3	69.79	O II	4	66.906	O II	3
4005.244			71.741			87.931	He I	8
09.33	He I	4	72.156	O II	3	4414.888	O II	3
26.18	He I, N	12	75.869	O II	3	15.125		
4143.759	He I	5	85.01	O II	2	16.974	O II	2
43.70			89.07	Si IV, O II	4	37.548	He I	2
53.310	O II	2	97.31	O II, N	1	47.035	O II, N	1
69.23	O II	2	4101.736	H I (δ)	16	71.477	He I	7
85.453	O II	2	04.90	O II		81.229	Mg II	4
89.793	O II	2	16.13	Si IV	2	4528.618		

²³ *Ap. J.* 55, 361, 1922 and 57, 57, 1923.

The seventeen violet spectrograms were measured and reduced in the manner previously described for the treatment of the Mills observations. The wavelengths of stellar and iron spark comparison lines adopted as the basis of the reduction are given in columns one and four of Table V, in which the values in italics are the lines of the iron comparison from the determinations by Babcock.²⁴ Columns two and five contain the identification of the stellar absorbing atom. The third and sixth columns give the intensities of the stellar lines as observed on the present spectrograms, the estimates being on an arbitrary scale in which 20 has been assigned to a line of maximum intensity. The effects of the diurnal and annual motions of the observer were removed from the velocity given for each plate, as usual. The measured position of each line was not corrected for the effect of the curvature of the lines resulting from the use of a straight slit, but a constant correction, corresponding to the curvature²⁵ at $\lambda 4200\text{\AA}$, has been added to each measured velocity given in Table II.

Velocity curves.—The velocities derived from the 95 Mills plates of 1926, and 1927 were reduced to the epoch of October 7, 1926, J. D. 2424796.700 with a period of 0.1904795 day. The phase given in the third column of Table I refers to this epoch and period. The velocities were then combined into 19 normal places each covering 0.01 day and including from two to eight measures. The combination of the velocities was made in accordance with their weights, the measures of each plate having been assigned a weight of 1, 2, or 3, depending upon the number of lines measured and the consistency of the results. Table VI contains the normal places and the sum of the weights in each case.

Phase	V	Wt.	Phase	V	Wt.
0.0045	+ 0.33	15	0.1037	-8.09	9
137	- 4.85	10	144	-4.60	12
272	- 9.42	13	268	+4.26	10
341	-14.04	8	339	+6.07	15
466	-18.52	5	428	+9.80	5
534	-19.78	10	539	+9.21	9
634	-20.79	9	631	+7.57	9
728	-17.94	5	739	+7.84	7
867	-15.53	11	831	+4.13	7
0.0908	-15.96	5	0.1945	+0.33	15

The mean square error of the velocity derived from a single plate of unit weight is about $\pm 2\frac{1}{4}$ km./sec. relative to the mean velocity curve. The middle error, or "probable error" of a normal place of weight 15 is then ± 0.38 km./sec. and for weight 5, ± 0.67 km./sec.

²⁴ *Cont. Mt. Wilson*, No. 343.

²⁵ *L. O. B.* 12, 138, 1926.

The velocities combined into normal places were first corrected for the effect of the change of velocity during each exposure. This correction was determined on the assumption that a sine curve of appropriate period and amplitude was a fair approximation to the true velocity curve. The error of the observed velocity would then be the difference between the ordinate at the middle of the exposure and the average ordinate for the time covered by the exposure. Or, the error, $\epsilon = \sin T - \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \sin \phi d\phi$, where T is the phase, ϕ , at the time of the middle of the exposure and t_1 and t_2 are the phases at the times of beginning and ending the exposure. Putting $(t_2 - t_1) = t$, it follows that $\epsilon = \sin T (1 - 2/t \cdot \sin t/2)$, or the true velocity at the recorded time, $V = V' + \Delta \cdot V'$, where Δ is the factor $(1 - 2/t \cdot \sin t/2)$ which can be tabulated against the length of the exposure time or put in the form of a graph. The resulting corrections range generally from 0.0 to 0.6 km./sec., both positive and negative.

Fig. 1. Upper curve: Velocity curve determined from measures of 95 Mills III-prism spectrograms. The normal places are plotted with circles of radius equal to the "probable error" in each case.

Lower curve: Light curve determined by photo-electric measures at Berlin-Babelsberg in Sept., 1925.

The normal places are shown in Figure 1, plotted as circles with radii equal to the probable errors. A smooth curve has been drawn to represent these points as closely as possible. The reality of the indentations in the curve near the maximum and minimum points seems established by the relative smallness of the middle errors for the plotted points. The chance of two such similar indentations, correspondingly placed on the curve, being the result of a combination of accidental errors is vanishingly small. The possibility of a correlation between a small variation in the phase

or shape and in the amplitude of the curve at different epochs has been considered as an explanation of the shape of the mean curve, but no such correlation could be discerned. The light curve obtained at Berlin-Babelsberg in 1925 has been included in Figure 1. A few of the velocity curves obtained at different epochs are given in Figure 2. The lack of evident indentations in these may be due to the fact that the time covered by the average exposure is about equal to that involved in the entire indentation. Provisional orbital elements derived from the curve of September 1928 are given in Table VII of the next section.

Fig. 2. Velocity curves from Mills spectrograms obtained at separate epochs. The epoch of curve (a) is June 23, 1926, of (b) is October 7, 1926, of (c) is October 24, 1926, and of (d) is January 7, 1927.

The radial velocities derived from the 17 violet spectrograms are plotted in Figure 3 after being reduced to the same phase by means of the period 0.1904795 day. The measures of each plate are plotted separately, symbols being used to designate the epoch of observation in each case. These velocities suggest a somewhat differently shaped curve than that derived from the Mills plates, but the points are too few in number, especially along the ascending branch of the former curve, to establish the reality of the observed difference in form. A real discrepancy between the two curves might be explainable in terms of a differ-

ence in effective stellar atmospheric levels at which the lines originate for the two different spectral regions. This phase of the subject will be taken up in a following section. Approximate orbital elements derived from the violet spectrographic velocities are included in Table VII of the next section.

Fig. 3. Upper curve: Velocity curve from observations with III-prism violet spectrograph on July 19, July 28, August 5, and August 14, represented by a diamond, circles, a square, and triangles respectively.

Lower curve: Light curve for ultra-violet region determined from spectro-photometric observations described on pages 146 to 150 of this paper.

Positive reproductions of the spectrum of β Cephei as obtained with the Mills spectrograph and with the violet spectrograph are shown in Plate IV, Figures (a) and (b) respectively. A few prominent lines are marked on each.

VARIATION OF ELEMENTS

It is clearly recognized that the short period velocity variation in the case of β Cephei is probably not due to orbital motion of this star as a component of a binary system in the period indicated. Frost²⁶ in 1906, called attention to the fact that the major axis of the orbit is small in comparison with the probable size of the star. The mean of five determinations of the parallax is 0".006, the largest being 0".018. Assuming the effective temperature to be 21000° K we find the radius of the star to be $1\frac{1}{2}$ or $3\frac{1}{2}$ million km. for the two values of the parallax quoted above. The values of $a \sin i$ found for this star are less than 50000 km. or 1/30 of its probable radius. Evidently recourse must be had to the pulsation hypothesis of Shapley and Eddington,²⁷ or to some modification of it such as proposed by Jeans.²⁸ However, the orbital elements

²⁶ *Ap. J.* **24**, 260, 1906.

²⁷ *M. N.* **79**, 2, 1918.

²⁸ *M. N.* **85**, 797, 1925.

determined from the velocity curves serve as a convenient means of describing quite closely the shapes of these curves. They will be used in this section as a means of studying the variation in shape of the velocity curves derived at separate epochs.

All published sets of orbital elements of β Cephei have been collected in Table VII, together with elements estimated by the writer in cases where only the velocities were given. In the first column are given the dates approximately covered by the observations upon which the elements are based. The next six columns contain the orbital elements other than the period which was assumed to be 0.1904795 day in nearly every case. The values of $a \sin i$ are given in units of 1000 km. The next, or 8th column gives the number of plates involved, those in parentheses being also included in other groups. The Roman numerals in the 9th column indicate the number of prisms in the spectrograph used. These are followed by the linear dispersion in Angstrom units per mm. at the center of the plates. The next two columns give respectively the names of the observers responsible for the measurements and reductions of the plates and the names of the computers who derived the orbital elements from the velocities. The elements that have been estimated by the writer are provisional, being derived from the velocity curve by the method of Lehmann-Filhes without a least square solution in any case.

Eighteen complete sets of elements have been tabulated, together with incomplete data in several cases. In a few instances more than one set of elements are based upon the same observational material. A considerable variation in the elements is evident from the tabulated values. Each of the first four elements and the period will be studied separately.

The Period, P.—Crump's²⁹ value of the period, 0.1904795 day, has generally been accepted as correct. Over 40,000 "revolutions" have been made by β Cephei from 1906 to 1928, so that the average period over this interval may be determined with high accuracy.

Fig. 4. Residuals, observed minus computed, for the times of "aphelion" using a period of 0.1904795 day and zero epoch in July, 1906.

²⁹ Pub. Detroit Obs. 2, 144, 1916.

Using as a reference point the intersection of the descending branch of the velocity curve with the axis representing the velocity of the system, i.e., the aphelion point, one finds the average period to have been 0.1904806 day—an increase of 11 in the seventh decimal place over Crump's period. The aphelion time has been computed for each epoch appearing in Table VII by means of Crump's period on the assumption that true period and average period coincided for the epoch July 6, 1906. A fairly smooth curve, as shown in Figure 4, is obtained by plotting the residuals, observed minus computed, for the time of aphelion at each epoch. No reason is apparent for the conspicuous disagreement of the residuals for the epochs of 1906 and 1917. However, it seems safe to conclude that the period of the velocity variation must itself be varying about an average value close to Crump's result of 0.1904795 day. Apparently the period was shortest in 1912 and longest in 1922. The average value of the period between 1906 and 1912 was 0.1904795 day. The average value between 1906 and 1922 was again 0.1904795 day. It may be seen from the curve that the period of the velocity variation may be approximately expressed as

$$P = 0.1904795 + 0.0000130 \cos \{18^\circ (t - 1922)\},$$

the coefficient of the second term being estimated from the slope of the curve at the points of inflection on the zero axis. By 1932 the period should have decreased so that the average value of the period from 1906 to 1932 will again be very nearly 0.1904795 day.

Fig. 5. Values of the systemic velocity, γ . The smooth curve was drawn to represent an assumed period of variation of γ of 20 years.

The Velocity of the System.—The values of γ given in Table VII are plotted in Figure 5. A considerable variation in γ is apparent from the figure. Excluding the observations of 1914, 16, and 17, from Pulkovo^{30, 31} however, the values remain rather close to -10 km./sec. The value of γ is, of course, dependent upon the wave-lengths used in the reductions. A com-

³⁰ Bull. de l'Obs. Central de Russie, Pulkovo, 7 No. 80.

³¹ Bull. de l'Académie des Sciences de Russie, 1918.

parison of the wave-lengths used by Belopolsky and by the writer indicates that the differences average close to zero, while a similar comparison for the wave-lengths used by Crump and by Henroteau gives a difference of only 1.4 km./sec. relative to those used by the writer. One concludes that the velocity of the system has probably varied as shown by the smooth curve drawn through the plotted points. Large discrepancies exist for several of the points relative to

this curve. The four results for γ that have been derived from Lick Observatory observations are all within 2 km./sec. of a constant value and deviate as much as 10 km./sec. from the curve as drawn. Moreover, the rapid change in γ shown by the Russian observations of 1916 is in the opposite direction from that given by the curve, being a decrease instead of an increase. Therefore this curve, and conclusions drawn from it, must be accepted with caution.

TABLE VII
ORBITAL ELEMENTS DERIVED FROM VELOCITY CURVES OF β Cephei

Dates	γ in km./sec.	K in km./sec.	e	ω	T in Julian days	$a \sin i$ 1000 km.	No. of plates	No. prisms and A/mm.	Velocity by	Elements by
					241+					
1902							11	III, 11	Frost and Adams ³²	
1903, Sept. 16 and 20							2	III, 11	H. D. Curtis	
1906, July 6 and Aug. 27	- 5	17	.0			45	34	II, 22	Frost ³³	Frost
July 6 and Aug. 27	- 5.7	18	.04	94		48	(24)	II, 22	Frost	Belopolsky
July 6	- 4.9	16.9	.03	165	7398.859		(12)	II, 22	Frost	Crump
July 6	- 7.8	16.3			7398		(12)	II, 22	Frost and Struve ³⁴ and Barrett	Mendenhall
1912, Apr. 29	- 10.2				9522				Frost ³⁵	Frost
Aug. 8–Nov. 21	- 14.1	15.8	.04	3	9638.812	41	163	I, 44	Crump ³⁵	Crump
Oct. 1	- 12.9	19.2	.05	17	9638.820		(18)	I, 44	Crump	Crump
					242+					
1914, Oct. 9–21	- 26.1	16.8	.16	184	0415.285	43	35	III, 30	Belopolsky ³⁶ and Gerasimovic	Gerasimovic
1916, Sept. 12–Oct. 31	- 32.4	19.0	.10	334	1119.363	48	55	III, 30	Belopolsky ³⁷	Belopolsky
Sept. 12–30	- 29.5	19.9	.0		1119		(32)	III, 30	Belopolsky	Mendenhall
Oct. 1–29	- 36.2	20.0	.0		1119		(23)	III, 30	Belopolsky	Mendenhall
1917, Sept. 16–Oct. 31	- 22.5	18.0	.07	305	1489.311	47	32	III, 30	Belopolsky ³⁷	Belopolsky
1918, May 15	- 6.3	18.3	.07	305	1729.837	48	6	III, 11	Henroteau; Hobe; and Mendenhall	Mendenhall
1919, Sept.	- 9.9	13.3					32		Boothroyd	Van Arn ³⁸
1919 and 1922	- 9.8	11.3	.05	59	2219.820	29			Boothroyd	Van Arn ³⁸
1920, Jan. 19–Feb. 3	- 10.5	14.3	.14	230	2343.741	37	71	33	Henroteau ³⁹ and Henderson	Mendenhall
1921, Nov. 6	- 3.6	14.8	.05	78	3000.622	39	16	III, 11	Hobe and Mendenhall	Mendenhall
1922, Feb. 16–Apr. 18	- 14.2	13.1	.09	82	3102.733	34	155	33	Henroteau ⁴⁰	Mendenhall
1923, Mar. 5–28	- 6.8	10.6	.00		3484	28	68	33	Henroteau ⁴⁰	Mendenhall
1926, June 23–Dec. 38	- 5.1	15.4	.05	274	4796.832	39	95	III, 11	Mendenhall	Mendenhall
July 19–Aug. 14	- 6.3	13.8	.08	220	4796.755	36	17	III, 22	Mendenhall	Mendenhall
1928, Sept. 10	- 5.0	17.0	.02	12	5500.874	45	8	III, 11	Mendenhall	Mendenhall

Belopolsky⁴¹ has combined the results of all the observations of β Cephei made at Pulkovo in 1914, 1916, and 1917 into a single set of elements by making allowance for the change in γ and ω , but he has evidently made an error of an even day at one point

in the process.⁴² His result for γ is given as $-19 \text{ km.} - 13 \text{ km.} \sin \{19^\circ (1911.32 - t)\}$ but the epoch 1911 is probably a misprint. The 19-year period may be correct although the observational data in Figure 5 will admit of any period whatever that is greater than about 15 years.

³² *Ap. J.* **15**, 340, 1902.

³³ *Ap. J.* **24**, 260, 1906.

³⁴ *Ap. J.* **64**, 68, 1926.

³⁵ *Pub. Detroit Obs.* **2**, 144, 1916.

³⁶ *Bull. de l'Obs. Central de Russie*, Pulkovo, **7** No. 80.

³⁷ *Bull. de l'Académie des Sciences de Russie*, 1918.

³⁸ *Pop. Ast.* **36**, 347, 1928. Definitive elements by Van Arn appeared in *Publ. DAO.* **4**, 171, 1929 after present paper was in press.

³⁹ *Pub. Dom. Obs.* **5**, 77, 1922.

⁴⁰ *Pub. Dom. Obs.* **9**, 36, 1925.

⁴¹ *Bull. de l'Académie des Sciences de Russie*, 1918.

⁴² *Id.* 1918, page 1798, line 8.

The Semi-amplitude.—The values of the semi-amplitude for each curve given by the different series of observations of β Cephei in the interval from 1906 to 1928 have been plotted in Figure 6, together with a smooth curve drawn through them. The departure of the plotted points from the curve as drawn is in every case less than 3 km./sec. and is comparable with the accidental errors involved in the determination of the exact value of K . The plot is suggestive of a 20-year period for the variation, but there is little reason to conclude that the period is not many times greater.

Fig. 6. Values of the semi-amplitude, K , of the velocity variation.

Henroteau⁴³ has found a short period (28-day) variation in K with an amplitude of 20 km./sec. from his observations of the velocity of β Cephei in 1922. The Lick observations of 1926–27 fail to show either this period or this range for K . The latter observations cover the period completely at seven different epochs. For these the minima remain roughly fixed, while the heights of the maxima vary little more than 5 km./sec. in a manner suggestive of a period of the order of 180 days. This period would also approximately represent the values of K determined from Crump's observations of 1912, but the range of possible periods is large in both cases.

Fig. 7. Values of the orbital eccentricity, e .

The Eccentricity.—The fifteen determinations of eccentricity made in the interval from 1906 to 1928 and plotted in Figure 7 give results ranging from 0.0 to 0.16. A comparison of the various velocity curves leaves no doubt as to the reality of a variation in

⁴³ *Pub. Dom. Obs.* 9, 36, 1925.

their shape greater than could be accounted for by accidental errors in the velocities or in the drawing of the mean curves. Occasional errors in the computation of e may account for the apparent lack of regularity in its variation. The published value⁴⁴ of e for Belopolsky's observations of 1916 is 0.10 whereas no eccentricity is discernible in a plot of his velocities for this date. It may be significant that the largest values of e appear during the interval covered by the rapid variation in γ and K .

Fig. 8. Values of the longitude of periastron, ω . The broken line represents a continuous variation of ω through 360° in about 8.2 years.

The Longitude of Periastron.—The tabulated values of the angle ω have been plotted in Figure 8 and at first sight are apparently without system. The small values of the eccentricity result in large uncertainties in the determination of ω and T so that probably only the quadrant in which ω is found has any real significance. The lines in Figure 8 represent a continuous change of ω through 360° in 3000 days or about 8.2 years. The majority of the points lie within 90° of these lines, which would be improbable if the distribution of the angles were entirely the result of accidental errors. It is necessary to conclude that a force is acting which produces an effect upon the velocity curves corresponding to a rotation of the line of apsides in a binary system.

Belopolsky⁴⁵ gives ω equal to $0^\circ + 76.5 (1917.19 - t)$, but a period of $4\frac{1}{2}$ or 5 years does not seem plausible in view of the data obtained since 1917.

In general, there seems to be some correlation between the accuracy of the observations and the variation of the orbital elements derived from them but the variability is greater and more regular than can reasonably be accounted for by accidental errors. The general character of the variation of P , γ , and K over long periods of time is fairly well established by the approximate consistency of the plotted values although several outstanding discrepancies remain unexplained. The nature of the variation in ω is somewhat more doubtful, while the apparent random

⁴⁴ *Bull. de l'Académie des Sciences de Russie*, 1918.

⁴⁵ *Bull. de l'Académie des Sciences de Russie*, 1918.

Fig. (a). Spectrum of β Cephei obtained with the III-prism violet spectograph and 36-inch refractor.

Fig. (b). Spectrum of β Cephei obtained with the Mills III-prism spectograph and 36-inch refractor.

Fig. (c). Spectrum of β Cephei between spectra of comparison star 19 Cephei obtained with a slitless quartz spectograph.

Fig. (d). Microphotometer tracings of the above spectra. Short vertical lines indicate regions that were compared in intensity.

variation in e is probably not real. The complexity of some of the velocity curves, such as the one derived from the measures of the Mills plates of 1926 and 1927 and shown in Figure 1, undoubtedly affects the accuracy of the foregoing results.

LEVEL EFFECT

Interesting results have been obtained by recent investigators⁴⁶ from a study of the relative displacements of the individual lines, in the spectra of the Cepheid variables, according to a method first proposed by Rufus.⁴⁷ The method consists in dividing the observed lines into from two to four groups, each of which corresponds to a low, intermediate, or high level in the Sun's atmosphere as given in Mitchell's⁴⁸ list of chromospheric heights combined with Rowland's intensities. The mean velocity curves for these groups are then compared and differences in phase, shape, or amplitude noted.

From an investigation of this type for *S Sagittae*, using 124 spectrograms made at the Detroit Observatory in the interval from 1913 to 1923, Aldrich⁴⁹ has found a lag in phase and a larger amplitude for the velocity curves derived from lines representing high levels relative to the curves given by lower levels. He moreover points out that the relative atmospheric motions would alternately produce a compression and an expansion of the star's outer atmosphere and that the observed epochs of light maximum and light minimum agree closely with the predicted times of maximum compression and maximum expansion, respectively.

Corresponding results were found by Rufus⁵⁰ from a similar investigation of *η Aquilae*. The prominent "Stillstand" in the mean velocity curve is shown to be emphasized in the high-level and low-level curves and to be absent for the intermediate levels. Curves of velocity differences were drawn, one of which—hydrogen minus intermediate—had an amplitude fully half as great as the mean velocity variation. This same difference curve resembles the photometric light curve of the star and also the curve for spectral variation.

In the case of *W Sagittarii*, R. H. Curtiss⁵¹ has observed marked differences between the curves for various groups of lines originating at different chromospheric heights. However, he finds the range of velocity variation to be less for greater chromospheric heights instead of the reverse, as found for *η Aquilae* and *S Sagittae*.

Velocity curves of *ζ Geminorum* have been studied by Rufus⁵² and these have likewise yielded systematic differences for displacements of lines from different atmospheric levels. In general the high level lines were found to give a higher and sharper maximum in the velocity curve than the maximum derived from the low level lines. Maximum velocity also occurs earlier for the lines of higher level than for the low level lines. Hydrogen was found to show a very high maximum and greater phase lag in its velocity curve than the other high level lines. Velocity difference curves—high minus low, and high minus intermediate—have indicated a sudden change from negative to positive values near the time of light minimum. The amplitude of these curves is found to be greater than 1/3 that of the velocity curve itself.

These investigators have found a correlation between the times of maximum and of minimum compression of the radiating gases and the times of light maximum and minimum. Their conclusions are that pulsations originate deep within the star and travel outward in the form of successive compressions and expansions of concentric layers of the star's atmosphere and that these are sufficient to produce both the observed radial motion of the absorbing gases and the observed variation in the rate of radiation due to the resulting change in temperature and absorbing power of the stellar atmosphere.

Fig. 9. Velocity curves derived from individual lines measured on the Mills spectrograms of 1926 and 1927. Each plotted point represents the mean of determinations of velocity from 4 or 5 plates with the He I line at 4387A (represented by crosses and the broken line curve) and the Si III line at 4567A (indicated by circles and the continuous curve).

The measures of the 95 plates of *β Cephei* obtained during the summer, fall, and winter of 1926-27 with the new Mills III-prism spectrograph were made the basis for a study of the relative displacements of the spectral lines. The eleven italicized lines given in Table VIII were measured with sufficient consistency throughout the series of 95 plates to yield individual velocity curves for the different lines. A superposition of these plots shows that, with a single exception, no certain difference in phase, amplitude, or shape exists between the curves for these lines. In the case of

⁴⁶ *Pop. Ast.* **32**, 471, 1924.

⁴⁷ *Pop. Ast.* **32**, 22, 1924.

⁴⁸ *Ap. J.* **33**, 407, 1913.

⁴⁹ *Pop. Ast.* **32**, 218, 1924.

⁵⁰ *Pop. Ast.* **32**, 228, 1924.

⁵¹ *Pop. Ast.* **32**, 547, 1924.

⁵² *Pop. Ast.* **34**, 242, 1926.

the He I line at $\lambda 4387$, the semi-amplitude is smaller by 3 or 4 km./sec. than that given by the curves for the other lines. The curves for this He line and for the Si III line at $\lambda 4567A$ are shown in Figure 9. Each normal place plotted in Figure 9 covers 0.01 day on the average and consists of the mean of measures of the displacement, on 4 or 5 plates, of the line $\lambda 4387A$, (indicated by crosses) and of the Si III line at $\lambda 4567A$, (indicated by circles). The measures of the latter line are typical of results obtained from the other ten good lines as far as the general characteristics of the velocity curves are concerned. The measures are not sufficiently numerous or reliable to merit a consideration of the deviations of normal points from the smooth curves as drawn, although the indentation in the curve suggested by the points for the Si line immediately following minimum is probably real.

Six other lines give such widely scattered measures, due to their faintness and diffuseness, as to render them of no value in this study except to show that no large differences exist in the character of the velocity curves. Table VIII gives the wave-length and identification of the 17 lines studied, together with the heights, where available, at which these lines originate in the solar chromosphere.

TABLE VIII
LINES INVESTIGATED FOR RELATIVE DISPLACEMENTS ON
THE MILLS PLATES

λ I. A.	Elem.	Height
4387.85	He I	2000 kms.
95.94	O II	
4414.88	O II	
16.90	O II	
37.49	He I	750 kms.
47.01	N II, O II	
52.51	O II	
69.66	O II	
71.52	He I	7500 kms.
81.23	Mg II	400 kms.
4552.59	Si III, N II	
67.84	Si III	
74.80	Si III	
90.97	O II	
96.18	O II	
4601.65	N II	
07.12	N II	

The region of the spectrum in good focus on the plates obtained with the violet spectrograph is about twice as extensive as that on the Mills plates, resulting in a corresponding increase in the number of available chromospheric lines. The same set of 35 lines was, in general, measured on each of the 17 violet spectrograms and the velocities derived from each line were plotted. Small differences between the velocity curves for the various lines are indicated but the measures are not of sufficient precision or number to permit an accurate determination of the relative displacements

of the individual lines. Table IX contains the lines that were investigated for relative displacements. The ionization potential is given for the absorbing atoms, and the estimated intensity is included in parentheses after each wavelength.

TABLE IX
LINES INVESTIGATED FOR RELATIVE DISPLACEMENTS ON
VIOLET SPECTROGRAMS

H I, I.P.	14 volts	N II, I.P.	30 volts
4340.47	(15)	4241.79	(2)
4101.74	(16)	4041.34	(1)
3970.08	(20)	4035.05	(2)
		3994.99	(3)
O II, I.P.	35 volts	Mg II, I.P.	15 volts
4416.97	(2)	4481.23	(4)
4414.89	(3)		
4366.91	(3)		
4351.27	(3)	He I, I.P.	24 volts
4349.44	(3)	4471.48	(7)
4347.43	(2)	4437.55	(2)
4345.57	(1)	4387.93	(8)
4319.65	(2)	4143.76	(5)
4317.16	(2)	4009.33	(4)
4189.79	(2)	3964.73	(3)
4185.45	(2)		
4169.23	(2)		
4153.31	(2)	Si IV, I.P.	45 volts
4119.22	(3)	4116.13	(2)
4104.90	(2)		
4075.87	(3)		
4072.16	(3)	C II, I.P.	24 volts
4069.79	(4)	4267.16	(3)
3973.27	(3)		

These lines were combined into groups and their velocity differences were plotted against the time. Several bases for grouping the lines were tried, including ionization potential and solar chromospheric height. The latter was available for the H and He lines only. These preliminary groupings and difference curves indicated that the hydrogen lines were in general displaced by the same amount as the oxygen lines. Figure 10 includes a plot of the velocity differences of hydrogen relative to oxygen. Although rather scattered, the points show that little or no relative motion exists. Therefore, these 22 lines were combined into a single group. Comparison of the other lines with the hydrogen and oxygen group showed that within the errors of observation the He, N, Mg, C, and Si lines could be combined into a single group in view of their similar displacements. This was done and the resulting group compared with the hydrogen and oxygen group. Figure 10 includes a plot of the velocity differences derived from these two groups.

It is obvious from this plot that no large relative motion exists between the gases responsible for the lines in the two groups. No larger relative displacements were found by means of other groupings. The plotted points are few in number and quite scattered,

which results in considerable uncertainty in the curve as drawn. Two maxima appear in the curve, but their reality is very questionable. In fact it would be possible to remove all appearance of relative displacement by shifting only one or two points by amounts no greater than the possible errors of observation.

Fig. 10. Upper curve: Velocity differences, H (γ , δ , ϵ) minus O (19 II lines) from measures of the 17 violet spectrograms.

Lower curve: Velocity differences, O and H minus He, N, Mg, C, and Si (13 lines), from measures of the violet spectrograms.

The following results have been derived from the present investigation of the relative displacements of lines in the spectra of β Cephei obtained in 1926 and 1927: (1) Among eleven accurately measured lines on the 95 Mills plates, only one indicates relative motions as large as 5 km./sec. (2) Among certain groups of lines measured on the 17 violet spectrograms, relative displacements of very doubtful reality appear, corresponding to velocities as high as 5 km./sec.

Thus it appears that no definite relative displacements of the spectral lines exist in the case of β Cephei such as have been found in the spectra of other variables by Curtiss, Rufus, and others.

DISCUSSION OF SPECTROGRAPHIC OBSERVATIONS

The spectrographic data considered in the preceding paragraphs, although not so complete as could be desired for a definite decision upon many of the questions raised, appear to justify the following conclusions concerning the system of β Cephei.

There is good reason for believing that the observed variation in the radial velocity, having a period of about 0.1905 day is not to be interpreted as arising from simple orbital motion of the star, but probably is to be ascribed to the motion of the stellar atmosphere caused by the star's alternate expansion and contraction. Observed changes in the form of the velocity curve are therefore to be considered as variations in the pulsations of the stellar atmosphere. It is to be noted that on this interpretation e and ω lose their usual physical significance and indicate merely a change in the form of the velocity curve. On the other hand, P and K refer to the period and semi-amplitude of the pulsation and have a perfectly defi-

nite meaning, while γ in this case denotes the radial velocity of the star as a whole. That γ or the radial velocity of the star is variable, with a period of 18 or more years and with an amplitude of about 32 km./sec. seems to be fairly well established by observations secured in the interval 1903-1928. From this it is concluded that β Cephei is the brighter component of a binary system having a period of the order of 20 years. With an assumed period of 20 years, the following approximate elements of the orbit for the brighter star have been derived from the velocity curve:

$$P_1 = 20 \text{ years (assumed)}$$

$$K_1 = 16.3 \text{ km./sec.}$$

$$e_1 = 0.55$$

$$\omega_1 = 180^\circ$$

$$T_1 = 1916.0$$

$$\gamma_1 = -13.0 \text{ km./sec.}$$

$$a \sin i = 9.14 \text{ A.U.}$$

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 1.9 \text{ Sun's mass.}$$

The data derived in this investigation show that the period P and semi-amplitude K of the pulsations of the brighter star are variable in a period which appears to be of the order of 20 years or the same as that tentatively assumed for the orbital revolution of β Cephei in the binary system. It is also to be noted that the maximum amplitude of the velocity curves attributed to pulsations occur at or near the epoch of periastron for the binary orbit. Since this orbit is quite eccentric there seems to be some evidence that the observed changes in the amplitude of pulsation may be the result of the varying tidal action upon the atmosphere of the bright star with its changing distance from the secondary. Variation in the apparent period of pulsation of a star moving in an orbit of the probable dimensions obtained above is readily calculable. One finds that the motion of the bright star would result in an apparent change in the period of 0.000,020 day. The observed variation in the period amounts to about 0.000,026 day. Similarly, the observed times of the short period velocity maxima do not in general agree with the times computed by means of an average value of the period. The observed maxima precede the computed near perihelion and follow near aphelion, the two being assumed coincident at periastron. The total observed difference amounts to 2.12 hours while computation shows that 2.04 hours would be required for the light to travel the distance $b \sin i$ (the minor axis of the large orbit corrected for the foreshortening effect of the orbital inclination). These approximate agreements between observation and computation tend to substantiate the existence and size of the orbit of the bright star described above.

In this connection it is of interest to examine the data for stars having short period variation in velocity with reference to the variability of the systemic velocity, γ , and to observed changes in the amplitude and period of the velocity curve. Five appear to have been sufficiently observed to furnish definite information on these points. Data for the five are the following:

	Period	Velocity of system	Amplitude and period
β Cephei	0.190 days	variable	variable
12 Lacertae	0.193	variable	variable
σ Scorpii	0.247	variable	?
β Canis Majoris	0.257	variable	variable
RR Lyrae	0.567	variable	variable

Two of these stars, β Cephei and RR Lyrae are light variables of the Cepheid type. All have variable velocity of the system, that is, they are probably long period binary systems. With the possible exception of σ Scorpii the velocity curves of all show changes in period and amplitude.

The following Cepheids of longer periods are definitely known to have variable velocity of the system: *Polaris*, *S Sagittae*, *Y Sagittarii*. There is also some evidence that the systemic velocities of δ Cephei and η Aquilae are variable. The forms of the velocity curves of *S Sagittae* and *Y Sagittarii* vary and changes of amplitude are also observed for *S Sagittae*. The periods of all are probably variable. *Polaris* has been shown to be the brighter component of a binary, revolving in a highly eccentric orbit in a period of approximately 30 years. The velocity curve for its short period variation shows marked changes in form analogous to those found for β Cephei.

It is a significant fact that many of the Cepheids are found to be the brighter components of a binary, and that the two which are known to have very eccentric orbits also exhibit changes in form of the short period velocity curves of such a nature as to suggest that these changes may be the result of tidal action.

SPECTROPHOTOMETRIC OBSERVATIONS

All photometric methods which have previously been employed—visual, photographic, and photo-electric—furnish light curves that represent the integrated variations in intensity over a considerable region of the spectrum. In the study of the variation in luminosity of the Cepheid type of variable it is important to know definitely to what wave-lengths of the spectrum the observations refer and to be able to compare the variation in different parts of the spectrum. The spectrophotometric method to be described appears to be well adapted to this purpose. Moreover, it possesses certain advantages over photometric methods previously employed in that it is free from any personal equation of the observer, avoids

certain difficulties which enter in the determination of magnitudes from photographic star images, and is not subject to the well-known practical difficulties encountered when the photo-electric cell is used in measuring stellar intensities.

The proposed method may be described briefly as follows: The spectrum of the star whose light intensity is to be investigated is photographed on a series of plates on each of which are obtained two or more apertured spectra of a comparison star of supposedly constant luminosity. Let it be assumed that such a series of spectrograms of a variable star, distributed throughout the period, have been obtained, and that upon each plate two spectra of known intensity difference have been impressed, the apertures of the telescope having been so adjusted that the intensity of the spectrum of the variable is intermediate between the intensities of the two comparison spectra. The exposure time is kept the same for the variable and for the comparison star spectra. On each spectrogram the relative intensities at corresponding points in the three spectra, for example at $\lambda 3900\text{\AA}$, are then determined from measures of the microphotometer record of the photographic blackening in the three spectral images at the chosen wave-length. Variation in the luminosity of the star in this particular spectral region is given directly as the result of these intensity measures. Where the range in luminosity of the variable is greater than 0.5 magnitude, three or more apertured spectra of the comparison star should be used.

This method is readily adaptable to the study of the ultraviolet region of the spectrum. Such a study was deemed of special interest in the case of β Cephei because of its early spectral type. For this investigation the two-prism quartz spectrograph was utilized in connection with the Crossley reflector. The instrument was used in its slitless form since with a slit spectrograph an unknown amount of light would be intercepted by the slit jaws. The amount of light entering the telescope was controlled by means of black cardboard diaphragms which fit over the end of the telescope tube. There is now available for the Crossley a large cardboard disk in which there are sets of four 10-inch, 6.3-inch, 4-inch, 2.5-inch, and 3-inch circular openings which give approximate magnitude intervals of 1, 1, 1, and -0.4 magnitudes respectively.

By choosing comparison stars that are close in the sky to the variable, one is able to eliminate largely the effects of changes in zenith distance and atmospheric transparency. Some difficulty was experienced in finding comparison stars sufficiently close to β Cephei both in the sky and in spectral type that are not themselves variable, but this difficulty would not be encountered for the more common spectral types.

Three good comparison stars were found at moderate distances from β Cephei, namely, 7 Cephei, type B₅, 14 Cephei, B₀, and 19 Cephei, O₉. A comparison was also made with α Cephei because of its well known constancy of light. Four series of plates were obtained, each series with a different comparison star and covering more than the $4\frac{1}{2}$ -hour period of β Cephei. The two largest diaphragms were used with the comparison stars except α Cephei, for which the 4-inch and 2.5-inch openings were utilized. The 3-inch diaphragm was used for β Cephei in all cases except with 7 Cephei, where the 2.5-inch diaphragm gave the desired intensity. Every spectrum received a $1\frac{1}{2}$ -minute exposure except that a 1-minute exposure was used when α Cephei was taken as the comparison star. The exposures were accurately timed by means of a stop watch.

The following procedure, adopted in making the exposures with 19 Cephei, is typical of that followed throughout. Set telescope on 19 Cephei; $1\frac{1}{2}$ -minute

exposure with 10-inch diaphragm; move tube slightly with slow motion; $1\frac{1}{2}$ -minute exposure with 6.3-inch diaphragm; set telescope on β Cephei so that spectrum will fall between the two comparison spectra already on the plate; $1\frac{1}{2}$ -minute exposure with 3-inch diaphragm; substitute unexposed plate; $1\frac{1}{2}$ -minute exposure with 3-inch diaphragm; set telescope on 19 Cephei; etc. The saving in time resulting from but a single shifting of the telescope (except by slow motion) for each plate was considered to outweigh the possible advantage of taking one of the two comparison spectra before and one after each exposure on β Cephei. The guiding of the telescope during these short exposures was left entirely to the driving clock of the Crossley which was adjusted for accurate following. It is evident that a considerable number of plates may be obtained by this method during a single epoch of even a short period variable.

TABLE X

RELATIVE MAGNITUDE OF β Cephei FROM SPECTRO-PHOTOMETRIC OBSERVATIONS

Comparison Star, 19 Cephei						Comparison Star, 14 Cephei					
Date 1928	J. D. 242+	Phase	Δ mag. 3900A	Δ mag. 3600A	Δ mag. 3300A	Date 1928	J. D. 242+	Phase	Δ mag. 3900A	Δ mag. 3600A	Δ mag. 3300A
Aug. 30	5489.490	.051	+.27	+.09	++ .04	Sept. 2	5492.491	.004	+.12	+.03	.00
	.522	.083	+.09	-.02	-.14		.512	.025	-.04	-.16	-.10
	.525	.086	-.08	-.06	-.04		.514	.027	+.29	+.11	+.09
	.552	.113	-.12	-.14	-.17		.532	.045	-.02	-.08	-.03
	.555	.116	-.12	-.02	-.01		.536	.049	+.02	-.09	-.06
	.576	.137	-.03	-.06	-.05		.554	.067			-.13
	.592	.153	+.07	+.07	+.01		.556	.069	+.27	+.08	+.08
	.617	.178	-.07	+.12	+.02		.577	.090	-.05	-.06	-.03
	.624	.185	+.08	+.08	+.08		.588	.101	-.07	-.07	-.04
	.646	.016	-.23	-.12	-.08		.607	.120	+.07	-.06	-.01
	.651	.021	+.02	-.06	+.06		.609	.122	-.07	-.16	-.22
	.676	.046	-.13	+.05	+.06		.628	.141	-.16	-.21	-.30
	.683	.053	+.02	.00	-.04		.630	.143	-.05	-.19	-.17
	.705	.075	-.05	-.08	-.10		.652	.165	-.27	-.23	-.21
							.665	.178	-.01	-.23	-.28
							.685	.008	-.06	-.16	-.13
Comparison Star, 7 Cephei						Comparison Star, α Cephei					
Aug. 31	5490.487	.096	-.14	-.05	-.12	Sept. 3	5493.485	.045	-.04	+.04	+.04
	.509	.118	-.34	-.10	-.10		.503	.063	+.09	+.13	+.11
	.517	.126	-.16	-.04	-.05		.505	.065	+.06	-.01	+.03
	.541	.150	-.02	.00			.521	.081	+.08	+.04	+.08
	.560	.169	-.12	-.04	-.06		.523	.083	+.01	+.06	+.10
	.567	.176	-.04	+.06	+.06		.537	.097	+.09	+.04	+.02
	.585	.003	-.05	+.06	-.01		.558	.118	+.07	+.09	+.08
	.592	.010	-.09	+.05	-.02		.575	.135	+.07	+.07	
	.610	.028	-.05	+.14	+.10		.596	.156	+.10	-.02	-.10
	.616	.034	+.10	+.06	+.02		.618	.178	+.07	+.02	
	.635	.053	-.13	-.06	-.12		.633	.003	-.02	+.02	-.01
	.642	.060	+.01	+.08	+.06		.647	.017	+.03	+.05	
	.661	.079	-.06	+.01	-.01		.649	.019	+.04	+.01	
	.668	.086	+.05	-.04			.665	.035	.00	-.02	
	.689	.107	+.07	-.06			.666	.036	-.03	-.13	
							.686	.056	-.04	-.04	
							.704	.074	-.02	+.05	

The time of the exposures should be arranged so that the stars come to the meridian near the middle of the series, thus reducing to a minimum the change in relative zenith distance of the variable and the comparison star. However, for the present investigation the telescope was not available until β Cephei was approximately on the meridian. The resulting change in zenith distance together with the rapid change in focus of the reflecting telescope, difficulties not foreseen by the writer, have combined to produce somewhat wider and more diffuse spectra toward the end of each series. While this may have produced a lower precision for the observations than that of which the method is capable, it is believed that neither the general form of the light curve nor the order of the amplitude of the observed light variation has been seriously affected.

The plates were developed in groups of 7 or 8 under as nearly identical conditions as possible. They were then run through the Moll self-registering microphotometer at Mt. Hamilton, with the slit in front of the thermocouple of sufficient length to cover the width of the spectrum and include some clear film on each side. The machine was run at maximum speed since the continuous spectra, rather than the absorption lines, are of interest here. The order in which the plates were run through the photometer was varied so as to eliminate any possible secular effect such as might result from variations in the instrument. The three tracings derived from the three spectra on each plate were recorded on a single half sheet of sensitized paper.

A positive reproduction of one of the plates secured August 30, 1928, is shown in Plate II, Figure (c). The spectrum of β Cephei appears between the two comparison spectra of 19 Cephei. Figure (d) is a reproduction of the microphotometer tracings of these spectra. The four regions at which measures were made are indicated by short vertical lines drawn across the tracings.

The measurement of the tracings consisted in determining the deflection of the galvanometer, or the height of the curve, in terms of the total deflection from clear film to opacity. This process was greatly facilitated by use of a "converging reseau" for which the writer is indebted to N. W. Storer. The reseau consisted of a piece of glass upon which were ruled numerous parallel equidistant lines in one direction and 21 slightly convergent lines in the other direction, the 21st line being perpendicular to the parallel lines. This provided an adjustable scale that could be used to divide the distance from clear film to opacity into 20 equal parts. The heights of the three curves, i.e., the blackening of the photographic plate, were measured to a tenth of a division of the reseau at four

points corresponding to the spectral region of $H\gamma$, $H\epsilon$, and $\lambda\lambda 3600, 3300A$, and are believed to be accurate to one per cent of the total deflection from clear film to opacity.

In Figure 11, the curve was obtained by plotting photographic blackening of a set of sensitometer images having intensity ratios of 1:2:4:8:16 against the logarithms of these relative intensities. For deflections over 20% and less than 80%, the relation of deflection, Δ , to $\log I$ is approximately linear. Since magnitude is a linear function of $\log I$, it is also a linear function of Δ over the interval quoted.

Fig. 11. Relation of photographic blackening to the intensity of the incident light from sensitometer images.

The curve of Figure 11 is not directly applicable to the measures of the spectral tracings because of the clear film included on both sides of the spectra. An infinite exposure would result in a deflection considerably less than 100%. The extent to which this linear relation holds for the tracings of slitless spectra is shown by the plotted measures represented by circles in Figure 12. These are measures of Δ corresponding to a series of exposures of β Cephei with the various diaphragms and exposure times indicated on the graph. The range of deflections obtained with these diaphragms and exposures more than covers the range of measures

to be reduced to magnitude intervals. If the relation between deflection and magnitude was perfectly linear for all values of the deflection obtained for the spectra, then Figure 12 would consist of five absolutely straight lines. An obvious deviation from a straight line occurs for small deflections of 5% and less. No measures for such small deflections have been included. A small deviation also appears for deflections between 70% and 85% due to the close approach to infinite blackening. The deflections for the region of $H\gamma$ all fall between or near these values, where the method becomes insensitive to small changes in intensity of the incident light. The measures of light variation at $H\gamma$ have therefore been discarded. It is evident from these graphs that the assumption of a linear relation between deflection and magnitude will be a very close approximation for deflections between 5% and 70%.

Fig. 12. Relation of photographic blackening to the intensity of the incident light for slitless spectra of β Cephei.

particularly where the variation in light is of the order of only 0.05 magnitude. The measures of the tracings have therefore been reduced on the assumption that the relation between magnitude and photographic blackening is linear.

Table X contains the measures of relative magnitude for β Cephei obtained in this way for the three

spectral regions $\lambda\lambda 3900, 3600, 3300\text{\AA}$, with reference to the four comparison stars. Each value represents the difference in height of the β Cephei spectrum and

Fig. 13. Light curves at (a) 3900A, (b) 3600A, (c) 3300A. The broken curves represent the mean of the three regions.

the weaker comparison spectrum divided by the difference in height of the two comparison spectra.

Fig. 14. Upper curve: Light variation in the ultraviolet from spectro-photometric observations.

Lower curve: Velocity curve from Mills spectrographic observations of September, 1928—ten days after photometric observations.

Table XI gives the normal places computed for each region measured after reducing the measures to one epoch with $P=0.1904795$ day. The corresponding curves for each region are shown in Figure 13. A mean light curve derived from the combined measures for the three regions, to each of which equal weight was assigned, is shown in Figure 14, together with the velocity curve obtained a few days later, reduced to the same epoch. Each normal is the mean of measures of 6 plates and covers about 0.02 day on the average. The mean square error of a single observation is found to be ± 0.12 magnitude, and the "probable error" ± 0.08 magnitude. The probable error of a normal place containing 18 individual measures comes out ± 0.02 magnitudes.

TABLE XI
NORMAL PLACES FOR RELATIVE MAGNITUDE OF β Cephei

Mean phase	3900A Δ mag.	3600A Δ mag.	3300A Δ mag.	Mean Δ mag.
.0075	-0.011	0.028	0.007	0.008
.0227	0.005	0.004	0.054	0.021
.0412	-0.009	0.020	0.063	0.025
.0538	0.025	0.017	0.016	0.019
.0722	0.053	0.051	0.049	0.051
.0873	-0.020	-0.008	0.050	0.007
.1087	-0.058	-0.038	-0.020	-0.039
.1273	-0.047	-0.053	-0.051	-0.050
.1503	-0.017	-0.030	-0.052	-0.033
.1723	-0.030	0.022	0.000	-0.003

The mean light curve in the ultra violet region, which has an amplitude of about 0.1 magnitude, indicates a rapid rise in luminosity, beginning about 0.070 day after the computed time of maximum positive velocity and ending approximately 0.055 day later. This is followed by a gradual decline in luminosity. The secondary variations indicated on the descending branch of the curve are probably real. This mean curve refers to an average wave-length of 3600A while the effective wave-length in the case of the photo-electric methods is about 4500A. Light curves of β Cephei have been determined by means of the photo-electric cell by Guthnick and Praeger in 1913-14⁵³ and 1917,⁵⁴ by Miss Cummings⁵⁵ in 1921, and by M. Gussow⁵⁶ in 1925. The normal points derived by the latter have been plotted in Figure 1. Previously determined light curves are similar to that obtained in 1925 except that the small secondary variations were found to lie on the *ascending* as well as the *descending* branch. The amplitudes of light curves obtained photo-electrically have ranged from 0.042 magnitude in 1925 to 0.063 magnitude in 1914.

The following conclusions may be drawn from the light curves shown in Figures 13 and 14. (1) The

light curve of β Cephei determined for the short wave-lengths in August and September 1928, differs in form from that in the photographic region obtained by earlier investigators. (2) The amplitude of variation in the ultra-violet is found to be at least 50% greater than in the region measured with the photo-electric cell. (3) There is some indication of a progressive lag in the phase of the maximum point with decreasing wave-length, shown by measures of $\lambda 3900$, $\lambda 3600$, and $\lambda 3300$ Å.

The probable error of a single observation obtained by the spectro-photometric method is apparently about seven times greater than that given for the Lick photo-electric measures of β Cephei,⁵⁷ but a single observation in the latter case represents 20 settings, or between 8 and 10 independent comparisons of intensity. Since the precision of the mean increases as the square root of the number of observations, the probable error of a single comparison would be roughly 3 times greater than for a "single observation" or about ± 0.036 magnitude. Hence, the precision of the photo-electric method is little more than twice as great as that of the new method. There is no doubt that improvement of the simple technique of observation with the spectrophotometric method at various points suggested by experience will result in measures that rival in accuracy those obtained with the photo-electric cell.

The chief advantages of the new method are its accuracy, the fact that it affords a method of studying light variations in limited and definite spectral regions, including the extreme ultra-violet which is otherwise inaccessible, and the permanency of the record.

SUMMARY OF RESULTS

(1) A mean radial velocity curve has been determined from measures of 95 Mills III-prism spectrograms of β Cephei obtained in 1926 and 1927. The mean square error for the individual measures of these plates is about ± 1.5 km./sec. The following orbital elements were estimated from the velocity curve shown in Figure 1:

$$\begin{aligned}
 P &= 0.1904795 \text{ (assumed)} \\
 \gamma &= -5.14 \text{ km./sec.} \\
 K &= -15.40 \text{ km./sec.} \\
 e &= 0.045 \\
 \omega &= 274^\circ \\
 T &= \text{J. D. } 2424796.832 \\
 a \sin i &= 39000 \text{ km.}
 \end{aligned}$$

The indentations in the curve near the maximum and minimum points are considered to be real.

⁵³ *Berlin-Babelsberg* 1, 23, 1914.

⁵⁴ *Berlin-Babelsberg* 2, 78, 1918.

⁵⁵ *L. O. B.* 11, 115, 1923-24.

⁵⁶ *A. N.* 5484, 201, 1925.

⁵⁷ *L. O. B.* 11, 115, 1923-24.

(2) The variation in the shape of the velocity curves derived from different absorption lines, or combinations of lines, is small and of doubtful reality.

(3) A comparison of the orbital elements derived in the interval 1906–1928 indicates that all of them vary in a more or less regular manner over a period of years. The general character of the observed variation of the period, P , and of K , the semi-amplitude of the velocity curves, are undoubtedly real. Variability in the velocity of the system, γ , appears to have been established. Comparatively large uncertainties in the determination of the values of e and ω tend to mask the nature of the true variation in these elements but some regularity seems to exist in the variation of ω .

Interpretation of the observed variation of γ has been made in terms of orbital motion of the bright star as a component of a binary system, and a possible connection between this orbital motion and the variation of the amplitude for pulsation of the star's atmosphere has been suggested.

(4) A new method of photometric observation, consisting in the measurement, by means of the Moll recording microphotometer, of the photographic blackening in a series of slitless quartz-prism spectrograms,

has been applied to β Cephei. Light curves have been obtained for the three spectral regions $\lambda\lambda 3900$, 3600 , and $3300\text{\AA}$, which show that the amplitude of light variation in the ultra-violet region is about 50% greater than that found for the photographic region at $\lambda 4500\text{\AA}$.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge my indebtedness to Director W. W. Campbell and Associate Director R. G. Aitken for the opportunity of using the observatory equipment in this investigation. I am, moreover, greatly indebted to the various members and former members of the Lick Observatory personnel and to the members of the Students' Observatory staff who have aided in securing a number of the observations and offered helpful criticism. In particular, to Dr. J. H. Moore, who suggested and outlined the problems, and to Dr. C. D. Shane is extended my most sincere appreciation of the valuable suggestions received from them and patient interest shown by them throughout the course of this study. Finally, this work was made possible, in part, by a grant of the Lick Observatory Fellowship in Astronomy in the University of California.

LICK OBSERVATORY,

October, 1929.

Issued January 7, 1930.