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ON THE USE OF A HALF COLOR FILTER FOR THE DETERMINATION OF
COLOR INDICES

BY

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The determination of magnitudes and color indices of stars in certain sample areas and the statistical discussion of such data furnishes at present the most promising method for studying the space distribution of the stars. In the Milky Way in particular it is necessary to extend the observations to faint stars in order to reach those distant star clouds. The determination of color indices of faint stars is therefore of considerable importance, and the method here described is intended to facilitate this difficult task.

Direct photographs taken through color filters in the primary focus of large reflectors are practically the only means for observing color equivalents of faint stars and exposures of an hour or more will be required. In such cases the principle of using two successive exposures of the same duration as being of identical effect must be abandoned. It is practically never possible to obtain uniform conditions for transparency of the atmosphere, for seeing and guiding, and the focus of the mirror as well as its optical shape will generally change during long periods of observation. Such changes not only affect the zero point of the magnitude scale but also the rate of increase in the size of the star images for successive magnitudes, and it becomes imperative to use for every long exposure a separate reduction curve for converting the estimates of image intensity into stellar magnitudes.

It is proposed to employ a filter of which one half transmits violet and ultra-violet light, the other half green and yellow light. On a photograph taken through such a filter the images in one half of the field measure the photographic magnitudes of the stars, in the other half the photovisual. A second exposure with the filter turned 180° will have the reversed arrangement; but the two exposures are quite

independent and may be taken on different nights. The idea is, that, if the photographic magnitudes in the area to be investigated are known, the stars photographed through the violet half serve to establish the relation between image intensity and magnitude (reduction curve) for that peculiar exposure, and that this relation can be used also to reduce the image measures of the yellow half. By such procedure, it is true, a systematic error is introduced similar to the Purkinje effect of visual observations. This error is due to the well known fact that the contrast of a plate, or the increase in the star images for a given increase in the light intensity is not the same for all wavelengths. It stands to reason, however, that this Purkinje error, which is of the order of a few tenths of a magnitude, is independent of the conditions peculiar to each exposure such as telescopic image definition, guiding, etc. For a given filter combination, plate emulsion, and development this error can be evaluated once for all, and we shall show that if desirable it can even be determined for each plate.

With Wratten gelatine filters cemented between two thin glass plates it is easy to make a combination filter as described above. Clear glass may be used for the violet half of the filter (both halves must have the same optical thickness on account of the change in focus setting), but then, for a given exposure, the violet images would be much stronger than the yellow ones for all but extremely red stars. It seems desirable that the violet and yellow images of a star of average color (F-G type) should be of nearly equal intensity. This is accomplished by using for the violet half a filter of suitable transmission; it is, moreover, advantageous to choose this filter so that the effective wavelength of the photographic magnitudes is shifted farther toward

the ultra-violet. This will increase the scale of the measured color equivalents and thus improve the accuracy of the color determination.

Experiments made by C. J. Krieger have shown that a K2 filter for the yellow half and a 35D filter (Wratten color filters of the Eastman Kodak Company) for the violet half form a favorable combination when used with Cramer Instantaneous Iso plates. Equal image intensity is obtained for a star of type G0, the color indices are 35% larger than those of the Mount Wilson North Polar Sequence, and the exposure required for this combination is only 3 to 4 times longer than for a direct photograph without filter on Eastman 40 plates.

The method requires in the first place that the photographic magnitudes in the area to be investigated should be known. If that is not the case they may be determined for a well distributed sequence of stars in the usual way by comparison with the North Polar Sequence. The photographs required for these comparisons may be made without filter in order to shorten the exposure time. Two photographs of the area are then obtained with the combination filter: one with the violet half east (south), and one with the violet half west (north). The two should have approximately the same exposure, but do not need to be taken on the same night.

All the images yellow or violet are measured by comparison with an arbitrary scale of images; these scale readings will be designated by S . According to Schwarzschild's law we have then:

$$S = f(I.t^p) = F(m - m_o - 2.5 p \log t)$$

where I is the brightness of the star, m its magnitude, t the exposure time, p the Schwarzschild exponent, a constant which depends on the plate emulsion and development but seems to be independent of the wavelength of the light.¹ Inverting the arbitrary function F we have:

$$(1) \quad m - m_o - 2.5 p \log t = \varphi(S)$$

The function $\varphi(S)$ is the equation of the reduction curve which relates the scale reading of each image to the magnitude of the star. For the reasons already stated, this function will be slightly different for each exposure and will differ further for the yellow and the violet images of any given half filter exposure. Using the subscript v for the images taken through the yellow filter which measure the photovisual magnitude, and the subscript p for the violet filter which furnishes photographic magnitudes, we have thus

$$(2) \quad \begin{aligned} m_p &= \varphi_p(S_p) + m_{op} + 2.5 p \log t = \Phi(S_p) \\ m_v &= \varphi_v(S_v) + m_{ov} + 2.5 p \log t \end{aligned}$$

The function $\Phi(S_p)$ is readily found by plotting the observed scale readings against the known photographic magnitudes for those stars of the area which were photographed through the violet filter. The reduction curve drawn through the plotted points will permit us to convert the measures S_p into photographic magnitudes m_p . If we use the same curve for the reduction of the measures S_v of the yellow images of the same exposure, we do not obtain true photovisual magnitudes, but quantities which we shall call quasi-visual magnitudes μ :

$$(3) \quad \mu = \Phi(S_v)$$

From the equations (2) we find the corrections which have to be applied to these quasi-visual magnitudes in order to convert them into true photovisual magnitudes m_v :

$$(4) \quad m_v = \mu + [\varphi_v(S_v) - \varphi_p(S_v)] + [m_{ov} - m_{op}]$$

The first correction term is a function of the image intensity S_v only; it removes the photographic Purkinje effect, and we shall designate it by $n(S_v)$. The last term is a constant c which depends on the transmission of the two filters, the sensitivity of the plate, and the selective absorption of the atmosphere; it has the character of a zero correction. We can then write equation (4) in the following form:

$$(4^1) \quad m_v = \mu + n(S_v) + c$$

For the determination of the color index we need two photographs of the area with the half filter; the first, with the violet half east, designated by $'$; the second, with the yellow half east, by $''$. The color index C is calculated from the measures S' and S'' according to the following formulae:

(5) Eastern half:

$$C = m_p - m_v = \Phi'(S'_p) - \Phi''(S'_v) - n''(S'_v) - c.$$

Western half:

$$C = m_p - m_v = \Phi''(S''_p) - \Phi'(S''_v) - n'(S''_v) - c$$

Φ' and Φ'' are the reduction curves of the two exposures. Both are established by means of the known photographic magnitudes of the sequence stars. If the effective wavelength of the latter is not the same as that of the violet filter images, a correction for color equation has first to be applied. This requires a knowledge of the approximate color-indices of the sequence stars; it is thus necessary to proceed in successive approximations. In the first approximation the color equation and the small Purkinje corrections n are neglected and approximate values of the color indices of the sequence stars derived. By means of the latter the known photographic magnitudes of the sequence stars are reduced to the color system of the violet filter. The

¹ See N. W. Storer, *L. O. Bull.* 14, 48, 1929.

reduction curves Φ' and Φ'' are then redetermined in second approximation and used for the reduction of all measures according to formulae (5).

We have now to take up the determination of the Purkinje correction n which removes a systematic error depending on the brightness of the star. It has been pointed out already that the function $n(S_v)$ is conditioned by the plate emulsion and its development and must be identical for two exposures taken on the same plate. Suppose we have a plate on which two exposures of the same area were made with the half filter: a long exposure which we shall designate with the subscript l , and a short exposure of about one tenth the duration of the former, designated with the subscript s . From the known photographic magnitudes of the sequence stars photographed with the violet filter the reduction curves Φ_l and Φ_s are derived separately for the two exposures, and the quasi-visual magnitudes are then determined for the stars photographed through the yellow filter according to formula (3). For each star which can be measured on both exposures we thus obtain two values for the quasi-visual magnitude:

$$\mu_l = \Phi_l(S_{vl}), \text{ and } \mu_s = \Phi_s(S_{vs})$$

while according to equation (4¹):

$$m_v = \mu_l + n(S_{vl}) + c = \mu_s + n(S_{vs}) + c$$

Each pair of images furnishes for the determination of n an equation of the form

$$(6) \quad n(S_{vl}) - n(S_{vs}) = \mu_s - \mu_l$$

From the differences in the values of the function n for given arguments, the function itself can be reconstructed, except for an arbitrary constant.

If the scale readings S are so chosen that they are approximately proportional to the magnitudes, then the differences $S_{vl} - S_{vs}$ are nearly equal for all stars and we can divide equation (6) by this quantity:

$$\frac{n(S_{vl}) - n(S_{vs})}{S_{vl} - S_{vs}} = \frac{\mu_s - \mu_l}{S_{vl} - S_{vs}}$$

For infinitely small intervals $S_{vl} - S_{vs}$ the left hand side would represent the differential quotient of the function n , and the function itself would result from numerical integration.

As n is a small correction and its relation to the image intensity is not likely to be complicated, it will generally be sufficient to use for n an interpolation formula of the second order:

$$(7) \quad n(S_v) = a.(S_v - S_o) + b.(S_v - S_o)^2$$

No constant term needs to be included, as such is already taken care of by the constant c in formula (4).

It is convenient to use for S_o a medium value of the scale interval, but the same value should be adopted for all plates, as otherwise the zero correction c of the observed color indices will slightly vary. The problem of finding the function n then resolves itself into the determination of the two numerical coefficients a and b . For this purpose we insert the interpolation formula (7) into equation (6):

$$a(S_{vl} - S_{vs}) + b[(S_{vl} - S_o)^2 - (S_{vs} - S_o)^2] = \mu_s - \mu_l, \text{ or} \\ (a - 2bS_o)(S_{vl} - S_{vs}) + b.(S_{vl} + S_{vs}).(S_{vl} - S_{vs}) = \mu_s - \mu_l$$

Again, if the scale readings S are approximately proportional to the magnitudes, we can divide by $S_{vl} - S_{vs}$, and our equation takes the simple form:

$$(8) \quad A + B \frac{S_{vl} + S_{vs}}{2} = \frac{\mu_s - \mu_l}{S_{vl} - S_{vs}}$$

where $A = a - 2bS_o$, $B = 2b$.

Every star photographed through the yellow half of the filter for which both the long and short exposure images have been measured furnishes such an equation, and the constants A and B are found either by a least squares solution or by the graphical method, and from these a and b are computed. While the determination of the correction $n(S_v)$ for each plate from an additional short exposure is simple enough, it will generally be found that this correction remains constant for the same instrument, material, and development. In densely crowded star fields or clusters it may be preferable to take the long and short exposure on separate plates of the same emulsion number, developing them together in the same tray.

After the corrections n have been applied according to formulae (5) there remains the task of calibrating the observed color equivalents against spectral types or other color-index systems. This can only be done by observing a number of stars of known spectral type or color-index and it implies the determination of the zero correction c which has to be chosen so that the color-indices of stars of spectral type A_0 become zero.

The star images falling into a narrow strip of the plate along the dividing line of the filter can not be utilized as it remains uncertain with which filter they were taken. If two determinations of the color indices are made, it is advisable to take the first pair of plates with the dividing line oriented north-south, the second with the dividing line east-west. It is also possible to combine a pair of half filter exposures with photographs taken through full filters of the same transmission as those selected for the half filter. The reduction curves of the full filter photographs are then established by comparison with the results of the half filter exposures.

The method here described for the determination of color-indices from photographs taken through a half color filter has the following advantages:

1. The method is differential.
2. It is economical in exposure time.
3. A separate reduction curve is derived for each individual exposure, which permits the use of long exposures.
4. The color equivalents measured are larger than the usual color-indices and are thus less affected by errors of observation.
5. Only one image of each star needs to be taken on each plate which is of special advantage in densely crowded star fields or clusters.

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