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THE RADIAL VELOCITY ORBIT OF α DRACONIS

BY

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 α Draconis, Type F8

The calendar date of the first observation is 1898
July 25, and of the last observation 1899 Oct. 16.

$$\alpha(1900) = 18^{\text{h}}22^{\text{m}}9^{\text{s}} : \delta(1900) = +72^{\circ}41'$$

The investigation of the radial velocity orbit of this
star is based upon the following observations:

SERIES I (1898-1899)

Obs. No.	Date J.D. 2414	Wt.	Vel. (observed)	Vel. ¹ (computed) _i	O-C $\Delta\text{Vel.} = v$
1	496.79	1	+45.3	+44.3	+1.0
2	538.68	1	+45.7	+46.0	-0.3
3	552.68	$\frac{2}{3}$	+42.4	+43.5	-1.1
4	589.89	1	+14.2	+14.4	-0.2
5	595.65	$\frac{2}{3}$	+11.6	+11.6	0.0
6	599.60	$\frac{2}{3}$	+11.0	+10.7	+0.3
7	606.59	$\frac{2}{3}$	+10.4	+10.7	-0.3
8	612.59	$\frac{2}{3}$	+10.4	+11.7	-1.3
9	631.59	$\frac{2}{3}$	+18.1	+17.1	+1.0
10	641.10	$\frac{2}{3}$	+20.8	+19.9	+0.9
11	641.59	$\frac{2}{3}$	+19.8	+20.0	-0.2
12	693.09	1	+32.1	+32.0	+0.1
13	708.95	1	+34.0	+34.8	-0.8
14	721.01	1	+36.5	+36.8	-0.3
15	749.03	1	+41.0	+40.9	+0.1
16	754.03	1	+40.5	+41.5	-1.0
17	776.90	1	+44.6	+44.3	+0.3
18	804.89	$\frac{2}{3}$	+47.0	+46.3	+0.7
19	813.98	1	+46.2	+46.3	-0.1
20	820.95	1	+46.2	+45.9	+0.3
21	833.92	$\frac{2}{3}$	+42.4	+43.3	-0.9
22	841.74	1	+41.3	+39.9	+1.4
23	847.87	1	+35.6	+35.8	-0.2
24	852.88	1	+31.0	+31.4	-0.4
25	867.82	1	+17.1	+16.5	+0.6
26	890.75	1	+10.6	+11.2	-0.6
27	909.72	$\frac{2}{3}$	+17.2	+16.3	+0.9
28	944.71	$\frac{2}{3}$	+26.2	+25.9	+0.3

$$\Sigma pvv = 10.8$$

SERIES II (1922-1923)

Obs. No.	Date J.D. 2423	Vel. (observed)	Vel. ¹ (computed) _{iv}	(O-C) $\Delta\text{Vel.} = v$
29	188.00	+43.3	+44.6	-1.3
30	245.78	+43.8	+44.6	-0.8
31	250.71	+42.1	+43.4	-1.3
32	257.72	+39.2	+40.8	-1.6
33	278.74	+25.5	+24.9	+0.6
34	295.76	+12.8	+13.1	-0.3
35	305.68	+11.4	+12.5	-1.1
36	306.67	+12.2	+12.6	-0.4
37	312.72	+14.2	+13.8	+0.4
38	320.73	+15.7	+16.2	-0.5
39	348.64	+24.5	+24.6	-0.1
40	388.59	+35.0	+33.6	+1.4
41	428.70	+39.5	+40.0	-0.5
42	472.05	+46.2	+44.9	+1.3
43	484.04	+44.0	+45.9	-1.9
44	496.03	+46.1	+46.5	-0.4
45	508.00	+48.3	+46.5	+1.8
46	522.00	+47.9	+45.4	+2.5
47	530.00	+44.6	+43.8	+0.8
48	532.89	+42.9	+42.9	0.0
49	550.91	+32.8	+32.7	+0.1
50	577.92	+12.8	+12.7	+0.1
51	595.81	+16.8	+14.5	+2.3
52	604.94	+16.6	+17.3	-0.7
53	628.74	+23.8	+24.4	-0.6

$$\Sigma vv = 32.8$$

The calendar date of the first observation of this
series is 1922 May 12, and of the last observation
1923 July 27.

SERIES III (1928)

Obs. No.	Date J.D. 2425	Vel. (observed)	Vel. ¹ (computed) _{iv}	(O-C) $\Delta\text{Vel.} = v$
54	287.05	+16.6	+16.6	0.0
55	325.05	+27.7	+27.6	+0.1
56	345.94	+31.9	+32.2	-0.3

The calendar date of observation No. 54 is 1928
Feb. 10, and of No. 56 April 9.

¹ The source of the computed velocities is explained later
in the text.

The velocities are expressed in kilometers per second as usual. Hereafter the "per second" is omitted in writing the velocities.

The observed velocities tabulated above include all of the known systematic corrections² to render the observations homogeneous. It will be noted that all of the observed velocities in Series I are systematically smaller, by 0.25 km., than the values given by Wright³ in his investigation of this orbit. Corrections for flexure (amounting to 0.2 km. in the maximum) are available for the observations of Series II and III. These have not been applied, however, as the flexure corrections for the observations of Series I are unknown, and it is desired here to have all of the observations as nearly homogeneous as possible. This flexure correction is +0.2 km. for the observations numbered 29, 33, 34, 35, 36, 37, 38, 39, 40, 50, and 52. It is -0.2 km. for the other observations of Series II. It is -0.15 km. for the observations of Series III.

The weights assigned in Series I are those given by Wright³ for reasons set forth in his determination of the orbit based upon this series of observations. All of the observations of the other two series are considered to be of weight unity.

In the interval between, and including, the first two series of observations (1898-1923) the binary system completed thirty-two revolutions, so that material is now available for a more accurate determination of the period. The steps taken in this investigation are:

1.—A differential correction of Wright's orbit to represent the observations of Series II. This gives a first approximation to the periastron passage time for the year 1923, which is hereafter designated T_{II} .

2.—Evaluation of a second approximation to the period, P , (Wright's $P=281^d.8$ being the first approximation) by $P = \frac{T_{II} - T_I}{32}$ where T_{II} is that of Step 1 and

and T_I is the periastron passage time of 1898 (the first approximation for which is that from Wright's elements).

The interval between periastron passage times is used for the determination of P since T is quite accurately determined in an orbit of such eccentricity as this (here $e > 0.4$). P thus determined is, of course, the anomalistic period.

3.—With the P of Step 2 constant, a differential correction of Wright's other elements to represent the observations of Series I for the purpose of obtaining an improved T_I on the basis of this new P .

4.—With the P of Step 2 constant, a differential correction of the other elements resulting from Step 1 to represent the observations of Series II for the purpose of obtaining an improved T_{II} on the basis of this P .

² *Publ. Lick Obs.*, 16, xxi, 1928.

³ *A.p.J.*, 11, 131, 1900.

5.—The final value of P obtained as in Step 2, using now the values of T_I and T_{II} resulting from Steps 3 and 4 respectively.

6.—Representation of the observations of Series III.

The differential correction of Step 1 resulted in $T_{II} = \text{J. D. } 2423565.8$. From Wright's orbit $T_I = \text{J. D. } 2414582.5$. Hence (Step 2)

$$P = \frac{8983^d.3}{32} = 280^d.728$$

Step 3 resulted in the following set of elements:

ELEMENTS I

$$T = \text{J.D. } 2414583.682 \pm 0^d.665$$

$$K = 17.97 \text{ km.} \pm 0.13 \text{ km.}$$

$$e = 0.428 \pm 0.006$$

$$\omega = 119^\circ.41 \pm 1^\circ.23$$

$$V = +32.21 \text{ km.} \pm 0.22 \text{ km.}$$

$$P = 280^d.728$$

$$r_o = \pm 0.46 \text{ km. (probable error of a single observation of weight unity)}$$

$$\Sigma pvv = 10.8$$

It is from these elements that the "computed velocities" given in the table of observations of Series I have been derived.

(It will be noted that this Σpvv is larger than Wright's by an amount greater than would be expected as the elements are made to differ only because of the imposition of $P = 280^d.728$. An error was found in his residual for observation No. 22. Instead of +0.5, as published, it should be +1.7. Making this correction his Σpvv becomes 9.7 instead of 7.1.)

Step 4 resulted in the following set of elements:

ELEMENTS II

$$T = \text{J.D. } 2423565.521 \pm 1^d.288$$

$$K = 17.17 \text{ km.} \pm 0.23 \text{ km.}$$

$$e = 0.419 \pm 0.012$$

$$\omega = 126^\circ.03 \pm 2^\circ.38$$

$$V = +33.64 \text{ km.} \pm 0.38 \text{ km.}$$

$$P = 280^d.728$$

$$r_o = \pm 0.85 \text{ km. : } \Sigma vv = 32.2$$

The final value of the anomalistic period resulting from this investigation is therefore (Step 5)

$$P = \frac{8981^d.839}{32} = 280^d.683 \pm 0^d.045$$

The differences between Elements I which represent the 1898-99 observations and Elements II which represent the 1922-23 observations seem to be too large to be accidental in a case such as this, where the probable error of a single observation of weight unity

is less than one kilometer. To test this a third set of elements was obtained by taking the weighted mean of Elements I and II with the period $P=280^d683$. This gives

ELEMENTS III

$$\begin{aligned} K &= 17.79 \text{ km.} \\ e &= 0.426 \\ \omega &= 120^\circ80 \\ V &= +32.57 \text{ km.} \\ P &= 280^d683 \end{aligned}$$

The periastron passage times are those of Elements I and Elements II. The observations of the two series are not at all satisfactorily represented by this set of elements. It was then considered that a satisfactory representation might be obtained by using these elements except V , and deriving a separate V for each series to render Σpv or Σv for each series equal zero. This resulted in $V=+32.31$ km. for the 1898-99 series, and $V=+33.27$ km. for the 1922-23 series. This procedure gave a fair representation of the first series of observations (Σpvv being 13.7 an increase of 2.9 over the $\Sigma pvv=10.8$ for Elements I). But the observations of Series II were very poorly represented (Σvv being 66.4, more than twice that for the representation by Elements II, viz. $\Sigma vv=32.8$). A least square solution to represent all of the observations undoubtedly would yield elements but little different from Elements III.

The evidence to date shows that, *unless there is some unknown systematic difference between the observations of*

Series I and those of Series II, all of the observations cannot be represented satisfactorily by one set of elements. This means that *the system of χ Draconis consists of more than two bodies*.

The differences between Elements I and Elements II are not necessarily due entirely to perturbations by a third body, but are probably due, in large part, to changing values of $\sin i$ arising from the changing inclination as the binary system swings about the center of gravity of the three (or more) body system. After another interval of twenty to twenty-five years the question of the possible multiple character of the system can be settled definitely.

At this epoch (1928) the best set of elements is that of Elements II with $P=280^d683$ instead of $P=280^d728$, i. e.

ELEMENTS IV

$$\begin{aligned} T &= \text{J.D. } 2423565.521 \\ K &= 17.17 \text{ km.} \\ e &= 0.419 \\ \omega &= 126^\circ03 \\ V &= +33.64 \text{ km.} \\ P &= 280^d683 \\ a \sin i &= 60,170,000 \text{ km.} \end{aligned}$$

It is from these Elements IV that the "computed velocities" tabulated with the observations of Series II and Series III were obtained. It will be noted that the Series III (1928) observations are most satisfactorily represented (Step 6).

Figure 1. Graph of Elements IV, χ Draconis.

The accompanying figure is the graph representing Elements IV with the observed velocities of Series II and Series III represented by the symbols $\circ$ and $\times$ respectively.

It will be instructive to see how observations made from now on are represented by these elements. For this purpose the following ephemeris has been com-

puted. This gives $\frac{dz}{dt}$, the velocity referred to the center of mass of the binary system, as a function of $(t-T)$, the interval of time from periastron passage.

Then $\text{Vel.} = \frac{dz}{dt} + V$.

EPHEMERIS FOR VELOCITIES REFERRED TO THE CENTER OF GRAVITY OF THE BINARY SYSTEM
(Computed from Elements IV)

$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km
-140.0	+ 5.92	-70.0	+12.80	0.0	-14.33	+ 70.0	-7.40
-137.5	+ 6.26	-67.5	+12.87	+ 2.5	-16.28	+ 72.5	-6.77
-135.0	+ 6.60	-65.0	+12.92	+ 5.0	-17.96	+ 75.0	-6.15
-132.5	+ 6.93	-62.5	+12.94	+ 7.5	-19.30	+ 77.5	-5.55
-130.0	+ 7.26	-60.0	+12.93	+10.0	-20.30	+ 80.0	-4.96
-127.5	+ 7.58	-57.5	+12.88	+12.5	-20.96	+ 82.5	-4.39
-125.0	+ 7.90	-55.0	+12.80	+15.0	-21.32	+ 85.0	-3.83
-122.5	+ 8.21	-52.5	+12.67	+17.5	-21.40	+ 87.5	-3.28
-120.0	+ 8.51	-50.0	+12.50	+20.0	-21.26	+ 90.0	-2.75
-117.5	+ 8.81	-47.5	+12.27	+22.5	-20.94	+ 92.5	-2.23
-115.0	+ 9.10	-45.0	+11.98	+25.0	-20.47	+ 95.0	-1.72
-112.5	+ 9.38	-42.5	+11.62	+27.5	-19.90	+ 97.5	-1.22
-110.0	+ 9.66	-40.0	+11.17	+30.0	-19.24	+100.0	-0.74
-107.5	+ 9.93	-37.5	+10.63	+32.5	-18.53	+102.5	-0.27
-105.0	+10.19	-35.0	+ 9.99	+35.0	-17.77	+105.0	+0.20
-102.5	+10.44	-32.5	+ 9.23	+37.5	-16.99	+107.5	+0.65
-100.0	+10.69	-30.0	+ 8.33	+40.0	-16.20	+110.0	+1.10
- 97.5	+10.93	-27.5	+ 7.28	+42.5	-15.40	+112.5	+1.53
- 95.0	+11.16	-25.0	+ 6.08	+45.0	-14.61	+115.0	+1.96
- 92.5	+11.38	-22.5	+ 4.70	+47.5	-13.82	+117.5	+2.38
- 90.0	+11.59	-20.0	+ 3.14	+50.0	-13.05	+120.0	+2.79
- 87.5	+11.79	-17.5	+ 1.38	+52.5	-12.29	+122.5	+3.19
- 85.0	+11.98	-15.0	- 0.58	+55.0	-11.54	+125.0	+3.59
- 82.5	+12.16	-12.5	- 2.74	+57.5	-10.81	+127.5	+3.98
- 80.0	+12.32	-10.0	- 5.03	+60.0	-10.09	+130.0	+4.36
- 77.5	+12.46	- 7.5	- 7.41	+62.5	- 9.39	+132.5	+4.73
- 75.0	+12.59	- 5.0	- 9.80	+65.0	- 8.71	+135.0	+5.10
- 72.5	+12.70	- 2.5	-12.14	+67.5	- 8.05	+137.5	+5.46
- 70.0	+12.80	0.0	-14.33	+70.0	- 7.40	+140.0	+5.82

Maximum = +12.94 at $(t-T) = -62^{\text{d}}12$

Minimum = -21.40 at $(t-T) = +17.13$

$\frac{dz}{dt} = 0$ at $\begin{cases} (t-T) = -15^{\text{d}}71 \text{ and} \\ (t-T) = +103.93 \end{cases}$

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