LICK OBSERVATORY BULLETIN

NUMBER 398

MOTION OF MATTER IN THE TAIL OF COMET 1908 III (MOREHOUSE)

BY

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Comet Morehouse showed an unusual activity in the formation of the tail. Many violent outbursts with complete changes in the shape of the tail were photographed at different observatories. They are commonly described as explosions, although in some cases a sudden stoppage of emission would explain the fact as well.

One of the most striking explosions occurred on October 14, 1908, with the consequent motion of cloud formations in the tail. It has been the subject of several investigations, but the problem of the motion of these cloud-like masses cannot be regarded as solved. Indeed, K. Pokrowski¹ and A. J. Orlov² who computed orbits for these masses treated them as a single point, whereas it is apparent that the condensations were moving along the tail with very unequal speeds and showed many peculiarities worthy of notice.

The idea of a more thorough investigation of the motion of the tail formations suggested itself to the present writer during his stay at the Yerkes Observatory in 1926. The late Professor E. E. Barnard collected there a remarkable series of the photographs of this comet of which only few have been discussed and published. The explosion of October 14 which forms the subject of this paper was described by Barnard in the *Astrophysical Journal* (28, 384, 1908) and three of the best plates were reproduced there.

DATA

The condensations out of which five were selected for detailed study are first noticeable on the plates of October 14. Barnard in his paper does not mention

¹ *A.N.*, 184, 3, 1910.

² *Publ. der K. U.-Sternwarte zu Jurjew*, 21, Heft 3, 1910.

them on this date, although they are clearly visible on the negatives in spite of overexposure and all peculiar features of their appearance on October 15 can be identified on the plates of October 14. They were followed on October 16 and 17 and they are still visible on the Lick plates of October 18, but identification is impossible. The notation is as shown in Fig. 1.

Fig. 1. Morehouse's Comet on October 15, 1908.
Astrophysical Journal, 28, plate 32, 1908.

It is at once apparent from the comparison of photographs that condensations I and II were moving much faster than III and IV. On October 14, I and II were between the nucleus and condensations III and IV. On October 15, all condensations were almost on a straight line perpendicular to the axis of the tail and on October 16, I and II were farther from the nucleus

than III and IV. A. Kopff³ has found different velocities for different condensations measured on these dates. In his notation condensations *d* and *e* apparently correspond to I and II of the present study. M. Wolf⁴ has also derived similar results.

There were altogether twenty-five measureable plates of this series obtained by Barnard on four nights. Some of them were taken simultaneously or almost simultaneously and, accordingly, the mean of the results of individual plates was taken. In addition to this four excellent reproductions of negatives obtained by F. Quénnisset⁵ at Juvisy were found among Barnard's papers and kindly placed at my disposal by Miss Mary R. Calvert, Professor Barnard's niece. They furnished intermediate points between the Yerkes plates and therefore were used, at some sacrifice of the homogeneity of the material. Thus nine points for the orbit of condensation I, sixteen points for II, III and IV, and five points for V have been obtained. The plates were denoted as follows: *a*—Bruce 10-inch telescope, *b*—Bruce 6-inch, *c*—Bruce 3.5-inch, *Q*—Quénnisset Voigtländer photographs. The instruments have been described in the original articles for which references are given.

The measurement presented many difficulties. The exposure was sometimes as long as two hours during which time the condensations were moving. This introduced fuzziness at the edges of the condensations. The stars on the plates were long, wavy lines and a precise bisection of them was impossible. In addition to this the condensations showed unquestionable rapid changes in shape and brightness and also frequent deviations from the smooth line of motion which were several times larger than the uncertainty of measurement. These deviations could be ascertained sometimes on two or even three plates taken simultaneously, which all gave the same result. The oscillatory motion of the condensations showed all the characteristics of the undulatory cometary tail, that is, increasing amplitude and period with the recession from the nucleus.

Finally, the following method was adopted. First, condensations were located among the stars on the B. D. charts and their positions to within one minute of arc were determined. Then for greater precision the position of the densest part of every condensation was measured in the measuring machine differentially with respect to the nucleus and thus $\Delta\alpha$ and $\Delta\delta$ were found. One-tenth of a minute of arc was the utmost precision of measurement to be hoped for, especially on later dates. The position of the nucleus was therefore computed from the elements by Kobold⁶ which gave more precise results than could possibly be obtained under

the conditions of measurement. This also gave at once the geocentric position for 1908.0 and thus spared considerable time and labor. The final results of the measurement are given in Table 1, the times being corrected for aberration.

The usual assumption has been made, namely, that the condensations were moving in the plane of the orbit of the nucleus. For the reduction to the plane of the orbit a method developed by A. J. Orlow⁷ has been employed. The necessary equatorial elements of the orbit of the nucleus were obtained from Kobold's elements as follows:

$$\begin{aligned} T &= \text{Dec. 26.7858 G.M.T.} \\ \omega' &= 202^\circ \ 9' \ 37'' \\ \Omega' &= 125^\circ \ 10' \ 32'' \\ i' &= 130^\circ \ 17' \ 7'' \\ \log q &= \bar{1}.97532 \end{aligned}$$

The results of this reduction are presented in Table 2. *R* is the heliocentric distance of the condensation and *w* is the true anomaly of the condensation in the orbit of the nucleus. *r* and *v* are the corresponding quantities for the nucleus.

DETERMINATION OF THE REPULSIVE FORCE OF THE SUN AND OF THE ORBITS OF CONDENSATIONS

Several methods for the computation of the repulsive force of the Sun acting on the particles in the comet's tail are in existence. They are either extremely laborious (Socolow's method) or are not reliable in the present case, as *w* varied but little (Jaegermann's method). A. J. Orlow⁸ has devised a short and efficient method which was used in the present paper. Since this method has been published only in Russian, in a hardly accessible publication, it seems desirable to reproduce it here in its main features.

In the following the quantities pertaining to the orbit of the nucleus will be denoted as usual by small letters, those pertaining to the orbit of a particle or a condensation in the tail by the same capital letters. Thus we shall have two sets of quantities for the orbit of a particle: one referring to the orbit of the nucleus, the other referring to the particle's own orbit.

Let *E* be the excentricity of the condensation's orbit, *P* the parameter, *M*_π the time of perihelion passage, *w*_π the true anomaly of the axis of the particle's orbit. Under assumptions that

- (1) the condensation is moving in the plane of the orbit of the nucleus;
- (2) it is acted upon by a central repulsive force *f* originating in the Sun and varying inversely as the

³ *A.N.*, **180**, 122, 1909.

⁴ *A.N.*, **180**, 1, 1909.

⁵ *Bull. Soc. Astron. de France*, **23**, 32, 1909.

⁶ *A.N.*, **179**, 273, 1908.

⁷ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, No. 4, p. 299, 1909.

⁸ *Publ. der K. U.-Sternwarte zu Jurjew*, **21**, Heft 3, 1910.

TABLE 1

1908.0

 $\alpha = 19^h +$

Point	Plates	Time Oct. G.M.T.	Nucleus		Condensation I		Condensation II		Condensation III		Condensation IV		Condensation V	
			δ	α	δ	α	δ	α	δ	α	δ	α	δ	α
1	161a	14.535	+51°37'3	30 ^m 7 ^s 0	+51°37'0	30 ^m 11 ^s 7	+51°37'0	30 ^m 13 ^s 7	+51°37'3	30 ^m 16 ^s 4	+51°38'6	30 ^m 21 ^s 1		
	162b													
2	163a	.583	33.4	29 57.0	33.2	8.1	32.2	10.1	33.9	13.0	34.7	18.9		
	164c													
3	Q ₁	15.359	50 10.7	27 12.1	50 10.7	29 21.5	50 12.2	29 41.3	50 15.3	0.7	50 21.4	30.1		
4	Q ₂	.430	4.7	26 57.2	7.3	40.5	9.4	30 9.2	12.4	18.7	18.5	47.6		
5	165b	.433	49 51.8	35.0	49 48.7	30 13.8	49 50.9	32.2	49 57.1	59.8	1.4	31 5.8	+50° 3'1	32 ^m 43 ^s 0
	166a													
6	167c	.560	48.8	31.3	45.4	13.3	47.9	34.5	54.6	31 10.1	49 59.0	12.1	1.1	49.3
7	168a	.598	45.2	23.3	41.7	5.3	43.7	47.7	50.5	7.7	56.2	9.7	49 57.8	55.3
	169b													
8	170c	.629	42.0	15.9			37.0	51.5	43.2	11.9	49.7	11.1		
9	171b	.700	37.0	3.4	31.6	41.4	32.8	31 4.6	42.8	26.2	48.6	25.4	50.2	33 30.2
	172a													
10	173c	.723	31.5	25 57.9			27.5	4.3	36.0	30.3	42.5	26.7	46.3	45.9
11	174a	.745	23.9	52.8			24.6	7.2	34.1	43.2	41.4	32.8		
	174b													
12	174 ¹ / ₂ a	.793	24.3	45.5			19.9	20.5	28.0	51.1	35.6	43.5		
	174 ¹ / ₂ b													
13	Q ₃	16.311	29.2	24 5.3	48 25.9	33 34.1	48 33.8	34 10.7	48 44.5	34 10.7	48 50.8	33 34.1		
	175a													
14	175b	.532	48 6.5	23 25.6			6.0	45.6	15.0	35 14.8	26.1	34 54.0		
	175c													
	176a	.641	47 52.2	2.3			47 53.8	57.9	1.5	14.3	17.3	31 1.9		
15	176b													
	176c													
16	Q ₄	17.448	46 27.9	20 41.7	46 34.3	40 31.8					47 3.3	37 26.5		
	Q ₄													
17	179a	.568	16.3	20.4			46 35.2	42 21.2	46 45.7	39 22.8				
	179b													
	180c													

TABLE 2

Point	Mid-Exp. G.M.T. October	Nucleus		I		II		III		IV		V	
		r	v	R	w	R	w	R	w	R	w	R	w
1	14.535	1.5368	-76°45'0	1.5372	-76°43'7	1.5374	-76°43'7	1.5372	-76°45'0	1.5377	-76°44'8		
2	.583	.5364	42.5			.5372	42.8	.5373	43.3	.5377	43.8		
3	15.359	.5268	15.4	.5367	14.3	.5382	14.8	.5393	17.3	.5409	21.4		
4	.430	.5259	12.9	.5380	14.4	.5400	15.7	.5404	17.8	.5421	21.9		
5	.433	.5246	9.3	.5417	5.0	.5420	12.9	.5445	10.7	.5446	13.8	1.5520	-76°14'2
6	.560	.5243	8.3	.5418	3.8	.5432	5.4	.5453	10.0	.5450	13.1	.5525	13.9
7	.598	.5239	7.0	.5414	2.4	.5445	3.5	.5454	8.2	.5450	12.3	.5531	12.7
8	.629	.5234	5.9			.5453	75 59.8	.5463	4.0	.5455	8.8	.5565	8.0
9	.700	.5227	3.5	.5446	75 58.6	.5468	57.0	.5474	4.7	.5463	10.6	.5580	8.9
10	.723	.5223	2.5			.5466	56.2	.5478	2.6	.5469	7.3		
11	.745	.5220	1.8			.5470	55.1	.5489	2.0	.5473	7.2		
12	.793	.5215	0.2			.5483	53.7	.5498	75 59.3	.5484	4.8		
13	16.311	.5152	75 41.9	.5615	33.0	.5636	38.4	.5607	51.3	.5549	2.6		
14	.532	.5126	34.1			.5697	25.8	.5697	32.0	.5666	75 40.7		
15	.641	.5112	30.2			.5698	21.2	.5704	26.7	.5674	38.5		
16	17.448	.5014	1.3	.6043	49.2					.5835	13.0		
17	.568	.5000	74 56.9			.6138	74 52.2	.5956	2.0				

square of the distance we have for the motion of the condensation the following equations:

$$(1) \quad \begin{cases} R^2 \frac{dw}{dt} = C \\ \frac{d^2 R}{dt^2} = \frac{f}{R^2} + \frac{C^2}{R^3} \end{cases}$$

where C is the constant of areas, equal to $\sqrt{fP}$ and R and w the quantities used in Table 2. From equation 1 we easily derive:

$$(2) \quad f = R^2 \frac{d^2 R}{dt^2} - \frac{C^2}{R}$$

The idea of the method is to determine for the mean moment $t=t_0$ of all observations the corresponding radius vector R_0 , its first and second derivatives, and the constant of areas from the whole series of observations. Since C is usually very small we can neglect it altogether in the first approximation and put

$$(3) \quad f = R_0^2 \frac{d^2 R_0}{dt^2}$$

For the determination of the derivatives of R at $t=t_0$ we have Taylor's series (4):

$$(4) \quad \begin{cases} R = R_0 + \frac{dR_0}{dt} t + \frac{1}{2} \frac{d^2 R_0}{dt^2} t^2 + \frac{1}{6} \frac{d^3 R_0}{dt^3} + \dots \\ w = w_0 + \frac{dw_0}{dt} t + \frac{1}{2} \frac{d^2 w_0}{dt^2} t^2 + \frac{1}{6} \frac{d^3 w_0}{dt^3} + \dots \end{cases}$$

Taking in the series for R only three first terms we can easily find approximate values of R_0 and its derivatives from three or more observations well distributed through the whole period of observation. The approximate value of f can be found then from (3).

Differentiating (1) in succession and putting the obtained values of the derivatives in (4) we have, neglecting the terms with the derivatives of higher order than three,

$$(5) \quad \begin{cases} R = R_0 + \left(t - \frac{ft^3}{3R_0^2} \right) \frac{dR_0}{dt} + \frac{1}{2} \frac{d^2 R_0}{dt^2} t^2 \\ w = w_0 + \frac{Ct}{R_0^2} - \frac{Ct^2}{R_0^3} \frac{dR_0}{dt} - \frac{Ct^3}{3R_0^3} \frac{d^2 R_0}{dt^2} \end{cases}$$

In the series for R the coefficient $t - \frac{ft^3}{3R_0^2} = t'$ differs only slightly from t , if the whole period of observation covers only a few days, since f is usually a small number. Its approximate value and R_0 can be taken from (3).

Now we have from the measurements n values of R and w . Subtracting from each equation,

$R = R_0 + \frac{dR_0}{dt} t' + \frac{1}{2} \frac{d^2 R_0}{dt^2} t'^2$, the mean of all equations and denoting the residuals with Δ , we have:

$$(6) \quad \begin{cases} \Delta R_1 = \frac{dR_0}{dt} \Delta t'_1 + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t'^2_1 \\ \Delta R_2 = \frac{dR_0}{dt} \Delta t'_2 + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t'^2_2 \\ \dots \dots \dots \\ \Delta R_n = \frac{dR_0}{dt} \Delta t'_n + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t'^2_n \end{cases}$$

From these equations $\frac{dR_0}{dt}$ and $\frac{d^2 R_0}{dt^2}$ are obtained by the method of least squares. R_0 is then easily found from the mean of all equations. Using in the same way the second equation of (5), $w = w_0 + \frac{Ct''}{R_0^2}$, where $t'' = t' - \frac{t'^2}{R_0} \frac{dR_0}{dt}$, we find C and w_0 . For the final check on the convergence of series (4), we have, neglecting only terms with derivatives of higher order than six,

$$(7) \quad R = R_\pi + \frac{1}{2} \frac{d^2 R_\pi}{dt^2} t_\pi^2 + \frac{1}{24} \frac{d^4 R_\pi}{dt^4} t_\pi^4$$

For $t = -t_\pi$, $R = R_0$ in equation (7); substituting $t = -t_\pi$ in the expression of R and its derivatives, we therefore obtain:

$$(8) \quad \begin{cases} R_0 = R_\pi + \frac{1}{2} \frac{d^2 R_\pi}{dt^2} t_\pi^2 + \frac{1}{24} \frac{d^4 R_\pi}{dt^4} t_\pi^4 \\ \frac{dR_0}{dt} = -\frac{d^2 R_\pi}{dt^2} t_\pi - \frac{1}{6} \frac{d^4 R_\pi}{dt^4} t_\pi^3 \\ \frac{d^2 R_0}{dt^2} = +\frac{d^2 R_\pi}{dt^2} + \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \end{cases}$$

From (1) we have by successive differentiations at $R = R_\pi$

$$(9) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{f}{R_\pi^2} + \frac{C^2}{R_\pi^3} \\ \frac{d^4 R_\pi}{dt^4} = -\frac{2f}{R_\pi^3} \frac{d^2 R_\pi}{dt^2} - \frac{3C^2}{R_\pi^4} \frac{d^2 R_\pi}{dt^2} \end{cases}$$

Heretofore we have not made any hypothesis concerning the kind of the conic section in which the particle or the condensation moves. Bredichin⁹ has shown that the kind of the orbit depends entirely upon the value

⁹ R. Jaegermann, *Bredichin's Mechanische Untersuchungen über Cometenformen*. St. Petersburg, 1903, pp. 283-291.

of the repulsive force $(1-\mu)$ of the Sun acting on the particle. For $(1-\mu) > 1$ it will be a hyperbola convex to the Sun. As a fact of observation we can assume that in the tails of the comets of Bredichin's first type, to which the tail of Comet Morehouse evidently belongs, $(1-\mu)$ is much larger than unity. Therefore the resulting orbit of the condensation should be the branch of a hyperbola convex to the Sun. Considering the equation of such hyperbola

$$(10) \quad R = \frac{P}{E \cos V - 1}$$

we derive for R_π

$$(11) \quad R_\pi = \frac{P}{E-1} \text{ or } E = 1 + \frac{P}{R_\pi}$$

Remembering that $C = \sqrt{fP}$ we obtain from (9) and (11):

$$(12) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{fE}{R_\pi^2} \\ \frac{d^4 R_\pi}{dt^4} = -\frac{f^2 E(3E-1)}{R_\pi^5} \end{cases}$$

In the last equation of (12) f is a small quantity and its approximate value from (6) will serve. P and therefore E are known from the relation $P = \frac{C^2}{f}$ where C is already known. The approximate value of R_π is also known. Indeed, we have from (5):

$$(13) \quad \frac{dR}{dt} = \frac{dR_o}{dt} \left(1 - \frac{f}{R_o^3} t^2 \right) + \frac{d^2 R_o}{dt^2} t$$

For $t = t_\pi$, $\frac{dR}{dt} = 0$, and we obtain:

$$(14) \quad t_\pi = -\frac{\frac{dR_o}{dt}}{\frac{d^2 R_o}{dt^2}} \left(1 - \frac{f}{R_o^3} t_\pi^2 \right)$$

Here $\frac{f}{R_o^3} t_\pi^2$ is a small factor and it is easy to find t_π by successive approximations or by solving the quadratic equation (14) directly for t_π . The choice between the roots is easy as they differ very much. Knowing t_π we find R_π from the mean of equations (5). Therefore $\frac{d^4 R_\pi}{dt^4}$ is known with sufficient accuracy from (12) and we have to solve (8) only for R_π , $\frac{d^2 R_\pi}{dt^2}$ and t_π . The result is:

$$(15) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{d^2 R_o}{dt^2} - \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \\ t_\pi = -\frac{\frac{dR_o}{dt} + \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^2}{\frac{d^2 R_\pi}{dt^2}} \\ R_\pi = R_o - \frac{1}{2} \left(\frac{d^2 R_\pi}{dt^2} + \frac{1}{12} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \right) t_\pi^2 \end{cases}$$

In the right side of equation (15) the approximate value of t_π should be taken from (14). Finally, from (12) we have

$$(16) \quad f = \frac{R_\pi^2}{E} \frac{d^2 R_\pi}{dt^2}$$

This value of f should coincide with that from equations (6) and (2) if the convergence of the series employed is rapid enough.

We have found already t_π and E . The remaining elements can be obtained by simple considerations in the following way:

$$(17) \quad \begin{cases} M_\pi = t_o + t_\pi \\ A = \frac{P}{E^2 - 1} \\ w_\pi = w_o - V_o \end{cases}$$

V_o , the true anomaly of the condensation in its own orbit, is computed for the moment t_o together with the ephemeris for all points used in the calculation. It is easy to compute the ephemeris using tables for hyperbolic motion (given also by A. J. Orlov). We have the following equations for the computation of the ephemeris:

$$(18) \quad \begin{cases} N = \frac{\sqrt{f}}{A^{\frac{3}{2}}} t & x = E - 1 \\ \tan F - \log \tan \left(45^\circ + \frac{F}{2} \right) = N - x \tan F \\ \tan \frac{V}{2} = \sqrt{\frac{E-1}{E+1}} \tan \frac{F}{2} \\ \frac{1}{R} = \frac{1}{R_\pi} - \frac{2E}{P} \sin^2 \frac{V}{2} \end{cases}$$

Since, however, the intervals of time in the case of motion of the condensations are usually small, a simplified formula for V may be used. Expanding w as a function of w_π and noting that $w - w_\pi = V$ we come by simple considerations to the formulae

$$(19) \quad \begin{cases} n = N \sqrt{\frac{E-1}{(E+1)^3}} \\ V = n - \frac{n^3 E}{3x} \end{cases}$$

where t is counted from the perihelion of the condensation.

Suppose now that the gaseous mass was ejected from the nucleus with an initial velocity g and under an angle G with the radius vector; counting G positive in the direction opposite to the motion of the nucleus. The vector of the initial impulse is the geometric sum of two vectors: one, of the velocity of ejection and the other, of the orbital velocity of the nucleus at that point. Considering the projection of these vectors on the prolongation of the radius vector and the perpendicular to it at the nucleus, we obtain:

$$(20) \quad \begin{cases} g \cos G = \frac{dr}{dt} - \frac{dR}{dt} \\ g \sin G = \frac{c - C}{R} \end{cases}$$

where $\frac{dr}{dt} = \frac{k \sin v}{\sqrt{2q}}$, $\frac{dR}{dt} = \frac{E\sqrt{f} \sin V}{\sqrt{P}}$, $c = k\sqrt{2q}$, $C = \sqrt{fP}$ at the moment of ejection. From equations (20) g and G may be found. The moment of ejection is determined graphically or by comparing the ephemeris of the nucleus and the condensation at their nearest approach.

This method is short and efficient in practice. Once the normal equations for (6) are formed, the equations for the following approximations are obtained in a very short time. If several condensations are observed at the same time the coefficients of the unknowns in the least squares solutions remain almost the same, which simplifies the work. The chief drawback of the method is a rather lengthy reduction to the plane of the orbit of the nucleus and the derivation of the heliocentric polar coordinates.

CONDENSATION I

In the first approximation, four equations, $R = R_o + \frac{dR_o}{dt}t + \frac{1}{2} \frac{d^2R_o}{dt^2}t^2$, were formed making use of point 1; mean from 5, 6, 7; point 13 and point 16. They gave:

$$\begin{aligned} R_o &= 1.5510 \\ \frac{dR_o}{dt} &= 0.0221 \\ \frac{d^2R_o}{dt^2} &= 0.0180 \\ \log f &= \bar{2}.63190 \end{aligned}$$

Comparing f with the Gaussian constant k , we find the effective repulsive force of the Sun $\mu = -145$ and its absolute value $(1-\mu) = 146$ in the units of k^2 , that is 146 times the force of gravitation acting on the same condensation. For the second approximation all nine equations, $R = R_o + \frac{dR_o}{dt}t + \frac{1}{2} \frac{d^2R_o}{dt^2}t^2$, were formed. They gave the following normal equations¹⁰ in the least squares solution:

$$\left. \begin{aligned} +99324 \frac{dR_o}{dt} + 37252 \frac{d^2R_o}{dt^2} &= 2546 \\ +41788 \frac{d^2R_o}{dt^2} &= 1490 \end{aligned} \right\}$$

$$50459 \frac{C}{R_o^2} = 585$$

which yielded:

$$\begin{aligned} R_o &= 1.5443 \\ \frac{dR_o}{dt} &= 0.0184 \pm 0.0006 \\ \frac{d^2R_o}{dt^2} &= 0.0192 \pm 0.0010 \text{ (probable error)} \\ w_o &= -75^\circ 58'.1 \\ \log C &= \bar{2}.44119 \\ (1-\mu) &= 154 \pm 8.1 \end{aligned}$$

The next approximation (16) gave the same value of $(1-\mu)$, which is also true for the rest of the condensations.

This results in the elements:

$$\begin{aligned} A &= 0.7632 \\ E &= 1.0110 \\ R_\pi &= 1.5357 \\ w_\pi &= -76^\circ 36'.1 \\ M_\pi &= \text{October } 14.764, 1908, \text{ G. M. T.} \end{aligned}$$

The orbit represents the observations as shown in Table 3. The subscripts mean: c , the value computed and o , the value observed. It was not considered necessary to compute α and δ for the condensations as the heliocentric coordinates give a good idea of the degree of correspondence. $1'$ in w corresponds approximately to 2.5 and 0.0001 in R would give about 0.5 on the original plates. It is seen from this that the orbit satisfies the observations fairly well, taking into account the difficulty of measurement.

¹⁰ In this and later normal equations a factor of 10000 has been introduced as a matter of convenience.

TABLE 3

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5362	1.5372	+0.0010	-76°45'2	-76°43'7	+1'5
2	.5392	.5667	- .0025	12.2	14.3	-2.1
3	.5400	.5380	- .0020	9.3	14.4	-5.1
5	.5401	.5417	+ .0016	9.2	5.0	+4.2
6	.5418	.5418	.0000	4.1	3.8	+0.3
7	.5425	.5414	- .0011	2.5	2.4	+0.1
9	.5443	.5446	+ .0003	75 58.4	75 58.6	-0.2
13	.5593	.5615	+ .0022	33.8	33.0	+0.8
16	.6076	.6043	- .0033	74 48.8	74 49.2	-0.4

CONDENSATION II

The first approximation gave $(1-\mu)=143$. The second approximation, including all sixteen equations, resulted in the following normal equations:

$$\left. \begin{aligned} +163510 \frac{dR_o}{dt} + 35781 \frac{d^2 R_o}{dt^2} &= +3970 \\ +58337 \frac{d^2 R_o}{dt^2} &= +1795 \\ +78067 \frac{C}{R_o^2} &= +823 \end{aligned} \right\}$$

which are satisfied by

$$\begin{aligned} R_o &= 1.5475 \\ \frac{dR_o}{dt} &= 0.0203 \pm 0.0003 \\ \frac{d^2 R_o}{dt^2} &= 0.0183 \pm 0.0006 \\ w_o &= -75^\circ 57'7 \\ \log C &= \bar{2}.40223 \\ (1-\mu) &= 148 \pm 4.7 \end{aligned}$$

The representation of observations is given in Table 4.

TABLE 4

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5367	1.5374	+0.0007	-76°42'0	-76°43'7	-1'7
2	.5365	.5372	+ .0007	40.2	42.8	-2.6
3	.5408	.5382	+ .0026	11.8	14.8	-3.0
4	.5416	.5400	- .0016	9.2	15.7	-6.5
5	.5417	.5420	+ .0003	9.1	12.9	-3.8
6	.5437	.5432	- .0005	4.4	5.4	-1.0
7	.5442	.5445	+ .0003	3.0	3.5	-0.5
8	.5445	.5453	+ .0008	1.9	75 59.8	+2.1
9	.5464	.5468	+ .0004	75 59.2	57.0	+2.2
10	.5467	.5466	- .0001	58.4	56.2	+2.2
11	.5471	.5470	- .0001	57.6	55.1	+2.5
12	.5481	.5483	+ .0002	55.9	53.7	+2.2
13	.5616	.5636	+ .0020	36.9	38.4	-1.5
14	.5690	.5697	+ .0007	28.9	25.8	+3.1
15	.5730	.5698	- .0032	24.9	21.2	+3.7
17	.6166	.6138	- .0028	74 51.7	74 51.2	-0.5

The elements were derived as follows:

$$\begin{aligned} A &= 0.7643 \\ E &= 1.0096 \\ R_\pi &= 1.5365 \\ w_\pi &= -76^\circ 37'6 \\ M_\pi &= \text{October } 14.669 \end{aligned}$$

CONDENSATION III

The first approximation gave $(1-\mu)=77$. From the second approximation, including all sixteen equations, there resulted the normal equations:

$$\left. \begin{aligned} +164516 \frac{dR_o}{dt} + 35994 \frac{d^2 R_o}{dt^2} &= +3258 \\ +58337 \frac{d^2 R_o}{dt^2} &= +1263 \\ +81402 \frac{C}{R_o^2} &= +812 \end{aligned} \right\}$$

which gave:

$$\begin{aligned} R_o &= 1.5488 \\ \frac{dR_o}{dt} &= 0.0174 \pm 0.0004 \\ \frac{d^2 R_o}{dt^2} &= 0.0109 \pm 0.0010 \\ w_o &= -76^\circ 2'1 \\ \log C &= \bar{2}.38830 \\ (1-\mu) &= 88 \pm 8.0 \end{aligned}$$

The following elements were obtained:

$$\begin{aligned} A &= 0.7612 \\ E &= 1.0151 \\ R_\pi &= 1.5353 \\ w_\pi &= -76^\circ 58'2 \\ M_\pi &= \text{October } 14.188 \end{aligned}$$

This orbit satisfies the observations in the following way:

TABLE 5

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5360	1.5372	+0.0012	-76°45'7	-76°45'0	+0'7
2	.5362	.5373	+ .0011	44.0	43.0	+1.0
3	.5430	.5393	- .0037	16.3	17.3	-1.0
4	.5436	.5404	- .0032	13.9	17.8	-3.9
5	.5437	.5447	+ .0010	13.8	10.7	+3.1
6	.5450	.5443	- .0007	9.3	10.0	-0.7
7	.5465	.5454	- .0011	7.8	8.2	-0.4
8	.5469	.5463	- .0006	6.7	4.0	+2.7
9	.5479	.5474	- .0005	4.3	4.7	-0.4
10	.5485	.5478	- .0007	3.4	2.6	+0.8
11	.5488	.5489	+ .0001	2.7	2.0	+0.7
12	.5492	.5498	+ .0006	0.9	75 59.3	+1.6
13	.5607	.5607	.0000	75 42.4	51.3	-8.9
14	.5665	.5697	+ .0032	34.5	32.0	+2.5
15	.5693	.5704	+ .0011	30.7	26.7	+4.0
17	.6009	.5956	- .0053	74 57.9	2.0	-4.1

CONDENSATION IV

The first approximation gave $(1-\mu)=79$. The normal equations of the second approximation are as follows:

$$\left. \begin{aligned} +156506 \frac{dR_o}{dt} + 26078 \frac{d^2R_o}{dt^2} &= +2542 \\ +46850 \frac{d^2R_o}{dt^2} &= +882 \\ +78330 \frac{C}{R_o^2} &= +707 \end{aligned} \right\}$$

which yielded

$$\begin{aligned} R_o &= 1.5473 \\ \frac{dR_o}{dt} &= 0.0145 \pm 0.0005 \\ \frac{d^2R_o}{dt^2} &= 0.0108 \pm 0.0010 \\ w_o &= -76^\circ 7'.7 \\ \log C &= 2.33294 \\ (1-\mu) &= 88 \pm 8.1 \end{aligned}$$

from which the following elements have been derived:

$$\begin{aligned} A &= 0.7812 \\ E &= 1.0118 \\ R_\pi &= 1.5377 \\ w_\pi &= -76^\circ 47'.7 \\ M_\pi &= \text{October } 14.424 \end{aligned}$$

This orbit represents the observations as shown in Table 6.

TABLE 6

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5376	1.5377	+0.0001	-76°44'.4	-76°44'.8	-0'.4
2	.5376	.5377	+ .0001	43.0	43.8	-0.8
3	.5420	.5409	- .0011	19.6	21.4	-1.8
4	.5427	.5421	- .0006	17.4	21.9	-4.5
5	.5428	.5446	+ .0018	17.3	13.8	+3.5
6	.5443	.5450	+ .0007	13.5	13.1	+0.4
7	.5449	.5450	+ .0001	12.3	12.3	0.0
8	.5452	.5455	+ .0003	11.4	8.8	+2.6
9	.5460	.5463	+ .0003	9.2	10.6	-1.4
10	.5462	.5469	+ .0007	8.6	7.3	+1.3
11	.5466	.5473	+ .0007	7.9	7.2	+0.7
12	.5473	.5484	+ .0011	6.4	4.8	+1.6
13	.5559	.5549	- .0010	75 50.8	2.6	-11.8
14	.5607	.5666	+ .0059	44.0	75 40.7	+3.3
15	.5630	.5674	+ .0044	40.8	38.5	+2.3
16	.5853	.5835	- .0018	16.6	13.0	+3.6

CONDENSATION V

Only five positions of this condensation could be measured with certainty on the plates of October 15. The orbit based on such a short arc is indeterminate. But since all four orbits computed above intersect the

orbit of the nucleus approximately in the same place, $R=1.5365$ and $w=-76^\circ 43'.0$, we may reasonably assume that orbit V also passed through this point. This gives us an additional point corresponding to $t=\text{October } 14.535$.

The resulting normal equations are as follows:

$$\left. \begin{aligned} +191220 \frac{dR_o}{dt} - 62200 \frac{d^2R_o}{dt^2} &= 3310 \\ +22420 \frac{d^2R_o}{dt^2} &= 1057 \end{aligned} \right\}$$

which give the approximate value $(1-\mu)=90$. This value, however, is rather uncertain owing to the small difference between two quantities in the numerator of the expression for $\frac{d^2R_o}{dt^2}$. This may influence $(1-\mu)$ for about ten units either way. Owing to this uncertainty and to the assumption mentioned above the orbit has not been computed.

GENERAL REMARKS ON THE ORBITS

The computed orbits agree with the observations satisfactorily. Indeed, taking into account the conditions of measurement, no better correspondence could be expected. Any method of variation of the elements might possibly improve them, but such a procedure is clearly not justified by the nature of the problem as all measurements were necessarily approximate. Moreover, this would not change the value of $(1-\mu)$ which depends only on the quantities derived from the least squares solution, principally upon R_o and its second derivative. The time of perihelion passage depends almost entirely on the ratio of the first and second derivatives of R at $t=t_o$, and therefore is also obtained from the least squares solution.

Figure 2 represents the orbits of condensations II and III and of the nucleus. The deviations from the smooth line are far too large to be explained by the errors of measurement as has been mentioned before. The path of the condensations resembles a cork-screw, which is typical for the structure of the whole tail of this comet.

If we consider the differences $R_o - R_c$ we see that at the beginning of the motion the values are positive which may be accounted for by the repulsive action of the nucleus on the condensations. It is difficult to ascertain the existence of such action between the condensations themselves, owing to the undulatory motion shared by all condensations. The fact that they drew apart from each other with recession from the nucleus

Fig. 2. Orbits of condensations II and III and of the nucleus. Heliocentric coordinates R and w . The insert shows the vectors of the velocity of ejection of condensations with the velocity of the nucleus. Perihelion passage is marked by short vertical lines.

cannot be taken as an evidence for mutual repulsion. It is of course a consequence of the different values of $(1-\mu)$ and the different initial conditions.

It is clear furthermore that on the last date, October 17, all condensations lag considerably behind the computed points although they do not deviate much from the orbit. This may be interpreted as a diminution of acceleration due to the dissipation of these cloud-like formations and consequent increase of resist-

ance on the part of matter filling the tail and moving with smaller velocities. Such a decrease of acceleration was actually found by M. Wolf¹¹ from his measurements of this comet. Analogous phenomena in the case of comet 1882 II in the motion of the so-called Schmidt's Clouds (*Die Schmidtschen Wolken*) has been studied in detail by Bredichin and others.¹²

¹¹ loc. cit.

¹² Jaegermann, *Bredichin's Mechanische Untersuchungen über Cometenformen*, St. Petersburg, 1903, p. 408.

INITIAL CONDITIONS

The moment of explosion cannot be determined with great accuracy on account of the perturbative action of the nucleus on the condensations immediately after the moment of explosion. This changes the moment of the closest approach of the condensations to the nucleus which is computed backwards from the ephemeris.

We cannot assume the moment of emission earlier than the first observation, that is October 14.535, although some condensations were later than this at the nearest distance from the nucleus. Apparently the first photograph was obtained almost at the moment of ejection. Comparison of different orbits fixes the moment of ejection at October 14.500, which cannot be far from the truth.

We can compute, however, the velocity of ejection g and the angle of projection G with considerable accuracy. Indeed, in (20), $\frac{dR}{dt}$ in our case is a small quantity

in comparison with $\frac{dr}{dt}$, as the explosion naturally occurred near the perihelion of the condensations. R also does not change appreciably in the limits of uncertainty as to the time of explosion. C does not depend immediately on the time of ejection but is determined from the whole orbit. Therefore even a large variation in the time of the explosion will not change the results materially. It is also clear from (20) that G can be determined with greater accuracy than g . Indeed, the change of the moment of explosion in Case II by as much as 0.340, which greatly exceeds the limits of uncertainty, gave $G=184^\circ$ instead of 186° previously obtained and $g=21$ km/sec instead of 12 km/sec.

Assuming then October 14.5 as the moment when the gases in the nucleus were suddenly released we have for g and G the values given in Table 7. The insert in Figure 2 shows the vectors of the initial velocities of the condensations and of the velocity of the nucleus on an arbitrary scale. It will be seen that the cometary gases were thrown from the nucleus in a very narrow fan (about 27°) in a direction almost opposite to the direction of the radius vector. The initial velocities fall into two groups: I and II with the mean 11.2 km/sec and III and IV with the mean 20.3 km/sec.

DISCUSSION OF RESULTS

There is little doubt that condensations I and II formed one group with the same physical properties, while condensations III, IV and V formed another. Table 7 gives a summary of the results obtained. Quantities Ψ are the asymptote angles of the hyperbolas.

TABLE 7

Condensation	I	II	III	IV	V
$(1-\mu)$	154	148	88	88	90
g in km/sec.....	10.0	12.4	22.8	17.8	
G	$200^\circ 48'$	$186^\circ 33'$	$198^\circ 21'$	$174^\circ 2'$	
A	0.7632	0.7643	0.7612	0.7812	
E	1.0110	1.0095	1.0151	1.0118	
ψ	$8^\circ 27'$	$7^\circ 53'$	$9^\circ 54'$	$8^\circ 48'$	
R_π	1.5357	1.5365	1.5353	1.5377	
w_π	$-76^\circ 36' 1$	$-76^\circ 37' 6$	$-76^\circ 58' 3$	$-76^\circ 47' 7$	
M_π 1908, Oct. 14.764 G.M.T.	14.669	14.118	14.424		

An examination of this table reveals several interesting facts. The mean of $(1-\mu)$ for condensations I and II is 151, exactly the value of the repulsive force of the Sun calculated recently for the CO^+ molecule by Baade and Pauli.¹³ The spectrum of the tail of this comet has been studied extensively by many observers and identified with the so-called Third Negative Carbon Group belonging to the CO^+ molecule. Indeed, the intensity of these radiations in the tail of Morehouse's comet constituted its peculiar feature. The spectrum of condensations such as have been discussed here was found to be identical with the spectrum of the tail,¹⁴ that is the condensations consisted almost exclusively of CO^+ , which is in perfect accordance with our results.

The good agreement of the theoretical value of $(1-\mu)$ with its dynamical value for condensations I and II shows that these condensations, at least, were moving in the plane of the orbit of the nucleus, that is, the assumption made at the beginning of this investigation is justified. On the other hand, it means also that the Comet Tail bands are a resonance radiation, since Baade and Pauli treated the CO^+ molecule as a resonator. This conclusion can be arrived at by a quite different way of reasoning.¹⁵ It may be of interest to note that the value $(1-\mu)=156$ has been obtained by S. V. Orlov¹⁶ for this comet in the case of motion of the condensations on October 1, 1908.

The question arises as to the significance of the other value of the repulsive force, namely $(1-\mu)=88$, obtained for condensations III, IV and V. We know that the only other constituent of the tail was the N_2^+ molecule which gave a band at $\lambda 3914$. However, this radiation was insignificant in comparison with the CO^+ group, whereas in the tail condensations III, IV and V form the bulk.

The possibility of the inclination of the plane of the orbits III and IV to that of I and II has been considered. By tipping the plane of the orbit III through rather large angles (about 40°) to the plane of the orbit

¹³ *Naturwissenschaften*, **15**, 49, 1927.

¹⁴ F. Baldet, *Ann. de l'Observ. d'Astron. Phys. de Paris*, **7**, 28, 1926.

¹⁵ Bobrovnikoff, *Astroph. Jour.*, **66**, 439, 1927.

¹⁶ *Russian Astron. Jour.*, **1**, 71, 1924.

II a fair degree of correspondence of the points of III with those of II may be reached. But such an inclination is hardly possible. Again, although orbit II is quite different in shape from orbit I, both give the same $(1-\mu)$. This is also true for the orbits of the second group as compared with each other. Condensations I and II on one side and III and IV on the other were symmetrically situated with respect to the axis of the tail, and it is difficult to see why I and II should move exactly in the plane of the orbit of the nucleus, while III and IV move far away from it.

As is well known, the stereoscope in such cases gives striking results. Some observers reported that the south part of the condensations (that is I and II) was nearer to us than the northern part (III and IV), the inference being that they did not lie in the same plane. This is, however, the natural result of the inclination of the orbit of the nucleus to the ecliptic and the situation of the comet in its orbit with reference to the Earth. On the other hand, the stereoscope, in the case of moving and rapidly changing condensations, may give quite erroneous information as has been pointed out by Barnard.¹⁷

There is another line of evidence which favors the view that all condensations were moving in the same plane. The number 88 is not due to chance. The numbers $(1-\mu)=151$ and $(1-\mu)=88$ are approximately simple multiples of 22, their ratio being 7 : 4. S. V. Orlov¹⁸ has determined by an approximate method the values of $(1-\mu)$ from the motion of receding masses in this comet as measured by M. Wolf.¹⁹ The frequency curves of $(1-\mu)$ show sharp maxima for the multiples of 22 or 23, among them being $(1-\mu)=89$. The value $(1-\mu)=89$ was also obtained by R. Jaegermann²⁰ for the motion of condensations in the tail of comet 1903 IV (Borrelly). This can hardly be a coincidence. It may depend on the peculiarities of the structure of the C_0^+ molecule in cometary tails.

The gaseous masses were ejected in the direction almost opposite to the Sun. The larger initial velocity of condensations III and IV and the smaller value of the repulsive force as compared with condensations I and II explains the deformation and twists in the tail in perfect agreement with observations. Indeed, since G is nearly 180° we can apply to the motion of condensations in reference to the nucleus the simple formula of uniformly accelerated motion with the initial velocity g . We have then for velocity v

$$(21) \quad v = g + ht,$$

where t is the time from the moment of ejection and h is the acceleration due to the repulsive force $(1-\mu)$. For condensations II and III we have the following values of v :

TABLE 8

Time	II	III
October 14.50	12	23 km/sec
14.75	26	32
15.00	41	41
15.25	72	58

We see that at the beginning III was moving faster and accordingly covered a greater distance than II. Condensation II soon overtook III owing to its larger $(1-\mu)$. The resulting figure was investigated in fuller detail by S. V. Orlov²¹ based on Kopff's measurements.²² The agreement between the computed and observed values of v is quite satisfactory. The mechanical theory of cometary forms can well explain the resulting shape of the tail without resorting to the hypothesis of a resisting medium in space or currents in the ether.

The results obtained here are hardly in favor of the Bessel-Bredichin theory of cometary forms. The values of $(1-\mu)$ and of g are not only much larger than ever admitted by Bredichin, but also there is no correlation between their values as is required by Bredichin's theory. According to Bredichin, the formation of the tail is intimately connected with the existence of envelopes. The envelopes are formed by continuous emission of matter from the nucleus on the sunward side. Different matter is ejected with different g 's forming successive envelopes in such a way that to each value of $(1-\mu)$ corresponds its own value of g . $(1-\mu)$ is approximately proportional to g , or conversely. Jaegermann²³ gives the results of Bredichin's investigations of more than fifty comets, which were divided by Bredichin into three types (Table 9):

TABLE 9

	First type	Second type	Third type
$1-\mu$	18	from 2.2 to 0.5	from 0.3 to 0
g (mean).....	6.5 km/sec	1.5	0.4

It is seen from Table 7 that in our case we have the converse relation, namely, the smaller values of g correspond to the larger values of $(1-\mu)$. We might derive a larger value of g for condensations I and II than that for condensations III and IV assuming only that I and II were ejected considerably later than III and IV, by at least $0^d.5$. This would be in flat contra-

¹⁷ *Mon. Not., R.A.S.*, **69**, 624, 1909.

¹⁸ *Publ. Russ. Astroph. Obs.*, **1**, 231, 1922.

¹⁹ *A.N.*, **180**, 1, 1909.

²⁰ *Mem. de l'Acad. des Sci. de St. Pétersbourg*, **16**, No. 12.

²¹ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, No. 18, p. 1800, 1916.

²² *Loc. cit.*

²³ *Bredichin's Mechanische Untersuchungen über Cometenformen*, p. 392.

diction with the computed orbits for the condensations as well as with the direct observation on October 14. There seems to be no escape from the conclusion that these condensations belong to the same isochrone, according to Bredichin's terminology (*Isochronengebilde*). Being ejected almost directly away from the Sun they never formed any envelopes around the nucleus. It may be added here that in this comet, as well as in other comets, notably Halley's, studied by the present writer, the CO^+ spectral images do not show the envelopes in contrast with the radiations of cyanogen and Swan.

The straight stem or streamer connecting the condensations with the nucleus was found to be exactly on the prolongation of the radius vector on all three days of observation. Measurements by Miss E. Glancy²⁴ show that even at a considerable distance from the nucleus (on the average 9°) the tail hardly deviated from the radius vector. This is what we might expect from the direction of the initial impulse and large $(1-\mu)$ as derived in this investigation for the condensations. This would show that the condensations did not differ essentially in their behavior from the rest of the matter in the tail, which was confirmed by the spectral analysis of the tail. Apparently the whole tail was formed in the same manner, that is by ejection of matter almost in the direction of the prolonged radius vector.

In such a case the very idea of explosion is not applicable. There was apparently a continuous emission of matter into the tail and the condensations were only more than usually copious emission. After this abundant emission the matter in the nucleus was spent to a considerable degree which was evidenced by a faint and slender tail until a new supply was formed.

The case under investigation gives us an idea of the magnitude of the internal forces in the comets. If the matter ejected on October 14 did not consist of molecules of CO^+ but of solid particles subject to Newtonian gravitation, the resulting orbit would have been a hyperbola concave to the Sun and a new comet would have been formed. Bredichin²⁵ pointed this out as a possible cause of the disruption of comets, which in many instances cannot be explained by the action of the planets. In the case of Comet 1899 I (Swift) he found²⁶ that the secondary nucleus, which afterward formed an independent comet, was expelled from the

main body with $g=12.5$ km/sec and $G=180^\circ$, that is under almost exactly the same conditions as condensations I and II in the present study.

We derive from this investigation one more surprising fact. Matter, in this case CO^+ , may show the same spectrum but possess quite different physical properties and experience different repulsive action of the Sun, which would mean some sort of isotope effect. This is in direct connection with recent observations²⁷ on comets having tails of different Bredichin types but showing the same spectrum for both tails.

SUMMARY

1. The motion of five condensations in the tail of Morehouse's comet on October 14, 15, 16 and 17 has been investigated. The resulting orbits are hyperbolas convex to the Sun. The values of the repulsive force of the Sun fall into two groups:

$$(1-\mu)=151 \text{ and } (1-\mu)=88.$$

2. The first value is exactly what has been found by Baade and Pauli for the CO^+ molecule under the radiation pressure of the Sun.

3. The second value cannot possibly be explained by the inclination of the orbits of the second group to that of the first group.

4. There exists a simple relation between these values: $7:4$, as both numbers are approximate multiples of 22. This is in agreement with the results of S. V. Orlov for this comet in general.

5. The computed orbits and the velocities derived therefrom well explain the observed motion of the condensations and the deformation of the tail without the aid of additional hypotheses, such as, for instance, a resisting medium in space.

6. The moment of ejection of the matter was approximately October 14.500, with the direction of impulse almost directly away from the Sun and initial velocities which fall into two groups: $g=11$ km/sec, and $g=21$ km/sec, corresponding to $(1-\mu)=151$ and $(1-\mu)=88$.

7. The results obtained here are not in agreement with the accepted ideas of the formation of cometary tails.

²⁷ Baldet, *loc. cit.*; Bobrovnikoff, *loc. cit.*

²⁴ *Lick Obs. Bull.*, No. 161, 1909.

²⁵ *Études sur l'Origine des Météores Cosmiques, etc.*, St. Petersburg, 1903, p. 222.

²⁶ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, 1901, p. 483.

LICK OBSERVATORY, February 14, 1928.

Issued March 28, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 399

ELEMENTS AND EPHEMERIS OF COMET *a* 1928 (REINMUTH)

BY

L. BERMAN and F. L. WHIPPLE

Comet *a* 1928 was detected on a Heidelberg plate of February 22 by Dr. K. Reinmuth. It has been described by some observers as having a round coma with a "starlike condensation." The estimated magni-

tude at discovery was 12.5. The comet has since become fainter, later estimates giving the magnitude as 13-14.

The following positions have been received:

1928 U. T.	$\alpha(1928.0)$	$\delta(1928.0)$	Observer	Authority
(1) Jan. 29.10757†	9 ^h 23 ^m 35 ^s .60	+17°41'19".1	Reinmuth, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(2) Feb. 22.96160	9 15 07.45	+21 44 54.7	Reinmuth, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(3) Feb. 23.96368	9 14 54.93	+21 52 39	Mündler, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(4) Feb. 24.82611	9 14 45.61	+21 59 02.3	Chofardet, Besancon	Circ. B. C. A. No. 191, 1928
(5) Feb. 24.95063	9 14 43.60	+22 00 00.1	Mündler, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(6) Feb. 24.95736	9 14 42.2	+22 00 06	G. Struve, Berlin—Babelsberg	Beob. Zirk. No. 9, 1928
(7) Feb. 25.07288	9 14 42.97	+22 00 46.4	Van Biesbroeck, Yerkes—40", visual	Letter
(8) Feb. 25.93181	9 14 33.93	+22 06 57	G. Struve, Berlin—Babelsberg	Beob. Zirk. No. 9, 1928
(9) Feb. 25.95344	9 14 33.79	+22 07 06.1	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(10) Feb. 26.17611*	9 14 31.2	+22 08 37	Crawford, Lick—12", visual	Telephone
(11) Feb. 26.18323	9 14 31.94	+22 08 44.2	Van Biesbroeck, Yerkes—40", visual	Letter
(12) Feb. 26.89764	9 14 26.0	+22 13 28	Volta, Turin	Circ. B. C. A. No. 191, 1928
(13) Feb. 26.99978	9 14 24.82	+22 14 19.5	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(14) Feb. 27.01899	9 14 24.60	+22 14 26.7	G. Struve, Berlin—Babelsberg	Circ. B. C. A. No. 193, 1928
(15) Feb. 27.02935	9 14 24.87	+22 14 27.3	Chofardet, Besancon	Circ. B. C. A. No. 193, 1928
(16) Feb. 27.17776	9 14 23.75	+22 15 31.1	Van Biesbroeck, Yerkes—40", visual	Letter
(17) Feb. 28.00729**	9 14 18.6	+22 21 03	Bianchi, Milan	Circ. B. C. A. No. 192, 1928
(18) Feb. 28.08804	9 14 17.09	+22 21 24.5	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(19) Feb. 28.19653*	9 14 18.96	+22 22 19.5	Crawford, Lick—12", visual	Telephone
(20) Feb. 28.92540	9 14 12.81	+22 26 40.7	G. Struve, Berlin—Babelsberg	Circ. B. C. A. No. 193, 1928
(21) Mar. 1.32824	9 14 07.40	+22 35 18.2	Van Biesbroeck, Yerkes—40", visual	Letter
(22) Mar. 2.19064	9 14 06.65	+22 40 11.2	Van Biesbroeck, Yerkes—40", visual	Letter
(23) Mar. 15.19074	9 16 22.41	+23 24 04.4	Van Biesbroeck, Yerkes—40", visual	Letter
(24) Mar. 15.27569	9 16 24.16	+23 29 13.5	Crawford, Lick—Crossley, phot.	Letter
(25) Mar. 17.35833	9 17 14.68	+23 32 35.4	Crawford, Lick—Crossley, phot.	Letter

* Position reported as unsatisfactory on account of unfavorable conditions.

† Found later on a plate taken before discovery.

** Apparent place.

A preliminary elliptic orbit was computed from the first three complete observations received, viz., (3) of February 23.96, (7) of February 25.07, and (10) of February 26.18. The last was reported as doubtful.

ELEMENTS, 1928.0

$T=1928 \text{ Feb. } 27.24 \text{ U. T.}$

$\omega = 18^\circ 43'.7$

$\Omega = 125 \ 44.8$

$i = 10 \ 55.9$

$e = 0.74$

$a = 8.6403$

$q = 2.269$

$P = 25.4 \text{ years}$

O-C	(3)	(7)	(10)
$\Delta\alpha$	+5"	0"	0"
$\Delta\delta$	0	0	-1

The elements and a short ephemeris were distributed on *Harvard College Observatory Card No. 57*.

An inspection of the critical determinant

$$N = (a')^3 \tan \delta - a'' (\tan \delta)' + a' (\tan \delta)''$$

revealed that the dominating term, $a'' (\tan \delta)'$, had only the first figure fairly well determined and that the resulting geocentric distance might be expected to be uncertain by 50%.

On receipt of the complete Heidelberg positions, (1) of January 29.11 and (2) of February 22.96 by telegram, and of (19) of February 28.20 by telephone, a shift to (2) as new middle place was made preparatory to a differential correction. The residuals to be removed were the following:

O-C	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29	+14' 3".8	-39' 40".9
(19) Feb. 28	+68.9	+13.0

The practically complete removal of the above residuals was effected in one differential correction by the use of Leuschner's approximate formulae¹.

ELEMENTS, 1928.0

$T=1928 \text{ Feb. } 13.78 \text{ U. T.}$

$\omega = 16^\circ 00' 01''$

$\Omega = 124 \ 15 \ 39$

$i = 7 \ 17 \ 39$

$e = 0.4742$

$a = 3.4084$

$q = 1.7920$

$P = 6.29$

O-C	(1)	(19)
$\Delta\alpha$	+2".1	-0".6
$\Delta\delta$	-5.9	+0.8

On account of the doubtful character of the third place no further reduction of the residuals was attempted.

¹ *Publ. Lick Obs.* 7, 266, 1913.

This orbit and an ephemeris were distributed on *Harvard College Observatory Announcement Cards* Nos. 59 and 60. While the elements so obtained on the 30 day arc suggested identity with Taylor's comet, 1916 I, the application of Tisserand's criterion now renders the identity doubtful.

On receipt of (24) of March 15 by Crawford, this observation was substituted in place of the approximate position (19) for a further differential correction of the orbit. The residuals to be removed were as follows:

O-C	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29	+ 2".1	- 5".9
(24) Mar. 15	-389.5	+90.6

ELEMENTS, 1928.0

$T = \text{Jan. } 31.0727 \text{ U. T.}$

$\omega = 8^\circ 39' 05".3$

$\Omega = 124 \ 56 \ 05.8$

$i = 8 \ 03 \ 45.1$

$a = 3.74157$

$e = 0.5028$

$q = 1.86036$

$P = 7.237$

CONSTANTS FOR THE EQUATOR, 1928.0

$$x = r[9.997110] \sin (223^\circ 51' 15".1 + v)$$

$$y = r[9.976493] \sin (136 \ 05 \ 49.7 + v)$$

$$z = r[9.531892] \sin (115 \ 12 \ 09.9 + v)$$

EPHEMERIS 1928.0

1928 U. T.	α	δ	$\log \rho$
Mar. 19.0	9 ^h 18 ^m 01. ^s 63	+23°34'24".8	0.0264
Mar. 27.0	9 22 52.71	+23 33 11.1	0.0541
April 4.0	9 29 31.00	+23 16 21.2	0.0830
April 12.0	9 37 40.91	+22 45 46.4	0.1124
April 20.0	9 47 07.32	+22 03 08.2	0.1418
April 28.0	9 57 36.36	+21 09 58.9	0.1710
May 6.0	10 08 53.86	+20 07 51.5	0.1995
May 14.0	10 20 47.36	+20 58 10.7	0.2273
May 22.0	10 33 07.93	+17 42 06.3	0.2543
May 30.0	10 45 48.42	+16 20 44.5	0.2803

These elements are in substantial agreement with those obtained by Crommelin² and by Ebell³.

The complete representation, with the omission of the approximate positions, is as follows:

1928 U. T.	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29.11*	0".0	0".0
(2) Feb. 22.96*	0.0	0.0
(3) Feb. 23.96	+1	+3
(4) Feb. 24.83	+3.2	+2.4
(5) Feb. 24.95	-1.6	+5.4
(7) Feb. 25.07	+2.2	-1.9
(8) Feb. 25.93	-1	+1
(9) Feb. 25.95	+1.0	+0.8
(11) Feb. 26.18	+4.2	+1.4
(12) Feb. 26.90	+2	-9

² *B.A.A. Circular No. 62*, 1928.

³ *Beob. Zirk. No. 10*, 1928.

1928 U. T.	$\Delta\alpha$	$\Delta\delta$
(13) Feb. 27.00	+1".0	+2".0
(14) Feb. 27.02	+0.8	+2.1
(15) Feb. 27.03	+6.2	-2.3
(16) Feb. 27.18	+3.1	+0.6
(17) Feb. 28.01	0	+6
(18) Feb. 28.09	-0.4	-3.2
(20) Feb. 28.93	-2.5	-6.2
(21) Mar. 1.33	-3.5	+2.8
(22) Mar. 2.19	+1.5	-1.1
(23) Mar. 15.19	+3.0	+1.3
(24) Mar. 15.28*	+0.3	+0.1
(25) Mar. 17.36	+2.5	+0.2

The starred observations are those on which the solution has been based.

Two subsequent observations received by letter from Van Biesbroeck as the bulletin was going to press are represented as follows:

1928 U. T.	$\alpha(1928.0)$	$\delta(1928.0)$	$\Delta\alpha$	$\Delta\delta$
(26) Mar. 21.17665	9 ^h 19 ^m 08 ^s .68	+23°35'44".8	+6".7	+1".9
(27) Mar. 24.18553	9 20 56.01	+23 35 28.3	-1.9	-0.5

BERKELEY ASTRONOMICAL DEPARTMENT, March 30, 1928.

Issued April 12, 1928.