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THE SPACE DISTRIBUTION OF THE STARS OF DIFFERENT
SPECTRAL CLASSES AS DETERMINED FROM STUDIES
IN THE CYGNUS REGION OF THE MILKY WAY

By

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I. INTRODUCTION

The absence of data concerning the distances of the fainter stars has been one of the most serious limitations encountered by statistical investigators. In view of recent work by Lundmark,¹ Lindblad² and others, indicating that a fair approximation of distance can be derived from a careful classification of spectra, the writer determined to make a spectroscopic study of some of the stars of Kapteyn's Selected Areas (systematic plan), with a view to studying the distribution in space of the stars in the Milky Way between *Vulpecula* and *Perseus* down to about magnitude 13.5 on the Harvard photographic scale. The slitless quartz spectrograph of the Crossley reflector seemed adapted to such work, but as the diameter of the field is rather small the study was restricted to twenty-four regions of about 20' diameter within six of the galactic Areas. The choice of galactic fields was made partly on account of instrumental limitations and partly because the galactic distribution is probably of greater interest than the non-galactic distribution of stars in space. For statistical purposes, of course, a maximum number of stars is imperative, and within a given interval of time this number is most readily obtained in the Milky Way. The Harvard-Gronigen *Durchmusterung of Selected Areas*³ together with systematic correc-

tions recently published by Seares and others⁴ gives us well-determined photographic magnitudes of all stars in these Areas down to magnitude about 16.5. A fairly uniform section of the Milky Way is found between galactic longitudes 30° and 110°, bordered by the constellations *Vulpecula* and *Perseus*. The six Areas in this region will be combined and discussed as a unit. The region has the additional advantage that it is easily observed over an interval of several months from Mount Hamilton.

The "Plan of Selected Areas," outlined by Kapteyn in 1906 in a booklet under that title, is a proposed method of attack on the problem of the structure of the sidereal system, which shall reduce the amount of work to a minimum. On the systematic plan⁵ 206 Areas are selected at intervals of fifteen degrees in declination from the north to the south pole and at intervals of one to four hours in right ascension according to the distance from the equator.

Kapteyn's plan embraced a *Durchmusterung* and a determination of the photographic magnitudes, visual magnitudes, parallaxes, proper motions, spectral classes and radial velocities of the fainter stars in these Areas. For details the reader is referred to the booklet just mentioned, and to subsequent reports on the progress of the Plan.

Spectra of stars brighter than magnitude 8.5 are classified in the *Henry Draper Catalogue*. For magnitudes 8.5 to 11.0, the Hamburger Sternwarte at Berge-

¹ *Publ. A. S. P.* 34, 150, 1922.

² B. Lindblad gives methods of distinguishing high luminosity stars from stars of low luminosity and of the same spectral type. *Mt. Wilson Contr.* 228, 1922. See also "Spectrophotometric Determinations of Stellar Luminosities" (first paper), *Nova Acta Reg. Soc. Scient. Upsala*, Ser. IV, 6, no. 5, 1925.

³ *H. A.* 101, 1918; 102, 1923; 103, 1924.

⁴ *Mt. Wilson Contr.* no. 289, 1924.

⁵ In this paper, by "Selected Areas" we refer to the Systematic Plan and not to the Special Plan proposed by Kapteyn.

dorf is planning the spectral classification. Mr. Humason at Mount Wilson has undertaken the classification of some of the stars in northern Areas from magnitudes 11 to 12. The present classification will doubtless overlap the latter to some extent, but we shall in some cases be able to extend the limit of brightness by over 1.5 magnitudes for the special regions under consideration. Furthermore, for reasons which will appear in the text, a classification of spectra based partly on the ultra-violet region is especially desirable.

It has become quite customary in modern investigations to deal with what may be called a "simplified stellar system." Usually the galaxy is studied as a whole, local irregularities are neglected, and the variation of density of stars in space in terms of the distance from the Sun is represented by a mathematical formula called the "density law." Various forms of the density function have been proposed and each investigator has used that which he has found most nearly accordant with the facts. The luminosity law and the frequency function of apparent magnitudes are related through the density law. The forms of these laws are usually assumed and the constants are evaluated from the data available. The assumptions must be made so that the mean parallaxes of groups of stars obtained in different ways, are well represented. Charlier⁶ enumerates six hypotheses as to the form of the density law, which, except for notation, are as follows:

- I. $\Delta(\rho) = \text{constant}; M = \text{constant}$ (Herschel)
- II. $\Delta(\rho) = \text{constant}; \varphi(M)$ arbitrary (Schiaparelli)
- III. $\Delta(\rho) = \gamma \rho^{-s} \quad (s > 0)$ (Seeliger)
- IV. $D(y) = \frac{N}{\sigma \sqrt{2\pi}} e^{-\frac{(y-y_0)^2}{2\sigma^2}}$ where $y = 5 \log_{10} \rho$ (Schwarzschild)
- V. $\Delta(\rho) = \frac{C}{\rho} e^{-k\rho^2}$ (Dyson and Eddington)
- VI. $\Delta(\rho) = \Delta(0) e^{-\frac{\rho^2}{2k^2}}$ (Charlier)

where M is the absolute magnitude, $\varphi(M)$ is the luminosity law, ρ is the distance from the Sun, and $\gamma, s, y_0, k, \sigma$, and C are constants. The first and second lead to a constant magnitude-ratio of 3.981 and the third to a magnitude-ratio $10^{0.2(3-s)}$. The fourth leads to a normal frequency function for the frequency function of the apparent magnitudes if a normal frequency function for the luminosity law is assumed. This is the form adopted by Kapteyn, van Rhijn, Seares and others.

The density law will be derived empirically in the present paper using the distances of stars derived from spectral types. Thus with the assumed distance and the apparent magnitude of each star, we shall be

⁶ "The Motion and Distribution of the Stars," *Memoirs of the Univ. of Calif.*, 7, 97, 1924.

independent of specific forms of the function. Furthermore the results will be of a special nature, referring only to a definite section of the Milky Way. In this we depart, to a certain extent, from the simplified form of the stellar system.

The distances of stars in statistical discussions are usually derived from mean secular parallaxes, from parallaxes derived as functions of both proper motions and apparent magnitudes, or from spectroscopic parallaxes. Trigonometric parallaxes, Cepheid parallaxes, and moving cluster parallaxes are selective in character. Parallaxes derived from spectral types—to which we shall hereafter refer as "spectral" parallaxes—are possible, as shown by Lundmark, if the giant or dwarf character of the spectrum can be ascertained. This is a direct consequence of the discovery of Hertzsprung and Russell that the dispersion in absolute magnitudes among stars of a given sub-class is small.

The present study is based on the following data:

(1) 557 stars, mainly between photographic magnitudes 10.0 and 14.0, observed and classified for this purpose and contained in six of the Selected Areas.

(2) 347 stars from the *Henry Draper Catalogue* with photographic magnitudes 5.0 to 8.0 contained in six areas of approximately 44 square degrees each and concentric with the observed Selected Areas.

(3) 1156 stars in 83 square degrees having the *Cygnus* cloud in the center. The photographic magnitudes are from 8.0 to 10.0. This is the region covered by the first and part of the second installment of the *Henry Draper Extension* which contain 807 of the stars. The remainder are from the *Henry Draper Catalogue*.

The following problems will be attacked:

(1) To determine, for this part of the Milky Way, the density law of each spectral type. In this we shall be limited to those distances to which our observations, for stars of given luminosity, are complete.

(2) In the nearer regions of space, to find the relative frequencies of yellow-red giant stars and yellow-red dwarf stars. By this we mean stars of spectral types G and K.

(3) For the giant stars (including types A and B), to determine the proportion belonging to each spectral class and to find how this proportion varies with distance.

II. INSTRUMENTS AND METHOD OF OBSERVATION

The spectrograms were made with the two-prism quartz spectrograph in its slitless form, attached to the Crossley reflector. The instrument was designed by Mr. W. H. Wright and is described by him in *Lick*

Obs. Bull. 9, 52, 1917. The dispersion is such as to separate H β and H ϵ by 7.4 mm. The prisms are set for minimum deviation at $\lambda 3600$. Eastman speedway plates were used throughout. A field of about 22' in diameter is obtainable and excellent spectra are found within 7.5 of the center. To avoid risking a change in focus, only two plate holders, lettered "A" and "B," were used in this work.

A standardizing sensitometer designed by Wright⁷ and described in its present form by Yü⁸ was used to facilitate measures of the photographic density of some of the spectra, to be described later. Before each plate was placed in the spectrograph it was exposed in the sensitometer for 20 seconds to the light of a carbon lamp operating at 220 volts and using two amperes. After passing through a blue glass the light is cut off by small circular windows whose areas are in the proportion 1:4:8:32, and these windows are made to illuminate certain areas on the photographic plate. An accurate system of diaphragms prevents reflection from the sides, so that the intensities of the incident light on the photographic plate are in the above proportions. The instrument is provided with windows whose areas, on the same scale, are in the proportions 2 and 16, but these were not used in the present work, because of the danger of intercepting some of the spectra. As these measures are only for the purpose of separating certain giant stars from dwarfs of the same spectral type, the variation of development factor with wave-length, as well as the large disparity of exposure time, is neglected.

All plates used in this work were developed in two ounces of a ten per cent solution of *Rodinal* at 65° F. for ten minutes. Uniformity of development, intended for the facilitation of spectrophotometric measurements, is also important in the direct estimation of spectral type. Approximately uniform conditions of fixing and washing were obtained.

Each of the areas in Table I was divided into four square sections and the telescope was pointed, as nearly as possible, to the center of each section. In this way, four exposures of five hours each were made on each Selected Area. The work was duplicated with one-hour exposures to obtain the spectra of the brighter stars.

TABLE I
TABLE OF THE OBSERVED SELECTED AREAS

Sel. Area No.	Center (1900.0) ($x=0, y=0$)	Limits
	α	δ in z in y
64	19 ^b 58 ^m	+30° 00' - 74' to 14' -20' to +20'
40	20 47	45 00 0 75 to 15 -20 to +20
18	21 24	60 10 +7 83 to 23 -20 to +20
19	23 23	60 00 -1 80 to 20 -20 to +20
8	1 00	60 10 -2 77 to 17 -20 to +20
9	3 04	60 20 +4 77 to 17 -20 to +20

⁷ *Publ. A. S. P.* 35, 281, 1923.

⁸ *Lick Obs. Bull.* 12, 109, 1926.

The Areas are arranged in order of the dates of observations. The columns contain data taken from *Harvard Annals* 101, giving the number of the area, the position of the origin of the rectangular coördinates x and y , and the limits of the areas in rectangular coördinates. When x is negative it has been increased by 100'. The extreme limits of galactic longitude for this set of Areas are 35° and 106° and those of galactic latitude, -2° and +6°. All of the Areas are of the same size (40' x 40').

Each plate gives forty to fifty very narrow spectra, depending on the richness of the region. The limit of completeness of the plates varies, but is about 13.5 photographic magnitude on the Harvard scale at the centers of the plates. Since fainter spectra are visible in the center than near the edge, the plates have arbitrarily been divided into two concentric zones, the outermost 22', and the inner zone 15' in diameter. For statistical purposes a certain limit of magnitude must be established, beyond which no stars may be included. By taking different limits for the two zones a greater number of stars may be included and the limit of magnitude is pushed farther for the central zone.

To facilitate spectral classification a list of comparison stars (given in Table II) was chosen from the Mount Wilson lists⁹ of spectroscopic parallax stars. The spectral types of these stars were then taken from the *Henry Draper Catalogue* and used as a basis of the classification in this paper. The same spectrograph was used as for stars in the Selected Areas and the standard squares of the sensitometer were impressed on each plate. Five spectra with exposures in the ratio 1:4:8:32, placed side by side on the plate, were secured for each star. The longest exposure was twice the star's computed normal exposure. The spectra were only very slightly broadened in order to be directly comparable with the spectra to be classified.

TABLE II
TABLE OF COMPARISON STARS

No.	H.D. Sp.	Star	Plate No.	Normal Exposure	α (1900.0)
1	B0	69 Cygni.....	M228B	14 ^a	21 ^b 21 ^m 7
2	B1	ϵ Persei.....	M248B	3 ^a	3 51.1
3	B2	12 Lacertae.....	M206B	7 ^a	22 37.0
4	B3	16 Pegasi.....	M204B	6 ^a	21 48.5
5	B3	6 Lacertae.....	M205A	3 ^a	22 26.2
6	B5	Brad. 325.....	M249A	20 ^a	2 14.2
7	B8	Groomb. 244.....	M215A	11 ^a	1 02.4
8	B8	φ Andromedae.....	M214B	2 ^a 1/2	1 03.7
9	B8	Pulk. 2680.....	M201A	15 ^a	18 54.6
10	B8	14 Cygni.....	M217A	10 ^a	19 36.2
11	B9	Brad. 138.....	M241B	11 ^a	1 05.0
12	A0	7 Trianguli.....	M252A	10 ^a	2 10.0

⁹ Stars of types F, G, K, and M were taken from *Mt. Wilson Contr.* no. 199; those of type A from *Mt. Wilson Contr.* no. 244; and the B-type stars from *Mt. Wilson Contr.* no. 262.

TABLE II—(Continued)

No.	H.D. Sp.	Star	Plate No.	Normal Exposure	α (1900.0)
13	A0	29 Cygni.....	M203A	3½ ^s	20 ^b 10 ^m 8
14	A2	ι Delphini.....	M218B	12 ^s	20 33.0
15	A5	δ Cassiopeiae.....	M243A	½ ^s	1 19.3
16	A5	ω Aquilae.....	M238A	10 ^s	19 13.1
17	F0	22 Andromedae....	M209A	12 ^s	0 05.1
18	F0	28 Andromedae....	M210B	12 ^s	0 24.8
19	F0	ρ Piscium.....	M232B	14 ^s	1 20.9
20	F0	7 Andromedae.....	M207A	6 ^s	23 08.0
21	F2	13 Pegasi.....	M229A	15 ^s	21 45.4
22	F5	Pi. 171.....	M226B	22 ^s	1 43.0
23	F5	ν Persei.....	M247A	3 ^s	3 38.4
24	F5	25 Monocerotis....	M253B	15 ^s	7 32.3
25	F5	Pulk. 2677.....	M227A	17 ^s	18 53.8
26	gG5	Pi. 128.....	M211A	31 ^s	0 32.0
27	gG5	ε Andromedae.....	M222B	10 ^s	0 33.3
28	gG5	η Piscium.....	M225A	3½ ^s	1 26.1
29	gK0	ζ Andromedae.....	M213A	10 ^s	0 42.0
30	gK0	η Ceti.....	M236B	4 ^s	1 03.6
31	gK0	χ Piscium.....	M224B	19 ^s	1 06.1
32	gK0	ξ Andromedae.....	M242A	20 ^s	1 16.4
33	gK5	δ Piscium.....	M223A	15 ^s	0 43.5
34	gK5	31 Lyncis.....	M255B	12 ^s	8 16.0
35	gK5	ν Pegasi.....	M219A	22 ^s	22 00.6
36	gMa	χ Pegasi.....	M220B	28 ^s	0 09.4
37	gMa	ρ Ursae Majoris....	M254B	31 ^s	8 53.5
38	gMb	47 Piscium.....	M221A	43 ^s	0 22.8
39	dG0	9 Ceti.....	M235A	60 ^s	0 17.7
40	dG0	Lal. 1966.....	M240A	5 ^m	1 03.3
41	dG0	Lal. 3153.....	M233A	3 ^m	1 37.4
42	dG0	δ Trianguli.....	M246A	14 ^s	2 10.9
43	dG0	τ Persei.....	M250A	4 ^s	2 47.2
44	dG5	μ Cassiopeiae.....	M216B	20 ^s	1 01.6
45	dK0	W.B. 1 ^h 161.....	M237A	10 ^m	1 13.5
46	dK0	τ Ceti.....	M234B	4 ^s	1 39.4
47	dK0	Pi. 159.....	M245A	43 ^s	1 40.5
48	dK0	A0 25685.....	M230B	2 ^m 25 ^s	23 26.5
49	dK2	Bu 12740S.....	M231A	20 ^s	1 36.8
50	dK2	A0 1867.....	M244B	12 ^m	1 36.8
51	dK2	Brad. 3077.....	M208B	43 ^s	23 98.5
52	dMa	Groomb. 34.....	M239B	16 ^m	0 12.7

Column two of Table II gives the spectral type in the *Henry Draper Catalogue*, giant and dwarf stars being indicated by prefixes "g" and "d" respectively to the spectral type, in accordance with the international notation. The giant or dwarf character is taken from the Mount Wilson spectroscopic absolute magnitudes. The absence of a few spectral subdivisions arises from making the choice of stars with regard to the Mount Wilson "estimated" type rather than the Harvard type, and from the fact that certain subtypes of the Harvard sequence such as A3, G2 and late M-types, are not recognized in our classification.

III. SCHEME OF SPECTRAL CLASSIFICATION

The principal points of spectral classification as applied to the particular dispersion and character of the spectra obtained are given in the following scheme:

Principal Criteria for Spectral Classification

H.D. Type

- B0-B1 Spectrum is long and of nearly uniform density. The hydrogen lines are faint.
- B2-B3 The hydrogen lines are stronger, and the helium line $\lambda 3820$ is conspicuous.
- B5-B8 Marked by increasing strength of the hydrogen lines, weakening of $\lambda 3820$, and the beginning of continuous absorption in the ultra-violet.
- A0 The hydrogen lines reach a maximum and the K-line is occasionally visible.
- A2 The K-line, though faint, is easily seen.
- A5 The K-line is nearly as strong as H δ or H ζ .
- F0 The K-line is somewhat stronger than H δ or H ζ .
- F2 The lines H and K are very conspicuous. The G-band begins to appear.
- F5 The G-band is nearly as strong as H γ . The hydrogen lines are rather weak, and the lines H and K are very strong.
- G0 The G-band is stronger than H γ . The hydrogen lines, though faint, are easily seen.
- G5 The G-band is quite pronounced. There is some contrast at the G-band. H γ is still visible and the doublet $\lambda\lambda 4383-85$ is just visible.
- K0 There is strong contrast at the G-band, the doublet $\lambda\lambda 4383-85$ is strong, the hydrogen lines are generally invisible, and in dwarfs $\lambda 4226$ is occasionally seen.
- K2 The line $\lambda 4226$ is easily seen, the triplet $\lambda\lambda 4030, 4033, 4034$ and the iron line $\lambda 4045$ are easily visible.
- K5 Same as K2, except that $\lambda\lambda 4226, 4030-34, 4045$ are stronger.
- Ma Spectrum is banded, $\lambda 4226$ is extremely intense.
- Mb The banded character of the spectrum is very conspicuous; $\lambda 4226$ is broad and strong.

The distinction between giants and dwarfs was made at type G0 and later. Only two criteria have been used for this purpose, (1) the relative strength of the cyanogen band at $\lambda 3883$ and (2) the fact that giants and dwarfs of the same spectral type have a somewhat different color index and hence a different distribution of spectral intensity. The cyanogen band $\lambda 3883$ is nearly absent in dwarf stars of type K0, but in giants of the same type it is usually very strong. This contrast diminishes as we go toward G0 or Ma,

reaching a maximum at K0. The extreme sensitiveness of this region of the spectrum to changes in absolute magnitude was pointed out by Lindblad.¹⁰ The effect is still present at type G0, though less pronouncedly. We shall see later that certain conclusions drawn from this paper depend mainly on an accurate division of K-type stars into giants and dwarfs. Thus the greatest accuracy is present at the point of greatest need.

The color index criterion was used only for class M stars and those stars of type K whose spectra are intercepted in such a manner as to render the application of the first criterion impossible. It is true that, for the former, the ordinary color index change is rather small; yet when properly chosen regions of the spectrum are photometrically examined it is easily possible to distinguish the dwarfs from the giants. In the next section we shall briefly describe the method by which this is effected. It should be noted that it has been applied only to stars whose classification was already accomplished by means of the spectrum lines.

IV. METHOD OF MEASUREMENT OF SPECTRAL INTENSITIES

To distinguish between giants and dwarfs in the case of a few late type stars, two regions of the spectra were measured with the Hartmann¹¹ equalizing wedge microphotometer. The standard squares of known relative intensity, already mentioned, were available on all plates, both of comparison star spectra and of spectra in the Selected Areas. These were used to calibrate the wedge readings whence the relative spectral intensities could be obtained. If, as stated above, we neglect variation of the development factor with wave length and neglect the effect of the wide disparity of exposure times, a given reading of the wedge may be converted into a number which signifies the photographic intensity in terms of the faintest standard square. Dividing this number for one wave-length by the same number for another wave-length we get a ratio which, when found for different spectrograms, will give us some indication of the giant or dwarf character of a late-type star if its spectral class is obtained from the lines and if the spectral regions are properly chosen. Dividing this ratio for one star by the same for another star, we eliminate, to a first approximation, the effects of atmospheric transmission, reflectivity of the silver mirror, transmission of the quartz spectrograph, dispersion, and sensitivity of the Eastman speedway plates.

A word may be said concerning the method of reduction. By plotting the intensities of the standard

squares expressed in terms of the faintest square against the wedge reading, a "reduction curve" is obtained from which the wedge readings on the spectra give the spectral intensities. In order, however, to reduce the errors of graphical work to a minimum, use was made of the fact that the reduction curve has, as a first approximation, the form:

$$I_0 = 10^{1.505 \frac{D-D_1}{D_6-D_1}}$$

where I_0 is a first approximation to the intensity of any unknown region, D is the wedge reading for the region, and D_1 and D_6 are the wedge readings for the faintest and brightest standard squares respectively. The small difference $I - I_0$ was then plotted for other standard squares, and the correction from this reduction curve was applied to the unknown region. The disadvantage is an increase in the amount of computation, but the advantage is the gain in accuracy and saving of care in the graphical part of the work. Figure 1 shows the reduction curve for plate M250A.

Fig. 1. Reduction curve for Plate No. M250A.

After some experimenting the regions in the neighborhood of the wave-lengths $\lambda\lambda 4360$ and 4050 were found most suitable for the giant and dwarf classification. The several exposures on each of eighteen plates

Fig. 2. Intensity ratio $\frac{\lambda 4360}{\lambda 4050}$ plotted against spectral type.

¹⁰ *Mt. Wilson Contr.* no. 228, 20, 1922.

¹¹ J. Hartmann, *Zeitschr. für Instrumentenkunde*, 19, 97, 1899.

of giant and dwarf comparison stars were measured. The intensity ratio $\frac{\lambda 4360}{\lambda 4050}$ was formed for each exposure and means were taken for all the exposures on a single plate. These means were then plotted against the spectral type. Figure 2 gives the resulting distribution. All stars above the inclined line are giants and all below are dwarfs.

By repeating the procedure for spectra whose complete classification was desired, the position on the diagram was used to determine the order of luminosity. Stars so classified are marked "measured" in the column headed "Remarks" of the Catalogue which is to follow. Eleven stars in the Selected Areas were so classified.

V. CLASSIFICATION OF STARS IN THE SELECTED AREAS

As already stated, each plate of the spectra of stars in the Selected Areas, as soon as the center was determined, was divided into two concentric zones. We shall, for convenience, call the innermost zone, Zone I, and the other, Zone II. Zone I is 7'.5 in radius and Zone II is bounded by Zone I on the inside, and a concentric circle of radius 11' on the outside. A limit of photographic magnitude on the Harvard-Groningen scale (*H. A.* 101) was determined for each zone of each plate. Every effort was then made to classify accurately all of the spectra of stars brighter than that magnitude. This "limiting magnitude" will be defined as the mean of the magnitudes of the faintest star included in the classification and of the brightest star not classified, taken to the nearest tenth of a magnitude.

Each spectrum was classified at least twice independently. If the agreement was satisfactory the mean of the two classifications was adopted; if not, the plate was examined a third time. In the first classification, after each area had been covered, the plates of the comparison stars, after thorough mixing, were examined and classified. The same process was repeated after the completion of each pair of Areas in the second classification. Thus nine complete classifications of the comparison stars were made at well-distributed intervals throughout the classification of the unknown spectra.

In order to study the systematic error of classification the means of all the Harvard types classified as a given type were taken. These means represent the Harvard type corresponding to the given estimated type. The intention was to correct the estimates accordingly, but in all cases except types dK5 and gK2 the means agreed exactly with the estimates. As these two exceptions are not represented among the com-

parison star spectra, the disagreement has little or no meaning, and the correction was neglected.

Mount Wilson⁴ investigators have shown that the photographic magnitudes given in the Harvard-Groningen *Durchmusterung of Selected Areas* have a systematic scale error as well as a large color equation when compared with the Mount Wilson system. We shall here attempt to reduce the magnitudes to the Harvard photographic scale which is, by definition, the same as the Harvard visual scale for a star of sixth magnitude and class A0, and which is greater by one magnitude than the Harvard visual scale when the star is of class K0 and the sixth visual magnitude.¹²

Let

HP_g = the Harvard photographic magnitude.

HV = the Harvard visual magnitude, used in *Harvard Annals* 71, and on the same scale as that of the *Henry Draper Catalogue*.

Pv_W = the Mount Wilson photo-visual magnitude.

Pg_W = the Mount Wilson photographic magnitude.

DM = the Harvard - Groningen magnitudes, given in *Harvard Annals* 101 for stars in the Selected Areas.

C_H = the Harvard color index.

C_W = the Mount Wilson color index.

In Table I of *Mount Wilson Contribution* 287, 1925, we find for a star brighter than magnitude 6.5 and class K0, $C_W = +1.48$. Since for the same star, $C_H = +1.00$, it follows that

$$C_H = \frac{1}{1.48} C_W \quad (1)$$

By definition of C_H , $HP_g = HV + C_H$ (2)

Seares¹³ gives, as the color-equation of the Harvard visual system

$$HV = Pv_W + 0.14 C_W \quad (3)$$

if we neglect the scale correction, which should not affect the photographic scales. By definition of C_W

$$Pv_W = Pg_W - C_W \quad (4)$$

The color equation of the Harvard-Groningen *Durchmusterung* relative to the Mount Wilson photographic system is:⁴

$$Pg_W = DM + 0.06(DM - 6) + 0.21 C_W \quad (5)$$

⁴ *Mt. Wilson Contr.* no. 289, 1924.

¹² *H. A.* 80, no. 9, 1917.

¹³ *Trans. of Int. Ast. Union*, 1, 78, 1922.

Two-fold Enlargement of a Plate of Selected Area SSE

Type dK0

Type gK0

Typical Giant and Dwarf Spectra of Class K0

A combination of these five equations yields the desired equation:

HPg=DM+0.06(DM-6)+0.026 Cw

The coefficient of Cw is so small that it will here be neglected, and our working equation is:

HPg=DM+0.06(DM-6) (6)

Thus there is no color correction and we need only correct for scale. The equation is only applicable to the brighter stars (m<10). For the fainter stars we shall use the approximately constant correction of +0.25 given in column 10, Table IV, Mt. Wilson Contr. 289. That no discontinuity might be introduced at magnitude 10, the values for the different magnitudes were plotted and the corrections were read from a smoothed curve. The following table gives the corrections adopted:

DM Mag.	Scale Corr.	DM Mag.	Scale Corr.
7.0	0.06	11.0	0.25
7.5	.09	11.5	.25
8.0	.13	12.0	.25
8.5	.16	12.5	.25
9.0	.19	13.0	.25
9.5	.22	13.5	.25
10.0	.23	14.0	.25
10.5	.24		

The first column in Table III gives the Groningen number from Harvard Annals 101, 1918. The second column gives the zone on the photographic plate, defined above. The third column gives the Harvard photographic magnitude as just defined, and the fourth column gives the estimated spectral type. Although the original classification was limited to the types outlined in the scheme, in taking the means intermediate types were introduced. Thus a star classified first as A0 and then as A2 would be called A1.

Sections of the Areas are designated NE, NW, SE or SW according to their positions. Thus 64 NE is used to designate the north-east section of Area 64. The center of each plate is given in rectangular coördinates. The origins of the coördinate systems are defined by Table I. The date of exposure of each plate is also given.

TABLE III
CATALOGUE OF STARS IN THE SELECTED AREAS WHOSE SPECTRA ARE CLASSIFIED

Plate M111A, 1926, Aug. 5. Center of Plate { x+100'=98'6
Area 64 NE { y = 7.1

Limiting photographic { Zone I 13.5
magnitudes { Zone II 12.8

Gron. No.	Zone	H Pg	Sp. Type	Remarks
853	II	11.88	gK0	Overlaps No. 1358
1275	II	11.08	A0	Plate M113A
1298	II	11.48	A0	Plate M113A

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1307	II	11.52	A2	Plate M113A
1320	II	12.39	A0	Plate M113A
1324	I	11.64	F5	Plate M113A
1328	I	11.88	A0	Plate M113A
1344	II	12.68	gG0	
1352	I	12.99	A0	
1353	I	13.09	F5	
1357	I	13.09	dG0	
1358	II	12.73	A0	Overlaps No. 853
1360	I	13.44	F5	Faint
1374	I	10.02	gK0	BD+29° 3860
1382	I	12.99	A2	
1387	I	12.68	F5	
1389	I	13.23	F5	
1394	I	11.49	A5	
1395	I	12.83	A0	
1412	I	12.13	F5	} Overlap
1414	I	11.18	F2	
1427	I	13.26	A5	
1429	I	12.26	gK0	
1434	I	12.68	gK0	
1435	II	12.62	dG8	
1446	I	13.44	dG2	
1451	I	12.68	dG5	
1467	II	12.68	gK0	Measured
1469	II	12.68	A8	
1483	II	11.88	B3	
1485	II	12.26	dG0	
1778	II	12.26		} Overlap BD+30° 3857
1779	II	10.98		
1781	II	12.00	F8	
1783	II	12.68	Pec.	Two spectra ?
1823	II	12.39	F5	
1838	I	11.39	A0	
1840	I	13.13	gK5	Spectrum is intercepted
1859	I	12.99	gK0	
1862	I	12.62	dG0	
1875	II	12.26	dG2	
1889	II	12.68	dG2	

Plate M113A, 1926, Aug. 7. Center of Plate { x+100'=82'2
Area 64 NW { y = 4.3

Limiting photographic { Zone I 13.5
magnitudes { Zone II 12.9

Gron. No.	Zone	H Pg	Sp. Type	Remarks
535	II	12.53	A0	
581	II	12.53	A2	Plate M117A
609	I	12.26	gK0	
646	II	11.30	gK0	
677	I	12.99	A5	
706	II	10.68	A5	Plate M117A; BD+29° 3847
733	II	11.29	A0	Plate M117A
749	I	12.68	dG2	
778	II	12.83	A0	
779	II	11.20	A2	Plate M117A; BD+29° 3850
788	II	12.83	A0	
808	II	12.83	A1	
1048	II	12.13	F8	
1062	II	12.39	A2	
1063	I	12.99	F2	
1074	II	12.68	gK0	Faint
1103	I	12.68	A8	
1104	I	11.76	A2	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1113	I	12.53	A0	
1114	I	13.44	F0	
1174	I	10.53	A2	BD+29° 3842
1176	I	12.15	A2	
1179	I	11.40	A5	
1186	I	11.12	A0	
1188	I	11.11	A1	
1203	I	13.09	dG5	
1210	I	12.99		Intercepted by No. 1665
1218	I	11.26	B9	
1232	I	13.16	F2	
1260	I	12.26	dK0	
1263	I	13.26	dK1	Faint
1265	I	12.83	A0	Lines are faint
1275	I	11.08	A0	Plate M111A
1280	I	13.26		Intercepted by No. 1282
1282	I	12.00	A2	
1288	I	13.16	A0	Lines are faint
1290	I	12.99	F2	
1298	II	11.48	A0	Plate M111A
1307	II	11.52	A2	Plate M111A
1320	II	12.39	A0	Plate M111A
1324	II	11.64	F5	Plate M111A
1328	II	11.88	A0	Plate M111A
1550	II	12.83		Spectra overlap
1553	II	12.83		
1645	II	11.39	gK0	BD+30° 3852
1665	I	12.74		Intercepted by No. 1210
1670	II	12.83		Intercepted by No. 1218
1680	II	12.68	A1	
1704	II	12.00	A1	

Plate M115A, 1926, Aug. 8. Center of Plate $\begin{cases} x = 0.9 \\ y+60' = 48.2 \end{cases}$
Area 64 SE

Limiting photographic $\begin{cases} \text{Zone I } 12^m75 \\ \text{Zone II } 12.26 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
209	II	12.13	A2	Plate M117A
225	II	10.46	A2	Plate M117A
239	II	9.53	gK8	Plate M117A
263	I	12.53	gK0	
296	I	12.68	F2	
347	I	12.13	F8	
375	I	11.76	A0	
382	I	10.91	dK1	BD+29° 3865
383	II	11.39	gG5	Spectrum overlaps No. 382
384	I	12.39	dG0	
396	I	11.76	F5	
404	II	11.25	gK1	BD+29° 3868
834	II	8.26	gG2	{ HD 189943, type K0, BD+29° 3857
841	I	12.39	gK0	
867	I	11.14	gK0	
911	I	12.39	A5	
936	I	9.27	A1	BD+29° 3864
1007	II	10.60	A0	

Plate M117A, 1926, Aug. 9. Center of Plate $\begin{cases} x+100' = 84.0 \\ y+60 = 48.0 \end{cases}$
Area 64 SW

Limiting photographic $\begin{cases} \text{Zone I } 12^m9 \\ \text{Zone II } 12.5 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
34	II	12.39	F5	Spectrum is intercepted

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
64	I	12.26	F0	
87	I	12.83	F0	
95	I	11.76	A2	
102	I	12.53	gG8	
110	I	12.39	F4	Intercepted by No. 111
111	I	10.89	A2	
122	I	10.72	A1	BD+29° 3843
127	I	12.83	F8	BD+29° 3846
143	I	11.76	A2	
148	I	12.68	gK0	
149	I	11.09	gK0	{ Intercepted by No. 148; measured.
174	I	10.87	A5	
189	I	10.87	A2	
204	II	9.25	A2	BD+29° 3852
209	I	12.13	A2	Plate M115A
213	II	11.29	A2	
222	II	11.88	F0	
224	II	11.18	A5	
225	II	10.46	A2	Plate M115A; BD+29° 3853
239	II	9.53	gK8	Plate M115A; BD+29° 3855
543	II	12.39	dG5	
581	I	12.53	A2	Plate M113A
583	I	12.00	A0	
646	I	11.30	gK0	
706	II	10.68	A5	Plate M113A; BD+29° 3847
710	I	12.83	A0	
733	II	11.29	A0	Plate M113A
748	I	12.83	A0	
767	II	11.88	dG0	
772	I	11.88	A5	
779	II	11.20	A2	Plate M113A; BD+29° 3850
782	II	11.88		Intercepted by No. 779
790	II	12.39	A8	
809	II	12.39	gG2	Faint
826	II	12.45	F8	Faint

Plate M119A, 1926, Aug. 12. Center of Plate $\begin{cases} x=1.2 \\ y=6.5 \end{cases}$
Area 40 NE

Limiting photographic $\begin{cases} \text{Zone I } 14^m1 \\ \text{Zone II } 13.4 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
203	II	9.75	gK0	BD+44° 3596
208	II	13.20	dK0	
216	I	11.46	A5	Faint
224	II	13.14		{ Overlap; Plate M126B
225	II	12.19	dK1	
235	II	13.30	A3	{ Faint, intercepted; Plate M126B
401	II	11.43	A8	
406	II	13.14	dK0	Faint
417	I	12.19	A0	
420	I	11.71	B9	
421	I	12.19	F5	
425	I	12.97	Nb	Henry Draper classification
428	I	12.81	B6	
444	I	13.97	F8	
448	I	14.04	A0	
451	I	12.34	dG5	
466	I	13.30	A1	
470	I	13.62	dG0	
475	II	12.97	dG8	
480	II	13.14	F8	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
495	II	11.38	gG0	
679	II	12.19	B9	
683	II	12.34	A0	
686	II	13.14	F5	
721	I	14.04	dG2	Faint
729	I	13.30	B9	
Plate M121A, 1926, Aug. 13. Center of Plate $\begin{cases} x+100'=79'.6 \\ y = 11.8 \end{cases}$				
Area 40 NW				
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 12^m.9 \\ \text{Zone II } 12.6 \end{cases}$				
Gron. No.	Zone	H Pg	Sp. Type	Remarks
294	I	12.81	F8	
295	II	11.39	gK0	
300	I	8.02	A0	BD+44° 3590
306	II	12.19	dG5	
323	I	12.04	F8	
337	II	12.49	A0	
516	I	12.49	gG5	
521	I	12.04	gK2	
529	I	12.19	A1	
537	I	12.19	A3	
548	I	9.53	dG2	BD+45° 3288
553	I	12.81	dG5	
564	I	12.81	dG5	Faint
582	I	12.65	dK2	Intercepted; measured
587	I	9.19	A2	BD+45° 3290
598	I	10.95	A5	
640	II	12.19	dG5	
Plate M126B, 1926, Aug. 15. Center of Plate $\begin{cases} x = 3'.7 \\ y+60'=48.0 \end{cases}$				
Area 40 SE				
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.6 \end{cases}$				
Gron. No.	Zone	H Pg	Sp. Type	Remarks
64	II	12.34	F5	
67	II	12.34		Intercepted by No. 197
70	II	11.05	F0	
71	II	13.14	F8	
75	II	10.90	gK0	BD+44° 3598
76	I	13.14	gG0	
81	I	13.46	dG5	
82	I	13.63	dG2	Faint
84	I	13.30	B9	
88	I	12.97	F2	
95	I	13.63	F0	Faint
96	I	12.81	F5	
101	I	13.46	dG0	
105	II	10.14	gK4	BD+44° 3602
108	I	13.30	gG5	Faint
112	I	13.46	dG8	
117	II	11.38	F5	
196	II	13.14		Intercepted by No. 67
197	II	12.19	F4	
205	II	12.34	gK0	
224	II	13.14		} Overlap; Plate M119A
225	II	12.19	dK1	
232	I	12.19	dG0	
235	II	13.30	A3	Plate M119A
248	I	13.46	A0	
254	I	12.34	dG0	
259	II	11.64	B8	
262	II	12.81	gG5	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
265	II	9.95	F0 F2	BD+44° 3603
Plate M124B, 1926, Aug. 14. Center of Plate $\begin{cases} x+100'=80'.8 \\ y+60=46.6 \end{cases}$				
Area 40 SW				
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.9 \\ \text{Zone II } 13.5 \end{cases}$				
Gron. No.	Zone	H Pg	Sp. Type	Remarks
6	I	12.34	dG8	
10	I	12.81	gK0	
25	I	12.65	A0	
38	I	13.80	dG5	
43	I	12.19	F1	
55	II	11.43	F4	
121	I	13.53	A0	Faint
122	II	12.81	dG0	
126	I	13.46		Intercepted
135	II	13.14	gG8	
142	I	10.08	F0	BD+44° 3592
145	I	13.73		Intercepted by No. 142
150	I	10.87	F1	BD+44° 3593
152	I	13.14	dG0	
175	II	12.49	dG8	Faint
Plate M136A, 1926, Sept. 8. Center of Plate $\begin{cases} x = 14'.0 \\ y+10'=23.0 \end{cases}$				
Area 18 NE				
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.2 \end{cases}$				
Gron. No.	Zone	H Pg	Sp. Type	Remarks
959	II	13.00	dG2	Faint
972	I	13.56	gK0	Faint
976	I	13.75	A0	
1030	II	11.78	F0	
1232	II	10.23	B8	Plate M130A; BD+60° 2243
1254	I	13.75	F1	Faint
1257	I	12.81	A0	
1258	I	13.75	A0	
1263	I	13.75	F4	Faint
1273	I	13.37	A1	
1277	I	12.26	gK0	BD+60° 2244
1284	I	11.63	gK0	BD+60° 2245
1286	I	11.22	gK0	BD+60° 2246
1309	I	11.56	gK4	BD+60° 2248
1313	I	11.79	A3	
1317	I	10.90	A3	BD+60° 2249
1324	I	13.18	F0	
1332	I	12.81	dG2	
1343	II	12.63	A2	
1347	I	13.00	A8	
1359	I	12.04	A0	
Plate M130A, 1926, Sept. 3. Center of Plate $\begin{cases} x+100'=95'.5 \\ y+10=24.0 \end{cases}$				
Area 18 NW				
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.3 \end{cases}$				
Gron. No.	Zone	H Pg	Sp. Type	Remarks
747	II	11.86	dG0	
759	I	11.39	F2	
777	II	12.18	A5	
838	II	12.44	F2	
844	II	12.04	F5	
862	II	12.81	A3	Faint
1082	II	12.63		Intercepted

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1116	II	13.18	gG0	Faint
1118	I	13.37	gG2	
1124	I	13.56	A1	
1130	I	11.07	A0	BD+60° 2238
1131	I	13.75		Intercepted
1150	I	13.00	dG2	
1154	I	13.18	dG8	Faint
1164	I	13.75	A3	
1165	I	13.18	A0	
1173	I	11.51	A0	
1175	I	13.56	dG5	Faint
1178	I	13.56	dG5	
1185	I	13.18	F5	
1189	I	13.00	gMa	Measured
1191	I	13.18	dG0	
1205	I	11.67	F0	
1209	I	12.63	F1	
1232	II	10.23	B8	Plate M136A; BD+60° 2243

Plate M132A, 1926, Sept. 4. Center of Plate $\begin{cases} x = 11'.9 \\ y + 70' = 56.3 \end{cases}$
Area 18 SE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m0 \\ \text{Zone II } 13.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
154	II	10.30	F0	Plate M134A; BD+59° 2379
171	II	11.64	dG4	Plate M134A
177	II	10.45	A1	Plate M134A; BD+59° 2382
206	I	12.44	A2	
222	I	11.85	gK2	
243	I	12.81	F1	
246	I	13.00	dK1	
259	I	13.56	dG5	Faint
265	I	13.37	dG0	
275	I	14.05	A0	
277	I	13.00	gK0	
279	I	13.37	F5	
289	I	13.75	F5	
296	I	13.75	dG2	
298	I	13.93	dG0	
299	I	13.56	F2	Double ?
313	I	13.93	A1	
330	I	12.81	gK8	Measured
341	II	12.81	gK2	
348	II	13.37	A5	Faint
349	II	12.44	A5	
374	II	8.98	B4	BD+59° 2387
606	I	13.18	A1	} Overlap
607	I	13.93		
618	I	13.37	F4	
664	II	11.79	gK1	

Plate M134A, 1926, Sept. 5. Center of Plate $\begin{cases} x + 100' = 94'.9 \\ y + 70' = 59.8 \end{cases}$
Area 18 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 13.3 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
22	II	13.00	dG8	Measured
27	II	12.63	A2	
40	II	10.06	gK1	BD+59° 2374
43	I	12.03	A0	
44	II	12.26	A0	
50	I	12.06	A8	
53	I	13.18	A3	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
60	I	11.23	A0	
80	I	12.07	F0	
96	I	11.95	A2	
112	I	13.07	gK0	
154	II	10.30	F0	Plate M132A; BD+59° 2379
171	II	11.64	dG4	Plate M132A
177	II	10.46	A1	Plate M132A; BD+59° 2382
412	II	12.44	A2	} Overlap
413	II	13.00		
429	I	13.00		Intercepted by No. 43
470	I	11.63	gK1	
492	I	12.26	F8	
508	II	12.81	dK1	Measured
513	I	13.00	F4	
540	II	9.90	gK4	BD+59° 2380

Plate M138A, 1926, Sept. 10. Center of Plate $\begin{cases} x = 10'.5 \\ y = 12.9 \end{cases}$
Area 19 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m1 \\ \text{Zone II } 13.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
623	II	12.79	gG8	Faint; Plate M140A
625	II	13.12	A5	Faint; Plate M140A
626	II	13.22	F5	Faint
632	II	13.12	A5	Plate M140A
635	II	12.46	A2	
639	II	13.12	A1	
650	I	14.06	gG2	Faint
653	II	12.79	A2	
669	II	13.29	dK0	
674	I	13.95	dG8	Faint
677	I	12.30	A5	
685	I	13.95	F0	Faint
709	I	13.29	A2	
736	II	13.29	gK2	Faint
911	II	12.46	A2	Plate M140A
925	II	10.21	A5	BD+60° 2546
927	II	13.29	F8	Faint
929	I	12.46	A2	
933	I	12.13	dG0	
940	I	10.02	A2	BD+60° 2552
952	I	12.46	A0	
964	I	14.06	A8	
965	I	10.19	F4	BD+60° 2554
976	I	12.30	A1	
981	I	13.29	A0	
983	I	12.96	B6	
986	I	11.33	A0	
987	I	13.88	A0	
988	I	13.45	gK0	
993	I	13.39	F4	
1003	I	13.55	A0	
1008	I	11.97	F1	
1053	I	12.79	A5	
1084	II	13.22	gG8	

Plate M140A, 1926, Sept. 11. Center of Plate $\begin{cases} x + 100' = 92'.2 \\ y = 13.8 \end{cases}$
Area 19 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m0 \\ \text{Zone II } 13.2 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
515	II	12.13	gK0	
521	II	11.64	B5	BD+59° 2719

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
544	I	12.96	A5	
563	I	13.62	dK0	
567	II	12.46	gK1	Plate M145A
571	II	10.83	F1	Plate M145A; BD+59° 2723
581	II	13.12	B6	Plate M145A
584	II	12.13	A2	Plate M145A
588	I	13.95	dG0	Faint
614	II	12.63	gK1	
615	II	12.96	F8	
618	II	12.96		Faint
623	II	12.79	gG8	Plate M138A
625	II	13.12	A5	Plate M138A
632	II	13.12	A5	Plate M138A
774	II	12.46	A5	
776	II	12.96	A3	
791	II	12.63	A1	
795	I	9.77	dG0	BD+60° 2538
798	I	11.22	gK1	BD+59° 2721
801	I	13.62	F2	
803	I	12.63	A1	
807	I	12.79	A5	
814	I	13.12	F5	
831	I	11.10	A0	
832	I	13.95	dK0	
839	I	14.02	A5	
848	I	13.95	A3	
850	I	13.45	B9	
854	I	13.95	A0	
856	I	13.95	A3	
862	I	13.62	F5	
870	I	13.45	F4	Faint
871	I	13.29	F5	
879	I	13.85	A0	
880	I	14.02	gG2	
883	I	12.96	A0	
888	I	11.97	F1	
899	I	13.12	dG0	
901	I	13.12	dG0	Faint
911	II	12.46	A2	Plate M138A

Plate M142A, 1926, Sept. 12. Center of Plate $\begin{cases} x = 10.3 \\ y+60' = 51.1 \end{cases}$
Area 19 SE

Limits of photographic magnitudes $\begin{cases} \text{Zone I } 14^m0 \\ \text{Zone II } 13.6 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
130	II	12.96	F4	
144	I	12.63	A0	
145	I	13.39	dK0	Intercepted; measured
147	I	13.72	dK5	Measured
179	II	11.80	A0	
187	II	11.43	F2	BD+59° 2730
196	II	12.96	A3	
208	I	13.95	dG5	
209	II	13.29	gG8	
213	I	10.33	A0	BD+59° 2734
219	II	12.63	F1	
224	II	13.55	F8	
235	II	12.30	A2	
237	II	13.45	A0	
354	II	10.44	A0	Plate M145A; BD+59° 2727
388	II	11.27	dG0	
391	I	13.85	dK0	Faint
393	I	13.29	gK0	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
398	I	13.39	gG0	
400	I	13.29	gMb	
414	I	12.79	A0	
415	I	11.97	dK0	
424	I	12.96	F8	
428	I	11.26	gK0	BD+59° 2731
431	I	13.12	gK2	
442	II	11.64	A2	
665	II	11.97	A5	

Plate M145A, 1926, Oct. 5. Center of Plate $\begin{cases} x+100' = 93.4 \\ y+60 = 55.4 \end{cases}$
Area 19 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m8 \\ \text{Zone II } 13.3 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
81	II	10.74	A0	BD+59° 2725
86	I	12.96	A8	
95	II	13.22	A0	
101	II	12.30	A5	
262	II	11.97	dG2	
277	II	12.96	A1	
280	I	12.79	A0	
284	I	13.72	gG0	Faint
293	I	13.55	dK5	Faint
308	I	11.97	A0	
311	I	12.63	A0	
312	I	13.62	A8	Faint
314	I	12.46	A2	
332	I	10.85	gK4	BD+59° 2724
333	I	12.30	gK0	
335	I	13.55	dG8	
345	I	14.02	A2	
347	I	13.78	A0	
354	I	10.44	A0	Plate M142A; BD+59° 2727
362	I	13.72		Intercepted by No. 621
363	I	13.78	A0	
567	II	12.46	gK1	Plate M140A
571	II	10.83	F1	Plate M140A; BD+59° 2723
581	II	13.12	B6	Faint; Plate M140A
584	II	12.13	A2	Plate M140A
606	I	11.80	gK1	
621	II	9.14	A2	BD+59° 2728
628	II	13.12	F5	

Plate M148A, 1926, Nov. 4. Center of Plate $\begin{cases} x = 6.1 \\ y+10' = 22.3 \end{cases}$
Area 8 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 13.1 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
580	I	11.27	A2	
583	I	12.30	F0	
599	I	13.32	A2	
604	I	11.73	F0	
607	II	12.60	F5	Plate M155A
619	II	11.73	A8	Plate M155A
622	I	12.38	F8	
625	II	12.80	dG0	Faint
629	II	11.73	A1	
639	II	12.60	gG0	Faint
718	II	11.20	F8	
724	I	11.20	A2	
731	I	13.49	A2	
734	I	12.68	F8	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
738	I	11.73	A8	
748	I	13.58	dK0	
758	I	11.54	gK1	
766	I	11.92	F0	
767	I	13.41	A8	Faint
768	I	12.00	A1	
774	I	12.60	A3	
777	I	13.49	A3	
781	II	12.11	A0	
783	I	12.95	A5	
786	I	12.11	A2	
788	II	11.39	A5	
800	II	12.68	F2	

Plate M150A, 1926, Nov. 6. Center of Plate $\begin{cases} x+100'=83.3 \\ y+10=22.2 \end{cases}$
Area 8 NW

Limiting photographic $\begin{cases} \text{Zone I } 13^m8 \\ \text{Zone II } 13.1 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
470	II	10.63	gK2	BD+60° 155
472	I	12.76	dG5	
480	I	13.41	gK0	Faint
486	I	13.49	dG2	Faint
488	II	12.76	A0	
493	I	12.87	A3	
495	I	12.30	A0	
499	I	11.73	A2	
518	I	13.63	A0	
523	I	12.95	A0	
545	II	11.73	A1	Plate M156A
644	I	10.81	A1	
650	I	12.49	A0	
659	I	12.80	F8	
671	I	13.58		Intercepted by No. 499
675	I	12.87	A2	
678	I	11.23	A0	
702	II	11.13	gK1	
707	II	12.11	dG2	
711	II	12.30	A2	

Plate M155A, 1926, Dec. 27. Center of Plate $\begin{cases} x = 4.3 \\ y+10'=4.6 \end{cases}$
Area 8 SE

Limiting photographic $\begin{cases} \text{Zone I } 13^m8 \\ \text{Zone II } 13.6 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
153	II	12.87	A5	
154	II	11.73	A0	
163	II	13.41	dG0	
166	I	13.72	A0	
184	I	13.41	gG0	
186	II	13.58	A5	
189	II	11.92	gK1	
190	I	13.11	F8	
237	II	12.76	F5	
380	I	10.19	A0	BD+59° 179; Plate M156A
387	I	12.68	gMa	Measured
391	I	11.45	F8	
398	I	12.30	A0	
399	I	12.45	A1	
402	I	12.91	A2	
404	I	13.58	F5	
407	I	13.07	F0	
409	I	11.04	A1	BD+59° 182

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
413	I	11.39	A3	
414	I	13.20	A2	
415	I	12.60	B5	
422	I	10.96	B3	} Overlap; BD+59° 184
423	I	13.58		
430	I	12.99	A0	
433	I	13.20	F4	
437	II	13.24	gK0	
438	I	13.80	gG0	
444	II	13.20	A0	
446	II	10.59	B3	BD+59° 186
452	II	12.80	A0	
573	II	11.92	F8	
584	I	12.87	A5	
607	II	12.60	F5	Plate M148A
619	II	11.73	A8	Plate M148A
621	II	12.41	F5	

Plate M156A, 1926, Dec. 28. Center of Plate $\begin{cases} x+100'=88.0 \\ y+10=4.4 \end{cases}$
Area 8 SW

Limiting photographic $\begin{cases} \text{Zone I } 13^m7 \\ \text{Zone II } 13.4 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
11	II	10.36	F0	} Two spectra; BD+59° 171
			F1	
31	II	12.11	A0	
63	I	13.41	A0	
65	I	13.49	gK5	Faint
72	I	11.92	A0	
88	II	12.80	gG5	
91	I	11.11	F5	
94	I	10.23	A1	BD+59° 177
105	II	13.28	F0	Faint
110	I	13.63	F8	
117	II	12.30	F0	
121	II	11.04	A5	
272	II	13.15	gK0	
283	I	12.11	gK8	Measured; BD+59° 173
284	II	12.95	gK2	Faint
287	I	13.63	gG5	Faint
291	I	13.41	dK0	Faint
293	I	12.41	dK0	
312	I	13.63	gG8	Faint
326	I	12.41	A0	
333	I	13.63	A8	
352	I	13.36	F1	
353	I	12.45	A5	
356	I	13.49	gG2	Faint
380	II	10.19	A0	BD+59° 179; Plate M155A
484	II	13.32	F8	Faint
488	II	12.76	A0	
516	II	13.40	A0	Faint
539	II	13.20	A5	
545	II	11.73	A1	Plate M150A
550	II	13.40	A8	

Plate M158A, 1926, Dec. 29. Center of Plate $\begin{cases} x = 5.6 \\ y+20'=32.4 \end{cases}$
Area 9 NE

Limiting photographic $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 12.9 \end{cases}$
magnitudes

Gron. No.	Zone	H Pg	Sp. Type	Remarks
246	I	13.47	gG5	
250	I	12.25	F2	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
252	I	12.47	A8	
253	I	8.86	A0	HD 19653, Type A0
264	I	12.80	A2	
265	II	12.80	F5	
275	II	12.80	F5	Faint
281	II	9.67	gK1	BD+60° 644
341	II	12.80	gK0	Faint
347	I	10.40	gK2	BD+60° 639
350	I	12.96	A0	
351	I	12.80	gK0	
355	I	12.96	A2	
359	I	12.09	A0	
361	I	9.93	A5	BD+60° 642
363	I	12.96	F4	} Overlap
364	I	13.30		
377	I	12.96	dG0	
379	II	12.47	dG2	Faint

Plate M160A, 1926, Dec. 30. Center of Plate $\begin{cases} x+100'=84.0 \\ y+20'=29.2 \end{cases}$
Area 9 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 12.8 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
199	I	12.19	A5	
203	II	12.27	dG5	Faint
207	I	11.11	A0	Plate M169A
210	I	11.81	A2	
214	I	13.47	dG5	
219	I	12.80	dG2	Faint
221	II	12.63	gG5	
300	II	12.63	dG8	
320	II	8.10	A5	{ BD+60° 633; HD 19501, Type A2

Plate M164A, 1927, Jan. 3. Center of Plate $\begin{cases} x = 5.1 \\ y+20'=11.5 \end{cases}$
Area 9 SE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m7 \\ \text{Zone II } 13.0 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
46	I	12.41	dG0	Plate M169A
57	I	12.63	A3	
128	II	11.84	A5	Plate M169A
130	II	7.09	A1	{ BD+60° 636; HD 19536, Type A0; Plate M169A
139	II	11.62	F5	
145	I	9.86	A5	} Overlap
152	I	13.47		
153	I	12.96	dK1	
163	I	11.91	A2	
164	I	13.65	F2	
169	II	11.09	F0	BD+60° 641
183	I	11.81	dG0	
184	II	12.80	dG8	
262	II	11.50	A4	

Plate M169A, 1927, Jan. 23. Center of Plate $\begin{cases} x+100'=88.4 \\ y+20'=11.6 \end{cases}$
Area 9 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 12.9 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
9	II	9.44	A0	BD+59° 597
10	I	13.47	gK0	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
15	I	12.41	gK0	
16	I	12.27	A2	
23	I	10.07	gK1	BD+59° 600
30	II	11.37	F5	
31	I	13.13	dG8	
32	I	10.22	gK1	BD+59° 602
38	I	13.47	gG2	
46	II	12.41	dG0	Faint; Plate M164A
104	II	10.90	A3	
107	I	11.91	F2	
117	I	13.13	dG5	
128	II	11.84	A5	Plate M164A
130	II	7.09	A2	{ BD+60° 636; HD 19536, Type A0, Plate M164A
207	II	11.11	A0	Plate M160A

Only one star, No. 425 of Area 40, was of type N. The classification is from the *Henry Draper Catalogue* in this case. The spectrum of No. 1783 of Area 64 is called "peculiar." The hydrogen lines $H\beta$, $H\gamma$ and $H\delta$ appear bright but the spectrum is too faint to warrant conclusions. The observed effect may be due to two overlapping spectra. Spectra which could not be classified because of overlapping are marked "intercepted." Such overlapping should not depend on spectrum type nor, to any great extent, on magnitude but only on the accidental position of the star in the sky. Thus we may safely omit these spectra without prejudicing our results. In one or two instances the star was omitted when too faint to classify. In these cases, however, the spectrum was much fainter than would be indicated by the photographic magnitude.

In seven instances, stars here classified were contained in the *Henry Draper Catalogue* and in twelve

TABLE IV

A COMPARISON OF THESE ESTIMATIONS WITH OTHER CLASSIFICATIONS

Area	Gron. No.	Plate Nos.	Lind.	Sp. Type	Est.
				HD	
64	535	M113A.....	A0		A0
64	581	M113A, M117A.....	B9		A2
64	583	M117A.....	A0		A0
64	646	M113A.....	gG8		gK0
64	677	M113A.....	A0		A5
64	706	M113A, M117A.....	A2		A5
64	710	M117A.....	A0		A0
64	733	M113A, M117A.....	A0		A0
64	748	M117A.....	A0		A0
64	767	M117A.....	dG5		dG0
64	834	M115A.....		K0	gG2
64	1007	M115A.....	A0		A0
64	1104	M113A.....	dA2		A2
40	300	M121A.....		A0	A0
40	587	M121A.....		B5	A2
18	374	M132A.....		B8	B4
9	130	M164A.....		A0	A1
9	253	M158A.....		A0	A0
9	320	M160A.....		A2	A5

cases the star has been classified at Mount Wilson by Lindblad. A comparison of these with the estimations in this paper is shown in Table IV. The latter are marked "Est."

The classification in the spectral sequence, as will appear later, is probably least accurate from G0 to K0 and most accurate from A0 to A5. In order to express the accuracy in a quantitative manner, the probable error of a pair of classifications was computed from the results for the standard comparison stars. This probable error, plotted in terms of spectral type, yielded the curve of Fig. 3. Thus the maximum probable error

Fig. 3. The computed probable error of a pair of estimates of spectral class, plotted as a function of the spectral type. The ordinates are given in decimals of a spectral division.

is well under 0.3 of a spectral division. In the A-type region the strength of the K-line gives a sharp criterion. It was here noted, however, that stars within a single Harvard sub-group seem to differ somewhat in type. This is probably due to the difference between the scheme used in this work and the Harvard scheme of classification. The extension of the spectrum into the ultra-violet has enabled us to use criteria not available to the Harvard observers.

VI. TREATMENT OF THE MATERIAL

The total area covered by the plates included 584 stars within the limiting magnitude of the plates. Due to overlapping, the spectra of 25 stars remain unclassified. Two stars have spectra not contained in the ordinary sequence, and eight stars are below magnitude 14.0 and will not be included in the discussion. There remain 557 stars of which 538 are between magnitudes 10.00 and 14.00. The distribution in spectral types is shown in Table V.

The limiting magnitude and the area covered on each plate being known, it was possible to reduce the results to stars per square degree. For this purpose it was necessary to group the stars into tenths of a magnitude and multiply each group by a factor depend-

TABLE V

B3-B8.....	11
B9-A5.....	221
A8-F5.....	107
gG0-gG5.....	24
gG8-gK5.....	73
dF8-dG5.....	82
dG8-dK5.....	33
dK8, gMa, gMb.....	6
Total.....	557

ing on the area covered to that magnitude. Care was taken to avoid counting overlapping areas twice, and to include each star but once. When the reduced frequencies were derived they were collected into classes of an entire magnitude. Stars whose spectra, for reasons purely accidental, could not be classified, were distributed over the spectral classes in the same proportion as the observed stars of that magnitude. They merely contributed to the total number of stars and not to the relative frequencies of spectral types.

The following table gives the number of stars per square degree for each main spectral division:

TABLE VI

Sp. \ m_{ph}	10.01-11.00	11.01-12.00	12.01-13.00	13.01-14.00
B3- B8	1.41	1.49	1.68	0.64
B9- A5	9.40	25.06	49.23	62.54
A8- F5	3.29	11.83	21.91	37.12
gF8-gG5		0.94	3.93	16.57
gG8-gK5	4.23	9.48	12.98	12.57
dF8-dG5		4.31	20.98	46.13
dG8-dK5	0.94	0.47	7.73	25.39
gK8, gMa, gMb			2.24	0.70
Peculiar			1.09	
Total	19.27	53.58	121.77	201.66

In the discussion which follows we shall make the customary assumption that there is no extinction of light in space. It admits of the use of the equation connecting distance in parsecs ρ , apparent magnitude m , and absolute magnitude at ten parsecs M :

$$5 \log \rho = m - M + 5 \quad (7)$$

without further discussion. We shall compute ρ for each star according to this formula. For M we can substitute the mean absolute magnitude corresponding to a given spectral type. Several investigators have given such values, derived in various ways. It seemed best, however, for the sake of uniformity, to confine ourselves to a single set of results. The Mount Wilson spectroscopic absolute magnitudes were accordingly chosen. Since the spectral types in this paper are on the Harvard system it is necessary to obtain the absolute magnitudes corresponding to Harvard spectral types.

The relations between the spectroscopic classifications of A and B type stars of Harvard and Mount Wilson are discussed by Lindblad¹⁴ and also by Miss Fairfield.¹⁵ The latter paper contains the mean absolute magnitudes, spectroscopically determined, for every sub-type of the Harvard classification and is based on a total of 666 stars. Means of the values for "nebulous" and "sharp" line spectra were adopted here. For stars of types F-M, the absolute magnitudes of Adams and Joy¹⁶ were averaged for each sub-type of the Henry Draper classification. The line of division between the giant and dwarf groups was estimated from Curtis's diagram.¹⁷ Stars above the line were grouped with giants and those below were grouped with dwarfs. Only stars in the *Henry Draper Catalogue* were used and stars falling on the line of division into giants and dwarfs as well as stars of types not considered in this paper, were omitted. The result is a set of mean absolute visual magnitudes for each sub-type of the *Henry Draper Catalogue* within the limits of types included in our discussion. For the late types 1501 stars were used, making 2167 stars in all. A few of these, however, are probably duplicates.

In order to coördinate the results obtained in the manner just described, the mean absolute magnitudes were plotted against spectral type, according to the Hertzsprung-Russell method, and a smooth curve was drawn to give greater weights to points representing greater numbers of stars. The result is the continuous curve of Figure 4.

As the limits of completeness of our catalogue of spectral types are referred to photographic magnitudes rather than to visual magnitudes a correction of the curve for color index was necessary. The color indices adopted for the spectral types are taken from the *Henry Draper Catalogue* for types earlier than G0, and for the giants and dwarfs of type G0 and later they are taken as proportional to the values given by Seares.¹⁸ In order that a giant K0 star may have a color index +1.0, the values of Seares were multiplied by $\frac{1}{1.48}$. The curve for the Harvard photographic magnitudes is the dotted curve of Figure 4. With the known spectral type the absolute photographic magnitude is taken from the curve and the distance in parsecs derived by equation (7), with the help of the apparent photographic magnitude. For those few stars whose spectra were found to be double, the luminosity has been divided equally between the components. This is partly justified by the fact that in

all such cases the intensities of the component spectra were found to be about equal.

A selection factor enters into the derivation of our curve, inasmuch as the stars upon which it was based are mostly brighter than magnitude 6.0. Malmquist¹⁹ has shown that if a simple exponential law of increase is assumed for the apparent magnitudes this correction is independent of the magnitude. Since the mean absolute magnitudes just derived are to be applied to stars selected with reference to magnitude, this effect on our own stars must be of second order.

Fig. 4. Adopted Hertzsprung-Russell Diagram.

Ordinates: Absolute magnitude from the Mount Wilson spectroscopic parallaxes.

Abscissae: Harvard spectral type.

□ Mean of "n" and "s" values for the absolute magnitude given by Miss Fairfield.

○ Mean of absolute magnitudes from the Mount Wilson list of 1646 stars.

The continuous curve is on the Harvard visual system, and the dotted curve on the Harvard photographic system.

The absolute magnitudes just derived agree fairly well with Lundmark's¹ values except for the type G0 for which Lundmark adopts a higher luminosity than that given by our curve. The same remark applies to the values derived by Lindblad²⁰ but to a lesser extent. Malmquist,²¹ basing his study on much the same material as the above, and including considerations of proper motion in his discussion, agrees very well with our adopted luminosities. Luyten²² finds a value -0.8 for the absolute magnitude of a giant M-type star,—considerably brighter than any of the foregoing authorities. For the B-stars, Gerasimovic²³ finds a somewhat higher luminosity than the above. These latter types, however, are not important in the study of the observa-

¹ *Publ. A. S. P.* 34, 150, 1922.

¹⁹ "A Contribution to the Problem of Determining the Distribution in Space of the Stars," *Lunds Medd. B.* 19A, no. 106, 1925.

²⁰ "Researches based on Determination of Stellar Luminosities" (second paper), *Medd. fran. Ast. Obs. Upsala*, no. 11, 15, 1926.

²¹ *Lunds Medd. Ser. III*, 1, no. 32, 1923.

²² *Harv. Circ.* 262, 1924.

²³ *Vjs. d. Ast. Ges.* 61, 219, 1926.

¹⁴ *Astroph. Jour.* 59, 305, 1924.

¹⁵ *Harv. Circ.* no. 264, 1924; See Table V.

¹⁶ *Mt. Wilson Contr.* no. 199, 1921.

¹⁷ "Stellar Luminosities," *Publ. A. S. P.* 34, 33, 1922.

¹⁸ *Mt. Wilson Contr.* no. 226, Table XII, 1921.

tional material of this paper, as they occur very infrequently in the list of spectral types here presented. It must be admitted, however, that uncertainties in luminosities at the ends of the spectral series may have an effect on the studies derived from spectral types of brighter stars.

Errors in the distances may be introduced in two ways:

- (1) From mis-classification of the spectra.
- (2) From an actual dispersion in absolute magnitude within each spectral sub-division.

The combined effect of the two errors, however, will be taken care of if we find the probable error resulting from the dispersion within a Harvard spectral sub-type. This was done for the stars upon which our Hertzsprung-Russell diagram was based. The results plotted in terms in spectral type form the curve of Figure 5.

Fig. 5. Ordinates are probable errors in absolute magnitude, computed from the dispersion in absolute magnitude within each sub-type as derived from the Mount Wilson spectroscopic parallax data. Abscissae are spectral types.

The small variations in the curve of Figure 5 are probably of accidental origin. The adopted probable error was 0.45 for types B-F and the dwarf branch. A weighted solution for all giant stars together gave 0.54. These correspond to dispersions σ_m , 0.67 and 0.80 respectively. The corresponding percentage errors in the distances are 0.21 and 0.25, fairly comparable with the accuracy of the spectroscopic parallaxes themselves, which are estimated at 18 per cent for types F-M.¹⁶ Systematic errors in the spectroscopic parallaxes affect our values to the same extent, of course, as these are based directly on the latter.

¹⁶ *Mt. Wilson Contr.* no. 199, 1921.

VII. MATERIAL FROM THE HENRY DRAPER CATALOGUE AND EXTENSION

To derive the densities of high luminosity stars near the Sun, data were taken from the *Henry Draper Catalogue* and the *Henry Draper Extension*. Areas concentric with the Selected Areas were chosen in such a way that they would be free from galactic irregularities and, for convenience, would be bounded by hour circles and declination parallels. The boundaries are as follows:

TABLE VII

Central Sel. Area:	Limits in α (1900.0)		Limits in δ (1900.0)	
64	19 ^h 42 ^m .4 to	20 ^h 12 ^m .8	+26° 42' to	+33° 18'
40	20 10.7	21 22.7	+43 17	+46 43
18	20 57.7	21 50.7	+56 52	+63 28
19	22 56.6	23 49.4	+56 42	+63 18
8	0 33.3	1 26.3	+56 52	+63 28
9	2 37.1	3 30.5	+57 02	+63 38

From these areas 347 stars whose photographic magnitudes are between 5.01 and 8.00 were taken from the *Henry Draper Catalogue*. The photographic magnitudes were computed from the measured visual magnitudes and mean color index of the spectral type. When given only to tenths of a magnitude, the division at the magnitude boundaries was made on the basis of an interpolation to hundredths. The following table gives the frequencies of the spectral sub-types reduced to stars per 10,000 square degrees:

TABLE VIII

HPg Mag	5.01-6.00	6.01-7.00	7.01-8.00
Sp.			
B0.....	76	249	131
B1.....		38	38
B2.....	76	38	152
B3.....	59	397	570
B5.....		215	490
B8.....	76	380	388
B9.....	76	325	730
A0.....		574	2586
A2.....	38	304	308
A3.....	76	97	194
A5.....	38	76	228
F0.....	38	38	114
F2.....		59	55
F5.....	59	169	498
F8.....	76	38	173
G0.....			152
G5.....			401
K0.....	59	245	574
K2.....			249
K5.....	76	38	152
Ma.....	38	21	93
Mb.....			38
Pec. (O, N, etc.).....	76	38	
Total.....	937	3339	8314

TABLE IX
REDUCTION TABLE FOR gK-STARS

$m \backslash y$	13.01	13.51	14.01	14.51	15.01	15.51	16.01	16.51	Total
	-13.50	-14.00	-14.50	-15.00	-15.50	-16.00	-16.50	-17.00	
>10.00									
10.01-10.50	28200								28200
10.51-11.00		9400	4700						14100
11.01-11.50			42300	9400					51700
11.51-12.00				43100					43100
12.01-12.50					38800	9800			48600
12.51-13.00						64400	18700		83100
13.01-13.50							55000	48000	103000
13.51-14.00								25500	25500
>14.00									
Total	28200	9400	47000	52500	38800	74200	73700	73500	397300

For the magnitudes 8.01 to 10.00, the region covered by nos. 225301-229300 of the first and second installments of the *Henry Draper Extension*,²⁴ was used. The *Henry Draper Catalogue* and *Extension* are complete in this region to about magnitude 10.75 on the photographic scale, according to Shapley.²⁵ The actual number of stars in this aggregation is not used. Instead, the total number of stars was interpolated from the foregoing group and the observed group by assuming a normal frequency distribution for the apparent magnitudes:

$$a(m) = \frac{N}{a\sqrt{2\pi}} e^{-\frac{(m-m_0)^2}{2a^2}} \quad (8)$$

where N , a , and m_0 are constants and $a(m) dm$ is the number of stars per 10,000 square degrees whose apparent photographic magnitude lies between m and $m+dm$. A weighted least square solution gave:

$$\text{Log}_{10} a(m) = -.00587 m^2 + 0.534 m + 0.263 \quad (9)$$

from which the number for the interval 8.01-10.00 was computed. By thus using relative frequencies of spectral types for this region, instead of actual frequencies, we eliminate the abnormal richness of the area covered by the *Extension*.

Systematic corrections to the spectral types of the *Extension*, to reduce them to the same system as the *Catalogue*, are given by Shapley²⁵ and have been applied to the former. No changes have been made in the magnitudes. The two groups were now combined, and in what follows, they will be discussed as a unit.

VIII. DISCUSSION AND RESULTS

Method of Derivation of Density Curves for the Observed Stars

Individual distances were derived for the observed stars according to the method already described. A table was then formed for each main spectral division,

²⁴ *H. A.* 100, nos. 1, 2, 1925.

²⁵ *Harv. Circ.* no. 278, 1925.

using m and $y = 5 \text{Log}_{10} \rho = m - M + 5$ as arguments. As an example, we reproduce herewith the table for the stars of type gK (gG8-gK5). The discussion in these paragraphs refers to only one main spectral division.

The frequencies are given in stars per 10,000 square degrees. The intervals in m and y are taken as 0.5 throughout, for the observed stars. If the table were constructed for but one sub-type the numbers would all fall on one diagonal corresponding to $m - y = \text{const.}$ The inclusion of several sub-types widens this to a band.

At the lower right hand end the transfer from completeness in *magnitude* to completeness in *distance* was effected by a partial extrapolation of the numbers, when necessary. This was not necessary for giant stars, however.

If the intervals were infinitely small and if the dispersion within a sub-type were zero the sums taken from left to right would give us $a(m)dm$ and those from top to bottom $n(y)dy$, the number of stars of all apparent magnitudes whose moduli of distance (y) lie between y and $y+dy$. In order to obtain this number law $n(y)$ two corrections must therefore be applied:

- (1) Correction for dispersion.
- (2) Correction to the median value of y .

The first might be applied by distributing each frequency over a normal error curve in the horizontal direction and taking the revised sums at the bottom. It is simpler, however, to make an approximate correction on the basis of an assumed law of increase. For this purpose let $\varphi_m(M)$ be the frequency function of the absolute magnitudes within a given apparent magnitude. We shall assume this function to be a normal error function for each sub-type. The adopted dispersion is to be that found earlier in this paper, viz., 0.67 for dwarfs and types A and B, and 0.80 for giants. Thus $\varphi_m(m-y)$ is a fraction by which each number in the diagonal is to be multiplied to give its effect on a neighboring column. The number of stars between

apparent magnitudes m and $m+dm$ and simultaneously between distances y and $y+dy$, we shall denote by $\Psi(m,y)dmdy$. The analytical expression of the latter is, then:

$$\Psi(m,y) = a(m) \cdot \varphi_m(m-y+5) \quad (10)$$

which is the relation from which the left-hand member is to be computed. Assuming for the first factor, the form

$$a(m) = ce^{\beta m} \quad (11)$$

and for the second a normal frequency distribution, it is easy to show that:

$$n(y) = \int_{-\infty}^{+\infty} \Psi(m,y) dm = e^{\frac{\sigma_m^2 \beta^2}{2}} a(M_m + y - 5) \quad (12)$$

where σ_m and M_m are dispersion and mode respectively of the frequency function $\varphi_m(M)$. Since y is originally computed from $y = m - M_m + 5$ the expression $a(M_m + y - 5)$ is the number originally tabulated, expressed in terms of y . According to (12), then, the effect of the dispersion is to multiply each of the original tabular values by $e^{\frac{\sigma_m^2 \beta^2}{2}}$. We shall simply multiply totals of each column by this factor.

Since our intervals are finite, the sums at the bottom of the table do not give the number law $n(y)$ corresponding to the median value of y , but are actually:

$$n_{\Delta y}(y) = \int_{y-\frac{1}{2}\Delta y}^{y+\frac{1}{2}\Delta y} n(y) dy \quad (13)$$

where Δy is the interval and y in the limits is the median y . Expanding in a Taylor's series and omitting higher order terms the above reduces to:

$$n_{\Delta y}(y) = n(y) \left[1 + \frac{(\Delta y)^2}{24} \frac{d^2 n(y)}{dy^2} \right] \Delta y \quad (14)$$

Assuming Seeliger's Law of density²⁶ over the interval for the evaluation of the corrective term, we find:

$$n(y) = \frac{n_{\Delta y}(y)}{\Delta y \left[1 + \frac{\beta^2}{24} (\Delta y)^2 \right]} \quad (15)$$

It is well known that the number of stars per 10,000 square degrees whose distance lies between ρ and $\rho+d\rho$ parsecs and whose apparent magnitude is between m and $m+dm$ is given by:

$$\psi(m,\rho)dmd\rho = \left(\frac{\pi}{1.8} \right)^2 \rho^2 \Delta(\rho) \varphi(m+5-5\log \rho) dmd\rho \quad (16)$$

where $\Delta(\rho)d\rho$ is the number of stars per cubic parsec whose distance lies between ρ and $\rho+d\rho$ divided by

²⁶ It is well known that Seeliger's Law leads directly to the exponential law of increase $a(m) = ce^{\beta m}$ where $\beta = \frac{0.2}{\log_{10} \epsilon} (3-s)$. For a demonstration, see Charlier, "The Motion and Distribution of the Stars," *Loc. Cit.*

the corresponding number in the neighborhood of the Sun, and $\varphi(M)dM$ is the number of stars per cubic parsec in the neighborhood of the Sun having absolute magnitudes M to $M+dM$. We shall also define N_0 by:

$$N_0 = \int_{-\infty}^{+\infty} \varphi(M) dM \quad (17)$$

or, in other words, the total number of stars in the neighborhood of the Sun, of all absolute magnitudes.

In place of ρ in (16), substituting

$$\rho = 10^{0.2y}$$

$$\text{and } D(y) = \Delta(\rho), \quad \Psi(m,y) = \psi(m,\rho)$$

the latter becomes:

$$\Psi(m,y) = 10^{0.147+0.6y} D(y) \varphi(m-y+5) \quad (18)$$

Integration with respect to m gives us the number law in terms of the density law:

$$n(y) = N_0 D(y) 10^{0.147+0.6y}$$

whence

$$N_0 D(y) = n(y) 10^{-0.147-0.6y} \quad (19)$$

Including, now, our two corrections we get, as our working equation:

$$N_0 D(y) = \frac{e^{\frac{\beta^2 \sigma_m^2}{2}} 10^{-0.147-0.6y} n_{\Delta y}(y)}{\Delta y \left[1 + \frac{\beta^2 (\Delta y)^2}{24} \right]} \quad (20)$$

Referring to equation (9) we see that β may be computed for all types together by:

$$\beta = \frac{d \log_e a(m)}{dm} = -0.027m + 1.23$$

Assuming that the mixture of spectral types does not vary greatly in going to fainter magnitudes, this value will probably be accurate enough for our corrective terms. We have, in fact, used only two values of β , one for the observed stars and another for the stars from the *Henry Draper Catalogue* and *Extension* group. The maximum value of the corrective term is about 33 per cent.

Dwarf Types.—We consider here two groups of stars classified as dwarfs, namely dG stars or stars with spectral types dF8–dG5, and dK stars or stars with spectral types dG8–dK5. Of the 82 stars in the former group 8 are outside of the limits of distance for this group:

$$10.51 < y < 14.00$$

and are therefore excluded. Similarly only 24 of the 33 dwarf K stars could be utilized. The loss of available material is a necessary consequence of the steep slope of the Hertzsprung-Russell diagram in these types.

Densities for the dwarf types were computed by formula (20) and plotted in terms of the modulus of distance, y . These densities represent nearly all of the available material for the dwarfs, except in the immediate neighborhood of the Sun. The dG stars, however, are well enough distributed to show a drop in the density occurring at about 250 parsecs. The points on the downward slope, however, are somewhat uncertain, as extrapolated values have entered with increasing influence.

TYPE dG

TYPE dK

Fig. 6. Density curves for the dwarf types.

Ordinates: $\log_{10} N_0 D(y) + 10$.

Abscissae: $y = 5 \log_{10} \rho$.

The dK-stars show no effect of density on distance, the data being too meagre for this purpose. We have, however, some idea of the relative frequencies of these types in a limited region of space. Thus the region from 125 to 200 parsecs distant is common to all types but type B in this work. At the risk of some anticipation the number of stars per 100,000 cubic parsecs are given below for this region:

TABLE X

dK.....	3800
dG.....	870
A.....	63
gK.....	44
F.....	19
gG.....	15
gM.....	3
B.....	3

Thus the dK-type is by far the most frequent in space of any in this discussion. It is interesting to note, in this connection, that Seares²⁷ and van Rhijn²⁸ find evidence of a continually increasing frequency curve as the luminosity is taken at successively smaller values. In any case, the vast majority of stars are left completely out of the present discussion.

To separate the dwarfs from the giants among the brighter stars a constant density was assumed for both of the foregoing types. Under this assumption, it is known that the number of stars $A(m)$ brighter than magnitude m is expressible by:

$$A(m) = A(0) 10^{+0.6m}$$

A weighted solution for $A(0)$ gave:

$$A(0) = .00312 \text{ for the dG-type}$$

and $A(0) = .00133$ for the dK-type

At the sixth visual magnitude this would give, for the whole sky, 102 and 87 dG and dK stars, respectively. According to Malmquist²¹ there are, respectively, 91 and 39 dG and dK stars brighter than visual magnitude 6.00. In this direction, therefore, we seem to have a greater density than Malmquist has found for all directions combined. Errors in the assumed density law are not serious, unless large, as the whole number of dwarfs to be subtracted is rather small. The above formula, therefore, was retained for this purpose.

Giant Stars and Early-Type Stars.—The stars from the observed list have been treated individually in deriving the density curves. The interval in both m and y is 0.5 magnitude. For the brighter stars (i.e. stars from the *Henry Draper Catalogue and Extension*) an interval of 1.0 magnitude was used and individual distances were not derived. Care was taken, however, to deal rigorously with each sub-type in the derivation of the density. The densities of each sub-type, referring originally to different values of y , were reduced to an adopted value; (i.e., 9.50, 10.50, 11.50, etc., for A-type stars) and then added to form the densities for a whole group. These were then plotted in the same way as the data derived from the observations of the fainter stars. The resulting curves are reproduced herewith, in Figs. 7 and 8. Except for type B the curves begin as early as 65 parsecs, or less. Thus the early part of the curves are really in the solar neighborhood.

The ordinates were chosen as the common logarithms of the density and the abscissae as five times the logarithms of the distances in parsecs. On the hypotheses of Herschel and Schiaparelli we would

²¹ *Lunds Medd. Ser. III*, 1, no. 32, 1923.

²⁷ *Mt. Wilson Contr.* no. 273, 1924.

²⁸ *Gron. Publ.* no. 38, 1925.

have, on this system, a horizontal straight line; Seeliger's hypothesis would give a straight line with a negative slope, Schwarzschild's an inverted parabola and Charlier's an exponential curve.

Fig. 7. Density curves for the early types.

Ordinates: $\log_{10} N_0 D(y) + 10$.

Abscissae: $y = 5 \log_{10} \rho$.

All of the curves show a drop in density at about 250 parsecs. The irregularities of the F-type curve are probably accidental. There are two reasons why this type should not be accurately represented:—the Hertzsprung-Russell diagram is steep at this point, and no subdivision into giants and dwarfs has been made. A more important reason may lie in a possible tendency of the *Henry Draper Extension* to avoid the F-type, putting the stars either into the A or the G types. The giant K stars show a more rapid drop than do the A-type stars. We shall revert to this point.

On the basis of the density curves of the different spectral types a total density curve for the types B, A, gG, gK, and gM was derived and is included herewith. Type F was omitted partly because of its uncertain nature and predominating influence, and partly because it belongs more properly with the dwarf group. An examination of the curve shows a dropping off of density of the giant stars at 200 to 400 parsecs. Pannekoek²⁹ from a study of the frequency gradient of

²⁹ B. A. N. 1, 54, 1922.

apparent magnitudes of all spectral types together in the *Cygnus* cloud, finds an agglomeration of stars at 200 to 600 parsecs. Although our curves show a falling off in density beyond this distance, they cannot be

Fig. 8. Density curves for the late giants.

Ordinates: $\log_{10} N_0 D(y) + 10$.

Abscissae: $y = 5 \log_{10} \rho$.

Fig. 9. Density curve of types B, A, gG, gK, and gM combined. The dotted portion is extrapolated.

Ordinates: $\log_{10} N_0 D(y) + 10$.

Abscissae: $y = 5 \log_{10} \rho$.

said to show any effect of such an agglomeration, in view of the fact that within this distance the density seems to be fairly constant. Of the local cluster hypothesis we shall speak later.

The preponderance of dwarf stars over giants is shown by Table X in which the relative numbers at 125 to 200 parsecs is given. The increase of this preponderance with spectral type is greater than that which is generally adopted. For instance, Malmquist²¹ gives for the solar neighborhood about 730 G dwarfs and 1840 K dwarfs per 100,000 cubic parsecs while the corresponding figures from our tables are 870 and 3800 respectively. It will be shown later, by deriving a provisional luminosity curve, that the same statement applies to a comparison with van Rhijn's values. As to the cause of this discrepancy we can only speculate. It may be due to a marked change in the giant-and-dwarf mixture with galactic latitude. This, however, seems unlikely in view of the fact that the mean color index of the Milky Way, as shown by Shajn,³⁰ is less than that of high galactic latitudes. The dwarfs are presumably of large color index and any great excess of them would make the effect opposite to that which is observed. A more probable explanation may lie in the supposition that dwarf stars have been very incompletely observed. This is in accord with the trend of results of recent investigators. Dwarf stars are usually picked up by their large proper motion. This is known to be a somewhat selective criterion, as stars whose cross-motion is parallel to the solar motion will be omitted. Incompleteness of material is also to be taken into account in discussions based on spectroscopic parallaxes.

The relative frequencies of giants of the different spectral types is readily obtainable from the density curves for the types and the total giant density curve. We give in Fig. 10 the curves of relative frequency in terms of the modulus of distance, y . The scale is such that the sum of all the ordinates at any point is equal to unity. Although many of the minor irregularities are probably accidental, one feature stands out so prominently that there can be little question of its reality. This is the increase of percentage of A-type stars with distance and the corresponding diminution of giant K-type stars. This was already noted in reference to the rapid drop in the gK-type density curve. The relative densities of the different types at the beginning of the curves ($y=10$) may be compared with Malmquist's²¹ values for the solar neighborhood:

Type	Malmquist	Density curves ($y=10$)
B	0.054	0.027
A	0.355	0.459
gG	0.064	0.149
gK	0.484	0.340
gM	0.043	0.025

The behavior of A and K types is in agreement with Shapley's³¹ observation that class A predominates in

²¹ *Lunds Medd. Ser. III*, 1, no. 32, 1923.

³⁰ *Mon. Not. R. A. S.* **86**, 382, 1926.

³¹ *Harv. Circ.* no. 248.

low galactic latitudes and class K in high latitudes, since the results of Malmquist are without reference to galactic latitude. Another possible influence of galactic latitude, even at these small distances, is suggested by the fact that our total number for these

Fig. 10. Curves of Relative Densities of Types B, A, gG, gK, gM.

Ordinates: fraction of total density; abscissae: $y = 5 \log_{10} \rho$.

types is 1312 stars per 100,000 cubic parsecs, while Malmquist's value reduces to 1056 stars. That our density for giant stars is larger than that found for all galactic latitudes together is confirmed by a comparison with van Rhijn's luminosity curve to be made presently. The former is systematically larger than the latter for giant stars. Van Rhijn²⁸ seems inclined to doubt the reality of any change in the luminosity curve with galactic latitude, but leaves the matter open to question. It must be remembered, too, that the absolutely nearest stars are left out of our derivation.

IX. MINOR RESULTS

A derivation of the luminosity curve for stars from 125 to 200 parsecs does not properly belong to the major results of this paper, for several reasons. The intention was not to deal with the luminosity function

²⁸ *Gron. Publ.* no. 38, 1925.

except as a means of getting the density curves. Thus for each spectral sub-type we have really assumed a luminosity function and a solution for it would lead to our primary assumptions. We may, however, for the limited regions just mentioned, derive a luminosity curve for all spectral types together. This is equivalent to multiplying that for each spectral type by a certain coefficient depending on the density and summing over all types. These coefficients are now available for the region from 125–200 parsecs for absolute magnitudes as faint as that corresponding to dK5. This makes it of interest to perform the derivation, bearing in mind that the material is rather limited and that too much emphasis should not be placed on the smaller details of the result.

We shall now briefly describe the method of deriving the luminosity curve. Results have already been presented for the relative frequencies of different spectral types. Let us examine only the spectral sequence beginning with B0 and ending on the dwarf branch with dK5. Summing the relative frequencies along the series we obtain:

$$\int_{B0}^{Sp} F(Sp) dSp$$

where $F(Sp)$ is the frequency of a spectral type denoted by Sp . The lower limit corresponds to B0, the beginning of our spectral sequence and the upper limit to the division points between main spectral types, viz., B8, A5, F5, dG5 and dK5. This integrated function could now be plotted in terms of the spectral type and five points obtained from which a curve could be drawn. Since the Hertzsprung-Russell diagram, however, for this group, is uni-directional in its variation, the abscissae may be replaced by corresponding absolute magnitudes, whence the ordinates would become

$$\int_{-\infty}^M \varphi(M) dM$$

if we neglect the number of stars brighter than average stars of type B0. Actually the common logarithms of the above were plotted and found to lie quite close to a smooth curve. By differentiation and multiplication of the derivative by the integral itself and by $\log_{10} e$ we get at once the desired function $\varphi(M)$. For the giants a separate and similar treatment was applied. The logarithm of the resulting luminosity curve is given by the continuous curve in Fig. 11. The luminosity curve of van Rhijn²⁸ is the dotted curve in the figure. The latter was derived by interpolating a fraction 1:1.48 between van Rhijn's photographic and visual curves. This is necessary because the latter's photographic curve is on the Mount Wilson system while ours is on the Harvard photographic system.

²⁸ *Gron. Publ.* no. 38, 1925.

A comparison of the two curves brings out two facts already anticipated; namely, the derived luminosity curve is systematically larger than van Rhijn's, and the increase in the dwarf branch is enormously more pronounced. In addition to the explanation already offered, the first of these may be partially accounted for, in the giant part, by assuming a systematic error in the assumptions of the mean absolute magnitudes of the giant spectral types. It should be stated that one such correction has already been

Fig. 11. The Luminosity Curve.

Ordinates: $\log_{10} \phi(M) + 10$.

Abscissae: M .

The continuous curve is derived from stars in our particular field of the Galaxy with distances from 125 to 200 parsecs. The dotted curve is that of Van Rhijn after reduction to the Harvard photographic system.

applied. The factor of selection in going from the mean absolute magnitude of a given apparent magnitude to the mean absolute magnitude in a given volume of space is, according to Malmquist,¹⁹ $\beta\sigma^2$, on the assumption of a simple exponential law of increase of the apparent magnitudes. This correction, on our assumed dispersion, is +0.68 for the giants and +0.48 for the dwarfs. It may be that this correction is slightly too large, but the discrepancy is more probably due to the fact that our curve is derived for the galactic regions, while that of van Rhijn is for all galactic latitudes together. Greater richness of the former would account for the difference.

The second discrepancy between the curves has, to some extent, already received our attention. It will here, however, play an important role in an attempt to decide on the bearing of our results on the problem of the galactic structure.

There are two prevailing theories as to this structure:

- (1) The older theory of a sidereal system of rather conservative dimensions, still re-

¹⁹ "A Contribution to the Problem of Determining the Distribution in Space of the Stars." *Lunds Medd. B.* 19A, no. 106, 1925.

tained by many astronomers. According to Seeliger, it has the form of an ellipsoid of revolution gradually diminishing in density. Seeliger considers the equatorial diameter to be about 20,000 light years³² (6100 parsecs).

- (2) The theory of a very extensive stellar system, brought about by the studies of Shapley on the globular clusters and assuming, moreover, that the globular clusters are intimately related to the stellar system. The dimensions of the system are about 15 times those of Seeliger's system. According to the proponents of this view the Sun is situated in a local cluster of stars.

The followers of the former hypothesis would interpret our density decrease as an actual and final fading away of the stellar system, in this particular direction of space. Those following the latter theory would interpret it as a falling off in density near the boundaries of the "local cluster" only, insisting that there are further condensations as we go into the supposedly denser and more distant galactic star clouds.

Assuming, for the moment, the latter idea to be the correct one, let us examine the situation in its relation to the results of other investigators. Our results for the density laws require, then, such an extent of the local cluster of giants that in the *Cygnus* direction the density does not begin to fall off appreciably until a distance of 200 to 400 parsecs is reached. The cluster derived by Charlier³³ for the B-type stars is probably identifiable with our supposed cluster. Shapley and Cannon³⁴ have found the same clustering for the A-type stars, the former³⁵ believing this B- and A-type cluster not related to the general structure of the Milky Way. The dimensions of the galactic system as given by Shapley and confirmed by Pannekoek³⁶ are many times those of the local cluster. The latter derived a distance of 40,000 parsecs for the *Cygnus* cloud but later corrected it to 18,000 parsecs.²⁹ Our density curves, therefore, would show no effect of the Milky Way as even for the B-type stars our limit is about 5000 parsecs. It is obvious that a sample, such as this, from one small region of space and so limited in extent as regards distance can produce no decisive evidence on this question.

If, on the other hand, the older theory is accepted, our limits must be almost comparable to the limits of

the galactic system. It is, therefore, natural to put the question "Is this density law of the giant stars, assuming the same law for the dwarfs, capable of representing the number of stars of very faint magnitude, which have been counted on photographic plates of long exposure?" More correctly, we want to know if it can represent the order of the number of stars counted. The Mount Wilson astronomers³⁷ have counted stars down to photographic magnitude 18.5 in the *Mount Wilson Catalogue of Selected Areas*. Other sources are utilized for the brighter stars. To test the above question a representation of this magnitude was attempted. The number law $n(y)$ is readily derivable from the density law. The number of stars brighter than photographic magnitude m is given by:

$$A(m) = \frac{1}{N_0} \int_{-\infty}^{+\infty} \Phi(m-y+5)n(y)dy$$

$$\text{where} \quad \Phi(M) = \int_{-\infty}^M \varphi(t)dt$$

The number therefore depends on the luminosity law. At first that of van Rhijn was used and found to yield a number far too small. After the derivation of our luminosity curve it was used and the density curve was drawn, rather arbitrarily, asymptotic to the line $y=20$ as shown by the dotted part of the curve in Fig. 9. The stellar system is thus arbitrarily limited to 10,000 parsecs in this direction of space. Numerical integration, based on these two curves gave:

$$A(18.5) = 1.31 \times 10^8$$

The value from the Mount Wilson counts is:

$$A(18.5) = 1.46 \times 10^8 \quad (\text{gal. lat. } 0^\circ)$$

In this comparison we have neglected the difference between the photographic system which we are using and the Mount Wilson system. The comparison shows, however, that it is possible to represent the number of galactic stars down to 18.5 magnitude. Assuming, therefore, the validity of the luminosity law here derived and that the density diminishes gradually to zero at 10,000 parsecs, we may say that there seems to be no direct evidence that further condensations do exist.

An attempt was made, incidentally, to represent the number of twenty-first magnitude stars as extrapolated by the Mount Wilson investigators. Assuming the luminosity curve to continue to increase as rapidly as it does at the last observed point, the representation was found to fall short of the extrapolated values.

The mean color index of both galactic and non-galactic stars is known to increase with apparent magnitude. This increase is usually represented as a

²⁹ B. A. N. 1, 54, 1922.

³² *München Ber.* 87-144, 1920.

³³ "The Distances and Distribution in Space of Stars of Spectral Type B," *Lunds Medd.* IV, 4, no. 7, 1916. Since the writing of this paper an improved study of the B-stars by Charlier has appeared. See *Lunds Medd.* no. 34, 1926.

³⁴ *Harv. Circ.* no. 229, 1922.

³⁵ *Mt. Wilson Contr.* no. 157, 1918.

³⁶ *Mon. Not. R. A. S.* 79, 500, 1919.

³⁷ *Mt. Wilson Contr.* no. 301, 1925.

linear equation between C and m . The spectra of the stars enable us to derive such an equation under the assumption that the color index is directly given by the spectral type. Such a method is considerably improved if, moreover, our classification distinguishes between giant and dwarf stars. For the stars discussed in this paper, we have derived the mean color index of each magnitude from 5.01 to 14.00, the grouping remaining on the same system of photographic magnitudes as we have heretofore used. The following table gives the mean color index on the Mount Wilson system, as determined from the spectral type. The values of Seares¹⁸ for each spectral type are used. For magnitudes 5.01–11.00 the constant density law for dwarfs was assumed. The weights are proportional to the number of stars used.

Med. m .	C_w	Wt.
5.5	0.277	1
6.5	0.131	4
7.5	0.300	9
8.5	0.269	10
9.5	0.334	31
10.5	0.498	2
11.5	0.476	4
12.5	0.525	9
13.5	0.565	3

A least square solution gave:

$$C_w = -0.16 + 0.054 m$$

Seares³⁸ gives the linear relation according to two groupings—visual and photographic. As, however, our photographic system is between these two Mount Wilson systems the equation with which to compare our results was interpolated from the two equations of Seares, giving:

$$C_w = +0.04 + 0.057 m \quad (\text{all gal. lat.})$$

$$C_w = +0.06 + 0.040 m \quad (\text{gal. lat. } 5^\circ)$$

It is thus seen that although there is some uncertainty in the zero point, the rate of change with m agrees very well. Shajn³⁰ gives 0.05 as the coefficient of m , both for the Milky Way and for high galactic latitudes. From the occurrence of the phenomenon among faint stars, the latter concludes that the proportion of late type giants is small in remote as well as in the nearer regions of space.

SUMMARY OF RESULTS

(1) From a classification of spectra of stars between magnitudes 10 and 14 in the *Cygnus* region of the Milky Way, together with data for the brighter

stars from the *Henry Draper Catalogue and Extension*, the density curves of the several spectral types extending to a maximum distance of 5000 parsecs, are derived for this part of space. All show a dropping off in density beginning at a distance of about 200 to 600 parsecs.

(2) The predominance of late type dwarfs over giants of the same spectral class seems to be greater than usually adopted for the nearer regions of space.

(3) The mixture of giant stars appears to vary with distance, the A-type stars showing an increase, while a corresponding relative diminution is shown by the gK-type stars.

(4) A provisional luminosity curve, derived for stars of all types between 125 and 200 parsecs distant, shows, in addition to a systematically higher density, an increase of frequency with absolute magnitude which for the intrinsically faint stars is more rapid than that given by other investigators.

(5) Although such limited observations are incompetent to make possible a decision on the problem of the structure of the galactic system, it is shown that the number of faint stars could be accounted for without exceeding dimensions of the order of 10,000 parsecs, provided that the frequency of dwarfs given by our luminosity curve is accepted.

(6) The mean color index of stars in this region increases with the apparent magnitude, the coefficient of increase being about 0.05 and therefore agreeing with results already adopted for the Milky Way as a whole.

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¹⁸ *Mt. Wilson Contr.* no. 226, Table XII, 1921.

³⁰ *Mon. Not. R. A. S.* **86**, 382, 1926.

³⁸ *Mt. Wilson Contr.* no. 287, 1925. Recently Kreiken (*Mon. Not. R. A. S.* **87**, 196, 1927) has found a somewhat larger coefficient in a few special regions near the Galaxy.