ELEMENTS OF (532) HERCULINA.

By J. C. HAMMOND.

This asteroid was discovered by WOLF at Königstuhl on April 20, 1904, and was unusually bright as compared with others discovered recently. Provisional elements were computed by the method given by TISSERAND in his *Leçons sur la Détermination des Orbites*. Two observations made by Doctor LUTHER at Düsseldorf on April 23 and May 7, and one by HAMMOND at Washington on May 23, were used.

These observations are as follows:

Date.	Place.	Local Mean Time.	Apparent α .	Apparent δ .	Log $p\Delta$.		Red. to 1904.0	
					α	δ	α	δ
1904		h m s	h m s	° ' "			s	"
Apr. 23	Düsseldorf....	11 18 43	13 50 27.86	+18 25 10.8	8.636 _n	0.676	-0.58	+0.5
May 7	Düsseldorf....	10 35 11	13 40 5.05	+17 53 50.9	7.789 _n	0.681	-0.67	+1.5
23	Washington ..	9 56 31	13 32 45.66	+16 1 28.9	8.777	0.532	-0.82	+2.5

Provisional Elements.

Epoch: 1904, May 8.0, Berlin Mean Time

$$\begin{aligned}
 M &= 18 \ 36 \ 30.6 \\
 \pi &= 182 \ 28 \ 38.4 \\
 \Omega &= 108 \ 10 \ 58.9 \\
 i &= 16 \ 22 \ 37.9 \\
 \varphi &= 10 \ 5 \ 24.3 \\
 \mu &= 767''.950 \\
 \log a &= 0.443116
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{Mean Equinox and Ecliptic 1904.0}$$

π is the longitude of perihelion counted from the equinox.

Residuals.	Apr. 23.	May 7.	May 23.
	"	"	"
O-C { α	+0.4	+0.4	-0.1
δ	+0.5	-1.5	-0.2

Heliocentric Coordinates 1904.0

$$\begin{aligned}
 x &= r [9.983829] \sin (273 \ 11 \ 33.8 + v) \\
 y &= r [9.979361] \sin (188 \ 13 \ 46.4 + v) \\
 z &= r [9.605330] \sin (144 \ 2 \ 26.1 + v)
 \end{aligned}$$

Ephemeris for 1904.

Berlin Mean Midnight.	Apparent α .	Apparent δ .	Log J .	Aberr. Time.
1904	h m s	° ' "		m s
Apr. 20	13 52 56.36	+18 23 4.4	0.141756	11 31
28	13 46 26.64	18 21 44.2	0.149818	11 44
May 6	13 40 41.73	17 58 8.3	0.161620	12 3
14	13 36 8.03	17 13 55.6	0.176565	12 28
22	13 33 4.16	16 11 46.7	0.193973	12 59
30	13 31 38.94	14 55 4.2	0.213141	13 34
δ				
June 7	13 31 52.27	13 27 6.2	0.233469	14 13
15	13 33 40.77	11 50 41.0	0.254476	14 56
23	13 36 58.35	10 8 17.7	0.275728	15 40
July 1	13 41 36.03	8 22 5.2	0.296884	16 27
9	13 47 25.29	6 33 37.6	0.317735	17 16
17	13 54 18.93	4 44 11.3	0.338099	18 6
25	14 2 9.63	2 54 52.9	0.357812	18 56
Aug. 2	14 10 50.58	+1 6 34.9	0.376798	19 47

The corrections to this ephemeris, as determined by observations made at various observatories, are given in the following table. A few observations have been omitted when, on account of poor comparison stars or for other reasons, the corrections seemed doubtful:

Corrections to the Ephemeris.

Date.	Place.	O-C		Date.	Place.	O-C	
		α	δ			α	δ
1904		s	"	1904		s	"
Apr. 22	Algiers.....	+0.01	+1.0	May 3	Kasan.....	-0.06	-1.5
22	Algiers.....	-0.07	+1.1	3	Marseille.....	-0.17	-1.4
23	Düsseldorf.....	+0.03	+0.7	3	Rome.....	-0.03	-1.7
23	Kasan.....	-0.01	+1.1	4	Kasan.....	-0.09	-1.6
24	Rome.....	+0.06	+1.0	4	Rome.....	-0.13	-0.2
24	Kasan.....	-0.19	+0.4	5	Marseille.....	-0.17	-2.5
25	Kasan.....	-0.19	+0.2	6	Kasan.....	-0.14	-2.9
25	Düsseldorf.....	-0.14	-0.4	6	Düsseldorf.....	+0.06	-0.9
26	Kasan.....	-0.25	+0.1	7	Düsseldorf.....	+0.03	-0.9
27	Kasan.....	-0.26	0.0	7	Marseille.....	-0.22	-2.9
28	Kasan.....	-0.12	-0.7	8	Kasan.....	-0.37	-2.1
28	Kasan.....	-0.25	-0.8	9	Kasan.....	-0.14	-2.0
May 2	Kasan.....	-0.14	+0.1	9	Marseille.....	-0.04	-2.8
2	Kasan.....	+0.01	-0.9	10	Kasan.....	-0.16	-3.5
2	Marseille.....	-0.18	-0.8	10	Rome.....	-0.05	-1.5
2	Vienna.....	-0.02	-2.0	10	Marseille.....	-0.14	-0.2
3	Kasan.....	-0.28	-0.3	11	Marseille.....	-0.08	+0.6

ORBITS OF ASTEROIDS.

B 81

Corrections to the Ephemeris—Continued.

Date.	Place.	O-C		Date.	Place.	O-C	
		α	δ			α	δ
1904		s	"	1904		s	"
May 16	Rome.....	-0.35	- 2.3	June 27	Rome.....	-0.60	+33.3
16	Düsseldorf.....	-0.30	- 1.5	30	Düsseldorf.....	-0.87	+32.4
19	Düsseldorf.....	+0.27	- 1.7	July 1	Arcetri.....	-0.71	+35.8
19	Washington.....	+0.16	- 2.2	1	Arcetri.....	-0.73	+37.9
20	Marseille.....	+0.04	- 2.1	1	Arcetri.....	-0.69	+35.2
20	Washington.....	0.00	- 1.8	1	Arcetri.....	-0.71	+36.1
21	Rome.....	+0.08	- 1.5	3	Arcetri.....	-0.96	+37.8
22	Washington.....	+0.15	- 1.2	4	Arcetri.....	-0.95	+38.7
23	Washington.....	+0.01	+ 0.6	4	Düsseldorf.....	-0.96	+42.4
25	Rome.....	-0.05	+ 0.5	6	Arcetri.....	-1.27	+41.7
25	Washington.....	0.00	+ 0.6	6	Rome.....	-1.24	+42.5
27	Washington.....	+0.08	+ 2.2	7	Vienna.....	-1.22	+44.4
28	Washington.....	+0.05	+ 2.9	7	Arcetri.....	-1.25	+43.3
29	Washington.....	+0.27	+ 4.9	7	Arcetri.....	-1.24	+44.3
June 3	Kasan.....	-0.12	+ 6.2	8	Arcetri.....	-1.06	+45.3
3	Düsseldorf.....	-0.09	+ 6.4	8	Arcetri.....	-1.26	+44.8
3	Washington.....	0.00	+ 5.0	9	Arcetri.....	-1.29	+46.1
4	Kasan.....	-0.04	+ 5.5	9	Arcetri.....	-1.23	+47.9
4	Jena.....	-0.15	+ 5.7	10	Arcetri.....	-1.18	+47.9
4	Düsseldorf.....	+0.07	+ 3.9	10	Arcetri.....	-1.31	+51.3
5	Düsseldorf.....	-0.05	+ 7.0	14	Rome.....	-1.85	+56.7
6	Rome.....	-0.04	+ 8.1	16	Arcetri.....	-2.09	+57.7
7	Kasan.....	+0.03	+ 8.4	17	Arcetri.....	-2.03	+59.2
7	Düsseldorf.....	+0.03	+ 8.1	17	Arcetri.....	-2.18	+60.6
8	Washington.....	+0.05	+ 9.6	19	Arcetri.....	-1.96	+67.0
9	Kasan.....	-0.02	+15.3	20	Arcetri.....	-2.11	+67.1
10	Kasan.....	-0.12	+16.3	21	Arcetri.....	-2.11	+67.8
11	Washington.....	-0.05	+10.5	22	Rome.....	-2.52	+68.3
12	Düsseldorf.....	-0.02	+12.0	23	Arcetri.....	-2.68	+72.6
12	Washington.....	-0.04	+14.5	24	Rome.....	-2.97	+70.6
13	Kasan.....	-0.36	+15.1	24	Arcetri.....	-2.76	+68.1
15	Rome.....	+0.20	+13.2	26	Arcetri.....	-3.09	+77.2
15	Rome.....	+0.03	+15.6	27	Arcetri.....	-3.02	+76.5
15	Kasan.....	-0.14	+17.0	28	Arcetri.....	-3.24	+77.9
15	Washington.....	-0.18	+15.6	29	Arcetri.....	-3.35	+78.4
18	Rome.....	-0.10	+18.7	30	Arcetri.....	-3.38	+79.0
18	Washington.....	-0.05	+18.7	30	Arcetri.....	-3.13	+78.1
20	Düsseldorf.....	-0.11	+23.5	Aug. 1	Arcetri.....	-3.66	+84.1
21	Rome.....	-0.15	+23.4	1	Arcetri.....	-3.89	+82.8
23	Rome.....	-0.53	+26.3	2	Arcetri.....	-3.76	+82.7
23	Rome.....	-0.53	+26.9				

In forming the normal places, the observations are given equal weight. For the first normal place, the 12 observations from April 22 to April 28, inclusive, are used; for the second normal place, the 22 observations from May 2 to May 11, inclusive, are used; for the third normal place, the 14 observations from May 16 to May 29, inclusive, are used; for the fourth normal place, the 17 observations from June 3 to June 13, inclusive, are used; for the fifth normal place, the 12 observations from June 15 to June 30, inclusive, are used; for the sixth normal place, the 22 observations from July 1 to July 17, inclusive, are used, and for the seventh normal place, the 16 observations from July 19 to August 2, inclusive, are used. The normal places, with the corresponding values of $\log \Delta$ derived from the provisional elements, are as follows:

Normal Places.	Berlin Mean Time.	True Equinox of Date.		Log Δ .
		α	δ	
		h m s	° ' "	
I	Apr. 25.5	13 48 49.04	+18 24 53.7	0.146326
II	May 6.5	13 40 41.62	+17 58 6.7	0.161620
III	22.5	13 33 4.19	+16 11 46.4	0.193973
IV	June 7.5	13 31 52.22	+13 27 15.4	0.233469
V	20.5	13 35 34.08	+10 47 36.7	0.267755
VI	July 7.5	13 45 50.49	+7 1 37.4	0.312563
VII	26.5	14 3 9.16	+2 42 31.2	0.360227

The provisional elements are corrected by the method of variation of geocentric distances so as to satisfy in the best possible manner these seven normal places. The first and last normal places are adopted as the base positions. With the coordinates α , δ , α'' , and δ'' of these places, and with the assumed geocentric distances, $\log \Delta = 0.146326$ and $\log \Delta'' = 0.360227$, a set of elements of an orbit is computed. This constitutes Hypothesis I. For Hypothesis II, $\log \Delta$ is decreased by 500 in the sixth place, while the value of $\log \Delta''$ is left unchanged. For Hypothesis III, $\log \Delta''$ is decreased by 500 in the sixth place and the original value of $\log \Delta$ is used. Each of the three resulting orbits of course passes through the two base positions.

The following formulæ for computing the elements are taken directly from BOCCARDI'S, Guide du Calculateur, Deuxième Partie. The equator is the fundamental plane.

$$\begin{aligned}
 & \left\{ \begin{aligned} \xi &= \Delta \cos \alpha \cos \delta = x + X \\ \eta &= \Delta \sin \alpha \cos \delta = y + Y \\ \zeta &= \Delta \sin \delta = z + Z \end{aligned} \right. \\
 & \left\{ \begin{aligned} r \cos l \cos b &= x \\ r \sin l \cos b &= y \\ r \sin b &= z \end{aligned} \right. \\
 & \left\{ \begin{aligned} \tan i \sin (l - \Omega) &= \tan b \\ \tan i \cos (l - \Omega) &= \frac{\tan b'' - \tan b \cos (l'' - l)}{\sin (l'' - l)} \\ \tan u &= \frac{\sin (l - \Omega) \cos i + \tan b \sin i}{\cos (l - \Omega)} \\ \tan u'' &= \frac{\sin (l'' - \Omega) \cos i + \tan b'' \sin i}{\cos (l'' - \Omega)} \end{aligned} \right.
 \end{aligned}$$

$$\begin{array}{l}
 \text{Checks.} \\
 3) \left\{ \begin{array}{l} \cos 2f = \sin b \sin b'' + \cos b \cos b'' \cos (l'' - l) \\ u'' - u = 2f \\ \sin^2 \frac{1}{2}(u'' - u) = \sin^2 f = \sin^2 \frac{1}{2}(l'' - l) \cos b \cos b'' \\ \quad + \sin^2 \frac{1}{2}(b'' - b) \end{array} \right. \\
 4) \left\{ \begin{array}{l} \tau = k(l'' - l) \text{ where } k = 8.2355814 \\ m = \frac{\tau^2}{(2 \cos f \sqrt{rr''})^3} \\ \tan (45^\circ + \omega) = \sqrt[4]{\frac{r''}{r}} \\ l = \frac{\sin^2 \frac{1}{2}f + \tan^2 2\omega}{\cos f} \end{array} \right. \\
 5) \left\{ \begin{array}{l} \alpha = \frac{2g - \sin 2g}{\sin^2 g} \\ \beta = l + \sin^2 \frac{1}{2}g \\ \alpha\beta + 1 = \eta \\ m = \eta^2 \beta \\ \tau = \eta \sqrt{\beta(2 \cos f \sqrt{rr''})^3} \end{array} \right. \\
 6) \left\{ \begin{array}{l} r^2 \sin \frac{1}{2}(F - G) \cos \frac{1}{2}\varphi = \cos \frac{1}{2}(f + g) \tan 2\omega \\ r^2 \cos \frac{1}{2}(F - G) \cos \frac{1}{2}\varphi = \sin \frac{1}{2}(f + g) \sec 2\omega \\ r^2 \sin \frac{1}{2}(F + G) \sin \frac{1}{2}\varphi = \cos \frac{1}{2}(f - g) \tan 2\omega \\ r^2 \cos \frac{1}{2}(F + G) \sin \frac{1}{2}\varphi = \sin \frac{1}{2}(f - g) \sec 2\omega \\ r^2 = \frac{\sqrt{2m \cos f}}{\eta} \} \text{Check.} \\
 7) \left\{ \begin{array}{l} v = F - f \\ v'' = F + f \\ E = G - g \\ E'' = G + g \\ \pi = u + \Omega - v = u'' + \Omega - v'' \\ \tan \frac{1}{2}v = \tan \frac{1}{2}E \tan (45^\circ + \frac{1}{2}\varphi) \\ \tan \frac{1}{2}v'' = \tan \frac{1}{2}E'' \tan (45^\circ + \frac{1}{2}\varphi) \} \text{Checks.} \\
 8) \left\{ \begin{array}{l} M = E - e'' \sin E \\ M'' = E'' - e'' \sin E'' \\ \mu = \frac{M'' - M}{l'' - l} \\ k'' = \mu a^{3/2} \\ p = \left(\frac{\eta r r'' \sin 2f}{\tau} \right)^2 = a \cos^2 \varphi \} \text{Check.} \end{array} \right.
 \end{array}$$

Instead of determining g by trial, as it is necessary to do if one makes use of equations (5) of the foregoing formulæ, this quantity may readily be obtained by means of TABLES VIII and IX of OPPOLZER'S, *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, Erster Band. With the argument $h = \frac{m}{5/6 + l}$, enter TABLE VIII and take from it $\log \eta^2$. With the argument $w = \frac{m}{\eta^2} - l$, enter TABLE IX and take from it ξ . Finally, with the argument $h = \frac{m}{5/6 + l + \xi}$, again enter TABLE VIII and take from it the final value of $\log \eta^2$.

Then

$$\sin^2 \frac{1}{2}g = w = \frac{m}{\eta^2} - l.$$

Base Positions and Values of Log Δ and Log Δ'' for the three Hypotheses.

Berlin Mean Time.	Base Positions.		Log Δ .		
	α (1904.0)	δ (1904.0)	Hyp. I.	Hyp. II.	Hyp. III.
1904	h m s	° ' "			
Apr. 25.5	13 48 48.45	+18 24 54.3	0.146326	0.145826	0.146326
July 26.5	14 3 7.57	+2 42 35.6	0.360227	0.360227	0.359727

Computation of the Elements.

		Hyp. I.	Hyp. II.	Hyp. III.	Corrected Elements. (1st approxima- tion).	Final Elements.
1)	x	0.301783 _n	0.301489 _n	0.301783 _n	0.301863 _n	0.301904 _n
	x''	0.148270 _n	0.148270 _n	0.147571 _n	0.149336 _n	0.149400 _n
	y	0.057289 _n	0.057023 _n	0.057289 _n	0.057361 _n	0.057398 _n
	y''	0.289763 _n	0.289763 _n	0.289462 _n	0.290222 _n	0.290250 _n
	z	9.324309	9.323260	9.324309	9.324593	9.324739
	z''	9.359355 _n	9.359355 _n	9.359591 _n	9.358992 _n	9.358971 _n
		0 / "	0 / "	0 / "	0 / "	0 / "
	l	209 39 43.9	209 39 49.6	209 39 43.9	209 39 42.2	209 39 41.4
	l''	234 10 21.4	234 10 21.4	234 11 51.1	234 8 4.5	234 7 56.4
	b	+5 13 45.3	+5 13 12.5	+5 13 45.3	+5 13 54.2	+5 13 58.7
	b''	-5 26 11.1	-5 26 11.1	-5 26 41.3	-5 25 25.0	-5 25 22.3
	r	0.364595	0.364302	0.364595	0.364675	0.364716
	r''	0.382816	0.382816	0.382384	0.383474	0.383513
		0 / "	0 / "	0 / "	0 / "	0 / "
2)	i	23 44 16.9	23 43 16.4	23 44 1.1	23 44 57.2	23 45 7.2
	Ω	41 40 27.3	41 39 51.0	41 40 36.4	41 40 23.2	41 40 27.3
	u	166 54 52.0	166 55 43.2	166 54 43.6	166 54 50.6	166 54 44.3
	u''	193 36 44.1	193 37 17.4	193 38 9.7	193 34 24.4	193 34 12.3
3)	$2f$	26° 41' 52''.1	26° 41' 34''.2	26° 43' 26''.1	26° 39' 33''.8	26° 39' 28''.0
	$\cos 2f$	9.951041	9.951059	9.950941	9.951187	9.951193
	$\left\{ \begin{array}{l} \sin b \sin b'' + \\ \cos b \cos b'' \cos(l'' - l) \end{array} \right.$	9.951041	9.951059	9.950941	9.951187	9.951194
	$\sin^2 f$	8.726774	8.726614	8.727608	8.725544	8.725494
	$\left\{ \begin{array}{l} \sin^2 \frac{1}{2}(l'' - l) \cos b \cos b'' \\ + \sin^2 \frac{1}{2}(b'' - b) \end{array} \right.$	8.726775	8.726614	8.727608	8.725545	8.725493
4)	τ	0.199369	0.199369	0.199369	0.199369	0.199369
	m	8.410215	8.410644	8.410935	8.409005	8.408883
	ω	0° 18' 1''.7	0° 18' 19''.1	0° 17' 35''.9	0° 18' 36''.2	0° 18' 36''.0
	l	8.146038	8.145989	8.146733	8.144997	8.144943
5)	h (number)	0.030350	0.030380	0.030400	0.030267	0.030258
	(Table VIII Opp.) η^2	0.027827	0.027853	0.027870	0.027755	0.027747
	(Table IX Opp.) ξ (number)	0.0000059	0.0000059	0.0000059	0.0000058	0.0000058
	w	8.005330	8.006358	8.005984	8.004060	8.003862
	g	11° 32' 57''.2	11° 33' 46''.6	11° 33' 28''.6	11° 31' 56''.3	11° 31' 46''.8
		0 / "	0 / "	0 / "	0 / "	0 / "
6)	F	36 27 13.9	37 9 41.0	35 41 21.7	37 24 51.8	37 22 50.9
	G	31 0 49.6	31 38 11.4	30 23 0.1	31 47 33.6	31 45 34.3
	φ	10 0 5.0	9 59 4.8	9 56 15.6	10 6 19.1	10 6 46.4
	r^2	9.335760	9.335964	9.336090	9.335210	9.335154
	$\sqrt{2m \cos f + \eta}$	9.335761	9.335966	9.336088	9.335210	9.335154

Computation of the Elements—Continued.

		Hyp. I.	Hyp. II.	Hyp. III.	Corrected Elements. (1st approxima- tion).	Final Elements.
		° ' "	° ' "	° ' "	° ' "	° ' "
7)	v	23 6 17.9	23 48 53.9	22 19 38.7	24 5 4.9	24 3 6.9
	v''	49 48 9.9	50 30 28.1	49 3 4.7	50 44 38.7	50 42 34.9
	E	19 27 52.4	20 4 24.8	18 49 31.5	20 15 37.3	20 13 47.5
	E''	42 33 46.8	43 11 58.0	41 56 28.7	43 19 29.9	43 17 21.1
	π	185 29 1.4	184 46 40.3	186 15 41.4	184 30 8.9	184 32 4.7
	$\tan \frac{1}{2} v$	9.310495	9.324014	9.295231	9.329050	9.328441
	$\tan \frac{1}{2} v''$	9.666718	9.673679	9.659219	9.675995	9.675659
	$\tan \frac{1}{2} E \tan (45^\circ + \frac{1}{2} \varphi)$	9.310494	9.324014	9.295231	9.329051	9.328441
	$\tan \frac{1}{2} E'' \tan (45^\circ + \frac{1}{2} \varphi)$	9.666717	9.673678	9.659219	9.675995	9.675659
		° ' "	° ' "	° ' "	° ' "	° ' "
8)	M	16 8 55.5	16 39 50.0	15 38 5.1	16 46 44.9	16 45 3.9
	M''	35 49 56.5	36 23 56.7	35 19 57.1	36 25 38.1	36 23 27.3
	μ	770'' .228	772'' .247	770'' .783	768'' .839	768'' .515
	a	0.442259	0.441501	0.442050	0.442781	0.442903
	$(\eta r r'' \sin 2j \div \tau)^2$	0.428956	0.428246	0.428920	0.429200	0.429302
	$a \cos^2 \varphi$	0.428958	0.428245	0.428920	0.429201	0.429302

The positions of the asteroid computed with the three sets of elements deduced from the three hypotheses and the normal places reduced to 1904.0 are as follows:

	Apr. 25.5	May 6.5	May 22.5	June 7.5	June 20.5	July 7.5	July 26.5
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
α_I	207 12 6.7	205 10 5.8	203 15 29.7	202 57 19.2	203 52 43.0	206 26 54.0	210 46 53.8
α_{II}	207 12 6.9	205 9 58.5	203 15 21.0	202 57 14.2	203 52 41.1	206 26 54.9	210 46 54.2
α_{III}	207 12 6.4	205 10 2.0	203 15 16.9	202 57 1.5	203 52 23.0	206 26 38.9	210 46 52.5
Normals= α_N	207 12 6.7	205 10 14.4	203 15 50.7	202 57 48.6	203 53 14.1	206 27 17.0	210 46 53.6
δ_I	+18 24 54.0	+17 58 23.1	+16 12 20.5	+13 27 55.6	+10 48 14.3	+7 2 3.5	+2 42 35.7
δ_{II}	+18 24 53.9	+17 58 21.0	+16 12 12.9	+13 27 44.8	+10 48 3.7	+7 1 56.3	+2 42 35.4
δ_{III}	+18 24 53.8	+17 58 34.9	+16 12 43.1	+13 28 21.2	+10 48 38.2	+7 2 18.7	+2 42 35.8
Normals= δ_N	+18 24 54.3	+17 58 8.1	+16 11 48.8	+13 27 18.7	+10 47 40.5	+7 1 41.6	+2 42 35.6

The equations of condition for determining the corrections to $\log \mathcal{A}$ and $\log \mathcal{A}''$ are of the form

$$(\alpha_{II} - \alpha_I) \cos \delta \frac{d(\log \mathcal{A})}{(-500)} + (\alpha_{III} - \alpha_I) \cos \delta \frac{d(\log \mathcal{A}'')}{(-500)} = (\alpha_N - \alpha_I) \cos \delta,$$

$$(\delta_{II} - \delta_I) \frac{d(\log \mathcal{A})}{(-500)} + (\delta_{III} - \delta_I) \frac{d(\log \mathcal{A}'')}{(-500)} = \delta_N - \delta_I.$$

The factor $\cos \delta$ is introduced in the first of the preceding equations to make it equal in weight to the second equation. Each normal place, with the exception of the two base positions, gives two equations for determining $d(\log \mathcal{A})$ and $d(\log \mathcal{A}'')$, and therefore, if we have more than three normal places altogether, we must resort to the method of *Least Squares*.

Equations of Condition.

$$\begin{array}{rcl}
 + 69 \cdot \frac{d(\log \mathcal{A})}{500} + 36 \cdot \frac{d(\log \mathcal{A}'')}{500} & = & + 82 \\
 + 84 & + 123 & = + 202 \\
 + 49 & + 172 & = + 286 \\
 + 19 & + 196 & = + 305 \\
 - 09 & + 150 & = + 228 \\
 + 21 & - 118 & = - 150 \\
 + 76 & - 226 & = - 317 \\
 + 108 & - 256 & = - 369 \\
 + 106 & - 239 & = - 338 \\
 + 72 & - 152 & = - 219
 \end{array}$$

Solving these, we obtain $d(\log \mathcal{A}) = +135$ and $d(\log \mathcal{A}'') = +763$, and the first corrected distances are $\log \mathcal{A} = 0.146461$ and $\log \mathcal{A}'' = 0.360990$. With these values of the geocentric distances, the set of elements in the column headed *Corrected Elements* (1st approximation) are deduced. A comparison of the positions of the asteroid, computed from the latter elements, with the five intermediate normal places gives the following residuals:

Residuals.

Date.	May 6.5	May 22.5	June 7.5	June 20.5	July 7.5
O-C { α	"	"	"	"	"
δ	+2.4	+2.4	+1.5	+2.3	+1.9
	+0.2	-0.7	-0.6	-1.2	-1.2

The first set of corrected elements is not sufficiently accurate, as shown by the residuals, and it is necessary to make another approximation. Strictly speaking, one ought to start again with this set as a first hypothesis and recompute new coefficients for $d(\log \mathcal{A})$ and $d(\log \mathcal{A}'')$. However, all the accuracy desired can be obtained by simply substituting these new residuals in place of the old ones in the former equations of condition, remembering to multiply the residuals in α by the corresponding $\cos \delta$. When this is done, we obtain, as the new corrections, $d(\log \mathcal{A}) = +70$ and $d(\log \mathcal{A}'') = +46$. The final corrected distances are then $\log \mathcal{A} = 0.146531$ and $\log \mathcal{A}'' = 0.361036$, and with these values the final set of corrected elements are computed. A comparison of the positions computed from this last set of elements, with the intermediate normal places, gives as residuals,

Residuals.

Date.	May 6.5	May 22.5	June 7.5	June 20.5	July 7.5
O-C { α	"	"	"	"	"
δ	+0.7	+0.5	-1.2	+0.2	+1.1
	+0.5	-0.2	+0.3	-0.6	-0.8

This seems as good a representation of the normal places as could be expected. The final elements referred to the ecliptic are

Epoch: 1904, June 7.5, Berlin Mean Time.

° ' "	° ' "
$M = 25 \ 55 \ 50.0$	
$\pi = 181 \ 18 \ 53.2$	$\pi = 181 \ 23 \ 55.1$
$\Omega = 108 \ 14 \ 32.0$	$\Omega = 108 \ 19 \ 24.6$
$i = 16 \ 22 \ 41.6$	$i = 16 \ 22 \ 40.4$
$\varphi = 10 \ 6 \ 46.4$	
$\mu = 768''.515$	
$\log a = 0.442903$	

Mean Ecliptic 1904.0 Mean Ecliptic 1910.0

If the magnitude be represented by $M = g + 5 (\log \Delta + \log r)$, we obtain from a discussion of the estimated magnitudes $g = 6^m.3$.

The perturbations of Jupiter and Saturn during the interval from June 7, 1904, to August 6, 1905, computed by the method of variation of parameters, produce the following changes in the elements:

' "
$dM = +8 \ 48.4$
$d\pi = -7 \ 37.1$
$d\Omega = -0 \ 14.4$
$di = +0 \ 10.8$
$d\varphi = +0 \ 2.9$
$d\mu = +0''.450$

The elements, with these perturbations included, are

Final Elements.

Epoch: 1905, August 6.5, Berlin Mean Time.

° ' "	° ' "
$M = 116 \ 48 \ 17.5$	
$\pi = 181 \ 16 \ 18.0$	
$\Omega = 108 \ 19 \ 10.2$	$\Omega = 108 \ 19 \ 10.2$
$i = 16 \ 22 \ 51.2$	$i = 16 \ 22 \ 51.2$
$\varphi = 10 \ 6 \ 49.3$	
$\mu = 768''.965$	
$\log a = 0.442734$	

Mean Ecliptic 1910.0

Heliocentric Coordinates 1905.0

° ' "
$x = r [9.983836] \sin (271 \ 55 \ 11.3 + v)$
$y = r [9.979408] \sin (186 \ 56 \ 59.4 + v)$
$z = r [9.605039] \sin (142 \ 44 \ 18.6 + v)$

With these elements the following ephemeris for 1905 was computed:

Ephemeris for 1905, including perturbations by Jupiter and Saturn.

Berlin Mean Midnight.		Apparent α .	Apparent δ .	Log J .	Aberr. Time.
1905		h m s	° ' "		m s
July	1	21 39 2.48	-22 1 10.8	0.333946	17 56
	5	21 37 31.90	22 32 51.3	0.328315	17 41
	9	21 35 39.87	23 5 51.7	0.323327	17 29
	13	21 33 27.64	23 39 52.0	0.319050	17 19
	17	21 30 56.66	24 14 29.9	0.315548	17 11
	21	21 28 8.87	24 49 21.1	0.312886	17 5
	25	21 25 6.32	25 24 0.0	0.311110	17 1
	29	21 21 51.82	25 58 0.6	0.310261	16 58
Aug.	2	21 18 28.54	26 30 54.5	0.310378	16 58
	6	21 14 59.98	27 2 18.1	0.311463	17 1
	10	21 11 29.69	27 31 49.1	0.313504	17 6
	14	21 8 1.49	27 59 7.9	0.316484	17 13
	18	21 4 38.71	28 24 0.9	0.320362	17 22
	22	21 1 24.73	28 46 16.8	0.325091	17 33
	26	20 58 22.55	29 5 47.8	0.330613	17 47
	30	20 55 35.26	29 22 28.9	0.336866	18 3
Sept.	3	20 53 5.57	29 36 21.8	0.343777	18 20
	7	20 50 55.60	-29 47 27.5	0.351260	18 39

The corrections to this ephemeris, as determined by observations made at Rome, Arcetri, and Washington, are given in the following table:

Corrections to the Ephemeris.

Date.	Place.	O-C		Date.	Place.	O-C	
		α	δ			α	δ
1905		s	"	1905		s	"
July	1 Rome.....	-0.58	-12.3	Aug.	4 Arcetri.....	-0.38	-14.8
	7 Washington.....	-0.66	-15.0		6 Washington.....	-0.74	-15.0
	8 Washington.....	-0.53	-14.7		8 Arcetri.....	-0.48	-15.0
	10 Rome.....	-0.37	-14.4		9 Arcetri.....	-0.32	-13.2
	11 Arcetri.....	-0.59	-11.7		10 Arcetri.....	-0.33	-14.0
	12 Arcetri*.....	-0.69	-14.4		12 Arcetri.....	-0.44	-10.9
	15 Washington.....	-0.67	-14.4		15 Washington.....	-0.63	-13.8
	20 Washington.....	-0.62	-13.0		17 Washington.....	-0.56	-12.5
	26 Arcetri.....	-0.41	-16.0		18 Washington.....	-0.49	-14.5
	26 Washington.....	-0.44	-13.7		22 Arcetri.....	-0.51	-13.7
	27 Arcetri.....	-0.32	-14.8		22 Washington*.....	-0.65	-11.2
	30 Arcetri.....	-0.56	-13.8		25 Arcetri.....	-0.63	-11.7
	30 Washington*.....	-0.59	-15.1		30 Washington.....	-0.43	-11.9
Aug.	1 Arcetri.....	-0.49	-13.8	Sept.	31 Washington.....	-0.47	-11.7
	3 Arcetri.....	-0.57	-17.0		6 Washington.....	-0.61	-13.5
	3 Washington.....	-0.65	-14.5				

* Two observations.

If we assign equal weights to these observations, we shall obtain as a normal place for 1905 [mean (O-C) = $-0^s.540$; $-13''.72$],

Berlin Mean Time.	True Equinox of Date.		
	α	δ	Log d .
1905 Aug. 4.5	h m s 21 16 44.16	$^{\circ}$ ' " -26 47 2.7	0.310800

We can combine the seven normal places for 1904 into one and have as our final normal places for 1904 and 1905 referred to the mean equator and equinox of 1910.0,

Epoch.	Mean Equator and Equinox, 1910.0		
	α	δ	Log d .
1904 June 7.5	h m s 13 32 9.02	$^{\circ}$ ' " +13 25 27.5	0.233469
1905 Aug. 4.5	21 17 0.37	-26 45 58.9	0.310800