ORBIT OF ENCELADUS.

BY

H. R. MORGAN.

FROM

OBSERVATIONS

MADE BY

T. J. J. SEE IN 1901.

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ORBIT OF ENCELADUS.

The observations used in computing this orbit were made by Prof. T. J. J. SEE, U. S. Navy, with the 26-inch equatorial of the United States Naval Observatory, in 1901. Since the observation of August 17 was incomplete, and that of June 18 gave a residual of 4'', they have been discarded. On nine nights of "bad" seeing the observations were given less weight than the other measures in the discussion.

The mass of Saturn was considered as known, and the distances of the satellites were derived therefrom. The peri-saturnium of Enceladus as derived from theory is $\Pi = 2 l_{Di} - l_{En}$, and no correction to this value has been introduced into the solution that follows.

The formulæ used are given by HERMANN STRUVE (Publications de Poulkova, Série II, Vol. XI) as are also the assumed elements, with the exception of the mean longitudes. The longitudes are reckoned on Saturn's equator as the fundamental plane, from its node, N_1 , on the Earth's equator, to the node of the satellite's orbit, θ , and thence along the orbit.

u = true longitude in orbit.

ϵ = mean longitude in orbit.

e, π = eccentricity, and longitude of peri-saturnium.

γ, θ = inclination, and longitude of node.

n = mean daily motion.

a = semi-major axis of satellite's orbit, at the mean distance ρ of Saturn from the Earth.

J_1, N_1 = inclination, and longitude of node of Saturn's equator on Earth's equator, reckoned from the equinox.

Assumed Elements.

Epoch: 1901.516 = July 8.0, Greenwich Mean Time.

Saturn's equator $\{ N_1 = 126^\circ 26'.6$

Mass = $\mu = 3495.3 \quad \{ J_1 = 6^\circ 55'.6$

Enceladus.	Tethys.
$u_0 = \epsilon_0 = 175^\circ.984$	$u_0 = \epsilon_0 = 36^\circ.694$
$\Delta\epsilon_0 = +0^\circ.268$	$\Delta\epsilon_0 = +0^\circ.062$
$e = \dots\dots$	$e = 0.0$
$\pi_0 = 257^\circ.28$	$\pi_0 = \dots\dots$
$\Delta\pi = +123^\circ.43 (t - 1901.516)$	$\Delta\pi = \dots\dots$
$\gamma = 0'.0$	$\gamma = 64'.36$
$\theta_0 = \dots\dots$	$\theta_0 = 177^\circ.0$
$\Delta\theta = -152^\circ.7 (t - 1901.516)$	$\Delta\theta = -72^\circ.5 (t - 1901.516)$
$n = 262^\circ.73199$	$n = 190^\circ.69795$
$a = 34''.416$	$a = 42''.605$

$$\begin{aligned}\text{Libration of Enceladus} = \Delta\epsilon &= \begin{cases} +11'.24 \sin\{143^\circ + 92^\circ.4(t-1889.25)\} \\ +20'.00 \sin\{75^\circ + 29^\circ.3(t-1889.25)\} \end{cases} \\ \text{Libration of Tethys} = \Delta\epsilon &= \begin{cases} +141'.6 \sin 5^\circ.1(t-1866.50) \\ +2'.5 \sin 15^\circ.3(t-1866.50) \end{cases}\end{aligned}$$

Formulae.

α, δ = coordinates of Saturn referred to Earth's equator and center.
 $\rho, U, -B$ = coordinates of Saturn referred to Earth's center and Saturn's equator.
 P = position angle of Saturn's pole.
 x, y = coordinates of satellite referred to Saturn's equator and center.
 s, p = distance and position angle observed.

$$\begin{aligned}x_1 - x_2 &= s \sin (p - P), \\ y_1 - y_2 &= s \cos (p - P),\end{aligned}$$

$$ds = \sin (p - P) d(x_1 - x_2) + \cos (p - P) d(y_1 - y_2),$$

$$\begin{aligned}\cos B \sin U &= \cos \delta \sin (\alpha - N_1) \cos J_1 + \sin \delta \sin J_1, \\ \cos B \cos U &= \cos \delta \cos (\alpha - N_1), \\ \sin B &= \cos \delta \sin (\alpha - N_1) \sin J_1 + \sin \delta \cos J_1, \\ \cos B \sin P &= -\sin J_1 \cos (\alpha - N_1), \\ \cos B \cos P &= \sin \delta \sin (\alpha - N_1) \sin J_1 + \cos \delta \cos J_1,\end{aligned}$$

$$\begin{aligned}x &= \nu r \sin (u - U), \\ y &= \nu r \cos (u - U) \sin B + \nu r \sin \gamma \sin (u - \theta_0 - \Delta\theta) \cos B, \\ \nu &= \frac{a(\rho)}{\rho} \cdot \frac{1}{1 + \zeta}, & \log(\rho) &= 0.97950, \\ \zeta &= \frac{a(\rho)}{\rho} \cdot r \cos B \cos (u - U) \sin 1''.\end{aligned}$$

Enceladus.

$$\begin{aligned}dx_1 &= +\nu_1 \cos (u_1 - U) \cdot d\epsilon_1 \\ &+ \nu_1 e_1 \{ \cos (u_1 - U) \sin (u_1 - \pi_1) + \sin (U - \pi_1) \}.\end{aligned}$$

$$\begin{aligned}dy_1 &= -\nu_1 \sin (u_1 - U) \sin B \cdot d\epsilon_1 \\ &- \nu_1 e_1 \sin B \{ \sin (u_1 - U) \sin (u_1 - \pi_1) + \cos (U - \pi_1) \} \\ &- \nu_1 \cos B \cos (u_1 - \Delta\theta_1) \cdot d(\gamma_1 \sin \theta_{01}) \\ &+ \nu_1 \cos B \sin (u_1 - \Delta\theta_1) \cdot d(\gamma_1 \cos \theta_{01}).\end{aligned}$$

Tethys.

$$dx_2 = +\nu_2 \cos (u_2 - U) \cdot d\epsilon_2.$$

$$dy_2 = -\nu_2 \sin (u_2 - U) \sin B \cdot d\epsilon_2.$$

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Observations.

Date.	Observed.			Computed.				O-C	
	Washington Mean Time.	p	s	x_1	x_2	y_1	y_2	x_1-x_2	y_1-y_2
1901	h m	°	"	"	"	"	"	"	"
Apr. 26	15 27.9	215.61	6.42	+28.21	+24.12	-7.67	-13.34	-1.03	-0.02
27	15 46.6	283.38	32.15	+16.92	-15.73	+11.98	+15.31	-0.68	-0.14
28	16 2.3	128.29	43.93	-30.74	+6.71	+5.95	-16.49	-0.17	+0.26
29	15 49.9	24.41	30.30	-9.38	-0.06	-13.32	+16.82	+0.35	+1.21
May 4	15 29.9	262.92	66.69	+31.44	-33.01	+5.66	-11.22	+0.17	-0.44
13	14 38.4	107.08	63.21	-34.47	+27.31	-2.04	-12.94	-0.50	-0.02
14	14 48.6	321.94	40.03	+8.22	-19.74	-13.83	+15.07	+0.46	+0.70
15	14 27.3	225.47	27.90	+31.72	+14.29	+5.96	-16.14	-0.14	-0.20
22	14 35.7	14.89	13.94	+33.83	+36.63	-4.05	+10.11	+0.93	+0.35
23	14 7.2	248.32	47.28	+2.41	-39.29	+14.40	-8.31	-0.28	+0.10
28	13 34.2	95.26	68.49	-27.94	+40.31	-8.99	-6.43	-0.20	+0.29
June 4	12 41.2	67.24	42.23	-35.40	+1.39	-2.09	+17.96	+0.18	-1.00
9	13 18.7	278.95	59.70	+21.77	-38.74	-11.80	-9.49	-0.83	+0.41
10	12 53.4	109.26	18.74	+23.53	+41.26	+11.22	+7.44	-0.59	+0.16
19	12 15.9	39.82	28.94	-31.11	-15.28	-7.65	+16.98	+0.19	+0.29
24	11 50.9	265.84	58.28	+34.30	-22.84	-4.88	-16.24	+0.01	+0.02
27	12 35.8	70.69	50.89	-3.46	+41.88	-15.07	+7.34	-0.24	-0.21
28	11 14.1	268.04	77.47	+34.76	-41.88	+4.34	-7.36	-0.14	+0.47
30	10 46.6	302.33	17.21	-29.35	-44.84	-8.92	-1.78	+0.07	-0.20
July 3	10 45.8	109.69	72.97	-30.71	+39.31	+8.12	-8.37	-1.18	-0.55
5	11 19.5	225.41	10.92	+33.16	+25.79	-6.21	-14.85	-0.59	-0.08
9	10 42.5	246.71	43.98	+35.57	-2.50	+3.04	-18.70	-0.11	+0.48
10	10 22.0	83.79	22.73	-13.83	+8.76	+14.12	+18.47	+0.47	-0.86
15	9 57.0	287.21	23.34	-16.48	-39.54	-13.63	-9.50	-0.09	+0.01
15	11 20.9	294.15	19.02	-24.43	-42.88	-11.31	-6.20	-0.27	-0.49
20	10 16.8	167.17	10.38	+36.02	+39.43	+1.42	-8.24	-0.11	+0.10
21	9 40.8	267.73	25.07	-11.82	-36.98	+14.48	+9.96	-0.42	-0.48
23	9 48.4	308.85	52.17	+19.20	-24.47	-12.98	+15.30	+0.65	+0.76
29	9 33.2	83.01	59.29	-35.21	+21.82	+3.15	+16.72	-0.50	-0.75
31	8 25.4	341.18	11.41	+34.70	+29.68	+3.95	+14.46	-0.05	+0.24
Aug. 1	10 30.1	248.89	47.98	+0.38	-41.51	+15.30	-7.36	+0.43	-0.07
2	9 26.0	88.18	77.73	-34.04	+42.06	-4.81	+6.67	-0.72	-0.41
7	9 12.1	291.90	68.90	+29.91	-36.91	-8.33	+9.68	-0.25	+0.25
8	10 2.1	169.48	27.90	+20.42	+27.96	+12.50	-14.02	-0.84	+0.09
9	9 7.8	47.38	13.12	-33.91	-25.75	+4.66	+14.75	-0.35	+0.10
18	10 1.1	281.18	62.01	+21.49	-40.43	-12.01	-7.66	-0.09	-0.23
19	8 35.7	109.72	23.59	+17.38	+40.20	+13.16	+7.83	-0.19	-0.11
20	8 28.7	230.27	13.51	-32.81	-42.42	+5.44	-4.84	-0.34	-0.45
21	8 39.8	80.98	55.64	-9.60	+43.45	-14.53	+0.94	-0.45	+0.18
22	7 55.3	277.38	78.12	+35.06	-43.28	-0.13	+0.63	-0.22	+0.15
26	7 37.5	280.60	60.24	+29.65	-30.69	+7.91	+12.47	-0.22	+0.71
27	7 17.4	123.03	56.46	-23.61	+26.11	+11.03	-14.24	-0.98	-0.43
28	7 48.9	24.16	28.80	-24.93	-16.38	-10.45	+16.76	+0.02	-0.30
29	8 12.3	253.76	20.40	+25.48	+6.41	-10.17	-18.10	-0.32	+0.09
30	7 34.3	292.78	19.09	+16.60	-2.17	+13.13	+18.32	-0.40	-0.03
Sept. 3	7 56.5	98.12	26.63	+2.15	+29.01	+14.85	+13.90	+0.22	-0.39
4	7 56.5	185.32	10.29	-34.34	-34.25	-0.96	-11.44	-0.20	-0.20
6	8 4.6	269.74	74.46	+32.85	-41.20	+4.27	-5.10	-0.15	-0.07
7	7 38.1	106.48	59.38	-16.54	+42.09	+12.95	+2.87	+0.09	-0.21
9	6 58.5	71.74	17.21	+26.41	+41.70	-9.35	-2.25	-0.28	-0.22
12	8 19.8	350.76	26.64	-18.16	-25.81	-12.42	+13.85	-0.24	+0.68

The corrections for the reduction of s to time of p : Aug. 18, +0".40, Aug. 21 -0".30 have been applied.

Weighted Equations of Condition in (x_1-x_2) .

Date.	$d\varepsilon_1$	e_1	$d\varepsilon_2$	v	$\sqrt{p}$
1901				"	
Apr. 26	0.9737 _n	1.3715 _n	1.2349	-0.42	1/2
27	1.4695	1.7030 _n	1.5912 _n	-0.56	1
28	1.1657	1.6277 _n	1.6192	+0.10	1
29	1.2148 _n	1.4660 _n	1.3243 _n	+0.19	1/2
May 4	1.1430	1.5308 _n	1.4287	+0.37	1
13	0.6998 _n	1.5188 _n	1.5233	-0.32	1
14	1.5303 _n	1.8302 _n	1.5841 _n	+0.52	1
15	1.1647	1.5617 _n	1.6108	+0.11	1
22	0.6951 _n	1.2921 _n	1.0740 _n	+0.49	1/2
23	1.5467	1.8361 _n	1.2814	+0.05	1
28	1.3408 _n	1.6583 _n	1.2431	-0.05	1
June 4	0.4056 _n	1.2511 _n	1.3448 _n	+0.09	1/2
9	1.1550 _n	1.4762 _n	1.0379	-0.32	1/2
10	1.4342	1.7404 _n	1.2214 _n	-0.41	1
19	1.2655 _n	1.6568 _n	1.6238 _n	+0.20	1
24	1.0683 _n	1.5992 _n	1.5868	+0.22	1
27	1.5577 _n	1.8592 _n	1.2089 _n	-0.11	1
28	1.0163	1.5992 _n	1.2099	+0.05	1
30	1.3292 _n	1.6982 _n	0.4067	+0.20	1
July 3	0.9864	1.3521 _n	1.0368	-0.47	1/2
5	1.1699 _n	1.6094 _n	1.5660	-0.39	1
9	0.8588	1.5837 _n	1.6519	+0.14	1
10	1.2246	1.5108 _n	1.3429 _n	+0.32	1/2
15	1.5090 _n	1.8233 _n	1.3263	+0.12	1
15	1.4278 _n	1.7660 _n	1.1200	-0.10	1
20	0.5259	1.5607 _n	1.3264	+0.07	1
21	1.5335	1.8084 _n	1.4013 _n	-0.22	1
23	1.4856 _n	1.7524 _n	1.5728 _n	+0.68	1
29	0.8691	1.5325 _n	1.5888 _n	-0.48	1
31	0.9664	1.5939 _n	1.5198 _n	+0.01	1
Aug. 1	1.5548	1.8368 _n	1.2010	+0.75	1
2	1.0515 _n	1.6105 _n	1.1504 _n	-0.64	1
7	1.2893 _n	1.5820 _n	1.3850 _n	-0.22	1
8	1.1646	1.4894 _n	1.2323	-0.25	1/2
9	1.0373	1.5176 _n	1.5534 _n	-0.32	1
18	1.4462 _n	1.6593 _n	1.2147	+0.05	1
19	1.4857	1.7963 _n	1.2249 _n	+0.02	1
20	1.1020	1.4945 _n	0.9862	-0.19	1
21	1.5287 _n	1.8143 _n	9.7053 _n	-0.30	1
22	9.4804 _n	1.4994 _n	0.4981 _n	-0.13	1
26	1.2639	1.6702 _n	1.4821 _n	-0.11	1
27	1.1072	1.2991 _n	1.2343	-0.36	1/2
28	1.3845 _n	1.7313 _n	1.5996 _n	+0.05	1
29	1.3728 _n	1.5654 _n	1.6284	-0.13	1
30	1.4834	1.7851 _n	1.6319 _n	-0.26	1
Sept. 3	1.5365	1.7841 _n	1.4952 _n	+0.40	1
4	0.3451 _n	1.4916 _n	1.4032	-0.04	1
6	0.9954	1.5657 _n	1.0114	+0.01	1
7	1.4769	1.6468 _n	0.7011 _n	+0.27	1
9	1.3353 _n	1.5035 _n	0.8357	-0.20	1
12	1.4583 _n	1.7581 _n	1.5212 _n	-0.19	1

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Weighted Equations of Condition in $(y_1 - y_2)$.

Date.	$d\varepsilon_1$	e_1	$d(\gamma_1 \sin \theta_{01})$	$d(\gamma_1 \cos \theta_{01})$	$d\varepsilon_2$	v	$\sqrt{p}$
1901						"	
Apr. 26	0.7589 _n	0.8561 _n	0.5262	1.1801 _n	0.6916	-0.07	1/2
27	0.8373 _n	9.7355 _n	1.4751	0.9304	0.8058 _n	0.00	I
28	1.0968	1.1403 _n	1.0245 _n	1.4662	0.4360	+0.49	I
29	0.2805	9.9255 _n	1.1345 _n	0.8778 _n	8.1091 _n	+0.51	1/2
May 4	1.1068 _n	0.5030 _n	1.4602	1.0929 _n	1.1280 _n	-0.46	I
13	1.1476	0.7307 _n	1.3711 _n	1.3310	1.0466	+0.11	I
14	0.5252 _n	0.8435 _n	1.2867 _n	1.4032 _n	0.9057 _n	+0.46	I
15	1.1119 _n	0.0860 _n	1.4548	1.1559 _n	0.7654	-0.20	I
22	0.8399 _n	0.6742 _n	0.7324	1.1806 _n	0.8744	+0.12	1/2
23	9.9943 _n	0.2776 _n	1.4380	1.2280	1.2061 _n	+0.27	I
28	1.0589	0.4920	1.4883 _n	1.0038	1.2182	+0.30	I
June 4	0.8624	9.8276 _n	1.0147 _n	1.1003	9.4551	-0.44	1/2
9	0.6526 _n	0.6426 _n	0.8107 _n	1.1771 _n	0.9028 _n	+0.07	1/2
10	0.9876 _n	0.7589	1.5059	0.8263 _n	1.2315	+0.24	I
19	1.1112	0.7994	1.4388 _n	1.2577	0.8025 _n	+0.30	I
24	1.1550 _n	0.5907 _n	0.4917	1.5159 _n	0.9784 _n	-0.22	I
27	0.1596	0.3331	1.4969 _n	1.0019 _n	1.2425	-0.35	I
28	1.1619 _n	0.7567	1.3217	1.4054 _n	1.2428 _n	+0.29	I
30	1.0890	0.9500	1.4476 _n	1.2393	1.2731 _n	-0.24	I
July 3	0.8085	0.3517 _n	0.5005	1.2089	0.9158	-0.12	1/2
5	1.1435 _n	0.4340 _n	1.3075 _n	1.5172 _n	1.0344	-0.35	I
9	1.1751 _n	0.8414	1.2248	1.4523 _n	0.0216 _n	+0.30	I
10	0.4642	0.1575 _n	1.0926	1.0353	0.2656	-0.30	1/2
15	0.8427	0.9713	1.5102 _n	0.7533	1.2226 _n	-0.14	I
15	1.0135	1.0581	1.4730 _n	1.1468	1.2579 _n	-0.57	I
20	1.1835 _n	0.8944	1.0644	1.4863 _n	1.2228	-0.07	I
21	0.7001	9.9614 _n	1.4328	1.2646	1.1953 _n	-0.29	I
23	0.9112 _n	0.3360 _n	1.3525 _n	1.3753 _n	1.0164 _n	+0.49	I
29	1.1759	0.7200	9.3899 _n	1.5126	0.9681	-0.51	I
31	1.1701 _n	1.0846	1.1619	1.4634 _n	1.1023	+0.07	I
Aug. 1	9.2120 _n	0.6682	1.5037	0.7767	1.2481 _n	+0.02	I
2	1.1622	1.1174	1.1991 _n	1.4519	1.2541	-0.25	I
7	1.1071 _n	0.3218	1.1170 _n	1.4695 _n	1.1984 _n	-0.06	I
8	0.6406 _n	0.8337	1.1598	0.8522 _n	0.7770	+0.03	1/2
9	1.1619	0.7718	0.7221	1.5017	1.0424 _n	+0.30	I
18	0.9654 _n	0.2267	1.3695 _n	1.3335 _n	1.2399 _n	-0.52	I
19	0.8735 _n	1.1345	1.4596	1.1263 _n	1.2376	-0.11	I
20	1.1496	0.8529	0.9613	1.4828	1.2612 _n	-0.26	I
21	0.6158	1.0188	1.4917 _n	0.8158	1.2717	+0.13	I
22	1.1787 _n	1.1062	0.2316	1.4997 _n	1.2702 _n	-0.16	I
26	1.1064 _n	1.2471	1.2434	1.4174 _n	1.1214 _n	+0.49	I
27	0.7066	0.2221	1.0506	1.0409	0.7502	-0.08	1/2
28	1.0313	1.2337	1.3495 _n	1.3424	0.8491 _n	-0.29	I
29	1.0409 _n	0.6205	1.3208 _n	1.3675 _n	0.4418	-0.16	I
30	0.8549 _n	1.1533	1.4406	1.1684 _n	9.9721 _n	-0.09	I
Sept. 3	9.9685 _n	0.9139	1.4910	0.4045 _n	1.0978	-0.29	I
4	1.1711	1.1949	0.0735 _n	1.4916	1.1700 _n	-0.06	I
6	1.1520 _n	1.2561	0.8877	1.4765 _n	1.2503 _n	-0.37	I
7	0.8540	0.6087	1.4427	1.1341	1.2597	+0.02	I
9	1.0574 _n	0.7970	1.3210 _n	1.3536 _n	1.2557	-0.44	I
12	0.8948	1.1989	1.3883 _n	1.2667	1.0474 _n	+0.65	I

Normal Equations.

$d\varepsilon_1$	e_1	$d\varepsilon_2$	$d(\gamma_1 \sin \theta_{01})$	$d(\gamma_1 \cos \theta_{01})$	$100 n$
29497.3	+ 1151.0	- 876.2	- 4136.9	+10566.9	- 9958.1
	110956.7	+ 7358.5	+ 89.4	- 297.2	+33900.4
		45332.4	+ 70.4	- 79.0	- 8983.9
			23084.5	- 2597.1	- 5770.6
				22733.8	-15826.9
					$[nn]=12.41$

Results

Epoch: 1901, July 8.0, Greenwich Mean Time.

*Corrections.**Corrected Elements.**Enceladus.**Enceladus.*

$$d\varepsilon = - 0^{\circ}.100 \pm 0^{\circ}.079$$

$$\varepsilon = 176^{\circ}.152$$

$$e = 0.0032 \pm 0.0006$$

$$e = 0.0032$$

$$\gamma \sin \theta_0 = - 12'.20 \pm 4'.89$$

$$\gamma = 25'.5$$

$$\gamma \cos \theta_0 = - 22'.43 \pm 5'.33$$

$$\theta_0 = 208^{\circ}.6$$

*Tethys.**Tethys.*

$$d\varepsilon = - 0^{\circ}.146 \pm 0^{\circ}.058$$

$$\varepsilon = 36^{\circ}.610$$

$$[pv^2] = 9''.697$$

$$[nn \cdot 5] = 9''.678$$

The probable error of one equation of weight unity $= \pm 0''.213$.

Owing to the Earth's elevation above the plane of the orbit, the above inclination has small weight. The longitudes agree closely with other determinations.