THE MASS OF TITAN.

BY

W. S. EICHELBERGER.

FROM

OBSERVATIONS OF HYPERION

MADE BY

ASAPH HALL IN 1884-1885.

B 5

THE MASS OF TITAN.

In the *Astronomical Journal*, Nos. 259-260, the author published a discussion of a series of observations of Hyperion made at nine oppositions from 1875 to 1890 and computed a set of elements for each opposition. From these nine sets the perturbations of the elements of Hyperion's orbit, together with the mean values of the elements referred to the equinox of 1884.0, were empirically deduced as follows:

Elements I of Hyperion.

$$\begin{aligned} E &= 295^{\circ}.92 + 16^{\circ}.92006 t + 9^{\circ}.05 \sin \left[48^{\circ}.07 + \frac{360^{\circ}}{639.5} t \right]. \\ P &= 8^{\circ}.46 - 18^{\circ}.442 T + 14^{\circ}.40 \sin \Pi - 1^{\circ}.40 \sin 2 \Pi + 1^{\circ}.11 \sin 3 \Pi. \\ \mu &= 16^{\circ}.92006 + 0^{\circ}.08892 \cos \left[48^{\circ}.07 + \frac{360^{\circ}}{639.5} t \right]. \\ e &= 0.1056 + 0.0258 \cos \Pi + 0.0008 \cos 2 \Pi. \\ t &= \text{time in days from Jan. 0.0, 1884.} \\ T &= \text{time in years from 1884.0.} \\ \Pi &= 263^{\circ}.54 + 18^{\circ}.942 T. \end{aligned}$$

As the elements assumed in the article referred to differed quite widely from those deduced, a comparison of the observations from December 4, 1884, to January 29, 1885, was made with an ephemeris based upon the elements just given, and a new set of elements was computed as given below;

Elements II of Hyperion.

Epoch: January 0.0, 1885, Greenwich Mean Time.

$$\begin{aligned} E &= 359 \quad 35.6 \\ P &= 337 \quad 25.3 \\ n &= 169 \quad 22.2 \\ i &= 27 \quad 12.5 \\ e &= 0.11143 = \sin 6^{\circ} 23'.9 \\ \mu &= 16^{\circ}.89572 \\ \Delta &= 214''.405 \end{aligned}$$

This discussion was finished in 1893, when work on the subject was discontinued. An ocean voyage to Sumatra to observe the Total Solar Eclipse of May 17, 1901, offered some leisure and it was determined to compute the special perturbations of Hyperion by Titan, obtain osculating elements of Hyperion and a mass of Titan. During the voyage it was the author's good fortune to have associated with him in all the computations Prof. F. B. LITTELL, U. S. Navy, for whose valuable assistance

he wishes to acknowledge his indebtedness. As no opportunity has presented itself since that time for continuing this work, it has been decided to publish what has already been done.

STRUVE's elements of Titan in Supplement I, aux Observations de Poulkova, were used with the mean distance given by Prof. ASAPH HALL, jr., Transactions of the Astronomical Observatory of Yale University, Volume I, Part II, and the mass by Dr. GEO. W. HILL, Astronomical Journal, No. 176.

Elements of Titan.

Epoch: January 0.0, 1885, Greenwich Mean Time.

$$\begin{aligned} E &= 74 \quad 46.9 \\ P &= 272 \quad 27.1 \\ n &= 167 \quad 45.4 \\ i &= 27 \quad 28.3 \\ e &= 0.029073 \\ \mu &= 22^\circ.5770116 \\ \Delta &= 176''.570 \\ m &= \frac{1}{4714} \end{aligned}$$

The special perturbations were computed by the method of the variation of the elements, and the interval employed was one day for about 6 days each side of opposition, $\frac{1}{4}$ day for about 3 days each side of conjunction, and $\frac{1}{2}$ day for the remaining time. Starting from Jan. 0.0, 1885, the perturbations were computed for ten dates. These results were then integrated, the integrated values applied to the original elements, and the perturbations computed for another ten dates, of which three would coincide with three of the previous ten. This method was continued throughout the entire work.

Perturbations of the Elements of Hyperion by Titan.

Greenwich Mean Time.	$\Delta\mu$	ΔE	ΔP	$\Delta\varphi$	Δi	Δn
1884						
Sept. 19. 75	-2.564	+137.51	+5 19.7	+5.8	-0.6	+0.1
20. 75	2.692	134.41	22.5	6.0	0.6	0.1
21. 75	2.841	131.22	25.9	6.0	0.6	0.1
22. 75	3.007	127.93	29.3	5.8	0.6	0.1
23. 75	-3.184	+124.55	+5 32.0	+5.4	-0.6	+0.1
24. 75	-3.369	+121.08	+5 33.6	+5.0	-0.6	+0.1
25. 75	3.558	117.50	33.8	4.5	0.6	0.1
26. 75	3.750	113.83	32.5	4.1	0.6	0.1
27. 75	3.944	110.05	30.0	3.7	0.6	0.1
28. 75	-4.139	+106.17	+5 26.5	+3.4	-0.6	+0.1
29. 75	-4.331	+102.18	+5 22.5	+3.3	-0.6	+0.1
30. 75	4.516	98.09	18.7	3.4	0.6	0.0
Oct. 1. 75	4.686	93.92	15.9	3.7	0.6	0.0
2. 75	4.832	89.66	14.6	4.0	0.6	0.0
3. 75	-4.942	+85.34	+5 15.1	+4.4	-0.6	0.0
4. 75	-5.005	+80.99	+5 17.3	+4.7	-0.6	0.0
5. 75	5.017	76.65	20.5	4.7	0.6	0.0
6. 75	4.977	72.37	23.7	4.6	0.6	0.0
7. 75	4.886	68.19	26.1	4.2	0.6	0.0
8. 75	-4.744	+64.19	+5 26.9	+3.8	-0.6	0.0

Perturbations of the Elements of Hyperion by Titan—Continued.

Greenwich Mean Time.	$\Delta\mu$	ΔE	ΔP	$\Delta\varphi$	Δi	Δn
1884						
Oct. 9.75	-4.545	+ 60.44	+5 25.4	+3.2	-0.6	0.0
10.75	4.277	57.03	21.2	2.6	0.6	+0.1
11.75	3.920	54.11	13.6	2.2	0.6	0.1
12.75	3.439	51.95	5 2.3	2.1	0.6	0.1
13.75	-2.786	+ 50.96	+4 46.6	+2.4	-0.6	+0.1
14.75	-1.960	+ 51.98	+4 25.7	+3.2	-0.6	+0.1
15.75	1.260	55.85	4 0.1	4.2	0.5	0.0
16.75	1.328	60.97	3 34.0	4.0	0.4	0.0
17.75	1.959	63.91	3 12.5	3.0	0.3	0.0
18.75	-2.538	+ 64.35	+2 56.8	+2.5	-0.3	0.0
19.75	-2.920	+ 63.36	+2 46.4	+2.4	-0.3	0.0
20.75	3.142	61.58	40.2	2.7	0.3	0.0
21.75	3.253	59.36	37.7	3.2	0.3	0.0
22.75	3.286	56.89	37.7	3.6	0.3	0.0
23.75	-3.267	+ 54.28	+2 39.5	+3.9	-0.3	0.0
24.75	-3.214	+ 51.63	+2 42.2	+4.0	-0.3	0.0
25.75	3.143	48.97	45.1	4.0	0.3	0.0
26.75	3.059	46.35	47.7	3.9	0.3	0.0
27.75	2.967	43.79	49.7	3.7	0.3	0.0
28.75	-2.862	+ 41.33	+2 50.8	+3.4	-0.3	0.0
29.75	-2.740	+ 38.97	+2 50.8	+3.1	-0.3	0.0
30.75	2.596	36.72	49.4	2.8	0.3	0.0
31.75	2.428	34.59	46.6	2.5	0.3	0.0
Nov. 1.75	2.238	32.56	42.8	2.4	0.3	0.0
2.75	-2.031	+ 30.66	+2 38.5	+2.5	-0.3	0.0
3.75	-1.814	+ 28.86	+2 34.7	+2.8	-0.3	+0.1
4.75	1.595	27.15	32.0	3.3	0.3	0.1
5.75	1.383	25.53	31.0	3.8	0.3	0.1
6.75	1.186	23.99	31.7	4.3	0.3	0.1
7.75	-1.011	+ 22.54	+2 33.7	+4.7	-0.3	+0.1
8.75	-0.862	+ 21.19	+2 36.7	+4.9	-0.3	+0.1
9.75	0.739	19.93	39.7	5.0	0.3	0.1
10.75	0.642	18.76	42.3	4.9	0.3	0.1
11.75	0.569	17.68	44.1	4.8	0.3	0.1
12.75	-0.516	+ 16.67	+2 45.0	+4.5	-0.3	+0.1
13.75	-0.481	+ 15.71	+2 44.8	+4.3	-0.3	+0.1
14.75	0.461	14.80	43.9	4.1	0.3	0.1
15.75	0.452	13.91	42.3	4.0	0.3	0.1
16.75	0.456	13.03	40.3	3.9	0.3	0.1
17.75	-0.472	+ 12.13	+2 38.2	+3.9	-0.3	+0.1
18.75	-0.503	+ 11.20	+2 36.3	+4.1	-0.3	+0.1
19.75	0.554	10.21	35.1	4.3	0.3	0.1
20.75	0.628	9.14	34.9	4.6	0.3	0.1
21.75	0.728	7.98	36.0	4.9	0.3	0.1
22.75	-0.855	+ 6.72	+2 38.3	+5.1	-0.3	+0.1
23.75	-1.004	+ 5.35	+2 41.6	+5.2	-0.3	+0.1
24.75	1.171	+ 3.89	45.3	5.1	0.3	0.1
25.75	1.351	+ 2.32	48.6	4.8	0.3	0.1
26.75	1.538	+ 0.65	51.1	4.5	0.3	0.1
27.75	-1.729	- 1.11	+2 52.4	+4.0	-0.3	+0.1
28.75	-1.922	- 2.98	+2 52.2	+3.5	-0.3	+0.1
29.75	2.115	4.94	50.5	3.1	0.3	0.1
30.75	2.307	7.02	47.6	2.7	0.3	0.1
Dec. 1.75	2.496	9.18	44.0	2.5	0.3	0.1
2.75	-2.677	- 11.43	+2 40.1	+2.5	-0.3	+0.1
3.75	-2.844	- 13.78	+2 36.7	+2.7	-0.3	+0.1
4.75	2.989	16.20	34.6	3.0	0.3	0.1
5.75	3.097	18.65	34.3	3.3	0.3	0.1
6.75	3.156	21.12	35.9	3.6	0.3	0.1
7.75	-3.156	- 23.57	+2 39.1	+3.8	-0.3	+0.1

MISCELLANEOUS ASTRONOMICAL PAPERS.

Perturbations of the Elements of Hyperion by Titan—Continued.

Greenwich Mean Time.	$\Delta\mu$	ΔE	ΔP	$\Delta\varphi$	Δi	Δn
1884						
Dec. 8. 75	-3.092	-25.92	+2 42.9	+3.7	-0.3	+0.1
9. 75	2.966	28.16	46.1	3.3	0.3	0.1
10. 75	2.780	30.19	47.8	2.8	0.3	0.1
11. 75	2.535	31.93	47.2	2.1	0.3	0.1
12. 75	-2.220	-33.30	+2 43.7	+1.4	-0.3	+0.1
13. 75	-1.820	-34.16	+2 36.6	+0.8	-0.3	+0.1
14. 75	-1.300	34.28	2 25.5	0.4	0.3	0.1
15. 75	-0.612	33.29	2 9.6	0.4	0.3	0.1
16. 75	+0.277	30.47	1 48.0	0.9	0.3	0.1
17. 75	+1.205	-24.78	+1 20.8	+1.7	-0.2	+0.1
18. 75	+1.521	-16.67	+0 52.3	+1.7	-0.2	0.0
19. 75	+1.016	9.78	+0 28.8	+0.5	-0.1	0.0
20. 75	+0.381	5.72	+0 11.5	-0.5	0.0	0.0
21. 75	-0.068	3.52	-0 0.7	-0.9	0.0	0.0
22. 75	-0.337	-2.31	-0 8.6	-0.8	0.0	0.0
23. 75	-0.474	-1.63	-0 12.7	-0.4	0.0	0.0
24. 75	0.521	1.26	13.8	0.0	0.0	0.0
25. 75	0.504	1.06	12.6	+0.3	0.0	0.0
26. 75	0.445	0.94	10.1	+0.6	0.0	0.0
27. 75	-0.359	-0.82	-0 7.1	+0.6	0.0	0.0
28. 75	-0.259	-0.66	-0 4.2	+0.5	0.0	0.0
29. 75	-0.149	-0.44	-0 1.9	+0.3	0.0	0.0
30. 75	-0.031	-0.10	-0 0.2	+0.1	0.0	0.0
31. 75	+0.097	+0.30	+0 0.3	-0.2	0.0	0.0
1885						
Jan. 1. 75	+0.241	+0.90	-0 0.4	-0.6	0.0	0.0
2. 75	+0.404	+1.60	-0 2.4	-0.9	0.0	0.0
3. 75	0.587	2.42	5.7	1.1	0.0	0.0
4. 75	0.788	3.36	9.9	1.1	0.0	0.0
5. 75	1.002	4.42	14.1	0.9	0.0	0.0
6. 75	+1.220	+5.58	-0 17.7	-0.6	0.0	0.0
7. 75	+1.437	+6.85	-0 19.9	-0.1	0.0	0.0
8. 75	1.642	8.19	20.4	+0.4	0.0	0.0
9. 75	1.829	9.63	19.2	+0.9	0.0	0.0
10. 75	1.992	11.15	16.8	+1.2	0.0	0.0
11. 75	+2.128	+12.76	-0 13.8	+1.4	0.0	0.0
12. 75	+2.237	+14.46	-0 10.9	+1.4	0.0	0.0
13. 75	2.319	16.25	8.6	1.3	0.0	0.0
14. 75	2.379	18.12	7.1	1.1	0.0	+0.1
15. 75	2.418	20.05	6.7	0.9	0.0	0.1
16. 75	+2.440	+22.03	-0 7.3	+0.6	0.0	+0.1
17. 75	+2.449	+24.05	-0 8.6	+0.5	0.0	+0.1
18. 75	2.445	26.06	10.4	0.4	0.0	0.1
19. 75	2.429	28.07	12.5	0.3	0.0	0.1
20. 75	2.401	30.05	14.5	0.4	0.0	0.1
21. 75	+2.356	+31.98	-0 16.2	+0.6	0.0	+0.1
22. 75	+2.292	+33.84	-0 17.0	+0.9	0.0	+0.1
23. 75	2.203	35.61	16.8	1.1	0.0	0.1
24. 75	2.089	37.29	15.2	1.4	0.0	0.1
25. 75	1.950	38.85	12.4	1.6	0.0	0.1
26. 75	+1.790	+40.31	-0 8.9	+1.6	0.0	+0.1
27. 75	+1.614	+41.67	-0 5.2	+1.5	0.0	+0.1
28. 75	1.428	42.92	-0 2.0	+1.2	0.0	0.0
29. 75	1.237	44.07	+0 0.2	+0.8	0.0	0.0
30. 75	1.044	45.12	+0 1.0	+0.3	0.0	0.0
31. 75	+0.850	+46.07	+0 0.2	-0.2	0.0	0.0
Feb. 1. 75	+0.657	+46.91	-0 1.9	-0.6	0.0	0.0
2. 75	0.467	47.70	5.1	0.9	0.0	0.0
3. 75	0.282	48.32	8.9	1.0	0.0	0.0
4. 75	+0.107	48.87	12.7	1.0	0.0	0.0
5. 75	-0.050	+49.36	-0 15.6	-0.8	0.0	0.0

Perturbations of the Elements of Hyperion by Titan—Continued.

Greenwich Mean Time.	$\Delta\mu$	ΔE	ΔP	$\Delta\varphi$	Δi	Δn
1885	/	/	° /	/	/	/
Feb. 6. 75	-0.180	+ 49.78	-0 17.1	-0.4	0.0	0.0
7. 75	0.269	50.17	16.7	-0.1	0.0	0.0
8. 75	0.304	50.57	14.4	+0.2	0.0	0.0
9. 75	0.277	51.01	10.8	+0.2	0.0	0.0
10. 75	-0.186	+ 51.57	-0 7.1	+0.1	0.0	0.0
11. 75	-0.033	+ 52.27	-0 4.2	-0.4	0.0	0.0
12. 75	+0.180	53.22	3.2	1.0	0.0	0.0
13. 75	0.458	54.48	4.9	1.7	0.0	0.0
14. 75	0.811	56.18	9.8	2.4	0.0	+0.1
15. 75	+1.264	+ 58.48	-0 18.5	-3.0	0.0	+0.1
16. 75	+1.855	+ 61.63	-0 31.7	-3.3	0.0	+0.1
17. 75	2.638	66.14	0 50.2	3.1	0.0	0.1
18. 75	3.613	72.91	1 14.8	2.4	0.0	+0.1
19. 75	4.446	82.83	1 44.2	1.7	+0.1	0.0
20. 75	+4.426	+ 94.17	-2 12.5	-2.2	+0.2	0.0
21. 75	+3.775	+103.17	-2 34.6	-3.5	+0.3	-0.1
22. 75	3.169	109.48	2 50.7	4.3	0.3	0.1
23. 75	2.769	114.26	3 1.8	4.5	0.3	0.1
24. 75	2.535	118.20	3 8.6	4.3	0.3	-0.1
25. 75	+2.421	+121.69	-3 11.8	-3.9	+0.3	0.0
26. 75	+2.389	+124.92	-3 12.2	-3.5	+0.3	0.0
27. 75	2.415	128.00	10.6	3.2	0.3	0.0
28. 75	2.478	131.03	7.8	3.0	0.3	0.0
March 1. 75	2.564	134.05	4.8	3.0	0.3	0.0
2. 75	+2.663	+137.14	-3 2.0	-3.1	+0.3	0.0
3. 75	+2.770	+140.29	-2 59.9	-3.3	+0.3	0.0
4. 75	2.887	143.55	58.5	3.6	0.3	0.0
5. 75	3.017	146.91	58.4	3.9	0.3	0.0
6. 75	3.164	150.40	2 59.5	4.3	0.3	0.0
7. 75	+3.330	+154.02	-3 2.0	-4.5	+0.3	0.0
8. 75	+3.517	+157.75	-3 5.7	-4.7	+0.3	0.0
9. 75	3.721	161.61	9.9	4.6	0.3	0.0
10. 75	3.936	165.57	14.0	4.4	0.3	0.0
11. 75	4.154	169.64	17.1	4.0	0.3	0.0
12. 75	+4.369	+173.81	-3 18.8	-3.5	+0.3	0.0
13. 75	+4.571	+178.06	-3 18.7	-3.0	+0.3	0.0
14. 75	4.755	182.41	17.0	2.5	0.3	0.0
15. 75	4.913	186.83	14.2	2.2	0.3	0.0
16. 75	+5.045	+191.35	-3 11.1	-2.1	+0.3	0.0
17. 75	+5.150	+195.97	-3 8.2	-2.1	+0.3	0.0
18. 75	5.229	200.69	6.1	2.3	0.3	0.0
19. 75	5.287	205.48	4.9	2.5	0.3	0.0
20. 75	+5.327	+210.32	-3 4.8	-2.7	+0.3	0.0

With *Elements II of Hyperion* and the perturbations tabulated above, an ephemeris was computed and a comparison made with the observations of Prof. ASAPH HALL, U. S. Navy, published in Washington Observations, 1884 and 1885. In computing the ephemeris places the following formulæ* were employed:

$$\begin{aligned}\xi &= r \frac{\rho_0}{\rho} \Delta \sin f \sin (F+u), & x &= s \sin p = \frac{\xi}{1+\zeta}, \\ \eta &= r \frac{\rho_0}{\rho} \Delta \sin g \sin (G+u), & y &= s \cos p = \frac{\eta}{1+\zeta}, \\ \zeta &= r \frac{\rho_0}{\rho} \Delta \sin h \sin (H+u),\end{aligned}$$

* See *Astronomische Nachrichten*, Nr. 193.

f, g, h, F, G , and H being given by the following:

$$\begin{aligned} \sin f \sin F &= -\sin (\alpha - N), & l \sin L &= \cos J \sin (\alpha - N), \\ \sin f \cos F &= \cos (\alpha - N) \cos J, & l \cos L &= \sin J, \\ \cos f &= -\cos (\alpha - N) \sin J, \\ \sin g \sin G &= -\sin \delta \cos (\alpha - N), & q \sin Q &= \sin J \sin (\alpha - N), \\ \sin g \cos G &= l \cos (\delta + L), & q \cos Q &= \cos J, \\ \cos g &= q \cos (\delta - Q), \\ \sin h \sin H &= \cos \delta \cos (\alpha - N), \\ \sin h \cos H &= l \sin (\delta + L), \\ \cos h &= q \sin (\delta - Q). \end{aligned}$$

The notation used in the above formulae and in other parts of this paper follows:

- E = Mean longitude of satellite.
- P = Longitude of peri-saturnium of satellite's orbit.
- N = Longitude of node on the equator.
- n = Longitude of node on the ecliptic.
- J = Inclination of orbit to equator.
- i = Inclination of orbit to ecliptic.
- e = Eccentricity of orbit = $\sin \varphi$.
- μ = Mean motion in longitude.
- Δ = Mean distance of satellite from planet, as seen at distance ρ_0 of planet.
- ρ_0 = Assumed mean distance of planet from Sun.
- ρ = Geocentric distance of planet.
- α = Right ascension of planet.
- δ = Declination of planet.
- ϵ = Eccentric anomaly of satellite.
- u = Longitude of satellite in orbit, starting from node on the equator.
- s = Observed distance of satellite.
- p = Observed position angle of satellite.
- w = Distance along orbit from node on equator to node on ecliptic.

The values of f, g, h, F, G , and H were computed for every 5 days during the period of observation; the values for each observation being found by interpolation.

Observations of Hyperion.

Washington Mean Time.			p	s	Δs	x_0	x_0	y_0	y_0	$x_0 - x_0$	$y_0 - y_0$
1884 Sept.			"	"	"	"	"	"	"	"	"
20	15	30.1	134.40	129.60	+0.17	+92.47	+87.35	-90.56	-91.36	+5.12	+0.80
22	16	34.6	105.30	206.89	-0.17	+199.39	+195.80	-54.55	-55.42	+3.59	+0.87
23	15	12.7	97.33	235.02	-0.12	+232.97	+229.43	-29.97	-32.77	+3.54	+2.80
23	16	25.2	96.98	235.20	-0.11	+233.35	+230.88	-28.57	-31.50	+2.47	+2.93
24	15	27.2	91.06	252.55	-0.05	+252.45	+251.24	-4.67	-6.40	+1.21	+1.73
24	16	22.7	90.75	251.81	-0.06	+251.73	+251.77	-3.30	-5.37	-0.04	+2.07
25	15	29.3	85.15	258.07	0.00	+257.15	+257.16	+21.82	+20.27	-0.01	+1.55
25	15	49.3	85.12	258.00	0.00	+257.07	+257.12	+21.95	+20.63	-0.05	+1.32
27	15	54.3	72.58	228.43	+0.26	+218.20	+220.10	+68.46	+68.70	-1.90	-0.24
28	15	0.2	63.98	197.31	+0.21	+177.50	+180.38	+86.65	+86.09	-2.88	+0.56

THE MASS OF TITAN.

B 13

Observations of Hyperion—Continued.

Washington Mean Time.			p	s	Δs	x_0	x_c	y_0	y_c	$x_0 - x_c$	$y_0 - y_c$
			$^{\circ}$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$
1884 Oct.	1	15 34.5	351.75	98.38	+0.02	-14.12	-9.51	+97.38	+97.70	-4.61	-0.32
	4	16 41.7	278.05	191.22	-0.15	-189.19	-186.30	+26.76	+29.38	-2.89	-2.62
	5	16 24.3	268.32	209.73	-0.07	-209.57	-208.18	-6.15	-3.91	-1.39	-2.24
	6	16 37.4	259.05	208.67	+0.05	-204.92	-205.00	-39.65	-37.52	+0.08	-2.13
	11	16 14.2	144.80	121.79	-0.12	+70.13	+66.32	-99.42	-98.96	+3.81	-0.46
	23	12 56.8	329.38	108.11	-0.11	-55.01	-51.86	+92.94	+93.57	-3.15	-0.63
	25	13 13.9	283.38	183.31	-0.18	-178.16	-175.26	+42.38	+44.65	-2.90	-2.27
	8	11 49.5	78.66	261.25	+0.08	+256.23	+259.69	+51.39	+53.23	-3.46	-1.84
	9	11 42.5	71.32	240.29	+0.13	+227.76	+228.04	+77.00	+76.68	-0.28	+0.32
	10	11 55.6	61.88	202.11	+0.18	+178.42	+179.70	+95.34	+95.06	-1.28	+0.28
Nov.	11	11 22.6	48.09	168.78	+0.27	+125.81	+119.68	+112.92	+105.98	[+6.13]	[+6.94]
	12	12 22.4	21.25	116.60	+0.21	+42.34	+45.40	+108.87	+108.46	-3.06	+0.41
	13	11 33.7	343.62	103.52	-0.04	-29.18	-27.66	+99.28	+101.25	-1.52	-1.97
	14	11 22.7	309.48	131.82	-0.14	-101.64	-99.97	+83.72	+84.01	-1.67	-0.29
	15	10 51.3	289.43	172.42	-0.20	-162.41	-160.72	+57.29	+58.15	-1.69	-0.86
	16	11 51.3	275.95	207.82	-0.12	-206.58	-205.39	+21.53	+23.49	-1.19	-1.96
	17	11 14.2	266.31	223.31	-0.04	-222.81	-222.52	-14.37	-11.88	-0.29	-2.49
	20	11 17.5	231.15	156.81	+0.19	-122.27	-123.30	-98.48	-97.26	+1.03	-1.22
	21	10 51.0	207.08	122.63	+0.10	-55.87	-57.14	-109.28	-108.36	+1.27	-0.92
	21	11 59.0	205.15	121.03	+0.09	-51.47	-53.77	-109.64	-108.66	+2.30	-0.98
Dec.	22	10 32.1	170.65	112.68	-0.03	+18.30	+14.66	-111.15	-109.91	+3.64	-1.24
	24	10 58.6	119.32	178.16	-0.22	+155.15	+152.89	-87.14	-86.64	+2.26	-0.50
	26	10 46.7	98.91	253.33	-0.14	+250.13	+247.98	-39.21	-39.11	+2.15	-0.10
	29	11 34.0	79.78	274.45	+0.09	+270.18	+269.86	+48.71	+47.94	+0.32	+0.77
	4	10 51.1	355.60	105.00	+0.02	-8.06	-7.32	+104.71	+106.41	-0.74	-1.70
	7	10 18.8	279.98	201.04	-0.18	-197.82	-197.14	+34.81	+35.80	-0.68	-0.99
	8	9 57.8	269.79	223.33	-0.05	-223.28	-222.71	-0.82	+0.08	-0.57	-0.90
	9	9 52.9	260.39	225.15	+0.04	-222.03	-221.45	-37.59	-36.03	-0.58	-1.56
	15	9 51.5	123.98	165.45	-0.21	+137.02	+135.95	-92.35	-92.37	+1.07	+0.02
	19	9 37.2	87.98	280.64	-0.02	+280.45	+280.28	+9.89	+10.66	+0.17	-0.77
1885 Jan.	26	8 14.0	327.06	105.08	-0.11	-62.53	-61.55	+96.49	+96.50	-0.98	-0.01
	2	8 45.7	221.65	139.34	+0.15	-92.70	-92.34	-104.23	-103.50	-0.36	-0.73
	8	8 40.9	95.62	261.80	-0.11	+260.43	+259.47	-25.63	-25.34	+0.96	-0.29
	9	9 2.5	89.27	274.91	-0.02	+274.87	+275.14	+3.50	+4.09	-0.27	-0.59
	10	8 39.0	83.28	275.38	+0.04	+273.52	+273.05	+32.23	+32.32	+0.47	-0.09
	12	7 20.0	70.01	234.38	+0.18	+220.42	+219.13	+80.18	+80.34	+1.29	-0.16
	13	8 6.1	59.55	192.88	+0.18	+166.43	+166.13	+97.84	+97.85	+0.30	-0.01
	17	10 50.2	301.92	145.24	-0.17	-123.13	-122.13	+76.70	+76.13	-1.00	+0.57
	21	8 39.9	255.15	209.22	+0.11	-202.34	-201.47	-53.65	-52.84	-0.87	-0.81
	22	7 59.4	244.00	180.68	+0.12	-162.50	-162.39	-79.26	-79.25	-0.11	-0.01
Feb.	29	7 1.6	97.40	247.11	-0.12	+244.93	+245.76	-31.81	-32.03	-0.83	+0.22
	1	6 55.7	78.88	258.23	+0.08	+253.46	+253.18	+49.82	+50.43	+0.28	-0.61
	6	7 4.9	320.32	171.92	-0.04	-109.75	-24.04	+132.28	+99.14	[-85.71]	[+33.14]
	10	7 7.0	267.70	213.24	-0.03	-213.04	-211.26	-8.56	-7.69	-1.78	-0.87
	11	6 46.6	258.70	208.01	+0.07	-204.05	-203.26	-40.77	-40.66	-0.79	-0.11
	20	8 51.6	92.47	251.14	-0.06	+250.85	+252.45	-10.82	-9.46	-1.60	-1.36
	21	7 7.6	86.85	259.36	0.00	+258.96	+257.93	+14.25	+15.86	+1.03	-1.61
	3	8 44.9	268.77	203.67	-0.04	-203.58	-202.31	-4.37	-3.09	-1.27	-1.28
	5	6 54.9	250.60								
	8	7 1.5	182.90	101.70	+0.01	-5.15	-4.25	-101.58	-100.59	-0.90	-0.99
Mar.	9	7 1.0	148.35	113.32	-0.20	+59.36	+60.59	-96.30	-96.01	-1.23	-0.29
	10	7 4.1	125.35	146.54	-0.20	+119.36	+120.74	-84.67	-83.97	-1.38	-0.70
	17	7 18.3	68.12	200.72	+0.15	+186.40	+185.18	+74.86	+76.52	+1.22	-1.66
	18	8 27.8	56.30	171.83	+0.15	+134.80	+133.30	+89.90	+90.85	+1.50	-0.95
	20	7 21.9	7.02	95.43	+0.06	+11.67	+10.34	+94.78	+95.40	+1.33	-0.62

The time given is the time of the observation in position angle and Δs is the correction to reduce the observed distance to that time.

In the equations of condition for correcting the assumed elements, the mean motion and the mean distance have been considered as independent, making seven unknowns, and the fractional part of itself by which the mass must be increased makes an eighth.

The form of the equation * arising from the residuals in x is

$$a_1 \cdot dE_0 + a_2 \cdot d\mu + b_1 \cdot edP + c_1 \cdot de + d_1 \cdot d\Delta + e_1 \cdot \sin J \, dN + f_1 \cdot dJ + g_1 \cdot m = n,$$

where

$$\begin{aligned} a_1 &= \frac{f'}{\cos \varphi} \left\{ \cos (F+u) + e \cos (F'+P) \right\}, \\ a_2 &= a_1 [t - \text{Jan. 0. 0, 1885}], \\ b_1 &= -\frac{f'}{\cos \varphi} \left\{ \cos (F+u) \left(\cos \varepsilon \cos \varphi + \frac{e}{1 + \cos \varphi} \right) + \cos (F'+P) \right\}, \\ c_1 &= \frac{f'}{\cos \varphi} \left\{ \cos (F+u) \sin \varepsilon - \sin (F'+P) \cos \varphi \right\}, \\ d_1 &= \frac{\xi}{\Delta}, \\ e_1 &= -f' \tan \frac{1}{2} i \frac{r}{\Delta} \cos (F+u) \cos w - \frac{\rho_0 r}{\rho \Delta} \cos f \cos u, \\ f_1 &= f' \tan \frac{1}{2} i \frac{r}{\Delta} \cos (F+u) \sin w + \frac{\rho_0 r}{\rho \Delta} \cos f \sin u, \\ g_1 &= \text{perturbation in } x \text{ produced by Titan,} \end{aligned}$$

in which

$$f' = \frac{\rho_0}{\rho} \sin f,$$

and

$$F' = F - n + w.$$

The corresponding equation in y is obtained by replacing f , F , and ξ by g , G , and η respectively, and where g_1 = perturbation in y produced by Titan.

In order to facilitate the solution of the normal equations the following transformations were made in the equations of condition:

$$\begin{aligned} dE &= dE' + \sin J \, dN - dJ = \frac{1}{2} (x' + w' - t') \\ dE' &= \frac{1}{2} x' & d\Delta &= \frac{1}{2} v' \\ d\mu &= \frac{1}{10} y' & \sin J \, dN &= \frac{1}{2} w' \\ edP &= \frac{1}{3} z' & dJ &= \frac{1}{2} t' \\ de &= \frac{1}{4} u' & m &= \frac{1}{10} m'. \end{aligned}$$

The transformed equation will then be

$$a_1' x' + a_2' y' + b_1' z' + c_1' u' + d_1' v' + e_1' w' + f_1' t' + g_1' m' = n,$$

where the coefficients are given in the following table.

* See *Astronomische Nachrichten*, Nr. 193.

The first column gives the Greenwich Mean Time corrected for the aberration time of the observation in position angle. The last column gives the final residuals resulting from the solution that immediately follows.

Coefficients of Equations of Condition in x .

Greenwich Mean Time.	a_1'	a_2'	b_1'	c_1'	d_1'	e_1'	f_1'	g_1'	n	v
1884										
Sept. 20.8086	+0.490	-0.993	+0.182	+0.490	+0.204	+0.443	-0.385	+1.032	+5.12	+0.07
22.8536	+0.330	-0.654	+0.236	+0.356	+0.457	+0.312	-0.273	+0.776	+3.59	+0.09
23.7968	+0.227	-0.446	+0.200	+0.302	+0.535	+0.226	-0.200	+0.579	+3.54	+0.95
23.8472	+0.221	-0.434	+0.197	+0.300	+0.539	+0.221	-0.196	+0.568	+2.47	-0.07
24.8070	+0.106	-0.206	+0.132	+0.268	+0.586	+0.122	-0.111	+0.340	+1.21	-0.34
24.8456	+0.101	-0.196	+0.129	+0.268	+0.587	+0.117	-0.108	+0.331	-0.04	-1.55
25.8086	-0.021	+0.041	+0.053	+0.267	+0.600	+0.011	-0.017	+0.096	-0.01	-0.49
25.8224	-0.023	+0.044	+0.052	+0.267	+0.600	+0.009	-0.015	+0.093	-0.05	-0.52
27.8261	-0.275	+0.518	-0.057	+0.357	+0.513	-0.217	+0.181	-0.326	-1.90	-0.23
28.7886	-0.383	+0.714	-0.055	+0.425	+0.420	-0.318	+0.270	-0.458	-2.88	-0.25
Oct. 1.8127	-0.559	+1.008	+0.199	+0.528	-0.022	-0.502	+0.442	-0.475	-4.61	-0.01
4.8597	-0.266	+0.464	+0.236	+0.305	-0.434	-0.264	+0.241	-0.137	-2.89	-0.25
5.8477	-0.075	+0.130	+0.108	+0.269	-0.485	-0.094	+0.091	-0.036	-1.39	-0.33
6.8569	+0.128	-0.219	-0.028	+0.297	-0.478	+0.090	-0.072	+0.063	-0.08	-0.59
11.8412	+0.527	-0.845	+0.164	+0.527	+0.155	+0.474	-0.411	+0.506	+3.81	-0.03
23.7051	-0.568	+0.776	+0.291	+0.514	-0.121	-0.517	+0.459	-0.250	-3.15	+0.59
25.7171	-0.355	+0.471	+0.302	+0.342	-0.408	-0.342	+0.310	-0.115	-2.90	-0.15
8.0594	[-0.187]	[+0.196]	[-0.014]	[+0.330]	[+0.605]	[-0.135]	[+0.110]	[+0.001]	[-3.46]	[-3.38]
9.0546	-0.318	+0.327	-0.048	+0.397	+0.532	-0.254	+0.215	-0.035	-0.28	+0.32
10.6037	-0.437	+0.440	-0.033	+0.474	+0.419	-0.366	+0.314	-0.049	-1.28	-0.14
12.6824	-0.588	+0.568	+0.143	+0.571	+0.106	-0.519	+0.455	-0.008	-3.06	-0.89
13.6487	-0.594	+0.563	+0.252	+0.552	-0.064	-0.535	+0.474	+0.028	-1.52	+0.95
14.6411	-0.541	+0.502	+0.327	+0.484	-0.233	-0.498	+0.445	+0.058	-1.67	+0.85
15.6193	-0.425	+0.386	+0.326	+0.390	-0.375	-0.402	+0.363	+0.070	-1.69	+0.52
16.6610	-0.240	+0.212	+0.230	+0.307	-0.479	-0.242	+0.221	+0.062	-1.19	+0.30
17.6353	-0.032	+0.028	+0.088	+0.285	-0.519	-0.057	+0.057	+0.045	-0.29	+0.27
20.6377	+0.492	-0.397	-0.095	+0.514	-0.288	+0.423	-0.369	+0.080	+1.03	-0.76
21.6194	+0.562	-0.442	-0.011	+0.571	-0.133	+0.491	-0.428	+0.120	+1.27	-0.74
21.6666	+0.564	-0.443	-0.006	+0.573	-0.126	+0.493	-0.430	+0.122	+2.30	+0.29
22.6063	+0.577	-0.443	-0.099	+0.581	+0.034	+0.511	-0.446	+0.156	+3.64	+1.66
24.6247	+0.475	-0.346	+0.258	+0.476	+0.357	+0.434	-0.378	+0.185	+2.26	+0.69
26.6165	+0.260	-0.179	+0.227	+0.335	+0.579	+0.256	-0.224	+0.146	+2.15	+1.09
29.6494	-0.157	+0.098	-0.014	+0.323	+0.629	-0.106	+0.087	+0.066	-0.32	-0.07
4.6198	-0.605	+0.319	+0.206	+0.574	-0.017	-0.540	+0.477	+0.226	-0.74	+0.46
7.5974	-0.304	+0.142	+0.259	+0.338	-0.460	-0.298	+0.270	+0.165	-0.68	+0.46
8.5828	-0.100	+0.045	+0.125	+0.291	-0.519	-0.118	+0.110	+0.055	-0.57	+0.12
9.5794	+0.118	-0.051	-0.024	+0.313	-0.516	+0.078	-0.065	-0.066	-0.58	-0.43
15.5784	+0.503	-0.155	+0.233	+0.504	+0.317	+0.455	-0.397	-0.136	+1.07	+0.58
19.5684	+0.025	-0.006	+0.093	+0.288	+0.654	+0.054	-0.049	-0.013	+0.17	-0.32
26.5105	-0.584	+0.052	+0.287	+0.534	-0.143	-0.527	+0.470	+0.007	-0.98	-0.19
1885										
Jan. 2.5323	+0.533	+0.027	-0.072	+0.546	-0.216	+0.460	-0.406	+0.001	-0.36	-0.54
8.5287	+0.191	+0.033	+0.188	+0.308	+0.605	+0.196	-0.170	-0.014	+0.96	+0.80
9.5436	+0.054	+0.010	+0.104	+0.285	+0.642	+0.078	-0.068	-0.025	-0.27	-0.66
10.5273	-0.084	-0.018	-0.020	+0.298	+0.637	-0.042	+0.036	-0.038	+0.47	-0.15
12.4723	-0.343	-0.086	-0.072	+0.411	+0.511	-0.274	+0.239	-0.075	+1.29	+0.41
13.5043	-0.458	-0.124	-0.048	+0.489	+0.387	-0.382	+0.335	-0.099	+0.30	-0.54
17.6180	-0.499	-0.176	+0.310	+0.455	-0.285	-0.459	+0.412	-0.128	-1.00	-0.68
21.5272	-0.230	+0.099	-0.089	+0.343	-0.470	+0.180	-0.160	-0.130	-0.87	-0.64
22.4990	+0.392	+0.176	-0.136	+0.432	-0.379	+0.327	-0.292	+0.201	-0.11	+0.07
29.4584	+0.220	+0.130	+0.189	+0.315	+0.573	+0.218	-0.190	+0.106	-0.83	-0.77
Feb. 1.4541	-0.178	-0.116	-0.040	+0.319	+0.590	-0.125	+0.110	-0.119	+0.28	-0.64
10.4612	-0.040	-0.034	+0.073	+0.274	-0.493	-0.062	+0.056	-0.029	-1.78	-1.40
11.4469	+0.160	+0.135	-0.058	+0.307	-0.474	+0.117	-0.104	+0.101	-0.79	-0.29
20.5329	+0.110	+0.113	+0.138	+0.272	+0.589	+0.124	-0.107	+0.001	-1.60	-1.54
21.4606	-0.011	-0.011	+0.065	+0.268	+0.602	+0.020	-0.016	-0.122	+1.03	+0.66
Mar. 3.5272	-0.068	-0.084	+0.107	+0.260	-0.472	-0.086	+0.078	-0.143	-1.27	-0.86
8.4549	+0.521	+0.703	+0.073	+0.525	-0.010	+0.455	-0.401	+0.931	-0.90	+0.29
9.4545	+0.502	+0.687	+0.170	+0.499	-0.141	+0.444	-0.390	+0.865	-1.23	+0.06
10.4565	+0.447	+0.621	+0.235	+0.442	+0.282	+0.402	-0.351	+0.742	-1.38	-0.12
17.4657	-0.338	-0.517	-0.040	+0.389	+0.432	-0.274	+0.241	-0.887	+1.22	-0.04
18.5139	-0.438	-0.678	0.000	+0.457	+0.311	-0.368	+0.325	-1.161	+1.50	+0.19
20.4680	-0.532	-0.846	+0.181	+0.504	+0.024	-0.470	+0.419	-1.493	+1.33	+0.45

Coefficients of Equations of Condition in γ .

Greenwich Mean Time.	a_1'	a_2'	b_1'	c_1'	d_1'	e_1'	f_1'	g_1'	n	v
1884										
Sept. 20. 8086	+0.093	-0.189	+0.187	+0.053	-0.213	-0.403	-0.243	+0.277	+0.80	+0.12
22. 8536	+0.179	-0.356	+0.266	+0.055	-0.129	-0.381	-0.050	+0.467	+0.87	-0.63
23. 7969	+0.201	-0.395	+0.294	+0.035	-0.076	-0.336	+0.051	+0.514	+2.80	+1.11
23. 8472	+0.202	-0.396	+0.295	+0.034	-0.074	-0.333	+0.056	+0.516	+2.93	+1.23
24. 8070	+0.211	-0.411	+0.308	+0.006	-0.015	-0.267	+0.156	+0.533	+1.73	-0.04
24. 8456	+0.212	-0.411	+0.308	+0.005	-0.013	-0.264	+0.160	+0.533	+2.07	+0.30
25. 8086	+0.208	-0.401	+0.303	-0.023	+0.047	-0.182	+0.251	+0.517	+1.55	-0.19
25. 8224	+0.208	-0.402	+0.303	-0.026	+0.048	-0.181	+0.253	+0.517	+1.32	-0.42
27. 8261	+0.162	-0.304	+0.244	-0.065	+0.160	+0.018	+0.391	+0.399	-0.24	-1.59
28. 7886	+0.120	-0.224	+0.206	-0.064	+0.201	+0.113	+0.425	+0.315	+0.56	-0.45
Oct. 1. 8127	-0.076	+0.136	+0.173	+0.021	+0.228	+0.304	+0.364	+0.055	-0.32	+0.37
4. 8597	-0.258	+0.450	+0.318	+0.030	+0.068	+0.188	+0.061	-0.037	-2.62	-0.17
5. 8477	-0.271	+0.460	+0.332	-0.007	-0.009	+0.091	-0.063	-0.038	-2.24	+0.37
6. 8569	-0.251	+0.427	+0.308	-0.043	-0.088	-0.019	-0.182	-0.022	-2.13	+0.31
11. 8412	+0.075	-0.121	+0.183	+0.047	-0.231	-0.376	-0.317	+0.151	-0.46	-0.70
Nov. 23. 7051	-0.133	+0.168	+0.203	+0.034	+0.218	+0.316	+0.335	+0.006	-0.63	+0.33
25. 7171	-0.251	+0.333	+0.313	+0.035	+0.104	+0.231	+0.122	-0.034	-2.27	-0.28
8. 6594	[+0.201]	[+0.210]	[+0.291]	[+0.057]	[+0.124]	[+0.086]	[+0.361]	[+0.142]	[-1.84]	[-2.44]
9. 6546	+0.166	-0.171	+0.251	-0.077	+0.179	+0.028	+0.422	+0.118	+0.32	+0.18
10. 6637	+0.116	-0.117	+0.209	-0.070	+0.222	+0.133	+0.457	+0.092	+0.28	-0.05
12. 6824	-0.022	+0.021	+0.171	-0.008	+0.253	+0.288	+0.435	+0.055	+0.41	+0.63
13. 6487	-0.098	+0.093	+0.193	+0.027	+0.236	+0.319	+0.380	+0.052	-1.97	-1.38
14. 6411	-0.176	+0.163	+0.242	+0.050	+0.196	+0.312	+0.293	+0.056	-0.29	+0.71
15. 6193	-0.239	+0.217	+0.300	+0.048	+0.136	+0.266	+0.183	+0.065	-0.86	+0.52
16. 6610	-0.280	+0.249	+0.340	+0.019	+0.055	+0.182	+0.047	+0.072	-1.96	-0.30
17. 6353	-0.286	+0.248	+0.351	-0.022	-0.028	+0.081	-0.086	+0.070	-2.49	-0.75
20. 6377	-0.128	+0.103	+0.206	-0.048	-0.227	-0.250	-0.370	+0.044	-1.22	-0.27
21. 6194	-0.050	+0.039	+0.176	-0.014	-0.253	-0.333	-0.386	+0.047	-0.92	-0.32
21. 6666	-0.046	+0.036	+0.176	-0.012	-0.254	-0.337	-0.385	+0.048	-0.98	-0.40
22. 6063	-0.025	-0.020	+0.176	+0.023	-0.257	-0.396	-0.359	+0.059	-1.24	-0.95
Dec. 24. 6247	+0.149	-0.109	+0.243	+0.065	-0.202	-0.444	-0.203	+0.094	-0.50	-0.60
26. 6105	+0.218	-0.150	+0.319	+0.039	-0.091	-0.382	+0.023	+0.117	-0.10	-0.34
29. 6494	+0.211	-0.132	+0.308	-0.056	+0.112	-0.121	+0.343	+0.099	+0.77	+0.53
4. 6198	-0.077	+0.040	+0.186	+0.021	+0.248	+0.311	+0.416	+0.096	-1.70	-1.40
7. 5974	-0.274	+0.128	+0.338	+0.037	+0.083	+0.220	+0.105	+0.202	-0.99	+0.02
8. 5828	-0.292	+0.131	+0.358	-0.002	+0.000	+0.126	-0.032	+0.214	-0.90	+0.22
9. 5794	-0.274	+0.117	+0.337	-0.041	-0.084	+0.015	-0.167	+0.202	-1.56	-0.46
15. 5784	+0.135	-0.042	+0.239	+0.067	-0.216	-0.441	-0.249	+0.005	+0.02	+0.28
19. 5684	+0.232	-0.053	+0.336	-0.021	+0.025	-0.262	+0.205	-0.010	-0.77	-0.51
26. 5105	-0.133	+0.012	+0.213	+0.040	+0.225	+0.311	+0.374	+0.000	-0.01	+0.41
Jan. 1885										
2. 5323	-0.093	-0.005	+0.189	-0.033	-0.242	-0.264	-0.392	+0.001	-0.73	+0.16
8. 5287	+0.224	+0.038	+0.327	+0.023	-0.059	-0.358	+0.048	+0.011	-0.29	+0.29
9. 5430	+0.230	+0.044	+0.334	+0.011	-0.012	-0.285	+0.166	+0.012	-0.59	-0.01
10. 5273	+0.220	+0.046	+0.319	-0.042	+0.075	-0.197	+0.270	+0.013	-0.09	+0.45
12. 4723	+0.160	+0.040	+0.248	-0.073	+0.187	+0.004	+0.421	+0.008	-0.16	+0.23
13. 5043	+0.106	+0.029	+0.204	-0.063	+0.228	+0.110	+0.462	+0.000	-0.01	+0.28
17. 6180	-0.196	-0.069	+0.257	+0.057	+0.177	+0.290	+0.293	-0.074	+0.57	+0.95
21. 5272	-0.243	-0.105	+0.301	-0.055	-0.123	-0.047	-0.217	-0.098	-0.81	-0.19
22. 4990	-0.184	-0.083	+0.245	-0.059	-0.139	-0.139	-0.315	-0.075	-0.01	+0.54
29. 4584	+0.215	+0.120	+0.312	+0.036	-0.075	-0.366	+0.007	+0.121	+0.22	+0.96
Feb. 1. 4541	+0.199	+0.129	+0.291	-0.054	+0.118	-0.128	+0.322	+0.114	-0.61	+0.04
10. 4012	-0.274	-0.227	+0.337	-0.009	-0.018	+0.111	-0.038	-0.182	-0.87	-0.57
11. 4469	-0.250	-0.213	+0.309	-0.043	-0.095	+0.013	-0.168	-0.166	-0.11	+0.21
20. 5329	+0.216	+0.222	+0.314	+0.003	-0.022	+0.305	+0.092	+0.167	-1.36	-0.35
21. 4606	+0.214	+0.224	+0.309	-0.029	+0.037	-0.236	+0.190	+0.171	-1.61	-0.63
3. 5272	-0.205	-0.331	+0.325	-0.015	-0.007	+0.117	-0.020	-0.582	-1.28	-0.88
Mar. 8. 4549	+0.002	+0.003	+0.158	+0.008	-0.235	-0.314	-0.359	-0.036	-0.99	-0.50
9. 4545	+0.066	+0.091	+0.175	+0.036	-0.224	-0.362	-0.318	+0.081	-0.29	+0.32
10. 4565	+0.120	+0.167	+0.209	+0.051	-0.196	-0.385	-0.248	+0.176	-0.70	-0.06
17. 4657	+0.135	+0.206	+0.210	-0.073	+0.178	+0.027	+0.384	+0.223	-1.66	-0.99
18. 5139	+0.082	+0.126	+0.173	-0.058	+0.212	+0.121	+0.415	+0.109	-0.95	-0.47
20. 4680	-0.047	-0.075	+0.159	+0.003	+0.222	+0.244	+0.390	-0.216	-0.62	-0.40

The preceding equations of condition give rise to the following normal equations:

$$\begin{aligned}
 &+10.6599x' - 4.9428y' - 0.7420z' - 0.5865u' + 0.4991v' + 5.7803w' - 5.8657t' + 6.5644m' = + 31.729 \\
 &- 4.9428 + 13.7942 + 0.4683 + 0.7065 - 1.2946 - 2.3888 + 2.8449 - 0.7958 = - 54.675 \\
 &- 0.7420 + 0.4683 + 6.0657 + 2.6029 + 0.5369 - 1.7429 + 1.8137 + 2.5627 = - 8.790 \\
 &- 0.5865 + 0.7065 + 2.6029 + 10.3640 + 2.0978 - 0.4960 + 0.2194 + 1.0209 = - 0.751 \\
 &+ 0.4991 - 1.2946 + 0.5369 + 2.0978 + 13.1182 + 3.0067 + 1.5943 + 0.6893 = + 16.096 \\
 &+ 5.7803 - 2.3888 - 1.7429 - 0.4960 + 3.0067 + 10.8764 - 3.7764 + 3.2694 = + 18.089 \\
 &- 5.8657 + 2.8449 + 1.8137 + 0.2194 + 1.5943 - 3.7764 + 10.0671 - 3.2194 = - 18.058 \\
 &+ 6.5644 - 0.7958 + 2.5627 + 1.0209 + 0.6893 + 3.2694 - 3.2194 + 13.4947 = + 18.583 \\
 &[m] = 310.52
 \end{aligned}$$

Solving these equations we obtain

$x' = +0.549$	± 0.250	$dE = +0$	1.2	± 2.6
$y' = -3.633$	± 0.152	$dP = -1$	36.4	± 10.0
$z' = -2.010$	± 0.242	$dN = -0$	26.8	± 12.9
$u' = +0.389$	± 0.167	$dJ = +0$	0.4	± 1.4
$v' = +0.871$	± 0.149			
$w' = -0.343$	± 0.191	$de = +0.00045$		± 0.00017
$t' = +0.055$	± 0.199	$d\mu = -0^{\circ}.00971$		$\pm 0^{\circ}.00035$
		$dA = +0''.436$		$\pm 0''.065$
$m' = +1.300$	± 0.183	$m = +0.1300$		± 0.0156
		$[nn.8] = 46.14$		
		$[vv] = 46.05$		

The final elements of Hyperion and mass of Titan are as follows:

Epoch: Jan. 0.0, 1885, Greenwich Mean Time.

$E = 359$	36.8	± 2.6
$P = 335$	48.9	± 10.0
$N = 124$	13.0	± 12.9
$J = 5$	53.5	± 1.4
$e = 0.11188$		± 0.00017
$\mu = 16^{\circ}.88601$		$\pm 0^{\circ}.00035$
$A = 214''.841$		$\pm 0''.065$

$$m = \frac{1}{4172 \pm 58}$$