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ORBIT OF HALLEY'S COMET.

The following observations have been made the basis of an orbit for the present apparition of Halley's Comet:

Date	Gr.M.T.	α 1909.0	δ 1909.0
1909 Sept. 17.85720		94° 44' 51".6	+17° 08' 59".2
1909 Dec. 16.77347		49 56 33.7	+13 59 51.8
1910 Feb. 28.63910		8 32 55.9	+ 7 53 50.0

The first of these was made by Barnard; the second is a normal place formed from four observations by Aitken; the third was made by Aitken. The coördinates given are those referred to the mean equinox of 1909.0.

The elements given by Cowell and Cromellin in the November, 1909, number of the *Monthly Notices* were used for a preliminary orbit. A second orbit was then derived which was based upon observations of September 17, October 23, and December 16. Upon the receipt of the observation of 1910 February 28, the middle place was shifted to December 16. With this accurately represented, the second orbit gave the following residuals:

		$\cos \delta \Delta \alpha$	$\Delta \delta$
1909 Sept. 17		+ 0".8	+ 0".8
1910 Feb. 28		-61.9	-20.2

By a single application of Leuschner's method of differential corrections, using his closed expressions for δf and δg , these were completely removed. The resulting elements and constants for the equator are:

ELEMENTS.

$$\begin{aligned}
 T &= 1910 \text{ April } 19.67760 \text{ Gr. M.T.} \\
 \omega &= 111^\circ 43' 09".32 \\
 \Omega &= 57 \quad 16 \quad 18.09 \\
 i &= 162 \quad 12 \quad 55.57
 \end{aligned}
 \left. \vphantom{\begin{aligned} \omega \\ \Omega \\ i \end{aligned}} \right\} 1910.0$$

$$\begin{aligned}
 \log e &= 9.985 \quad 5082 \\
 \log a &= 1.252 \quad 5224 \\
 \log q &= 9.768 \quad 6346
 \end{aligned}$$

CONSTANTS FOR THE EQUATOR 1910.0.

$$\begin{aligned}
 x &= r [9.9851681] \sin (145^\circ 44' 10".60 + v) \\
 y &= r [9.9884654] \sin (239 \quad 17 \quad 47.94 + v) \\
 z &= r [9.5354893] \sin (189 \quad 02 \quad 16.88 + v)
 \end{aligned}$$

The perturbations due to the action of Mars at its nearest approach to the comet, during January, 1910, were computed and found to be ineffective.

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April 8, 1910.

PARABOLIC ELEMENTS AND EPHEMERIS OF COMET *a* 1910.

Owing to pressure of other work, the computation of an orbit of Comet *a* 1910 had not been contemplated in the Berkeley Astronomical Department. Nevertheless, the difficulties encountered by various computers in deriving a satisfactory orbit from three observations at moderate intervals by the usual methods (*cf.* the preliminary orbits in A.N. 4385, 4386, 4387, and A.J. 609, 611) seemed to call for a test of Professor Leuschner's short methods in their present form. When the computation of an orbit

was decided upon, the following six observations were available at the Students' Observatory: the visual observations by Innes, January 16; by Hussey, January 24, both of which were received by telegraph; by Aitken, February 5, received by mail; and the three photographic positions secured by Curtis with the Crossley reflector on February 1, 2, 5 (*L. O. Bulletin*, 174), also received by mail. At the Students' Observatory the trees surrounding the domes precluded observations of the comet.

The photographic observations were selected for the computation of an orbit partly for lack of information concerning the first available visual observation and partly because they were made by the same observer and with the same instrument. The shortness of the first interval would probably have proved an advantage in any method, but the difference in the intervals, which are in the ratio 3 to 1, afforded a severe test of the convergency of the short methods.

The direct solution for a parabola, or first approximation, gave for the geocentric distance at the middle date: $\log \rho_0 = 0.14796$, and the following residuals:

O-C	I	III
$\cos \delta \Delta \alpha$	+3"	-55"
$\Delta \delta$	+2	-79

The geocentric distance is obtained graphically and without the usual cumbersome approximations. In the present case the scale was so chosen that the graphical method gave a direct solution for ρ_0 correct to four decimal places.

This value of ρ_0 agrees to two decimals with the value of ρ for the same date in Kobold's orbit (A.N. 4387), which is based on four places at nearly equal intervals between January 20 and January 30. It is therefore evident, without proceeding to the computation of the elements, that the direct solution yielded a fair approximation to the orbit and that the residuals given above could have been easily removed by a single approximate differential correction, as provided for in the short methods. It was decided, however, to leave this orbit in its approximate form and to compute a new orbit from an arc of considerable length, so that a rigorous numerical test might be had on the convergency of the method for correcting a roughly approximate orbit on the basis of an arc of *any* length and under the most unfavorable circumstances, with or without hypothesis regarding the eccentricity.

Aitken's observation of March 13, which was kindly communicated by mail, increased the arc to the desired length and the selection of an observation near perihelion at the minimum of heliocentric distance and at the maximum velocity in the orbit, introduced as unfavorable conditions as were available for testing the convergency of the formulae.

The correction of the approximate orbit derived from the photographic positions was based on the following observations:

1910 Gr.M.T.	Comet's Apparent			Observer
	α	δ		
Jan. 18.1287	20 ^h 14 ^m 09 ^s .70	-20° 53' 40".8	Zappà (Rome)	
Feb. 5.6211	21 46 40.73	+ 5 34 54.1	Aitken	
March 13.0440	22 24 43.41	+15 38 40.9	Aitken	

The geocentric distance and the heliocentric velocities at the new middle date in the approximate orbit were:

$$\log \rho_0 = 0.177170, \log x_0' = 0.052100, \log y_0' = 0.026952, \\ \log z_0' = 9.813836,$$

and the residuals for the first and third places corresponding to these values and to the new middle date were:

O-C	I	III
$\cos \delta \Delta \alpha$	-52.9	-14.1
$\Delta \delta$	-57.6	-24.5

These residuals would have been considerably smaller, if the residuals in the approximate orbit for February 1 and 5 had been removed. But the larger residuals were preferred so as to render the test on the convergency of the formulae more severe.

A first differential correction, on the assumption $e = 1$, reduced the residuals as follows:

O-C	I	III
$\cos \delta \Delta \alpha$	-292.9	+ 3.4
$\Delta \delta$	-302.5	+31.2

Without recomputing the differential coefficients, the substitution of these residuals in the absolute terms of the equations resulted in:

O-C	I	III
$\cos \delta \Delta \alpha$	+0.7	+ 6.0
$\Delta \delta$	-5.1	+21.0

In order to determine whether these residuals were due to a slight deviation of the orbit from a parabola or to the use of linear differential relations, the differential coefficients were recomputed on the basis of the corrected values of the geocentric distance and of the heliocentric velocities at the middle date and the last set of residuals introduced in the absolute terms. This differential correction produced the following final parabolic residuals:

O-C	I	III
$\cos \delta \Delta \alpha$	-1.9	-3.0
$\Delta \delta$	-1.2	+0.5

The removal of these residuals without assumption regarding the eccentricity would produce fictitious results, as they are comparable with the errors of observation.

ELEMENTS.

$$\begin{aligned} T &= 1910 \text{ January } 17.0888 \text{ Gr. M.T.} \\ \omega &= 320^\circ 54' 40".3 \\ \Omega &= 88 \ 47 \ 24.1 \\ i &= 138 \ 46 \ 47.9 \\ q &= 0.128980 \end{aligned} \left. \vphantom{\begin{aligned} T \\ \omega \\ \Omega \\ i \\ q \end{aligned}} \right\} 1910.0$$

CONSTANTS FOR THE EQUATOR 1910.0.

$$\begin{aligned} x &= r [9.876399] \sin (322^\circ 31' 10".6 + v) \\ y &= r [9.981395] \sin (\ 67 \ 42 \ 17.5 + v) \\ z &= r [9.856390] \sin (354 \ 32 \ 23.5 + v) \end{aligned}$$

EPHEMERIS FOR GREENWICH MEAN MIDNIGHT.

1910		True α	True δ	Log. Δ	Br.
April	9.5	22 ^h 38 ^m 41 ^s .6	+20° 59' 01"	0.4208	0.04
	13.5	39 48.0	21 44 04		
	17.5	40 38.8	22 28 51	0.4294	0.04
	21.5	41 13.0	23 13 24		
	25.5	41 30.3	23 57 40	0.4354	0.03
April	29.5	41 29.6	24 41 38		
May	3.5	41 10.4	25 25 15		
	7.5	40 30.5	26 08 22	0.4404	0.02
	11.5	39 29.9	26 50 55		
	15.5	38 07.2	27 32 42	0.4415	0.02
	19.5	36 21.4	28 13 31		
May	23.5	22 34 11.7	+28 53 10	0.4413	0.02

Brightness February 5 = 1.00.

The solution of the preliminary and of the final orbit was accomplished without difficulty and without arbitrary variation of the geocentric distance or of any other quantity.

While this bulletin was in press, the following position by Aitken was received by telegraph:

1910 April 12.9998 Gr. M.T., α 22^h 39^m 39^s.5, δ +21° 38' 23"

According to this observation the correction to the ephemeris on April 13.0 was $\cos \delta \Delta \alpha = -0^s.9$, $\Delta \delta = -4''$.

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April 8, 1910.

ELLIPTIC ELEMENTS OF COMET *e* 1909 (DANIEL).

In illustration of Professor Leuschner's short methods of computing orbits from three observations, we have computed two orbits for Comet *e* 1909 (Daniel), the one from a moderate, the other from a long arc. The first orbit is based on the following observations by Aitken, which were kindly communicated by mail:

1909 Gr. M.T.	α	Comet's Apparent δ
Dec. 11.74000	6 ^h 17 ^m 39 ^s .13	+38° 06' 15".4
15.70788	18 04.46	+41 12 21.0
Dec. 18.75589	6 18 07.80	+43 26 00.0

It will be observed that the geocentric motion is unfavorable to an accurate derivation of the required initial velocities and accelerations in right ascension and declination when the third and higher differences are neglected so that this example furnishes a good test of the convergency of the formulae.

The direct solution, first approximation, for a parabola resulted in the following residuals:

O-C	I	III
$\cos \delta \Delta \alpha$	+ 26".7	+ 0".1
$\Delta \delta$	+631.8	+350.5

Professor Leuschner has shown in his investigations, which are soon to be published, that the middle place and the initial velocities and accelerations can always be represented by a parabola, no matter how erroneous the latter may be, but that residuals will appear in the first and third places, either if the initial velocities and accelerations are inaccurate or if the orbit is not parabolic. It is possible, in general, by the application of a simple criterion, to decide at this stage of the computation to which of these two causes the residuals are to be ascribed and to complete the solution of the elements without hypothesis regarding the eccentricity, if the residuals are not found to be wholly due to the neglect of third and higher differ-

ences in deriving the initial velocities and accelerations from the three observations.

Although in the present case the criterion clearly established the periodicity of the orbit, it was evident that the residuals were due in part to the neglect of the third and higher differences of the observed coördinates. The computation of the elements was therefore not immediately completed without hypothesis regarding the eccentricity, but a very approximate differential correction of the parabolic geocentric distance and of the heliocentric velocities at the middle date was undertaken so as to reduce the residuals, on the one hand with, and on the other without hypothesis regarding the eccentricity. This double solution was readily accomplished on account of the convenience with which it is possible in the short methods to pass from one class of orbit to another. The resulting residuals were:

	PARABOLA.		ELLIPSE.	
	I	III	I	III
$\cos \delta \Delta \alpha$	+ 9".2	+ 1".4	+ 5".6	- 0".9
$\Delta \delta$	+416.1	+317.9	+54.2	-15.2

These residuals confirm the conclusion arrived at by the criterion referred to above, that the observations cannot be satisfied by a parabola. The removal of the residuals for the ellipse by a single application of the differential formulae produced the following elements:

ELEMENTS.

$T = 1909$ November 28.1924 Gr. M.T.
 $\omega = 03^{\circ} 02' 01''.7$
 $\Omega = 71 \quad 09 \quad 32.0$
 $i = 19 \quad 54 \quad 39.0$
 $e = 0.62524$
 $\mu = 496''.1411$
 $\log a = 0.569601$
 Period = 7.15157 years

The publication of these elements was deferred so that they might be tested and improved on the basis of later observations. In the meantime they were verified by the results published by Ebell in A. N. 4384.

The long arc orbit is based on the following observations:

Dec. 7.66050	6 ^h 16 ^m 42 ^s .10	+34° 44' 22".0	Barnard (Yerkes)
Dec. 18.75589	6 18 07.80	43 26 00.0	Aitken
March 3.47726	7 25 03.4	+52 54 42	Rambaud (Algiers)

The first and third of these positions are represented as follows on the basis of the first orbit:

O-C	I	III
$\cos \delta \Delta \alpha$	-1".5	+319".4
$\Delta \delta$	+9.3	-735.8

The application of the differential formulae on the basis of closed expressions for δf and δg so as to remove the residuals resulted in

O-C	I	III
$\cos \delta \Delta \alpha$	-0".7	+ 3".6
$\Delta \delta$	-0.7	-11.7

Without recomputation of the differential coefficients the substitution of these residuals in the absolute terms of the equations produced the following results, which represent the three given positions:

ELEMENTS.

$T = 1909$ November 28.7238 Gr. M.T.	
$\omega = 03^\circ 28' 43".9$	} 1910.0
$\Omega = 70 59 43.4$	
$i = 19 26 48.1$	
$e = 0.602481$	
$\mu = 547".5362$	
$\log a = 0.541063$	
Period = 6.48030 years	

CONSTANTS FOR THE EQUATOR 1910.0

$x = r [9.977341] \sin (163^\circ 24' 50".1 + v)$	
$y = r [9.944549] \sin (83 43 06.4 + v)$	
$z = r [9.755574] \sin (44 49 24.1 + v)$	

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