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THE RELATION BETWEEN THE SEPARATION AND THE MAGNITUDE
OF VISUAL DOUBLE STARS.

In carrying out the survey of the sky to secure data for statistical studies relating to double stars I have been impressed with the fact that the number of close pairs discovered far exceeds the number of pairs separated by moderate distances. More than 70 per cent. of the 3500 pairs so far discovered in the course of this work have measured angular distances of 2" or less, and about 50 per cent., distances of 1" or less.

This is obviously due in large part to the facts, (1) that the survey is carried on systematically, pains being taken to examine every star as bright as 9.0 B.D. magnitude on nights good enough to realize nearly the full power of the telescope, and (2) that a larger percentage of the easier pairs have been discovered by earlier observers. But the question arises whether the closer double stars are not in reality more numerous than the wider ones, and, further, whether the numbers do not show a progressive increase with diminishing angular separation, even when parallax¹ is taken into account.

It was in the latter form that the question was put to me by Dr. Campbell about a year ago.

When the double-star survey mentioned is completed, data will be at hand for a thorough discussion of this question, but at that time the best material available for its investigation was Burnham's "General Catalogue of Double Stars within 121° of the North Pole," supplemented by the lists of new pairs published later in the L. O. Bulletins.

A count was made under my direction by the Secretary of the Observatory of all the double stars in Part I of Burnham's Catalogue and the pairs listed according to angular distance and the assigned magnitude of the brighter component. Stars fainter than

9.5 magnitude, or wider than 5"0, were not included, and no attention was paid to those of Sir John Herschel's stars whose distance he only estimated.² To the figures thus obtained I subsequently added the stars Hu 1201-1327, and A 1251-1900. Table I shows the result, the rows giving the numbers for each 0.5 of angular separation to 5"0, the columns the numbers according to magnitude. The first column includes all pairs for which the principal component is as bright as 6.0 magnitude, the next three those for which the brighter component lies between 6.1-7.0, 7.1-8.0, and 8.1-9.0 respectively; and the fifth column, those in which both components are as faint as 9.1.

It is of course true that such a classification is merely approximate, as the magnitude estimates are those of the discoverers, as a rule, and are by no means systematic. However, it will appear that this does not materially affect our main conclusion.

TABLE I.
DOUBLE STARS UNDER 5"01 AND HAVING ONE COMPONENT AS
BRIGHT AS 9.5 MAGNITUDE IN BURNHAM'S GENERAL
CATALOGUE OF DOUBLE STARS.

	5.5 Mag.	6.5	7.5	8.5	9.5	Total	Total corrected for parallax. ³
Under 0"51	43	88	257	609	279	1276	1048
0.51-1.00	51	110	260	599	236	1256	1119
1.01-1.50	43	70	220	502	157	992	1079
1.51-2.00	37	66	159	405	145	812	717
2.01-2.50	25	51	160	308	93	637	495
2.51-3.00	32	42	122	266	72	534	469
3.01-3.50	24	48	81	237	66	456	306
3.51-4.00	30	30	94	161	52	367	221
4.01-4.50	17	35	69	188	39	348	188
4.51-5.00	15	28	60	167	25	295	167
Total	317	568	1482	3442	1164	6973	5809

² No appreciable error is introduced by the omission of the John Herschel stars, for, as I have found by actual comparison with the sky, very few indeed of them are as bright as 9.0 magnitude, except the very wide pairs, 15" to 60" or more.

³ The column 9.5 is not included in the summation given in the final column.

¹ It is clear that we should expect to find a much greater number of close pairs than of wide pairs assuming an approximately uniform distribution in space of binary systems and neglecting parallax.

Inspection of the table shows that under each magnitude, as well as in the column headed "Total," the number of pairs increases with decreasing angular distance. That the increase is less marked when we pass from the second row (distance 0^h51 to 1^h00) to the first (distance under 0^h51) is only what might be expected from the far greater difficulty of detecting the very close pairs. It will also be noticed that the number of double stars with distances under 5^h0 increases rapidly with the numerical increase of magnitude, up to the magnitude 8.5 inclusive. The falling off at magnitude 9.5 is due simply to the fact that the stars fainter than 9.0 have not been systematically examined by double-star observers.

To correct our results for parallax we may assume that Kapteyn's average parallaxes represent the *relative* distances of the stars of different magnitudes. The absolute parallaxes, it is obvious, are of no importance in this connection. To reduce the observed values to what they would be if all of the double stars were removed to the distance of the average 8.5 magnitude star, we may assume that the angular separation of the stars in the first row averages 0^h3 and in the following rows 0^h75, 1^h25, etc., and divide these values by the factors taken from the following table:

Magnitude	Factor
5.5	2.8
6.5	2.0
7.5	1.4
8.5	1.0

Summing the stars after this division, we obtain the last column of the table. The stars of 9.5 magnitude are not included in this summary because we know that the figures given do not at all adequately represent the numbers of pairs of this magnitude.

To check the results obtained by this investigation I have recently made an accurate count⁴ of the double stars under 5^h0 distance in the region of the sky north of +36° declination, omitting a small area in the second quadrant of Right Ascension which has not yet been fully examined in my survey. The sky area examined includes 44634 stars of 9.0 B.D. magnitude or brighter.

Table II shows the distribution of the double stars in this area according to angular distance and to the B.D. magnitude of the stars. The observers' magnitude estimates are thus eliminated, and only the known systematic errors of the B.D. magnitudes affect our results. I have not considered it necessary in this preliminary investigation to take these into account.

The next to the last column in this table shows the results corrected for parallax, *i.e.*, the numbers of double stars of the various distance classes if all were removed to the distance of the average star of 8.75 magnitude. It was assumed that the average magnitude of the pairs in the first column is 5.5 and the average separation of the stars in the first two rows combined, 0^h3, the mean values of magnitudes and of angular distances being taken for the rest of the table. The parallax factors employed are:

Magnitude	Factor
5.5	3.1
6.25	2.4
6.75	2.0
7.25	1.7
7.75	1.4
8.25	1.2
8.75	1.0

These factors suit Kapteyn's parallax values of the I type stars as well as those for the II type.

TABLE II.

DOUBLE STARS UNDER 5^h01 IN THE ZONE +36° TO +90°.

	6.0 Mag. or less	6.1 to 6.5	6.6 to 7.0	7.1 to 7.5	7.6 to 8.0	8.1 to 8.5	8.6 to 9.0	Total	Total corrected for parallax	Corrected total from Table I divided by 2.68
0 ^h 25 or less	7	8	13	16	24	40	38	146	558	391
0.26-0.50	7	7	32	37	62	94	157	396		
0.51-1.00	16	12	16	31	50	85	164	374	449	418
1.01-1.50	10	14	19	22	45	71	105	286	343	403
1.51-2.00	13	8	16	26	39	47	75	224	212	268
2.01-2.50	5	5	7	16	25	29	73	160	211	185
2.51-3.00	12	5	11	19	28	45	56	176	128	175
3.01-3.50	5	1	8	8	16	21	50	109	93	114
3.51-4.00	7	5	7	13	21	26	38	117	69	83
4.01-4.50	3	4	4	7	14	19	46	97	71	70
4.51-5.00	2	0	7	3	12	25	35	84	35	62
Total	87	69	140	198	336	502	837	2169	2169	2169

⁴ The count was made directly from the charts used in the survey of the sky; hence it includes every pair under 5^h0 and brighter than 9.1 B.D. magnitude (both the previously known pairs and those newly discovered) that were visible with the telescopes of this observatory.

The final column of Table II is derived from the last column of Table I by dividing by the factor 2.68, in order that the results of the two tables may be compared directly. It will be seen that the more accurate magnitude (and distance⁵) values of Table II bring out more clearly than the figures of Table I the marked increase in the number of double stars as the angular distance diminishes, and, moreover, the progressive nature of this increase. This general conclusion is, however, also to be derived from Table I, and seems therefore to be definitely established.⁶

Table II also shows the increase in the number of double stars of every distance class under 5"0 with increasing numerical magnitude, but the increase is not as rapid as that of the total number of stars after we pass the magnitude 7.0. This will appear from Table III. The second line in this table is formed on the assumption that the magnitude distribution of the stars in the area examined is the same as that in the Northern Hemisphere as a whole. Since the stars in this area are 44 per cent. of all those to 9.0 B.D. magnitude in the Northern Hemisphere, the assumption seems legitimate.

TABLE III.

	6.0 Mag. or brighter	6.1 to 6.5	6.6 to 7.0	7.1 to 7.5	7.6 to 8.0	8.1 to 8.5	8.6 to 9.0	Total
Double Stars under 5"01	87	69	140	198	336	502	837	2169
B.D. Stars	937	893	1741	2678	4910	10132	23343	44634
Ratio	1:11	1:13	1:12	1:14	1:15	1:20	1:28	1:20.6

From this table we may infer either that the number of close visual double stars (under 5"0) is relatively greater among the brighter stars, or that a larger percentage of the latter has been discovered. If the second explanation is the true one, the progressive increase in the number of double stars as the angular separation diminishes is still greater than that shown by Table II, for it is only the very close and difficult pairs that will have escaped detection.

⁵ In forming the second table the distances used are the most recent ones that I could find. They are therefore all referred to approximately the same epoch.

⁶ That Mr. Gavin J. Burns (*Jour. B. A. A.*, 12, 245, 1902) arrived at a different conclusion from his investigation of the Struve and Burnham stars is due to the incompleteness of the data these afford.

The fact that one star in about $13\frac{1}{2}$ of those as bright as 8.0 B.D. magnitude in this area is a double star under 5"0, while among the stars between 8.0 and 9.0 magnitude the ratio is only 1 to 25, is exceedingly interesting and possibly significant. It will be recalled that observations by Campbell, Frost and others show that one star in 5 or 6 among those with well determined radial velocities is a spectroscopic binary, and that the known spectroscopic binaries, excluding the variable stars, have an average magnitude of 3.9.

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