ON THE COMPUTATION OF THE MOON'S SPECTROGRAPHIC VELOCITY NEAR FULL MOON

The determination of the moon's spectrographic velocity from American Ephemeris data involves the use of the cosine of the angle (E) at the earth's center between the sun and moon, and also the product $\sin E \frac{dE}{dt}$. E and the reciprocal of its rate of change, $\frac{dt}{dE}$, are regularly tabulated in the Nautical Almanac except for seven or eight days at full moon, when our satellite is often most favorably situated for spectrographic observation. As a result, the facility of computation of the moon's radial velocity is somewhat impaired, though the problem presents no difficulty. $\cos E$ is computed directly and $\sin E \frac{dE}{dt}$ is obtained by numerical differentiation of $\cos E$, or more simply by means of differential formulae.

A complete discussion of the five components of velocity to be considered in this case has been given by Professor Campbell.* Components V_3 and V_4 only are to be considered here and they will be expressed invariably in kilometers per second. V_3 is the component of V_2 (the radial velocity of the moon with reference to the earth's center) in the line joining the sun and moon. V_4 is the component in this same line of the moon's velocity normal to the radius vector drawn from the moon to the earth. The formulae expressing these quantities are as follows:

$$V_3 = -V_2 \cos E,$$

$$V_4 = [4.6856] D_2 \sin E \frac{dE}{dt},$$

where D_2 is the moon's distance in kilometers from the earth's center and $\frac{dE}{dt}$ is expressed in seconds of arc per second of time. $\cos E$ and $\sin E \frac{dE}{dt}$ may be computed as follows:

Let

- A = the sun's right ascension,
- α = the moon's right ascension,
- D = the sun's declination,
- δ = the moon's declination.

Then

$$\begin{aligned} \cos E &= \sin \delta \sin D + \cos \delta \cos D \cos(A - \alpha), \quad (1) \\ \sin E \frac{dE}{dt} &= \sin \delta \cos D \left[\cos(A - \alpha) \frac{d\delta}{dt} - \frac{dD}{dt} \right] \\ &\quad + \cos \delta \sin D \left[\cos(A - \alpha) \frac{dD}{dt} - \frac{d\delta}{dt} \right] \\ &\quad + \cos \delta \cos D \sin(A - \alpha) \frac{d(A - \alpha)}{dt}. \quad (2) \end{aligned}$$

First Method.—This involves the computation of three values of $\cos E$, and a simple numerical differentiation. It is most convenient to compute $\sin E \frac{dE}{dt}$ in radians per hour by numerical differentiation of $\cos E$.

Then

$$V_3 = -V_2 \cos E,$$

$$V_4 = [6.4437] D_2 \sin E \frac{dE}{dt} = 278 \times 10^{-6} \times D_2 \sin E \frac{dE}{dt}.$$

Example.—Determine $\cos E$ and $\sin E \frac{dE}{dt}$, in radians per hour, for 1903 November 7^d 19^h 58^m.

November 7	19 ^h	20 ^h	21 ^h
A	222° 13' 57"	222° 16' 27"	222° 18' 57"
D	—16 15 14	—16 15 59	—16 16 43
α	83 54 1	84 31 11	85 8 22
δ	18 16 14	18 17 01	18 17 41
$A - \alpha$	138 19 56	137 45 16	137 10 35
$\log \sin \delta$	9.49624	9.49655	9.49680
$\sin D$	9.44699 _n	9.44732 _n	9.44764 _n
$\cos \delta$	9.97753	9.97750	9.97747
$\cos D$	9.98228	9.98226	9.98223
$\cos(A - \alpha)$	9.87333 _n	9.86939 _n	9.86537 _n
$\cos \delta \cos D \cos(A - \alpha)$	9.83314 _n	9.82915 _n	9.82507 _n
Add log	0.05264	0.05317	0.05370
$\sin D \sin \delta$	8.94323 _n	8.94387 _n	8.94444 _n
Diff. log	0.88991	0.88528	0.88063
$\log \cos E$	9.88578 _n	9.88232 _n	9.87877 _n
$\cos E$	—0.76874	—0.76264	—0.75643
$\frac{d \cos E}{dt} = -\sin E \frac{dE}{dt}$		+ 0.00610	+ 0.00621
$\frac{d^2 \cos E}{dt^2}$			+ 0.00011
[19 ^h 58 ^m] $\cos E$		— 0.763	
[19 ^h 58 ^m] $\sin E \frac{dE}{dt}$		— 0.00615	radians per hour. The equivalent of — 0'353 per second.

* Astrophysical Journal, 11, 141, 1900.

Second Method.—Both formulae (1) and (2) are employed. The hourly change in A and D , and the velocity in α and δ per minute are tabulated in the Almanac. It is convenient to reduce all these quantities mentally to seconds of arc per minute or per second of time, varying the factor in V_4 accordingly. All computations are accomplished with Crelle's Rechen tafeln and three-place tables. The formulae for V_3 and V_4 when $\frac{dE}{dt}$ is expressed in seconds of arc per second are:

$$V_3 = -V_2 \cos E,$$

$$V_4 = [4.6856] D_2 \sin E \frac{dE}{dt} = 485 \times 10^{-8} \times D_2 \sin E \frac{dE}{dt}.$$

Example.—Determine $\cos E$ and $\sin E \frac{dE}{dt}$ in seconds of arc per second of time, for 1903 November 7^d 19^h 58^m.

		1903	November 7 ^d 19 ^h 58 ^m .	
A	222° 16'		$\frac{dA}{dt}$ per second	+0.042
D	—16 16		$\frac{d\alpha}{dt}$ " "	+0.619
α	84 30			
δ	18 17		$\cos(A-\alpha) \frac{d\delta}{dt}$	—0.009
$A-\alpha$	137 46		$\frac{dD}{dt}$ per second	—0.012
$\sin \delta$	0.314		$\frac{d\delta}{dt}$ " "	+0.012
$\sin D$	—0.280		$\cos(A-\alpha) \frac{dD}{dt}$	+0.009
Product	—0.088			
$\cos \delta$	+0.960		$\cos(A-\alpha) \frac{d\delta}{dt} - \frac{dD}{dt}$	+0.003
$\cos D$	+0.950		$\sin \delta \cos D$	+0.3
$\cos(A-\alpha)$	—0.740		I Product	+0.001
Product	—0.675		$\cos(A-\alpha) \frac{dD}{dt} - \frac{d\delta}{dt}$	—0.003
$\cos E$	—0.763		$\cos \delta \sin D$	—0.3
			II Product	+0.001
			$\frac{dA - d\alpha}{dt}$	—0.577
			$\sin(A-\alpha)$	+0.672
			$\cos \delta \cos D$	+0.912
			III Product	—0.354
			[I+II+III] $\sin E \frac{dE}{dt}$	—0.352
			per second of time.	

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