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I.—A PROPOSED METHOD FOR THE MEASUREMENT AND REDUCTION OF SPECTROGRAMS FOR THE DETERMINATION OF THE RADIAL VELOCITIES OF CELESTIAL OBJECTS

II.—APPLICATION TO A STUDY OF THE VARIABLE STAR W SAGITTARI*

INTRODUCTION

Beginning thirty-five years ago, when Huggins and Vogel first made application of Doppler's principle to the determination of radial velocities of stars, the methods employed in this work have been developed until at present an efficient and practically uniform system has been adopted in the various astrophysical observatories. In 1888, when the attempt was made at Potsdam to record photographically the displacement of lines in the stellar spectra, the problem was placed on a firm basis. Subsequently, through improvement in the construction of spectrographs, a more accurate knowledge of the wave-lengths of spectral lines, and signal advances in the methods of measurement and reduction of the spectrograms, the results have attained their present degree of accuracy. The probable error of 22 km. per second for an average case of velocity determination with the older methods has been reduced to a few tenths of a kilometer at the present time.

Flexure and temperature change during exposure may be said to play no part in the performance of the modern properly-equipped spectrograph, and other observational sources of error are well under control. Undoubtedly, the spectrograms of to-day contain information regarding the velocities of celestial light-sources more precise than our present methods of measurement and reduction are capable of bringing out. In these methods there exist certain recognized sources of accidental and systematic error which

must be eliminated if the coveted hundredth of a kilometer can ever have any significance.

Evidences of the extent of uncertainty which characterizes the spectrographic results of the last eight years can be obtained by comparison of determinations of velocity from the same star with different instruments. In earlier work strong systematic differences were encountered. In measures of the star Zeta Herculis* the difference Campbell (5 plates) — Bépolsky (5 plates) = -3.5 km. Two more plates by Bépolsky greatly diminish this difference, but increase the probable error of his mean so signally that their inclusion is hardly consistent. For Zeta Geminorum† the residual Campbell (44 plates) — Bépolsky (15 plates) = -6 km. For Alpha Persei‡ the residual Campbell (4 plates) — Vogel (13 plates) = -0.8 km. In the case of Polaris§, Vogel finds: Campbell — Belopolsky = -3 km.; and for Campbell — Frost = $+2.2$ km. For Capella||, the difference Campbell (31 plates) — Newall (31 plates) = $+3$ km. For purposes of this comparison the investigations of standard velocity stars have not progressed far enough as yet. But from observations made at Yerkes Observatory Frost and Adams conclude that "there would appear to be a slight tendency toward a systematic difference between our results and those of other observers in the direction of a larger positive, or a smaller negative, value for our

* Astrophysical Journal, Vol. VIII, page 157.

† Ibid., Vol. XIII, page 90.

‡ Ibid., Vol. XIII, page 320.

§ Ibid., Vol. XIV, page 52.

|| Ibid., Vol. XII, page 251.

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velocities.”* That any large systematic discrepancies will be brought out by these results from standard stars, is highly improbable. The instruments involved have been in use long enough to be thoroughly under control, and the choice of lines from the star and check-plates would naturally be governed by their performance in previous comparisons with the theoretical velocities of the moon and planets. An uncertainty of at least several tenths of a kilometer, which, I think, is largely attributable to sources of error in the treatment of the plate, undoubtedly remains in spectrographic velocity determinations.

PART I.

METHODS OF MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

The plan of procedure that has been widely adopted, subject to slight variation in the case of different observers, may be briefly described. The spectrogram is measured twice, direct and reversed, on an engine supplied with a microscope of adjustable magnifying power, rigidly mounted over a plate-carriage which is moved by a delicate screw and micrometer system in one co-ordinate only, in the direction of extension of the spectrum. A set of measures includes the list of micrometer readings which correspond to the positions of the plate-carriage when the various spectral lines, both bright and dark, occupy in turn a given position with reference to some fixed point in the focal plane of the microscope. These readings, together with the laboratory determinations of the wave-lengths of the spectral lines employed for the measurements, constitute the data necessary for the determination of the radial velocity of the source emitting the light, in terms of the accepted standards of velocity. From the micrometer readings and wave-lengths corresponding to not less than three known lines in the comparison or solar spectrum, the constants of the Hartmann-Cornu interpolation formula† are easily computed. By this formula micrometer readings are readily transformed into approximate wave-lengths, and vice versa. If, now, each plate is to be reduced independently, as at Yerkes Observatory,‡ the constants of the empirical interpolation curve are computed as above from three or more comparison lines. Corrections to this curve are determined by comparison of the adopted and interpolated wave-lengths of the comparison lines, and are plotted as functions of the micrometer readings or the λ 's themselves. It then remains to compute by means of this corrected interpolation formula the

wave-lengths corresponding to the micrometer settings on the lines of the spectrum to be measured. We thus obtain the actual wave-lengths of these lines as affected by the motion of their source relatively to the observer. From the known wave-lengths of these lines, occurring in the spectrum of a light-source at rest with reference to the observer, a simple application of Doppler's principle will yield the desired radial velocity of the star. If, on the other hand, the dispersions of one or more solar plates* be taken as standards, by measures on the comparison lines of a stellar spectrogram, the micrometer readings on the star lines themselves are reduced to the selected dispersion and may be compared directly with the computed readings of the same lines in the fundamental table. In point of accuracy, there seems to be no choice between these two methods, but the latter one is undoubtedly shorter. The sources of error which are shared by both, I shall briefly review.

SOURCES OF ERROR IN THE REDUCTION AND MEASUREMENT OF SPECTROGRAMS.

(a) *Change in film.*—The actual shape of the photographic film while the plate is being exposed is never reproduced, owing to distortions ascribable to temperature change and the processes of development and drying, but the effect of these distortions on velocity determinations is minute and intangible, especially when many well distributed comparison lines are employed.

(b) *The measuring engine.*—(1) Errors in screw.—Errors due to inequalities in the pitch of the micrometer screw, whether periodic or otherwise, are usually negligible in modern engines. They may, of course, be eliminated by application of small corrections to the measures, but are essentially inoperative in most velocity work.

(2) Temperature change during measures.—The effect of temperature change on the measuring engine during measurements may play an important part unless considerable care is exercised. Measures should not be begun until the micrometer-head has been in contact with the hand for a short time, and during the measures the room temperature should be kept nearly constant. In my own work, after completing a plate, I have tested the temperature shift by setting back on the lines first measured, and have invariably found some discrepancy. In one solar plate in particular, which had required considerable time for measurement, this shift amounted to about .003 mm., or the equivalent of seven kilometers' velocity. This led me to test the character of the shift by remeasurement of some of the remaining comparison lines on

* Astrophysical Journal, Vol. XVIII, page 276.

† Astrophysical Journal, Vol. VIII, page 218. Astronomische Nachrichten, Nr. 3702.

‡ Publications of the University of Chicago, Vol. VIII, page 150. Astrophysical Journal, Vol. XV, No. 1.

* Astronomische Nachrichten, Nr. 3702. Astrophysical Journal, Vol. VIII, page 124.

the plate. The plot of differences thus obtained approximated a straight line which gradually approached the original curve. Evidently the temperature change was progressive, and was entirely taken up in the first curve. Here, again, the advantage of an even distribution of star and comparison lines is apparent. Evidently, also, the "smoothing out" of curves of residuals from interpolation formulæ should not be practised without due caution, for temperature change during the measures may produce very appreciable distortions in such a plot.

(c) *Subjective errors.*—Discrepancies arising from accidental errors of setting and from personal equation are among the most important to be considered. The former can be reduced only by long practice and the employment of a greater number of lines; the latter have been largely eliminated by the reversal of the spectrogram on the engine,* but it should be noted that in the reversal of a plate the spectrum is simultaneously inverted. Thus, in general, it would hardly be said that the effect of personal equation is entirely compensating in the two measures, for the appearance of a line after inversion might be so changed as seriously to interfere with the duplication of the conditions which existed before reversal. Further, in case of a straight-slit spectrograph the curvature† of the spectral lines would seriously interfere with the elimination of personal equation in this way.

(d) *Errors due to the assumption of wave lengths.*—

(1) An instrumental defect.—A source of error by no means inappreciable is found in the relative displacement of comparison and absorption lines due to imperfect focal conditions which obtain outside of given narrow regions of the spectrum, corresponding to the rays which pass through the prism train at minimum deviation, or corresponding to the intersections of the focal curve with the plane surface of the photographic plate. The so-called wings, which seriously impair the definition of the outlying lines of the comparison

spectrum, undoubtedly persist far into the region of apparently sharp focus, giving rise to errors in the measured positions of the bright lines. The corresponding displacements of the absorption lines of the stellar spectra, due to the same cause, are certainly much smaller, if at all appreciable. This difference is due to the fact that much light is concentrated in the bright lines, many of which are strongly exposed in order to bring out the fainter features of the comparison. Thus the wings, though they share a relatively small part of the light, may become quite as dense as the sharp part of the lines, for the density of the sharp part of the lines is but slightly affected by light after a certain time, while the wings continue to spread.

The nature of the effect of this source of error is difficult to predict, for it depends entirely upon the optical parts of the spectrograph and the character of the adjustments. If the focal curve of the camera lens is tangent to the plane of the surface of the photographic plate in the region of the spectrum corresponding to minimum deviation, the error arising would certainly differ from that which exists when the focal curve intersects the plate in two lines, though the character of the difference would be very uncertain. The results of a few investigations of actual measures with various instruments appear in the accompanying table. For each set of measures, the means were formed of the velocity in kilometers as obtained from a given number of lines to the violet, to the red, and at minimum deviation. These means are indicated by V , R , and M respectively, and the means of the differences between these quantities for an entire set of plates are given under the heads, $V-R$, $V-M$, and $R-M$. The number of lines used on each plate in the formation of the means V , R , and M is given in another column, and the approximate range in wave-length between the means V and R appears under the head, *Range*. The probable errors of the quantities determined are also added.

TABLE 1.

Spectro- graphs.	$V-M$	$R-M$	$V-R$	No. lines on each plate.	No. of measures of plates.	Range.
Bruce			$+0.16 \pm 0.08$ km.	6	84	98μ
Pulkowa			-0.85 ± 0.20 km.	6	35	200μ
Mills	-1.76 ± 0.16 km.	$+1.10 \pm 0.20$ km.	-0.66 ± 0.14 km.	30	20	180μ

The results for the Bruce and Pulkowa spectrographs were obtained from data recently published in connection with the velocity determinations of standard stars. The measures of Frost and Adams for each plate were reduced separately. The results for the

* L. O. Bulletin, No. 15, and Astrophysical Journal, Vol. XV, page 208.

† Curved slits are used on the Mills spectrographs. They give straight spectral lines, which is a decided advantage.

Mills spectrograph were obtained from a set of measures of negatives of Polaris, which afforded better material than the data used for the other instruments. The range covered by the Yerkes measures is so small that no marked effect could be expected, but for the other instruments there seems to be a fairly consistent difference between measures of lines in different parts of the spectrum. These results may be masked by

errors in wave-lengths employed, but the discrepancies are strong enough to call for further investigation.

(2) Physical differences.—Aside from subjective errors of measurement, there is no greater source of uncertainty in the reduction of spectrograms than that which arises from the assumption of wave-lengths for the star and comparison lines from Rowland's solar tables. It is doubtful if the conditions of density and internal motion in the sun and spark are sufficiently alike to warrant this procedure. But until better determinations of spark wave-lengths are available, there seems to be no alternative. A careful discussion of the matter by Frost and Adams* has led to the conclusion that "In the present state of our knowledge we . . . cannot say with any certainty how much our results are affected by the use of solar wave-lengths for our Ti lines; but presumably by an amount corresponding to less than 0.02 tenth meters, or about 1.4 kilometers, and perhaps very much less." Wave-lengths in star and sun may differ by small amounts, due to pressure, etc., but it seems practicable to assume agreement in the case of solar-type stars. In other cases the chances seem favorable.

(3) Errors in Rowland's table.—Rowland's determinations of solar wave-lengths are probably relatively accurate to the nearest hundredth of an Ångström Unit. Their absolute value need be known only approximately for velocity work.

(4) Errors in practice.—In actual performance some apparently good lines are found to yield consistently poor results, which usually leads to their final rejection. Errors in wave-lengths or intensities in Rowland's table, or close lines in the star, may be responsible for the trouble. However that may be, there are undoubtedly among the lines retained many which are similarly in error, but in a smaller degree. It simply remains to eliminate such lines as soon as they can be detected, though the uncertainty of proceeding in the dark in this regard is far from satisfactory.

There is a possibility of actual introduction of this form of error in the use of blends formed by weighting lines according to their intensities in Rowland's tables. The occurrence of enhanced lines is one great obstacle. But, further, in determining wave-lengths by blending, it must be remembered that the intensities of Rowland's lines were intended only for purposes of identification, and were not prepared with sufficient care to warrant more exacting applications. The characteristics as regards sharpness of lines comprising a blend would undoubtedly affect the apparent blended wave-length, though such characteristics could not be included in intensity estimates without considerable

difficulty. Again, an intensity a applies to all lines between $a + .5$ and $a - .5$ in Rowland's scale, or, in other words, the uncertainty of any intensity in Rowland's table amounts to ± 0.5 of a unit. The possible error in a blend of this kind is well illustrated by the well-known combination of Ti, 2, 4427.266 and Fe, 5, 4427.482 into a line of wave-length 4427.420. If, however, the two intensities are 1.5 and 5.5, or again 2.5 and 4.5, the resulting wave-lengths are 4427.436 and 4427.405 respectively. Thus, under the assumption that the intensities in Rowland's table are as accurate as possible, an uncertainty of 0.015 A.U. (the equivalent of 1 km. in velocity) must be recognized in the blend. It would therefore seem advisable with present methods to confine one's choice to the 75 or 80 good single lines that are available in most high-dispersion instruments, rather than to include tempting groups whose wave-lengths are subject to uncertainties in addition to those incident to the laboratory determinations.

(e) *Elimination of these errors.*—For errors due to change of film and inaccuracies in the measuring engine, including temperature change during measures, adequate remedy has been proposed, but no elimination of the remaining uncertainties incident to present methods seems possible without a radical change in the system employed. As long as the wave-lengths determined from measures with the grating or interferometer by an observer of one personal equation, are employed by another observer of a different personal equation to represent his measures of corresponding lines as produced by a different spectroscope, it can hardly be expected that the results of various observers will be entirely consistent. The problem demands that the conditions which obtain in the production, measurement, and reduction of a spectrographic plate of a star should be exactly duplicated in the production, measurement, and in general the reduction, of the plate from which the fundamental data for velocity determinations are secured. The same spectrograph should produce both plates. They should be measured by the same observer with the same measuring engine. And, finally, they should be reduced in parallel. These requirements are clearly most exacting, but as a suggestion of a means toward this end, I propose the following method, which, as far as I know, has never been applied in this way.

A PROPOSED METHOD FOR THE MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

Proceeding as in case of a stellar exposure with the spectrograph in final adjustment, from which it must not be disturbed, a source of accurately known velocity, such as the sky or sun, is photographed with the comparison in the usual manner. Then the measures upon

* Publ. Univ. of Chicago, Vol. VIII, page 155.

SPECTROGRAPH I.

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the comparison and continuous spectra of this plate with some engine by any observer will constitute his fundamental standard table for the engine and spectrograph used. These measures of the bright and dark lines of a plate of the sky or sun and spark will fix the relative positions of the Fraunhofer and comparison lines for a known velocity, and will constitute a velocity-standard table. For the determination of the velocity of any celestial object of the same characteristics as the standard source, it is only necessary to duplicate the exposure and measures of the standard plate and compare directly the relative positions of corresponding lines of the two sources referred to the comparison spectrum of each plate. In the reduction of a stellar plate for velocity determination, we first reduce the star measures to the dispersion of the standard plate by forming a simple plot with micrometer readings as abscissæ and differences in settings on comparison lines as ordinates. For the star lines the reductions to fundamental dispersion are read directly from this plot. If these reductions are applied to the readings on the stellar spectrum, the measured positions of any line on the sun and star plates will differ by an amount proportional to the relative velocity toward the earth of the star and sun.

The assumptions in this method are fundamental in every case of velocity determination. Further assumptions treated above as incident to other methods are here eliminated by the comparison of artificial lines with artificial lines and dark lines with dark lines. The advantages of this simple system need only be mentioned. Errors in the screw of the measuring engine are eliminated for small displacements, such as are found in stellar spectra, if the standard and star plates occupy identical positions on the plate-carriage during measures. Personal equation is well controlled without the reversal of the spectrogram, and may be closely followed by frequent measures of standard plates. It may even seem preferable not to reverse the plate, for, as I have suggested, the curvature of lines may introduce a variation of this unaccountable source of error. The magnitude of accidental errors of setting is not affected by the use of this method, but with the greater number of lines available in solar-type stars particularly, the reliability of the means from any plate is clearly increased. Uncertainties due to imperfect focal conditions are not only largely eliminated if the exposures on comparison and star are consistent in star and standard plates, but the region of spectrum available for measurement can be extended without fear of encountering difficulty. The number of lines available is further augmented for solar-type stars by the possibility of including blends, which are clearly reliable under this method. Until tests are made with high-dispersion instruments, it is difficult

to predict the extent of the advantage of this increase of the measurable lines for this case; but for low-dispersion instruments in my own experience with a magnifying power of twenty-five, the number of available lines increased from twenty to one hundred and seventy; and with a power of ten, though practically no lines could be used with methods depending on wave-lengths, sixty-five lines gave good results for velocity determinations. But for the present the chief advantage of the method lies in its independence of accurate relative or absolute values of the wave-lengths or intensities of the spectral lines of the spark or continuous spectrum. For the determination of the factors necessary to convert micrometer displacements into kilometers, rough relative values of the wave-lengths of three favorable spark lines are needed, but, aside from this, there is no necessity for data of this kind. It is possible to make final definitive measures at once.

In the application of this method any degree of refinement can be introduced at the will of the observer. The number of plates employed to form a standard table can be increased until the full possibilities of the spectrograph and measuring engine are realized. By comparison of standard plates any errors due to temperature change in the instrument can be detected, and tables can be prepared for different temperatures, if desired. Standard plates can be prepared occasionally and measured, along with the regular observing list, as checks on the constancy of the results. For absolute checks of the performance of the method lunar and planetary spectrograms are of course available, but the determination of the sky or solar velocities furnish equally reliable checks, and at the same time afford data for strengthening the original standard tables. In general, three sky or sun plates should furnish sufficient data for a fundamental table, though the conditions obtaining in any particular case might call for a greater or smaller number.

As a source of known velocity for the standard plates the sun is perhaps superior to the sky, but if exposures on the sky are more convenient there should be little, if any, hesitation in their use. The objection to the use of sky light lies in the fact that the lines in its spectrum are subject to a slight broadening due to the sun's rotation. This would hardly exceed $0.020 \text{ \AA.U.}$, which would occasion no impairment of the measures of lines which, in most favorable cases, are many times this amount in width. As star-light itself possesses the integrated characteristics of skylight, it may seem preferable to use the sky in the preparation of standard plates.

This velocity-standard method is best applicable to solar-type stars, but it may, if desired, be modified to suit the requirements of hydrogen or other stars with-

out the introduction of errors greater than those incident to present methods. Thus a standard table for any class of stars may be prepared by introducing in the spark, in addition to the regular comparison, those elements which produce good lines in the stellar spectra. The assumption is made that the relative positions of the spark and dark lines of the elements used are not affected differently either by the dissimilar physical conditions in the spark and star or by the peculiarities of the spectrograph; but this is an assumption commonly made in all spectrographic work.

PART II.

AN APPLICATION OF THE VELOCITY-STANDARD METHOD.

(a) *Apparatus employed.*—Necessity for some departure from the present methods of measurement and reduction of spectrograms first arose in the attempt to improve the accuracy of relative velocity determinations from spectrum plates made with *Spectrograph I* of the Lick Observatory. The performance of this instrument is well known in connection with the spectrographic studies by Campbell, Wright, H. D. Curtis, and Stebbins, of the spectra of Nova Persei, Nova Geminorum, and α Ceti, and with velocity determinations by Campbell of 1830 Groombridge. As the result of this work, Professor Campbell was led to remark that "The greatest interest in the observation lies in the fact that fairly accurate determinations of stellar velocities are shown to be possible down to the eighth or ninth photographic magnitudes, provided the spectra contain well-defined lines."

The importance of permanent adjustment of this spectrograph was realized from the first, but the use of nearly all its parts in the Mills Spectrograph necessitated a practical reconstruction of the single-prism instrument whenever it was used. The Mills Spectrograph was recently rebuilt, and all the parts of Spectrograph I were made permanently available with the exception of the collimator lens, the slit head and the comparison-spectrum apparatus. Professor Campbell asked me to design these parts in order to complete the instrument.* A slit-head† was adapted from Keeler's star spectroscope, and the comparison apparatus was patterned closely after the device proposed by Wright.‡ The new collimator lens was essentially a counterpart of the old one. The focal plane of the camera lens was found to be nearly flat. No wedge was required under the plate holder with $H\gamma$ light at

minimum deviation, and excellent definition for velocity work was secured from 3800 - 4600 Å.U.

A brief description of Spectrograph I appears in Bulletin No. 8 of the Lick Observatory. A more detailed description of some of the parts is given in Director Campbell's article on the Mills Spectrograph, but for the sake of completeness the principal constants will be repeated here.

Focal length collimator,	720mm
Aperture collimator,	37mm
Refracting angle prism,	60°
Focal length camera lens,	406mm
Dispersion per t. m. at $H\gamma$,	8"

The performance of this spectrograph is exceptionally good. The limit of resolving power with lantern-slide plates is about 0.50 Å.U. at $H\gamma$, which is all that could be expected of an instrument of one-fifth the dispersion of the Mills. However, with rapid emulsions, the increased size of silver grains interferes seriously with the definition. Careful tests by Dr. J. Stebbins and Dr. J. H. Moore have shown that all effects of flexure have been successfully eliminated in the construction of this instrument. But the effect of temperature change on the positions of lines is very marked. This is largely due to the unequal expansion in the triangle formed by the steel collimator tube, the brass camera tube and the brass tie-rods which extend from the collimator tube nearly to the plate holder. The extent of this shift has been determined by Dr. Moore. It is equivalent to approximately 36 kilometers in velocity per degree Centigrade change in temperature. The limitations of the instrument are thus readily appreciated. In my own work, in the absence of a temperature case, the spectrograph has been wrapped in several thicknesses of woolen blanket just before the exposure began. At the same time all the windows in the dome were closed. These precautions, together with the frequent introduction of the comparison, have made good results possible, but it has been almost invariably the case that the best velocity determinations have been made from plates for which the temperature range was a minimum. Recently a temperature case and automatic thermostat have been constructed for this instrument. These may be expected to result in very material improvement in its performance.

Further disadvantages are inherent in an instrument of low dispersion and resolving power. Linear defects in lines produce five times the error that they would occasion with the Mills Spectrograph. Probably no more than fifteen or twenty single lines could be measured with any power on any one plate of a solar-type star. Stebbins found only six coincidences of solar and stellar lines for α Ceti in a region of 340 Å.U., while in a region covering 120 Å.U. he found

* All these improvements, as well as a constant temperature case and thermostat, have been supplied by a grant from the Draper Fund of the National Academy of Sciences.—W. W. C.

† Described in Publications of the Lick Observatory, Vol. III, page 174.

‡ Astrophysical Journal, Vol. XII, page 274.

Twenty such lines on Mills plates. With a power of ten, absolutely no single lines would be available for velocity determinations. In view of these facts, it would seem that few tests could be more exacting than the application of the velocity-standard method to this case. Spectrograms made with an instrument of this kind, with no greater protection against temperature change, are affected by errors which accurate measurements serve to bring out; but it was one aim of the author to determine the degree of accuracy to be expected from an instrument of low dispersion, for its field of usefulness is practically inexhaustible, as it reduces the exposure time required with the Mills Spectrograph by ninety per cent.

(b) *The fundamental velocity tables.*—Following the plan above described, Zero-Standard TABLES II and III have been prepared from three sky and iron plates with a Toepler measuring engine, whose least reading is 0.00025 millimeters. These tables will be directly useful only to myself in the reduction of plates made last year, but will serve as a complete illustration of the method, if not as guides to other observers in the choice of lines. Corrections for radial velocity and diurnal rotation of the earth have not been applied, as they were considered negligible. The fundamental dispersion was obtained from a star plate at the mean temperature for the series to be measured. To this dispersion all the plates have been separately reduced. The results for all lines of wave-length greater than 4400 Å.U. are inferior, for the reason that beyond this point the sensitiveness of the lantern-slide emulsion drops suddenly, and the character of the iron lines suffers marked decline, but it has been considered wise to include them in the measures, as the three comparison lines at 4900 Å. U. afford a good control for the dispersion curve.

Wave-lengths of all the lines, sufficiently accurate for purposes of identification, appear in the first column of both tables. The second column contains the factors to convert micrometer displacements into kilometers. The quantity, V_s , which enters into these factors was taken directly from Frost's "Scheiner's Astronomical Spectroscopy."

Columns three and four of TABLE II contain the micrometer readings on the star and iron lines, respectively. The number of measures on which each reading depends is given in column six. In addition to the three standard plates, five other star plates were included in the determination of the iron lines.

All measures for TABLE II were made with a magnifying power of twenty-five, while TABLE III was prepared for a power of ten or fifteen. In forming the original low-power table, the measures of two standard plates with a power of ten were separately reduced to the fundamental dispersion by comparison with the

high-power table of comparison lines, which could then be adopted for the comparison-line readings of the low-power table. On comparison of this table with measures of star plates some discrepancies were found. These would arise from actual errors in the table or a difference in the wave-lengths of the lines in sun and star. Several measures in TABLE II which came into agreement with the low-power star measures were then introduced into the low-power table. Seventeen of the best of the remaining new or discordant lines in the star were determined from thirty plates and are included in TABLE IV.

In the measurement of thirty plates of W Sagittarii, made in 1903, the one aim was to obtain the best possible relative velocities without any sacrifice of absolute determinations. As the mean error in the setting of any line in the original table was found to be equivalent to about 2.3 km. velocity, it was conceivable that a velocity determination from the lines measurable on any one plate would differ slightly from a corresponding determination from any other plate, due to errors in the fundamental table. To eliminate this possibility a rather laborious method of reduction was employed. In the first place, all the plates were reduced by the original table. The residual for each line was then formed for every plate, and the mean of all the residuals for any line was applied as a correction to the reading for that line in the original table.

The weighted mean of the readings on any line occurring in the original table and in the table thus prepared from thirty plates was adopted as the final setting for that line. These means for all the lines appear in column three of TABLE III. The relative accuracy of these values thus depends upon measures of thirty-three plates, while their absolute accuracy depends upon the three original standard plates. With this new table, all the plates were reduced again.

As a further move toward greater relative accuracy, the seventeen additional star lines which were determined from thirty plates were used with the original standard lines in the reductions. In TABLE IV, the results for these seventeen lines are given under headings similar to those in TABLE III.

The settings for the iron lines in column four of TABLE III depend upon low-power measures of thirty-one plates which were reduced to the fundamental dispersion, and averaged. Column five contains the probable errors of the quantities appearing in the two preceding columns as determined from the observations on star plates only. The number of these observations is given in column six.

In column five of TABLE II and column seven of TABLE III a brief description of each line is given. The character of the line is indicated by the letters vG

(very good), G (good), F (fair), and P (poor). Then follows the intensity according to Rowland's scale. The remaining comments are self-explanatory, but include the following arbitrary abbreviations: b, broad; L, line; Gr, green; V, violet; inc, included; shp, sharp; gp, group; B, bright; Cl, close; I, intensity; and f, faint.

Column eight of TABLE III contains the differences between the settings for the various lines in the original solar table and the corresponding values in the final table in column three. Regarding these as residuals for the measures of the original table, the probable error of the absolute velocities as affected by the errors in the original low-power table is seen to be ± 0.24 km.

No correction for curvature has been introduced into either of these tables. It should be noticed that this correction is entirely differential in its nature with this method of measurement and is negligible if the slit-length is not altered. The formulæ for the computation of this small correction, based on the assumption of a parabolic curvature of the spectral lines, are as follows:

For Table II, $dV = -(y^2 - 1.2) 0.5$ km.,

For Table III, $dV = -(y^2 - 1.1) 0.46$ km.,

where y is the distance in units of $\frac{1}{4}$ millimeters from the center of the spectrum to the measured part of the comparison line.

STANDARD TABLE II. (Magnifying Power, 25.)

Wave Lengths.	r V s	Micrometer Readings. Sky.	Iron.	Notes.	Observations.
3969.4			13.879	G λ 3969.413	7
82.0	760	14.928		F-G10 b	3
87.0	767	15.634		G 20	3
94.7			16.668	F	3
4005.3			17.421	G λ 4005.370	8
05.3	778	17.423		G 20	3
06.8	781	17.562		F 15 b	3
12.5	781	18.100		F 10 b. Shp Ls to V	3
13.9	782	18.252		F 10 b	3
14.2	782	18.288		F 20. Mean	3
14.6	783	18.338		P 8	3
18.0	784	18.625		F 18	3
21.8			19.009	F	2
22.0	787	19.023		G 10 b. L F 6 at $-.070$	3
25.0	789	19.301		F 15. f Ls to V	3
28.6	791	19.634		F-P 8 b	2
29.8	792	19.748		F 6	2
30.9	792	19.845		G 15	3
30.9			19.847	F-G	8
32.0	793	19.978		P 8	2
33.0	793	20.057		F-G 10 b	3
34.6	794	20.201		G 9	3
35.8	795	20.313		G 8	2
40.2	797	20.711		F 5	2
40.9	798	20.778		F 7	2
41.7	800	20.853		F 8 b	3
44.0	801	21.083		G 8	3
44.8	801	21.162		F 6	3

Wave Lengths.	r V s	Micrometer Readings. Sky.	Iron.	Notes.	Observations.
46.0			21.249	G λ 4045.975	8
46.0	802	21.252		G 20	3
48.8	804	21.550		F 7	3
52.4	806	21.871		F 18 b	2
55.3	807	22.090		F 18 b	3
57.6	809	22.330		G 10	3
63.7			22.878	G λ 4063.705	8
63.9	812	22.882		G 17. L at $-.075$	3
71.9			23.613	G λ 4071.895	8
72.0	818	23.615		G 15. L at $-.075$	3
77.9	822	24.174		G 10	3
78.6	822	24.252		P 6 b	3
79.4	823	24.313		F 8 b. f Ls to Gr	3
92.7	832	25.463		G 12	3
96.2	836	25.787		F 8	3
98.5	836	25.978		F 10	3
4100.2	838	26.115		F 7. Mean 2 cl. Ls	3
04.4	840	26.524		F 6. L F 8 b at $+.060$	3
06.5	842	26.688		F 5	3
07.6			26.777	F	4
07.7	842	26.782		F 7	2
15.0	847	27.418		F 12 b	3
16.7	848	27.561		F 10 b	2
18.8			27.730	G λ 4118.806	8
18.9	850	27.741		G 12	3
21.8	851	27.997		F-P 10. Mean 2 Ls	3
23.9	853	28.163		F 8 b	3
26.0	854	28.339		G 7	3
26.2	854	28.360		G 15 b. Mean	3
26.5	855	28.392		G 7	3
28.0	856	28.518		F 12 b	3
32.5			28.881	G	8
32.6	859	28.892		G 15	3
34.0	859	29.043		F 8	3
34.5	860	29.077		G 18	3
34.7	860	29.101		F 10 b	3
43.9	867	29.865		G 15	3
44.0			29.866	G λ 4143.955	7
50.0	871	30.358		P 20	3
52.3	872	30.553		G 10	3
54.1	873	30.715		F 7	3
54.6			30.739	F λ 4154.571	6
54.4	873	30.741		G 15 b. Mean	3
54.8	874	30.771		F 7	3
56.5	874	30.893		F 8	3
56.8	875	30.947		G 15	3
57.0	875	30.923		G 8	3
57.9	876	31.044		P 8	2
60.7	878	31.275		F 8 b	3
61.2	878	31.310		G 12	3
61.5	878	31.336		F 8 b	3
63.8	879	31.539		F 8	3
65.7	880	31.654		F 8 b	3
67.4	882	31.819		v G 15	3
71.2	884	32.118		F 7 Shp	2
72.9	886	32.264		F 7	3
75.1	887	32.438		F 6 Shp	3
77.8	888	32.652		F 15. Mean	3
82.2			32.985	G λ 4182.155	8
82.2	891	32.988		G 10. L F 5 at $+.067$	3
87.3	894	33.414		G 8	3
87.6	894	33.433		G 17. Inc L6 λ 4186.9	3
87.6			33.438	F-G. λ 4187.572	8
87.6	894	33.438		G 15. Mean	2

Wave Lengths.	r V s	Micrometer Sky.	Readings. Iron.	Notes.	Observations.	Wave Lengths.	r V s	Micrometer Sky.	Readings. Iron.	Notes.	Observations.
87.9	895	33.465		G 8	3	52.0	1007	45.611		G 8	3
88.9	896	33.562		G 5	3	52.3	1007	45.641		G 18. 0.600 — 0.685	3
91.7			33.746	F λ 4191.678	6	67.8	1019	46.711		F 10 Shp	3
91.6	897	33.766		G 10 b. L P 6 + 0.080	3	70.0	1021	46.844		F-P 8	3
98.5	901	34.317		G 10	3	71.2	1022	46.930		F 8	2
98.6	901	34.326		G 16 λ 4198.5 + λ 4199.2	3	74.7	1024	47.144		F 8 b	3
98.7			34.351	G	8	75.4		47.189		F 18	3
4202.2			34.594	G λ 4202.185	8	76.1		47.230		F 8	2
02.2	903	34.603		G 10. L F 7 at —0.080	3	83.7		47.758		G 25	3
07.0	906	34.992		F 9 b	3	83.7			47.764	G λ 4383.67	8
16.0	912	35.676		G 20	3	85.2		47.866		F 9	3
19.5			35.955	F	8	99.9		48.844		F 6	3
19.5	914	35.956		F 8	3	4400.6		48.887		F 10 b	3
22.3			36.174	F	5	00.8		48.904		G 25	3
22.3	916	36.175		P 7. Cl Ls	2	01.5		48.939		G 8	3
26.9	919	36.524		F 15 "g"	3	03.3		49.060		G-F 7	3
27.6			36.578	G	8	04.9		49.163		G 12	3
27.6	919	36.580		F 9	3	04.9			49.164	G λ 4404.93	8
29.8	920	36.758		G 10	3	15.3			49.836	G-F λ 4415.29	8
33.5	923	37.031		G 15	3	15.4		49.841		G 15	3
33.6			37.040	G-F	8	22.8		50.303		F 10 b λ 0.245 — 0.320	3
36.1	925	37.228		G 12	3	27.5		50.620		G 8 b	3
36.1			37.230	G λ 4236.11	8	30.8		50.823		F-G 15	3
43.0	930	37.765		G 30. Shp Edges	3	35.5		51.140		F-P 20	3
50.4	936	38.307		G 8	3	47.5		51.908		G 15	3
50.8	936	38.337		G 17. Mean	3	51.2		52.093		F 12	3
50.6			38.343	G λ 4250.62	8	53.3		52.267		F 5	3
51.1	936	38.368		G 8	2	55.1		52.381		G 12	3
54.4	939	38.635		G 10	3	58.4			52.650	G-F	7
60.5			39.083	G λ 4260.50	8	62.0		52.808		G 12	3
60.4	942	39.084		G 12	3	66.9		53.105		F 7 b	2
71.7	950	39.910		G 18	3	66.7			53.106	F-G λ 4466.73	8
71.8			39.916	G λ 4271.76	8	76.2			53.688	G λ 4476.18	8
75.2	952	40.152		F 25. 0.120 — 0.180	2	82.4		54.060		F 8	2
80.5	957	40.541		F 15. Cl gp	3	82.4			54.063	F-G λ 4482.39	8
82.6			40.700	G λ 4282.56	8	94.7			54.813	G λ 4494.74	8
83.0	959	40.709		G 10 b	3	94.8		54.815		F 10 b	3
89.7	962	41.229		F 20. 0.180 — 0.270	3	4501.4		55.230		F 20	3
94.3			41.555	G λ 4294.30	8	18.0		56.209		F 8 b	3
94.2	967	41.567		F 8	3	20.4		56.342		F 7	3
97.0	968	41.765		G 12. 0.730 — 0.800	3	22.9		56.495		G 8	3
99.4			41.924	G λ 4299.41	8	25.3		56.642		F 6	2
99.6	969	41.939		F 206	3	26.8		56.735		F 8	3
4301.7	970	42.035		F 7	2	26.9			56.846	F-G	8
08.1			42.546	G λ 4308.8	8	29.0		56.852		F 8 b	3
08.1	976	42.553		F 15	3	31.3		56.987		G 9	3
14.1	980	43.002		F-P 7	3	45.0		57.795		F 8 b	3
14.5	980	43.033		G 20. Gp. 0.970 — 1.090	3	49.8			58.046	F	8
15.2	981	43.062		F-P 7	3	49.8		58.048		G 10 b	3
18.8	983	43.301		G 10	2	52.8		58.228		F 8	2
21.0	984	43.440		F 10	2	54.3		58.317		F 8 b	2
21.9	985	43.497		F 5	3	56.2		58.436		G 12 b	3
23.6	987	43.654		G 20	3	63.9		58.866		F 6	3
25.3	988	43.776		F 8	3	71.8		59.333		F 15	3
25.8	989	43.808		G 20	3	84.0			60.009	F-G λ 4584.02	8
26.4			43.811	G λ 4326.40	8	90.2		60.340		F 8	3
26.6	990	43.828		F 8	3	98.1		60.778		F 8 b	3
31.2	992	44.159		F 10. b gp	3	4611.3		61.656		G 7	3
37.2	997	44.608		F 9 b	3	4630.0			62.580	F	8
37.6	998	44.634		F 18	3	4859.9			73.896	F λ 4859.93	2
38.0	998	44.657		F 8	3	71.9			74.430	G λ 4871.88	5
40.7	999	44.835		G 20 "H γ "	3	91.4			75.301	G λ 4891.37	5
44.1	1002	45.093		F 12 b	3	4920.7			76.560	F λ 4920.68	1
46.7	1003	45.254		G 8	3	4957.7			78.156	G λ 4957.67	5

STANDARD TABLE III. (Magnifying Power 10 — 15.)

Wave Lengths.	$r V s$	Micrometer Readings.		P. E.	No. Obs.	Remarks.	Reduction from Sky Plates.
		Sky and Star.	Iron.				
3969.4			13.8790	± 0.0003	—	G	
4005.3			17.4189	02	—	G	
05.3	780	17.424		08	26	G 25. Close BL to Gr	+ 0.001
12.5	780	18.104		10	25	F 13. 0.080 — 0.150. Cl Ls to V	+ 0.004
24.8	790	19.289		16	19	G 30 b	+ 0.003
41.0	800	20.795		14	24	F 25. 0.740 — 0.840 = 3 Ls	+ 0.002
46.0			21.2472	03	—	G	
45.9	800	21.252		09	27	G 20. Cl Ls to V	— 0.005
63.7			22.8785	03	—	G	
63.9	810	22.881		09	20	F-G 17	— 0.001
66.8	810	23.187		12	18	G 15	+ 0.002
67.2	810	23.211		10	18	F 30 Mean	+ 0.001
72.0	820	23.612		09	22	G 20	— 0.007
71.9			23.6127	03	27	G	
92.8	830	25.462		09	14	G 12. L F 12 at — 0.160	— 0.001
4101.7	840	26.270		11	25	G 30. 0.220 — 0.350 H δ	± 0.000
18.8			27.733		2	F	
18.9	850	27.747		09	23	G 12. L + 0.100 F 6	— 0.004
28.0	860	28.523		15	16	F 12 b. Ls F 15 at — 0.150 and at + 0.144	+ 0.002
32.5			28.8831	03	28	G	
34.5	860	29.086		16	17	G 20	+ 0.001
37.0	860	29.315		13	20	G 15. In b region	— 0.008
43.6	870	29.840		10	9	G 25. Weight I. LF 12 at — 0.142	— 0.001
43.9	870	29.863		13	25	G 12	— 0.003
44.0			29.8655	02	28	G	
50.0	870	30.364		11	25	G-F 20	— 0.002
52.3	870	30.556		13	18	G 10	± 0.000
54.4	870	30.741		09	28	G 15	+ 0.002
56.8	870	30.923		08	28	G 15	— 0.002
61.2	880	31.319		09	28	G 12	+ 0.005
63.6	880	31.521		08	20	G 11	± 0.000
78.5	890	32.712		08	28	G 40. L + 0.130 not inc	+ 0.004
82.2			32.985		8	F	
82.3	890	32.995		09	22	G 10	± 0.000
84.7	890	33.178		16	17	F 20. Mean of 2 Ls	+ 0.003
87.6	890	33.434		10	27	G 17. L G 5 + 0.125 not inc	— 0.003
91.7	900	33.771		18	12	G-F 10	— 0.001
96.0	900	34.109		13	26	G-F 25	+ 0.004
98.5	900	34.325		09	22	G 20	+ 0.004
98.6	900	34.339		13	5	G 25	— 0.001
98.7			34.3524	03	28	G	
4202.2			34.5927	04	28	G	
02.2	900	34.603		11	22	F 25	+ 0.001
16.0	910	35.673		09	28	G 25. L 4217.7 Å U not inc	± 0.000
27.0	920	36.539		11	23	F-G 30	+ 0.001
27.6			36.584		6	F	
33.5	920	37.028		08	28	G-F 15	— 0.005
33.6			37.042		8	F	
36.1			37.233		9	F	
36.2	920	37.234		11	16	F 20. Whole L. Weight I.	— 0.004
36.3	930	37.252		14	9	Middle of L	± 0.000
50.6	940	38.341		13	25	G 17	± 0.000
50.6			38.3417	03	27	G	
60.5			39.0834	03	28	G	
60.4	940	39.089		11	25	G 12. LF 10 at — 0.140	+ 0.001
71.7	950	39.911		14	29	G 20	— 0.003
71.8			39.9139	03	29	G	
75.1	950	40.154		07	18	F-G 25. 0.120 — 0.180	± 0.000
82.6			40.6989	03	20	G	
83.2	960	40.721		11	24	F 10 b	± 0.000
90.0	960	41.231		11	6	F 20. 0.180 — 0.270	+ 0.002
94.3			41.5562	04	28	G	

Wave Lengths.	<i>r V s</i>	Micrometer Readings. Sky and Star.	Iron.	P. E.	No. Obs.	Remarks.	Reduction from Sky Plates.
94.3	970	41.562		15	13	F 10	— 0.005
94.4	970	41.580		22	12	G 20. Cl L to Gr	— 0.002
99.4			41.9266	04	25	G	
4300.4	970	41.964		11	28	G 30. L P 18 to Gr not inc. Wide gp	— 0.003
06.2	970	42.382		13	27	F-G 10	+ 0.004
08.1			42.5472	03	28	G	
26.4	990	43.806		10	27	G 20	— 0.006
26.4			43.8096	04	29	G	
31.2	990	44.154		10	27	F-G 10	+ 0.001
40.7	1000	44.839		09	27	G 20 H γ	+ 0.003
44.1	1000	45.095		10	29	F 15 b	+ 0.002
75.4	1020	47.186		08	30	G 20	$\pm$ 0.000
83.7			47.7651	03	29	G	
95.0	1040	48.544		11	27	G-F 15. Ls to V	— 0.004
4401.2	1040	48.895		11	16	Dense part	— 0.004
04.9	1050	49.161		13	24	G 12. L P 8 at — 0.070	+ 0.003
04.9			49.1636	02	29	G	
15.3			49.8347	03	27	G	
15.3	1050	49.837		10	28	G-F 15	+ 0.002
22.8	1060	50.308		12	18	F 15 b. 0.246 — 0.360. L F 10 at + 0.100	+ 0.001
43.0	1070	51.647		10	28	F 25. 0.590 — 0.720	$\pm$ 0.000
51.3	1080	52.102		11	21	G 18. G 12 in V. Measure whole L	— 0.001
55.4	1080	52.397		14	22	F 25. Mean of 2 Ls	+ 0.003
62.0	1090	52.806		14	24	F 12	+ 0.001
66.7			53.1055		10	F	
76.2			53.6887	07	21	G	
82.0	1100	54.002		15	22	F 25	— 0.002
94.7	1110	54.806		24	12	F 10 b	+ 0.002
94.7			54.8121	07	25	G	
4501.3	1120	55.220		11	22	F 20. Measure shp middle	— 0.005
28.9			56.8404	14	7	F-G	
41.0	1150	57.552		17	9	F 15 b. Ls to V not inc	— 0.002
49.8	1150	58.056		10	23	G 20	+ 0.003
71.8	1170	59.332		09	24	F 15	+ 0.005
84.0			60.0115	$\pm$ 0.0008	23	F-G	
4630.—			62.5809		16	F	
4871.9			74.4299		16	G	
91.4			75.3012		17	G	
4957.7			78.1556		17	G	

TABLE IV.—ADDITIONAL LINES IN STAR (Low Power).

Wave Length.	<i>r V s</i>	Micrometer Readings.	P. E.	No. of Obs.	Remarks.
4063.6	810	22.860	$\pm$ 0.0011	8	F 25. Weight I
4132.7	860	28.905		12	G 15
37.3	860	29.338		09	25. Whole L
91.4	900	33.751		13	G-F 20. L to V inc
4204.7	900	34.834		16	F 18. L F 12 at +0.120
90.1	960	41.257		10	F 30. Weight I. L P 12 at —0.140
97.0	970	41.745		15	F-G 12. 0.700 — 0.800
4314.4	980	43.026		09	G 20. 0.970 — 1.090
34.7	990	44.372		14	F-G 18. L at +0.100 not inc
37.8	1000	44.648		10	F-G 25
52.0	1010	45.611		11	G 30. L at —0.150 not inc
60.0	1010	46.113		11	F-G 20
84.5	1030	47.814		09	G 38
4401.0	1940	48.883		06	G 25. L at —0.170 not inc
17.9	1050	50.002		14	F 20. Mean of 2 Ls
66.4	1090	52.974		17	F 10
4583.7	1180	59.988	$\pm$ 0.0011	17	F-G 20

(c) *Application to the study of the variable star, W Sagittarii.*—Exhaustive studies of the light curve of W Sagittarii were made by Herr J. H. F. Schmidt, who discovered the variability of this star, then known as γ' Sagittarii, at Athens in 1866.* From a series of eight hundred and ninety observations, covering ten years and extending over one hundred and ninety-five maxima and one hundred and ninety-three minima he constructed a light curve, which is drawn in detail in Plate 8 of this paper. In addition to the strong irregularities in the curve, he suspected a perturbation in the light period running through a cycle of eight years, affecting the time of maximum and minimum several tenths of a day. But his observations were hardly extensive enough to warrant this last conclusion. The spectrographic observations at present available do not cover a period long enough to confirm or to disprove this result. As presented in the Chandler and Harvard Catalogues, the important data regarding this star, with some additions, are as follows:

CHANDLER'S THIRD CATALOGUE.

Max., 4^m8; Min., 5^m8; M-m, 3^d00; Period, 7^d59460*E*.
Epoch of Max., 1866 Sept. 4; Julian, 2,402,849^d45.

HARVARD CATALOGUES.

Max., 4^m3; Min., 5^m1; Class IV; Sp., G5K.
 α 1900.0, 17^h 58^m6; δ 1900.0, $-29^{\circ}35'$

ADDITIONAL DATA.

λ 1902.0, 269^o44'; β 1902.0, $-6^{\circ}08'$
Photo. Max., 5^m5; Photo. Min., 6^m5.

The spectrum of W Sagittarii approximates very closely to the solar type. The fact that sixty-five blends ranging in intensity from twelve to forty gave good results, as exhibited in the last column of TABLE III, establishes the close resemblance of this star to our own central body. The seventeen lines in TABLE IV suggest some differences, the most striking of which is the strong line at λ 4335.7, which is not found in the sun, but rises to an intensity of 12 in the star. But, on the whole, the identities are far-reaching enough to warrant an extensive system of blending.

This fact justified the use of a power of ten in the measures of the plates of W Sagittarii. But the selection of such a low power was not decided upon without considerable experiment with a higher one. With a power of ten, the width of the micrometer thread (about 0.50 Å.U.) interfered with settings on lines of intensity

12 to 15, while the density of the comparison lines was so great that it became difficult at times to distinguish the black vertical wire when set on them. After experimenting with powers of ten, twelve, fifteen and twenty-five, I am inclined to think that a power in the neighborhood of fifteen would perform most satisfactorily in the case of solar-type stars.

From preliminary measures in August, 1903, of three spectrograms of W Sagittarii, a very considerable variation in the radial velocity of this star was brought to light. Subsequently a series of thirty-three spectrograms distributed uniformly in the light period, was completed with Spectrograph I. These have been measured and reduced for the purpose of determining the character of the orbit of this interesting variable. I have not been able to recognize upon these plates any effect attributable to the light of the fainter companion now known to exist, but it should be remembered that a difference of one magnitude is nearly sufficient to obliterate the effect of the darker star.

The details of the production of these plates are recorded in the Journal of Observations, TABLE V. The longer exposures of the comparison were made with a diaphragm occulting all lines to the violet of λ 4415, while during the shorter exposures the entire plate was uncovered. When the comparison was introduced four times, it was photographed once with the diaphragm and once without on each side. When introduced six times, four exposures were made without and two with the occulting device. With ten introductions of the comparison, three exposures were made on each side with the diaphragm removed and two on each side with the diaphragm in position. The slit-width is expressed in terms of the divisions of the micrometer head. The unit is 0.025 millimeters. To illustrate the methods employed, complete measures and reductions of Plate 56A are published in TABLE VI. The columns from left to right contain the wave-lengths of the spectral lines, the micrometer readings on the star, those on the comparison, the reduction to fundamental dispersion, the displacements at fundamental dispersion of the star lines from the solar lines of the zero-standard table, and the equivalents of these displacements in kilometers per second. Referring to the description of the plates in the last two columns in TABLE VII and to the temperature range in TABLE V, it is evident that 56A is about an average plate.

* *Astronomische Nachrichten*, Nr. 2071, page 103.

2. Solar Spectrum $\left\{ \begin{array}{l} \text{Iron} \\ \text{Sky} \\ \text{Hydrogen} \end{array} \right.$
3. W Sagittarii, 1903, September 16. 77 A.

4. R Scuti, 1903, July 19. 13 E.
5. R Scuti, 1903, September 21. 84 B.
6. W Cygni, 1903, August 3. 24 C.
7. W Cygni, 1903, October 12. 111 A.

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TABLE V—JOURNAL OF OBSERVATIONS (W Sagittarii).

1904

Plate.	Astro- nomical Date.	Mt. Hamilton Sidereal Time, Mid-Exposure.	Duration.	Comparison. Times. of Length of Exposure.			Observing Room Temperature. Beginning. End. Range.			Slit Width.	Seeing.	Remarks.
1903												
20 B	Aug. 3	17 ^h 47 ^m	45 ^m	4	5 ^s and 25 ^s		18°3 C	18°2 C	0°1 C	1.4	Fair	
25 A	9	17 40	40	4	4	20-30	24.5	24.2	0.3	1.3	Good	
36 D	17	17 52	45	4	5	25-30	20.8	20.7	0.1	1.4+	Poor	
46 A	Sept. 2	18 21	30	6	3	25	24.8	24.4	0.4	1.3	Good	
47 B	4	18 38±3	35±5	6	3	25		12.8		1.4	Poor	Wind 30 miles
56 A	7	18 9	45	6	3	27	17.2	16.4	0.8	1.3+	Fair	
57 B	7	18 49	28	6	3	26	16.4	16.3	0.1	1.3+	Fair	
58 A	8	18 9	25	6	3	26	19.7	19.2	0.5	1.4	Fair	
59 D	8	18 38	30	6	3	26	19.2	18.8	0.4	1.4	Good	
60 A	9	18 9	15	6	2.5	26	21.7	21.4	0.3	1.3	Good	
61 D	9	18 32	24	6	2.5	26	21.4	21.1	0.3	1.3	Fair	
63 A	10	18 7	20	6	2.5	26	21.4		0.5	1.3	Fair	
64 B	10	18 28	19	6	3	26		20.5	0.4	1.3	Fair	
65 A	11	18 4	15	6	3	26	16.0		0.4	1.3	Fair	
66 B	11	18 23	20	6	3	26		15.3	0.3	1.3	Fair	
75 A	14	18 33	35	6	2-3	25	13.6	12.8	0.8	1.3	Fair	
76 B	14	19 1	21	6	2-3	20	12.8	12.6	0.2	1.3	Good	
77 A	16	18 25	24	6			19.2	18.5	0.7	1.3+	Good	
78 B	16	18 54	32	6	2-3	20	18.5	18.0	0.5	1.3+	Good	
82 A	20	19 1	83	6	2-3	25	18.9	17.5	1.4	1.3+	Good	Clouds cut off ½ light
83 A	21	19 15	85	6	3-4	23	18.4	17.8	0.6		Poor	Smoky sky cuts off ½ light
86 A	23	19 11	65	6	3	25	19.4	19.3	0.1	1.3+	Bad	
91 A	24	18 45	30	6	3	25	22.5	21.8	0.7	1.3+	Fair	
92 B	25	18 56	43	6	3	25	19.1	18.7	0.4	1.3+	Good	Smoke reduces light ½
93 B	26	18 51	35	6			19.8	19.8	0.0	1.3	Good	
94 D	27	19 3	53	6	3	25	16.7	16.6	0.1	1.3+	Poor	
95 B	Oct. 3	19 24	25	6	3-6	25-40	11.0	10.4	0.6	1.3+	Fair	
99 F	4	19 46	76	6	3-4	26	11.7	11.0	0.7	1.3	Changeable. Image faint.	Objective fogged?
100 A	5	19 47	45	6	3.5-4	25	11.8	11.3	0.5	1.3	Poor	
102 A	6	19 36	39	6	3-4	25-35	14.6	14.2	0.4	1.3	Good	
103 B	6	20 13	33	6	3-4	25-35	14.2	13.8	0.4	1.3	Changeable	
1904												
136 F	June 7	17 16	52	10	1.5	11	14.6	14.0	0.6	1.3+	Poor	
137 A	7	18 1	31	10	1.5	11	14.0	14.0	0.0	1.3+	Fair	

TABLE VI—MEASURES AND REDUCTION OF PLATE 56A, W SAGITTARI.

λ	Star.	Iron.	Red.	Disp.	V	λ	Star.	Iron.	Red.	Disp.	V
3969.4		13.876				44.0		29.8665			
4005.3		17.418		+ + km.		50.0	30.399			35	30
05.3	17.455		+1	32	25	52.3	30.582			26	23
12.5	18.136			33	26	54.4	30.763			22	19
24.8	19.316			28	22	56.8	30.955			32	28
41.0	20.839			45	36	61.2	31.338			19	17
46.0		21.246				63.6	31.547			26	23
45.9	21.273		+1	22	18	78.5	32.746			34	30
63.7		22.879				84.7	33.198			20	18
63.6	22.881		±0	21	17	87.6	33.463			29	26
67.2	23.233			22	18	91.7	33.790			18	16
72.0	23.635			23	19	96.0	34.143			34	31
71.9		23.6125				98.5	34.351			26	23
4101.7	26.287			17	14	98.7		34.3525			
18.9	27.773			26	22	4202.2		34.592			
28.0	28.558			35	30	02.2	34.630			27	24
32.5		28.882				04.7	34.852			18	16
32.7	28.935			30	26	16.0	35.694		±0	21	19
34.5	29.112			26	22	27.0	36.558		—1	18	17
37.0	29.325			10	9	33.5	37.047		—2	17	16
43.6	29.896			21	18	50.6	38.363		—3	19	18
43.9	29.861			33	29	50.6		38.343			

λ	Star.	Iron.	Red.	Disp.	V	λ	Star.	Iron.	Red.	Disp.	V
60.5		39.089				60.0	46.130		—3	14	14
60.4	39.110			18	17	75.4	47.213			24	24
71.7	39.945			31	29	83.7		47.7665			
71.8		39.9195				84.5	47.840		—3	23	24
75.1	40.177		—3	20	19	95.0	48.567		—4	19	20
82.6		40.6995				4401.2	48.912			13	14
83.2	40.753		—2	30	29	04.9	49.188		—4	23	24
90.0	41.257			24	23	04.9		49.170			
90.1	41.285			26	25	15.3		49.840			
94.3		41.5555				15.3	49.870		—5	28	29
94.4	41.608			26	25	17.9	50.037			30	31
99.4		41.930				43.0	51.673		—5	21	22
4300.4	41.986			20	19	55.4	52.422		—6	19	21
06.2	42.400			16	16	62.0	52.845		—6	33	36
08.1		42.5515				76.2		53.699			
14.4	43.048			20	20	94.7		54.815			
26.4	43.823			15	15	4501.3	55.240		—7	13	15
26.4		43.808				71.8	59.350		—4	14	16
31.2	44.180			24	24	84.0		60.015			
34.7	44.395			21	21	4630.—		62.588		Mean + 22.1 km.	
37.8	44.680			30	30	4871.9		74.436			
40.7	44.869			28	28	91.4		75.320			
44.1	45.110			13	13	4957.7		78.172			
52.0	45.639		—2	26	26						

TABLE VII includes the results of the measures of the plates whose numbers on the observing list occur in column one. Under the heading, V' , the radial velocities with respect to the observer are given. All the plates were measured without reversal with low power, in order to make the results strictly comparable with those of the standard plates. Eight of these velocities depend upon two measures, but the probable errors in column seven depend upon one measure only, unless the results from separate lines were combined before the mean was taken. The probable errors of the first thirty-one plates, excepting 25A, were obtained from a preliminary set of reductions. Corrections for the motion of the observer are combined in

column five. The resulting values of the radial velocity of the star with reference to the sun appear under the heading, V , in column six.

Assuming the identity of the light and velocity periods for this star, the quantities in column three have been formed from the following data:

MAXIMA OF W SAGITTARII.

1903	August,	2.865
		10.559
	September,	2.243
		9.838
		17.432
		25.027
	October,	2.621
1904	June,	1.649

TABLE VII—TABLE OF VELOCITY DETERMINATIONS FROM 33 PLATES OF W SAGITTARII.

Plate.	Date. G. M. Time.	Interval after Maximum.	V'	Red. to Sun.	V	P. E.	r'	r	Star.	Comparison.
1903										
	d	d	km	km	km	km	km	km		
20 B	August	3.713	0.848	—19.8	—19.3	—39.1	± 0.7	+4.9	+0.7	Fair
25 A		9.693	6.828	—21.0	—21.5	—42.5	1.5	—4.2	± 0.0	Fair to poor
36 D		17.678	7.219	—21.4	—23.9	—45.3	0.6	—1.5	+0.1	Fair
46 A	September	2.655	0.412	—10.8	—27.6	—38.4	0.7	+7.4	+3.7	Fair. Overexposed
47 B		4.661	2.418	—10.6	—28.1	—38.7	0.6	—5.7	—0.4	Good
56 A		7.633	5.390	+22.1	—28.4	— 6.3	0.5	+0.6	—0.2	Fair
57 B		7.660	5.417	+23.8	—28.4	— 4.6	0.8	+2.4	+1.8	Fair
58 A		8.631	6.388	— 2.8	—28.5	—31.3	0.6	—5.5	± 0.0	Good
59 D		8.651	6.408	— 4.3	—28.6	—32.9	0.4	—5.8	—0.3	Good
60 A		9.628	7.385	—15.7	—28.6	—44.3	0.7	+0.7	+0.7	Fair to good
61 D		9.643	7.400	—16.3	—28.7	—45.0	0.7	± 0.0	± 0.0	Fair to good
63 A		10.624	0.786	—12.6	—28.7	—41.3	0.6	+2.8	—1.4	Good
64 B		10.637	0.799	—11.3	—28.7	—40.0	0.6	+4.3	+0.1	Good
65 A		11.620	1.782	— 9.6	—28.9	—38.5	0.5	+0.4	—0.2	Fair
66 B		11.632	1.794	— 9.4	—28.9	—38.3	0.5	+0.6	—0.4	Good
75 A		14.631	4.793	+21.3	—29.2	— 7.9	0.8	+2.5	—1.4	Fair to poor
76 B		14.651	4.813	+23.9	—29.2	— 5.3	1.8	+4.9	+1.0	Underexposed
77 A		16.620	6.782	—10.7	—29.3	—40.0	0.5	—5.8	—0.6	Good
78 B		16.640	6.802	—11.1	—29.4	—40.5	0.5	—6.0	—0.8	Fair

Plate.	Date. G. M. Time.	Interval after Maximum.	V'	Red to Sun.	V	P. E.	r'	r	Star.	Comparison.
	d	d	km	km	km	km	km	km		
82 A	20.634	3.202	+ 3.2	-29.5	-26.3	0.7	-0.2	+3.1	Fair	Fair
83 A	21.640	4.208	+18.9	-29.6	-10.7	0.5	+5.5	+1.9	Fair	Fair
86 A	23.632	6.200	+ 4.6	-29.6	-25.0	0.6	-2.6	+0.1	Good	Fair
91 A	24.612	7.180	-11.9	-29.6	-41.5	0.5	+2.1	+3.9	Good	Good
92 B	25.617	0.590	-15.2	-29.6	-44.8	0.7	+0.4	-3.7	Fair	Fair
93 B	26.611	1.584	-10.0	-29.5	-39.5	0.7	+0.3	-0.5	Good	Good
94 D	27.617	2.590	- 7.3	-29.5	-36.8	0.7	-5.3	+0.2	Fair	Fair
95 B	October 3.615	0.994	-12.5	-29.2	-41.7	0.5	+1.6	-2.3	Fair	Poor
99' F	4.627	2.006	- 9.7	-29.2	-38.9	0.7	-2.5	+0.1	Overexposed	Poor
100 A	5.625	3.004	- 4.9	-29.1	-34.0	0.6	-4.7	+0.4	Good	Fair
102 A	6.615	3.994	+11.6	-28.8	-17.2	0.6	+1.1	-1.4	Fair	Fair
103 B	6.640	4.019	+12.4	-28.9	-16.5	± 1.0	+1.8	-0.8	Underexposed	Fair
1904										
136 F	June 7.845	6.196	-31.2	+ 6.6	-24.6	± 0.8	-4.6	+0.5	Good	Fair
137 A	7.877	6.228	-30.3	+ 6.5	-23.8	± 1.0	-2.9	+2.3	Fair	Good

PLATE 8. VELOCITY AND LIGHT CURVES OF W SAGITTARII.

Employing the quantities in columns three and six as abscissæ and ordinates, respectively, the plot of velocity determinations was formed which appears in Plate 8 of this Bulletin. Ample verification of the adopted value for the period was found in the close agreement between results obtained from plates covering more than forty maxima of the curve, or an interval of 309 days. This agreement was at once evident in TABLE VII from a comparison of Plates 86A and 136F. But when the attempt was made to pass an elliptic velocity curve through these observations, it was found that the plotted points oscillated above and below this curve with a period of 3.8 days, or one-half that of the light variation. After repeated trials of various ellipses with different values of periastron time, longitude of periastron, eccentricity, maximum positive and negative velocity, and the velocity of the system, I selected the velocity curve which is drawn on Plate 8 with the narrower line. (The residuals from this ellipse appear under the head, r' , in TABLE VII.) In selecting this conic it was assumed that the actual

PLATE 9. THE SECONDARY CURVE.

observed velocities followed a superimposed curve with a period of 3.8 days and with nearly equal amplitudes for crests and troughs. For the better determination of the secondary curve the residuals of all plates from the ellipse have been plotted on Plate 9

after being reduced to one complete period of 3.8 days, employing the well established nodal point at 1.7 days after the light maximum. A sine curve was then passed through these points with an amplitude of 4.2 km. at the crest and 5.5 km. at the trough, as shown in the diagram (Plate 9). This final curve was superimposed on the velocity curve and is represented by the heavy line of the upper curve of Plate 8. The residuals from this curve for all the plates appear in TABLE VII under the heading, r . Including all these residuals, and assigning equal weights to each, the resulting probable error of a single plate is ± 0.90 km. It can be seen by consulting the temperature range in TABLE V and the last two columns of TABLE VII that all the plates showing large residuals are undoubtedly inferior to the average plate. The long exposure and great temperature variation for 82A and the poor comparison spectrum of 95B would justify the rejection of these plates. Plate 91A shows good spectra, but it should be noticed that its temperature range is twice that of the average range for all the plates. Excluding five inferior plates whose velocities could be given very small weight in the construction of the curve, the probable error of any one of the remaining twenty-eight plates is ± 0.55 km. The occurrence of relatively larger residuals along the crest of the secondary curve finds ready explanation when it is noticed that, of the two crests which obtain in one complete revolution of the system, the one occurs at the light maximum where the greater activity of the star could lead to wide ranges in velocity, while the other occurs at light minimum where, with increased exposure time, the effect of temperature change in the instrument becomes a maximum.

The elements of the orbital or primary curve of the brighter component of W Sagittarii together with those of the superimposed secondary curve, are as follows:

ELEMENTS.

		Primary Curve.	Secondary Curve.
Apparent period,	P'	7.59460 days.	3.80 days.
Longitude of periastron,	ω	$70^\circ.0$	
Eccentricity,	e	0.320	0.0
Time of periastron after light maximum,	T	6.20 days	
Projection of semi-major axis on plane of sight,	$a \sin i$	1,930,000 km.	
Projection of periastron distance on plane of sight,	$q \sin i$	1,310,000 km.	
Projection of apastron distance on plane of sight,	$q' \sin i$	2,550,000 km.	
Ratio of masses,	$\frac{m^3 \sin^3 i}{(m + m_1)^2}$	0.00499 $\odot$	
Amplitude of velocity curve at crest,	A	+21.6 km.	+4.2 km.
Amplitude of velocity curve at trough,	B	-17.4 km.	-5.5 km.
Velocity of center of mass of the system,	V	-28.6 km.	

On the basis of these elements, I have constructed on Plate 10 the elliptical orbit of the brighter component of W Sagittarii, indicating the line of nodes by a horizontal line and the line to the sun by a vertical line drawn downward. Further, at the various points where they occur, I have indicated the light maximum and light minimum and the cross points of the primary and secondary velocity curves.

PLATE 10. ORBIT OF W SAGITTARI.

Our present knowledge of the elements of W Sagittarii makes possible the introduction of two small corrections to the period of this star depending upon the velocity of light. The first is due to the change in the distance between the earth and the bright component of the binary system occasioned by their orbital motions, and may be regarded as the combined effect of the equations of light of the two bodies. Its operation is not systematic, but may be taken into account in the computed times for maxima used in plotting the velocity observations by changes never exceeding 0.01 days. The maximum variation from the mean in the present problem is about 0.002 days, which is equivalent to a displacement of 0.007 cm. in the abscissæ of my original velocity diagram. It is therefore evident that this correction is negligible. The second correction to the period of this variable is based upon the continual approach of the system

toward the sun at the rate of 28.6 km. per second. As this correction is constant, the use of the apparent period for the velocity diagram does not affect the determination of the elements which characterize the form and position of the orbit. But the epochs of the curve referred to the time of maximum, and all dimensions of the orbit, are direct functions of the true period. The correction to the apparent period amounts to +62 seconds, or +0.00072 days. Accordingly, the true value of the light and velocity periods of W Sagittarii is

$$P = 7.59532 \text{ days.}$$

As the remaining elements of the velocity curve are published to three places only, this change in the fifth significant figure of P does not appreciably affect their values.

The angle i , which measures the inclination of the orbital plane to a plane tangent to the celestial sphere at the center of mass of the system, cannot be determined without micrometrical measures of the two components. The absolute scale of the ellipse is therefore indeterminate. For various assumptions of i the values of q are given below.

TABLE VIII.

i	Periastron Distance.
15°	5,060,000 km.
30°	2,620,000 km.
45°	1,850,000 km.
60°	1,510,000 km.
75°	1,360,000 km.
90°	1,310,000 km.

In the absence of any evidence regarding the radial velocity of the fainter star it is not possible to determine the masses of the system, but the constant of attraction furnishes data for the determination of these quantities when the angle i and the ratio of the masses are known. On the basis of various assumptions for i and m_1/m , I have tabulated below (TABLE IX) the corresponding values of the masses, in units of the sun's mass, of the two stars (m_1 and m) together with their distance apart at perihelion ($q_1 + q$): m_1 and q_1 refer to the bright star. The computations are based upon the equation

$$\frac{m^3 \sin^3 i}{(m + m_1)^2} = 0.00499 \odot.$$

TABLE IX.

$\frac{m_1}{m}$	i	15°	30°	45°	60°	75°	90°
0.25	m	0.45 ☉	0.062	0.022	0.012	0.009	0.008
	m_1	0.11 ☉	0.016	0.005	0.003	0.002	0.002
	$q + q_1$	6,320,000 km.	3,270,000	2,310,000	1,890,000	1,700,000	1,640,000
0.50	m	0.65 ☉	0.090	0.032	0.017	0.012	0.011
	m_1	0.32 ☉	0.045	0.016	0.009	0.006	0.006
	$q + q_1$	7,590,000 km.	3,930,000	2,770,000	2,260,000	2,040,000	1,960,000
0.75	m	0.88 ☉	0.121	0.043	0.024	0.017	0.015
	m_1	0.66 ☉	0.091	0.032	0.018	0.013	0.011
	$q + q_1$	8,850,000 km.	4,580,000	3,240,000	2,640,000	2,380,000	2,290,000
1.0	m	1.15 ☉	0.16	0.057	0.031	0.022	0.020
	m_1	1.15 ☉	0.16	0.057	0.031	0.022	0.020
	$q + q_1$	10,120,000 km.	5,240,000	3,700,000	3,020,000	2,720,000	2,620,000
2.5	m	3.5 ☉	0.49	0.17	0.095	0.068	0.061
	m_1	8.8 ☉	1.23	0.43	0.237	0.170	0.153
	$q + q_1$	17,700,000 km.	9,170,000	6,470,000	5,280,000	4,760,000	4,580,000
5.0	m	10.3 ☉	1.44	0.51	0.28	0.20	0.18
	m_1	51.7 ☉	7.20	2.54	1.38	0.99	0.90
	$q + q_1$	30,400,000 km.	15,700,000	11,100,000	9,060,000	8,160,000	7,860,000
7.5	m	20.8 ☉	2.9	1.38	0.56	0.40	0.36
	m_1	155.7 ☉	29.2	10.3	4.17	3.01	2.71
	$q + q_1$	43,000,000 km.	22,300,000	15,700,000	12,800,000	11,600,000	11,100,000
10.	m	34.7 ☉	4.8	1.7	0.93	0.67	0.60
	m_1	347. ☉	48.2	17.1	9.29	6.69	6.03
	$q + q_1$	55,700,000 km.	28,800,000	20,300,000	16,600,000	15,000,000	14,400,000
15.	m	73.6 ☉	10.2	3.6	2.0	1.4	1.3
	m_1	1104. ☉	153.	54.3	29.6	21.3	19.2
	$q + q_1$	81,000,000 km.	41,900,000	29,600,000	24,200,000	21,800,000	21,000,000

In determining the most probable value of the true scale of the orbit from TABLE IX, there is opportunity for unlimited conjecture. However, in view of what is known and of what in addition will be accepted as most reasonable, the range of such speculation can be greatly diminished. In view of the fact that the spectrum of the fainter star is of vanishing order, the ratio of the light intensities of the two stars would probably be greater than one to three. Assuming, then, that the apparent surface brightness of the two stars is the same, we are led to conclude that the ratio of the masses is greater than or equal to one to five, since this ratio varies as the three-halves power of the ratio of the apparent discs for equal densities of the two stars. We are thus quickly restricted to the last four rows in TABLE IX. Further, the secondary curve of Plates 8 and 9 possesses many features that point to its origin in the rotation of the brighter star. The varying velocity could be ascribable to a difference in brightness between two hemispheres of the star. Let us assume that the ratio of brightness of the two hemispheres is, roughly, 1 to 2, and take the amplitude of the secondary curve as 5 kms. per second; then, on

the basis of the rotation interpretation for the secondary curve of W Sagittarii, the radius of this body would appear to be about that of the sun if the inclination of the equatorial plane to the line of sight be about 90°. For other values of this inclination, the corresponding values of the radius of this star, together with its mass in terms of the sun's mass, appear in TABLE X, assuming equal density for the sun and this solar-type star.

TABLE X.

i	R	m_1
15°	2,700,000 km.	57.5 ☉
30	1,400,000	8.0
45	1,000,000	2.8
60	800,000	1.5
75	700,000	1.1
90	700,000	1.0

Reasoning from conditions that exist in our own planetary system, it seems most consistent to assume that the inclination of the equatorial plane of the brighter star to the orbital plane is approximately 0°. Comparing TABLES IX and X on this basis, it will be seen that the mass values in the latter table for each inclination (i) would be duplicated in TABLE IX with

a value of m_1/m , equal to five or six. Confining our attention to the sixth row of TABLE IX, we find the inclination still undetermined. However, as the maximum orbital velocity of the smaller mass assumes the large values of 500 km. per second for an inclination of 15° , 250 km. for 30° , and 160 km. for 45° , we naturally turn to the higher angles. I therefore suggest the following probable limiting values of the quantities involved:

Inclination,	i $45^\circ - 90^\circ$
Mass of brighter star,	m_1 $0.9 \odot - 2.5 \odot$
Mass of fainter star,	m $0.2 \odot - 0.5 \odot$
Periastron distance of brighter star,	q_1 1,300,000 km. — 1,800,000 km.
Periastron distance of fainter star,	q 7,000,000 km. — 9,000,000 km.
Radius of brighter star,	R_1 700,000 km. — 1,000,000 km.
Radius of fainter star,	R 400,000 km. — 600,000 km.

As the existence of a strong variation in a star's light may be considered an exception to the general rule, it is possible that the internal conditions of this system are correspondingly extraordinary and the assumptions above are far from the truth. However, they will illustrate the outcome of one continuous train of reasoning.

The failure of double-star observers to detect any duplicity in this star leads to the conclusion that the angular distance between the components is not greater than $0''.15$. On the assumption that the greatest apparent distance between the two is 10,000,000 km., the inference may be drawn that the distance of the system from the earth is greater than three and a half light years.

For purposes of comparison among spectroscopic studies of Cepheid variables, there exist in addition to the present investigation the excellent results of Wright and Blopolsky in connection with their researches regarding the orbits of δ Cephei* and of η Aquil.† A strong resemblance between the elements of these stars is at once evident. All show a pronounced eccentricity. Also, the time of closest approach of the bodies of the system occurs about a day before maximum brightness. The most marked analogies exist between Wright's elements of η Aquil and those of W Sagittarii; indeed, with the exception of a small difference in eccentricity, the uncertainty of inclination, and a difference of 0.5 days in the period, the two orbits are essentially identical even in regard to the positions of principal maximum and minimum. It is interesting to notice that Mr. Wright has obtained no evidence of a secondary curve for η Aquil, but such a curve would not have been recognized if its amplitude did not exceed 2 km., and in

some positions no great distortion of the ellipse would occur with an amplitude of five kilometers for the superimposed curve. Referring to Mr. Wright's article, it should be noticed that the velocity curve reproduced there is the empirical curve which was drawn for the purpose of determining the elements. His actual ellipse passes slightly below observation 11, not above it. It will also be noticed that his observations tend above the published curve from 1 day to 2.8 days after maximum and fall below from 2.8 to 4.5 days. It therefore seems probable that some curve oscillating about an ellipse would represent the observations better.

Mr. Wright's observations extend over four months, and my own over a period of less than a year. It is therefore probable that any influence connected with the long-period irregularities in the light recurrence of these stars would escape detection. Further study of these stars should reveal such phenomena.

The observed uniform correspondence between the light variations and the orbital conditions of these three stars suggests the mutual dependence of the two phenomena. Whatever may be the cause or causes of variation in the Geminids, they may not operate in the same manner in the Cepheids. It therefore seems advisable to treat each group separately in any attempt to construct a theory for the explanation of their light changes. I shall therefore advance some data regarding the most probable conditions that obtain in the system of W Sagittarii.

Darwin's expression for tidal potential is:

$$V = \frac{3}{2} \frac{m}{r^3} \rho^2 (\cos^2 z - 1/3),$$

where r is the distance between the masses, ρ is the radius of the disturbed body, and z the angle between r and ρ ; m is the mass of the disturbing body. Confining our attention to the line joining the two bodies, we place $z = 0^\circ$ and differentiate with reference to the direction of displacement and arrive at the expression, $F = 2m\rho/r^3$, where F is the tidal force. As the ratio of apastron to periastron distance in this system is 2 to 1, it is evident that the tidal force varies in the ratio of 1 to 8 for these two positions. Using the rough conclusions as to the most probable data for the system arrived at above, we find that the intensity of the tidal force acting on the brighter body is, roughly, 50,000 times that of the moon acting on the earth. Further, by the introduction of another term usually negligible in tidal computations, we find the ratio of the intensities of the tidal forces on the side toward the disturbing body to the corresponding forces on the opposite side of the brighter star is 15 to 10 at periastron and 13 to 10 at apastron.

While the increase of light may be due to enormous tidal disruptions in the molten matter of the star's

* Astrophysical Journal, Vol. I, page 160.

† Astrophysical Journal, Vol. VI, page 393. Ibid., Vol. IX, page 59.

surface, accompanied perhaps with a liberation of heat from below, it should be noted that the displacements of the absorption lines in the spectrum are caused by the motions of the star's atmosphere with and relative to the regions emitting the light. In the atmospheric regions above the high tidal area there would undoubtedly be an uprush of gases due to atmospheric tides, convection currents, increased light pressure, and explosive outburst. Again, in the belt of low tide ninety degrees from the two high tides, there would probably be a corresponding recession of the atmosphere toward the star's surface. This would clearly give rise, in the actual velocity curve, to an oscillation with respect to a true ellipse. These oscillations would fall below the mean at conjunction and above the mean at elongation, for at conjunction the high tidal areas are presented to the observer, and at elongation the low tidal areas. Consulting the diagrams, this is found to be the case. We can therefore depart from the rotation theory to account for the secondary curve, and indeed this seems justifiable, for it is possible with the operation of such immense tides through long periods of time that the rotation periods of the stars involved should be brought to identity with the revolution period, as in case of our own satellite. Further, on the basis of the tidal theory there would not seem to be sufficient variation in brightness due to difference in intensity of the tidal force between the two hemi-

spheres (the ratio is about 7/5) to produce by rotation the oscillation observed, unless we assume much greater masses for the system. The occurrence of the light maximum when the tidal forces have fallen one-third in intensity presents an anomaly, but it is possible that the effect of these forces lags to this extent.

Returning again to the rotation interpretation, it is easy to construct a plausible explanation for the light and velocity curves of W Sagittarii on the assumption that the system is pervaded by a resisting medium which enhances the brightness of that side of the star which faces the direction of motion. Or, again, a third body might be present in the system and give rise to the perturbations observed by Schmidt in the period. Until more data are available, to follow out such theories would be premature.

Considering all evidence, it seems reasonably certain that the star's variations in brightness, and particularly the principal variations, are attributable to the action of external forces.

It is a pleasure to acknowledge my indebtedness to Director Campbell, who placed the necessary apparatus at my disposal and gave continual counsel and encouragement during the prosecution of the work, and to Dr. H. D. Curtis and Dr. J. H. Moore for valuable advice and assistance.

RALPH H. CURTISS.

July, 1904.

THE RADIAL VELOCITIES OF S SAGITTAE AND Y SAGITTARII

Measures of seven plates of S Sagittæ, employing the low-power, sky standard-table published above, have resulted in the detection of a wide range in the radial velocity of this star. As a whole these plates are much below the average excellence, but are sufficiently reliable to establish the binary character of this variable. The accompanying table contains the number of the plate, the Greenwich date, the interval since maximum, the velocity referred to the sun, the num-

ber of lines used for each plate, and the temperature change during the exposure. If the velocities are plotted in the usual way, assuming the identity of the light and velocity periods, they are seen to follow a curve in every way similar to those of η Aquilæ and W Sagittarii, and the elements will approximate closely to those determined for W Sagittarii above. There is also some evidence pointing to a composite character for the curve.

No. Plate.	Date.	Interval since Maximum.	V.	Number of Lines.	Temperature Range.	Remarks.
28F	1903 August	9.9	1 ^d .4 —20.3 km.	20	0°.4C	Underexposed.
33E		14.9	6.4 —32.2	44	0.1	
35D		16.9	0.0 —30.2	45		
42B		26.9	1.6 —20.2	20	0.3	Underexposed.
54B	September	6.9	4.2 + 3.9	41	0.1	Comparison poor on one side.
72B		13.8	2.8 —17.3	37	0.0	
3350C	1904 July	20.9	3.7 — 3.9	9	0.0	Underexposed.

As the result of relative measures of four lines on two plates, Dr. Stebbins found a range of 9 km. in the radial velocity of Y Sagittarii. My own measures of nine plates of this star indicate that it is approaching our system, but not at a constant rate. The range of velocity so far observed amounts to 17 km. The

form of the curve seems quite different from that of other Cepheid variables, but the character and number of the plates available are not such as to warrant any definite conclusions on that point.

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July, 1904.

ON THE SPECTRA OF R SCUTI AND W CYGNI

Observations of the spectra of R Scuti and W Cygni are particularly valuable, as these stars occupy a unique position between the short-period variables on the one hand and the α Ceti variables on the other. Visual observations of R Scuti by Espin have led him to suspect the presence of bright lines in its spectrum, but he seems to have been unable to identify them. As far as I know, W Cygni had not been observed with the spectroscope.

Exposures upon both these stars with Spectrograph I were begun in the middle of July, 1903. They were continued until November 11, 1903, in case of R Scuti, and December 28, 1903, in case of W Cygni. R Scuti was examined visually with the spectrograph until December 7. In the meantime, R Scuti rose to maxima toward the end of July and the beginning of October, while maxima of W Cygni occurred early in August and in the middle of December. During this period about 25 spectrograms of R Scuti and 20 of W Cygni were secured with Spectrograph I.

On plate 13E of R Scuti (see Plate 4), $H\beta$, $H\gamma$, and $H\delta$ shone out strongly as bright lines, but they faded

quickly until at minimum the spectrum of the star departed but little from the solar type, with dark hydrogen lines of customary intensity. At the two subsequent maxima the bright hydrogen lines were not seen, but the intensity of the absorption line at $H\gamma$ seemed to have decreased very much as the star approached its maximum of October (See Plate 5). Judging from rough measures of a few plates, the radial velocity of this star appears to be constant and about +42 km. in actual value.

W Cygni shows a banded spectrum of a type characteristic of long-period variables. At the maximum of August, 1903, the hydrogen lines mentioned above appeared as strong bright lines (see Plate 6) which faded gradually to the star's minimum (see Plate 7), while the absorption line at g broadened greatly during the same period. Other suspected bright lines in the spectrum this star remain to be identified.

A detailed study of these plates will be made at the earliest opportunity.

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