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PARALLAX OF THE BINARY SYSTEM DELTA EQUULEI

The object of this communication is to present a value of the parallax of Delta Equulei obtained by using as data the elements of the orbit derived from micrometrical measures and the relative velocity of the components in the line of sight furnished by spectroscopic observations. While the result given is merely provisional, one to be improved when better elements of the orbit are available, as they no doubt will be after two or three years more, yet it is thought that it has already a measure of certainty which places it in the list with the more accurately determined stellar parallaxes. Moreover, the manner of its derivation differs from previous determinations. The use of comparison stars and the uncertainties of their assumed distances are eliminated. In theory the method leads to an absolute value of the parallax and not to a relative one as is necessarily the case when comparison stars are employed.

The mathematical relations connecting the elements of the orbit of a binary, the relative velocity of its components in the line of sight, and the parallax of the system, have long been known. Applications have hitherto been wanting for the reason that no double stars had been found which furnished all the data needed. Delta Equulei has proven an exception. The elements of its orbit are approximately known, and in 1901, at the time of the last periastron passage, the relative velocity of the components in the line of sight was so great that the spectra of the two stars were displaced to a measurable extent.

In the Monthly Notices of the Royal Astronomical Society for March, 1890, Professor Arthur A. Rambaut has given the following simple formula for computing the parallax of a binary from the data here considered:

$$\pi = \frac{la^2 \sqrt{1-e^2} \sin(\phi - \lambda) \sin i}{VPr \sqrt{1-e^2 \cos^2 \phi}}.$$

In this equation P denotes the periodic time, a the mean distance, e the eccentricity, i the inclination, r the radius vector, λ the angle between the line of nodes and the major axis of the true ellipse, ϕ the angle between the tangent to the orbit and the major axis, V the relative velocity of the components in the line of sight as furnished by the spectroscopic observations, l the velocity of the earth in its orbit, and π the parallax of the system.

The elements of the orbit of Delta Equulei are still imperfectly known. A nearly continuous record of the motion of the companion during the next three years is necessary to remove effectively the uncertainties which still exist in their values. Eight times since the discovery of this binary the companion has passed over the arc which it will describe during the next three years, and numerous observations relate to this portion of the orbit. But the measures are more or less discordant, and on this account additional ones are required to fix with certainty the direction and magnitude of the greatest apparent distance. When these measures are obtained and the elements of the orbit revised we shall have the data which will give an accurate value of the parallax of the system. A consideration of the material at present available appears to show that the changes which will probably take place in the elements due to the revision of the orbit will not greatly alter the value of the parallax which may now be derived, and it is on this account that a provisional value is here presented.

During the past three years this binary has been followed closely at the Lick Observatory and much has been done to advance our knowledge of its orbit. It has been shown that the periodic time is about 5.7 years; that the last epoch of periastron passage was about 1901.5; that the apparent distance at periastron is about 0".15; that the inclination is greater than

given by the trial elements which I printed in the Publications of the Astronomical Society of the Pacific for December, 1900, and afterwards in Volume V of the Publications of the Lick Observatory, being perhaps as much as 82° or 83° ; and that the mean distance is probably a little larger than given by those elements. Further, it is known that the line of nodes and the projection of the major axis of the orbit are nearly coincident.

If we retain the line of nodes and the angle between the line of nodes and the major axis the true ellipse as given by my published elements, and put the mean distance equal to $0''.28$ and the eccentricity equal to 0.46 , we get $0''.151$ and $0''.409$ as the periastron and apastron distances.

The relative velocity of the components in the line of sight at the epoch of periastron passage, as determined by the observers using the Mills Spectrograph,* was about 20.5 miles per second. At the same time the earth's orbital velocity was about 18.2 miles per second. At periastron ϕ becomes 90° or 270° , and with $\lambda = 179^\circ$, we have $\phi - \lambda = 91^\circ$. Substituting these values in Professor Rambaut's formula we get

$$\pi = 0''.071.$$

It is evidently impossible to give the probable error of this result. Some of the quantities upon which it depends are much better determined than others. It happens that those which appear to be the most uncertain may vary considerably without greatly affecting the resulting value of the parallax. For example, considerable variations may take place in the values of i and λ without greatly changing the values of $\sin i$ and $\sin(\phi - \lambda)$ which appear in the formula. Of the other quantities those which appear the least certain are the mean distance and the eccentricity. From a consideration of all the past measures

* Bulletin of the Lick Observatory, No. 4, and Publications of the Lick Observatory, Vol. V, p. 211.

it appears highly improbable that the apastron distance can be as much as $0''.05$ greater or less than the value used above. If we vary the apastron distance and eccentricity together so as to keep the already well-determined periastron distance constant, and further retain the other elements of the computation as used above, we shall find that the apastron distance and parallax increase together, but the former at about four times the rate of the latter. From this consideration it appears improbable that the parallax will be altered so much as $0''.012$ owing to changes which take place in the values of a and e due to the revision of the elements of the orbit.

The mass of the system corresponding to the parallax given above, and the mean distance and the periodic time used in its derivation, is 1.89, the mass of the sun being taken as unity. The components of this pair are slightly unequal in brightness, and perhaps also in mass. One may be as massive as the sun, but it cannot much exceed it.

The mean distance at which these stars perform their revolutions is about four times that of the earth from the sun. But owing to the eccentricity of the orbit the range in distance is enormous. At periastron the stars are separated by a space only a little greater than two astronomical units, while at apastron it is five of these units.

It would form an interesting analytical problem to investigate the effect of tidal action upon the relative orbit in this system. The data are pretty well ascertained. Broadly speaking the stars have spectra of the solar type, and being comparable with the sun in mass, it would seem not unreasonable to assume that their densities are approximately the same as that of the sun.

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